FRAGILITY OF MONEY MARKETS

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We provide the first comprehensive theoretical model for money markets encompassing unsecured and secured funding, asset markets, and central bank policy. Capital-constrained, leveraged banks invest in assets and raise short-term funds by borrowing in the unsecured and secured money markets. Our model derives how funding liquidity across money markets is related, explains how a shock to asset values can lead to mutually reinforcing liquidity spirals in both money markets, and shows how borrowers’ flight-to-safety and risk-seeking behavior impacts their liability structure. We derive the social optimum and show which combination of conventional and unconventional monetary policies and regulatory measures can reduce money market fragility.
Being a major source of funding for financial intermediaries, money markets are at the heart of the financial system. Well-functioning money markets are crucial for financial stability and disruptions can have severe consequences even for the real economy. In times of immediate liquidity needs, financial institutions obtain funding in the secured (or “repo”) market\(^1\), borrow in the unsecured money market, or liquidate assets (Freixas, Laeven, and Peydró, 2015). If liquidity dries up in all of these markets simultaneously, bank failures can occur, which cause contagion and spillover effects throughout financial markets and urge central banks to intervene as the lender of last resort.

This paper provides the first comprehensive theoretical model that includes all three sources of liquidity jointly. In our model, banks can borrow in the secured market, subject to margins (haircuts to the value of the collateral securities) or in the unsecured market, in which the interest rate increases with the borrower’s credit risk. We show that even when banks have access to both secured and unsecured funding, the market can become fragile, in the sense that a small shock to the fundamental value of an asset can lead to a large, discontinuous drop in its market price with adverse feedback effects on funding markets. Our model provides a unified framework that explains (i) the intricate dynamics and contagion channels between unsecured and secured funding, (ii) cross-sectional differences and time-series variation in banks’ share of unsecured and secured money market funding, (iii) comovements (“commonality”) of money market liquidity, (iv) commonality across funding and market liquidity, (v) the extent of market fragility, and (vi) the effects of central bank and regulatory policy on money market fragility.

Having a comprehensive model of money markets is important for at least two reasons. First, a thorough understanding of funding risk requires an integrated view and joint modeling approach of all short-term liquidity sources. Since money market liquidity is determined by both secured and unsecured funding liquidity, fragility crucially depends on the interrelation between money markets, and their interaction with the securities’ market liquidity. Second, the fragility of money markets contributed significantly to the global financial crisis (see, e.g., French et al., 2010). In its aftermath, central banks introduced various (unconventional) policies to alleviate funding strains and liquidity risk is at the center of regulators efforts to reform the financial system to reduce its fragility. To perform such important tasks, policy makers need a comprehensive model of money markets to assess not only the immediate impact of new policies on financial markets, but also the

\(^1\)A repurchase agreement or “repo” is essentially a collateralized loan based on a simultaneous sale and forward agreement to repurchase securities at the maturity date. Throughout this paper, we use the terms secured funding, collateralized funding, and repo interchangeably.
effects on future liquidity risk and market fragility.

Our model is similar in spirit to the models of Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009), but it is augmented by key features to gain a comprehensive understanding of money markets. Similar to the existing models, borrowers provide market liquidity to customers, facing a demand shock. The key novel feature of our model is that borrowers finance themselves in the secured and unsecured money market. Furthermore, our model includes a central bank, conducting conventional and unconventional policy. Borrowers are capital constraint and subject to liquidity risk due to the risk of higher margins in the secured market or higher interest rates in the unsecured market. We derive equilibrium asset prices and short-term interest rates of the economy by maximizing customers’, lenders’, and borrowers’ objective functions. In equilibrium, money market rates are linked to the central bank’s benchmark rate and the spread between unsecured and secured interest rates is a risk premium which increases with the borrower’s leverage.

We analyze the consequences of a shock to the fundamental value of borrowers’ assets on funding liquidity in both the unsecured and secured money market. Such shocks occurred, for instance, in mortgage-backed securities during the subprime mortgage crisis in the United States or sovereign bonds during the European sovereign debt crisis. First, we investigate unsecured funding in isolation and show that asset shocks can lead to unsecured funding problems. We identify two (unsecured) liquidity spirals, namely an interest rate and a loss spiral. The former spiral describes the increase in the interest rate due to higher leverage and counterparty credit risk, making it more costly to roll over existing positions. The loss spiral describes the eroding effect on capital following a reduction in the market value of the borrower’s assets, enforcing a deleveraging process and further downward pressure on prices. Consequently, the loss spiral reinforces the interest rate spiral. Considering the secured money market in isolation, Brunnermeier and Pedersen (2009) show similar patterns called the margin and loss spirals.

How do these spirals interact, when including both funding markets in the model? We show that liquidity spirals do not arise, if borrowing in one of the funding markets is unconstrained. In this case, a loss of funding liquidity in the constraint market can be substituted in the unconstraint market. Thus, it is not sufficient to analyze funding sources in isolation. In contrast, we show that markets can be fragile, when borrowers face funding problems in both the secured and the unsecured money market at the same time. In such a scenario, liquidity spirals arise in both funding markets simultaneously, mutually reinforce each other, and induce commonality
in funding illiquidity across money markets. For instance, when an asset shock increases market illiquidity and margins, leading to further downward price pressure, this increases leverage and thus unsecured interest rates. These mechanisms and interrelations in our model are summarized in Figure 1, which shows the interest rate spiral in the unsecured money market, the margin spiral in the secured money market (Brunnermeier and Pedersen, 2009), and the combined loss spiral including feedback effects from both money markets.

Figure 1. Liquidity Spirals in the Money Market
The figure shows the margin spiral in the secured money market (Brunnermeier and Pedersen, 2009, inner circle), the interest-rate spiral in the unsecured money market (outer circle), and the combined loss spiral, including feedback effects from both money markets. Spirals start when an asset shock leads to funding problems, i.e., borrowers’ funding constraints are binding in both the unsecured and the secured money market.

In equilibrium, these liquidity spiral dynamics lead to a re-allocation of money market liquidity from secured to unsecured funding. Since banks are (exogenously) capital-constrained, deleveraging occurs until the (endogenous) financial constraint in the unsecured market allows a substitution of liquidity. Our model brings to light a dual role of haircuts for money market fragility. As higher margins reduce the funding volume in the secured money market, they relax the funding constraint in the unsecured money market by lowering leverage and counterparty credit risk. By alleviating the interest rate spiral, higher margins eventually cause an increase in the equilibrium share of unsecured funding.
Indeed, we show that the substitutability of funding liquidity critically depends on borrowers’ initial leverage. The higher is a borrower’s leverage at the time of the shock, the more difficult it is to substitute a loss of secured funding liquidity in the unsecured market. This is because the shadow costs of capital in the secured market represent the marginal funding costs in the unsecured market when the capital constraint is binding. In other words, funding illiquidity spills over across funding markets if high leverage impairs the borrower’s ability to compensate a reduction in secured funding by raising more unsecured debt. In extremis, leverage following a shock can become so high that capital is eroded before leverage can be reduced sufficiently to raise unsecured funding, and the market freezes.

Another important outcome from our analysis is that the funding structure and liquidity dynamics critically depend on the liquidity risk of borrowers’ asset portfolios. Assets with higher liquidity risk are funded more in the unsecured market, whereas low liquidity-risk assets are funded more in the secured market. Hence, when market participants re-allocate their asset positions towards safer assets, the corresponding effect on the liability side is a flight-to-secured-funding. In turn, risk-seeking behavior of borrowers trying to exploit the opportunity to profit from illiquidity and higher expected returns on distressed assets is mirrored on the liability side by an increase in the share of unsecured funding.

Having established the market dynamics, we derive the socially optimal leverage ratio, which is characterized by equal shadow costs of capital in the secured and unsecured market. We show that central bank monetary policy can restore the socially optimal level of funding liquidity by an efficient combination of conventional policy (i.e., the interest-rate policy), and unconventional measures, namely purchasing assets (quantitative easing) or changing the haircuts for collateralized borrowing from the central bank. While central bank policy can prevent present fragility, it does not address banks’ excess leverage, creating future fragility by leaving banks unable to flexibly adjust their funding structure.

In addition to our analysis of monetary policy, we analyze the effects of the main, recently proposed regulatory measures on money market fragility, namely countercyclical capital buffers, leverage ratios, and liquidity coverage ratios. Our model suggests that countercyclical measures, that are inversely related to haircuts, reduce money market fragility.

Our model is consistent with well-known stylized facts and provides several new testable implications, including that (i) an increase in margins leads to a higher share of unsecured funding,
(ii) flight-to-safety induces flight-to-secured-funding, (iii) higher initial leverage causes a stronger reduction in total asset holdings after a shock, and (iv) central bank liquidity provision counteracts asset sales. We perform a simple empirical analysis using bank-level data and the European sovereign debt crisis as an example of an asset shock. Despite the limited number of observations, the empirical results support the main predictions of our model.

Our paper is related to two streams of the literature. First, prior theoretical research proposes models for secured funding, e.g., Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009), Acharya, Gale, and Yorulmazer (2011), and Martin, Skeie, and von Thadden (2014). In this stream, some models incorporate the relation between secured funding and central bank liquidity, e.g., Ashcraft, Gârlanu, and Pedersen (2011) and Koulischer and Struyven (2014). The second stream of the literature focuses on unsecured money markets, e.g., Acharya and Skeie (2011) and Acharya and Viswanathan (2011). Connection between unsecured funding and central bank liquidity is modeled in, e.g., Freixas, Martin, and Skeie (2011), Allen, Carletti, and Gale (2009), and Acharya and Tuckman (2014). We contribute to the previous literature by modeling unsecured and secured funding jointly and by incorporating central bank policy. None of the previous papers proposes a unified theory encompassing unsecured and secured money markets, asset markets, and central bank policy.

The remainder of this paper is structured as follows. Sections 1 and 2 presents the model setup and the market equilibrium, respectively. In Section 3, we analyze liquidity spirals and highlight the relation between unsecured and secured funding liquidity. In Section 4, we investigate this relation further by discussing substitution effects between funding markets, the extent of fragility, and the relation between funding and portfolio holdings. In Section 5, we derive the social optimum outcome of our model and assess the consequences and efficacy of central bank and regulatory policy for money market liquidity. Section 6 contains a simple empirical exercise. Section 7 concludes.
1. The Model

1.1. Assets

There are \( J \) assets traded at times \( t = 0, 1, 2, 3 \). Each asset \( j \in J \) pays a final cash flow at time \( T = 3 \) equal to the sum of income \( \omega^j_t \) generated in each period, \( \sum_{t=0}^{T} \omega^j_t \), evolving as

\[
\omega^j_t = \omega^j + \Theta^j_t,
\]

where \( \omega^j > 0 \) is a random variable defined on a probability space \( (\Omega, \mathcal{F}, \mathbb{P}) \). Shock \( \Theta^j_t = \sigma^j_t \varepsilon^j_t \) is normally distributed with volatility \( \sigma^j_t \) and \( \varepsilon^j_t \overset{iid}{\sim} \mathcal{N}(0, 1) \).\(^2\) Assets are in zero aggregate supply and the risk-free rate is normalized to zero. Hence, the fundamental value of each asset is the conditional expected value of the final payoff, \( \nu^j_t = \mathbb{E}_t \left[ \sum_{\tau=0}^{T} \omega^j_{\tau} \right] \), and evolves according to

\[
\nu^j_{t+1} = \mathbb{E}_{t+1} \left[ \sum_{\tau=0}^{T} \omega^j_{\tau} \right] = \nu^j_t + \omega^j + \Theta^j_{t+1}.
\]

Fundamental volatility \( \sigma^j_t \) has ARCH dynamics

\[
\sigma^j_{t+1} = \sigma^j + \delta^j |\Theta^j_t|,
\]

with \( \sigma^j > 0 \), and \( 0 < \delta^j < 1 \) implying that future volatility increases after a shock to the fundamental value and the process is stationary.\(^3\)

We let market illiquidity be the absolute deviation of an asset’s market price \( p^j_t \) from its fundamental value, \(|\Lambda^j_t| = |p^j_t - \nu^j_t|\), and denote \( r^j_t = \frac{\Delta p^j_t}{p^j_{t-1}} \), such that an asset’s expected market return \( \phi^j_t \) in the next period is given by \( \phi^j_{t+1} = \mathbb{E}_t \left[ r^j_{t+1} \right] \).

\(^2\)This notation reflects dirty prices of coupon bonds, zero-coupon bonds, or any other income-paying security (e.g., Gromb and Vayanos, 2002). All \( \varepsilon^j_t \) have a standard normal cumulative distribution function \( \Phi \) and are uncorrelated across time and assets.

\(^3\)The model can easily be generalized using other (G)ARCH dynamics. Without loss of generality, we chose the dynamics in Equation (3), in line with Brunnermeier and Pedersen (2009).
1.2. Agents

There are three types of agents in the economy, namely customers, banks with liquidity surplus ("lenders"), and banks with liquidity deficit ("borrowers").

Customers. There are three risk-averse customers \( k = 0, 1, 2 \) in the economy, with initial cash \( W_0^k > 0 \), asset holdings \( y_t^k = 0 \) before arriving in the market, and risk aversion coefficient \( \gamma > 0 \). At time 0, each customer learns about a random endowment shock of \( z^k = \{z_{1,k}, \ldots, z_{J,k}\} \) shares of an asset that he will face at time 3. These shocks represent binding orders which the customers must execute in the market until time \( T \) and are tied to the payout of \( \omega_t^j \). Since aggregate supply is zero, all shocks across customers aggregate to zero, \( \sum_{k=0}^2 z_{j,k} = 0 \).

With probability \((1 - a)\), all customers arrive in the market at time 0, and with probability \( a \), customer \( k = 1 \) arrives at time \( t = 1 \) and customer 2 at time 2. Without loss of generality we assume that when there is sequential arrival, aggregate order imbalance at dates \( \tau = 0, 1 \) is such that \( Z_\tau := \sum_{k=0}^\tau z^k > 0 \), and customer 2 takes the opposite position at time 2 and \( Z_2 = 0 \). Customers have exponential utility \( U(W_T^k) = -\exp(-\gamma W_T^k) \) and choose their optimal positions \( y_t^k \) by maximizing utility over final wealth \( W_T^k \), i.e.,

\[
\max_{y_t} -E_t \left[ e^{-\gamma W_T^k} \right],
\]

subject to their budget constraint

\[
W_{t+1}^k = W_t^k + (\Delta p_{t+1} - \omega_t)'(y_t^k + z^k).
\]

Intuitively, customers’ total position is associated with “inventory costs” when \( y_t^k + z^k > 0 \) and the market is illiquid such that \( \Delta p_t < \omega_t \). Conversely, customers make a profit if they sell assets at a higher price than the required payout. Finally, we consider liquid securities for which aggregate order imbalances are fairly rarely observed, meaning that \( a \to 0 \).

\[4\] In Brunnermeier and Pedersen (2009), lenders and borrowers are referred to as “financiers” and “speculators”, respectively. In Gromb and Vayanos (2002), “arbitrageurs” are borrowers and customers are called investors. We use the terms “borrowers” and “lenders” to highlight the generality of our model.
Lenders. Banks are risk-neutral and differ with respect to their cash endowment. Lenders (indexed “l” where necessary to distinguish them from borrowers) have capital $W^l_0 > 0$ and face a positive liquidity shock that translates into surplus funds $D_0 > 0$. In reality, $D_0$ could represent a sudden inflow of deposits, for which lenders have to pay liquidity costs $c^l_t = c_{lb}$, equal to the central bank’s refinancing interest rate.\footnote{Empirical evidence suggests that retail deposit rates are related to central bank rates (Borio, Gambacorta, and Hofmann, 2015). The refinancing rate serves as the benchmark rate in the economy.}

We assume lenders maximize expected wealth by allocating $D_0$ into a pool of liquid investments for one period.\footnote{For example, in the spirit of Allen and Gale (2000) or because lenders cannot verify the payoff of riskier and potentially more illiquid projects.} Such investments include secured and unsecured money market loans $M_t = M^s_t + M^u_t$, “near-money” securities $b$, or storage at a central bank’s deposit facility for an interest rate $i_d = 0$ equal to the economy’s risk-free rate. Near-money assets exhibit the lowest risk among all assets $J$, $\sigma^b_t = \inf \{ \sigma^j_t : j \in J \}$, and are traded in fully liquid markets, allowing lenders to readily convert them into cash.\footnote{Equivalently, “near-money” can mean eligible for refinancing involving almost no capital requirement.}

Interbank loans are either collateralized or unsecured, with corresponding interest rates $i^s_t$ and $i^u_t$, and expose the lender to risk. For secured loans, lenders protect themselves against the risk of the security by subtracting a margin $0 < m^j_t \leq p^j_t$ from the collateral’s market value to determine the size of the loan.

**Lemma 1.** Lenders are uninformed about fundamental values and have information set $\mathcal{F}_t = \sigma \{ p_0, \ldots, p_t, \omega \}$, and set margins based on observed prices and volatility. As $a \to 0$, margins are set to cover the collateral’s $\pi$-value-at-risk,

$$\pi = \Pr ( - (\Delta p^j_{t+1} - \omega^j) > m^j_t | \mathcal{F}_t), \quad (6)$$

such that margins are

$$m^j_t = \sigma^j + \delta^j |\Delta p^j_t - \omega^j| = \sigma^j + \delta^j |\Theta^j_t + \Delta \Lambda^j_t|, \quad (7)$$
where we define

\[ \sigma^j = \sigma^j \Phi^{-1}(1 - \pi), \]
\[ \delta^j = \delta^j \Phi^{-1}(1 - \pi). \]

As in the real world, margins reflect an asset’s past volatility and unexpected price movements. The intuition is that lenders expect \( \Delta p_t = \omega \) in every period, because they do not observe fundamental dynamics \( \omega^j_t \). When price changes deviate from \( \omega \), lenders assume that this is due to a change in fundamentals, and increase margins to protect against higher risk. In relative terms, margins are defined as haircuts, \( h^j_t \equiv \frac{m^j_t}{p^j_t} \), and since \( 0 < h^j_t \leq 1 \), shocks to the fundamental value or market illiquidity increase the over-collateralization of secured loans and the protection against risk.

Unsecured loans expose lenders to counterparty credit risk. Banks in our model have limited liability and are bankrupt when capital becomes zero. We denote the probability of a counterparty’s default as \( \theta \in [0, 1] \), which is a continuously increasing function of a borrower’s leverage ratio \( L_t \), i.e., \( \frac{\partial \theta}{\partial L_t} > 0 \). This specification is in line with empirical evidence that default probabilities, implied by CDS prices, increase with leverage (e.g., Collin-Dufresne, Goldstein, and Martin, 2001). Leverage is computed as the borrower’s total assets \( A_t \) over capital \( W_t \), \( L_t = \frac{A_t}{W_t} \). Overall, lenders’ wealth evolves as

\[
W_{t+1}^l = W_t^l + (1 - \theta)[i^s_t M_t^s + i^u_t M_t^u] - \theta M_t^u + \phi^b_t (D_0 - M_t) - i_{cb} D_0. \tag{8}
\]

Equation (8) states that lenders earn the interest on their money market loans if the borrower survives. We assume that in case of counterparty default, the lender loses the interest on the secured loan, but gets back the principle \( M_t^s \). Since \( \pi \) is low, margins effectively internalize the costs of liquidating collateral assets in distressed markets. In contrast, unsecured loans to defaulted borrowers have a zero value to the lender in the short-run (Acharya and Skeie, 2011). Thus, a

\[ \text{\textsuperscript{8}} \text{Note that our results hold irrespective of the exact value lenders receive from selling collateral in the market as long as secured creditors can expect higher reimbursement than unsecured creditors in case of counterparty default. For instance, when secured loans are traded, e.g., via a central clearing house (CCP), lenders are protected by several layers of defense undertaken by the clearing party to guarantee repayment (for a detailed description see Mancini, Ranaldo, and Wrampelmeyer, 2015). In contrast, unsecured loans are unavailable in the short-run until the counterparty’s bankruptcy case is filed and claims are negotiated.} \]
lender’s optimization problem is given by

$$\max_{M^s_t, M^u_t} \mathbb{E}_t \left[ W^t_{t+1} \right],$$

subject to the wealth dynamics in Equation (8).

**Borrowers.** Each borrower has initial capital $W_0 > 0$ and zero positions before entering the market at time 0. Borrowers act as financial intermediary and trade assets with the customers. They finance their positions by borrowing from lenders in the money market. Hence, the liquidity surplus $D_0$ is redistributed in the economy after endowment shocks $z^k$ have realized at time 0 and trades are settled.\(^9\)

Borrowers can fund their asset holdings either in the unsecured money market or in the secured money market. Positions for securities funded in the secured and unsecured market are denoted by $x^{j,s}_t$ and $x^{j,u}_t$, respectively. The total holding of a security is $x^j_t = x^{j,s}_t + x^{j,u}_t$.

Positions funded in the secured market are constrained by the amount of capital, $W_t$, that a borrower has available to satisfy the margin requirements set by the lenders in Lemma 1:

$$\sum_j x^{j,s}_t m^j_t \leq W_t.$$  \(10\)

The secured funding volume totals $M^s_t = \sum_j (1 - h^j_t) x^{j,s}_t p^j_t$. In addition to the capital constraint, borrowing involves funding costs. The costs for each asset funded in the secured market are given by the haircut-weighted average of the interest rate $i^s_t$ and equity costs $e$:

$$c^{j,s}_t = (1 - h^j_t) i^s_t + h^j_t e.$$  \(11\)

For $e > i^s_t$, we have that higher haircuts impose higher funding costs, $\frac{\partial c^{j,s}_t}{\partial h^j_t} > 0$.

In the unsecured market, the funding volume is given by $M^u_t = \sum_j x^{j,u}_t p^j_t$, and funding costs are equal to the interest rate, $c^{j,u}_t = i^u_t$. Taking secured and unsecured positions together, borrowers’

\(^9\)Thus, borrowers can be seen as speculators/arbitrageurs with trading capital $W_t$ in the spirit of Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009), or, alternatively, as (commercial) banks managing their liquidity needs after facing a negative liquidity shock at time 0 (e.g., Martin, Skeie, and von Thadden, 2014).
have wealth dynamics

\[ W_t = W_{t-1} + (r_t - c_t)(x_{t-1}^s \circ p_{t-1}) + (r_t - c_t^u)(x_{t-1}^u \circ p_{t-1}) + \eta_t, \]  

(12)

where \( \eta_t \) is an independent wealth shock, e.g., from other business units. Each borrower maximizes expected wealth by choosing the optimal security positions \( x_{t}^{j,s} \) and \( x_{t}^{j,u} \),

\[ \max_{x_{t}^{s},x_{t}^{u}} \mathbb{E}_t [W_{t+1}], \]  

(13)

subject to the capital constraint (10) and wealth dynamics (12). To summarize, an equilibrium in the economy is defined as follows.

**Definition 1.** A competitive equilibrium consists of a price process \( p_t \) and interest rates \( i_t^s \) and \( i_t^u \), such that (i) \( x_t \) maximizes the borrowers’ expected final wealth, subject to the capital constraint, (ii) \( y_t^k \) maximizes customer \( k \)’s expected utility after arrival in the market place and is zero beforehand, (iii) money market loans \( M_t^s \) and \( M_t^u \) maximize lenders’ expected wealth as given by Equation (8), (iv) margins are set according to the VaR specification in Lemma 1, and (v) markets clear, \( x_t + \sum_{k=0}^{2} y_t^k = 0 \).

As described, the dynamics comprise price processes in the asset market as well as in the money markets, and the market clearing condition ensures that all markets are in equilibrium simultaneously. In the next section, we analyze the equilibrium outcome of the economy.

### 2. Equilibrium

There are two simultaneous decisions at each time \( t \), namely, lenders interact with borrowers in the money market, and borrowers trade assets with customers.

#### 2.1. Money Market

We begin with the lenders’ maximization problem as it is identical for all time periods. Solving Equation (9) with respect to \( M_t^s \) and \( M_t^u \), the equilibrium condition between short-term interest rates yields

\[ (1 - \theta)i_t^s = \phi_t^b + i_{cb} = (1 - \theta)i_t^u - \theta. \]  

(14)
The spread between money market interest rates is determined by the borrower’s default probability \( \theta(L_t) \), which is a function of the borrower’s leverage, i.e.,

\[
i_t^u = i_t^s + \frac{\theta}{1 - \theta} = i_t^s + \mu(L_t).
\]

Equation (15) has interesting implications, which we summarize in Corollary 1.

**Corollary 1.** The equilibrium dynamics of short-term interest rates are such that

(i) the spread between \( i_t^u \) and \( i_t^s \) is a credit risk premium, which increases with leverage, i.e.,

\[
\mu(L_t) = i_t^u - i_t^s > 0 \quad \text{with} \quad \frac{\partial \mu}{\partial L_t} = \frac{1}{(1-\theta)^2} \frac{\partial \theta}{\partial L_t}, \quad \text{since} \quad \frac{\partial \theta}{\partial L_t} > 0,
\]

(ii) money market interest rates are positively affected by the central bank’s refinancing rate \( i_{cb} \), and

(iii) the spread of \( i_t^s \) to the expected return of the near-money asset, \( \phi_t^b \), moves with \( i_{cb} \), reflecting the opportunity costs of holding money.

All these patterns find empirical support. First, the spread between unsecured and secured rates is commonly used to proxy money market risk premiums and it is well-known that it tends to increase in times of crisis. Second, (conventional) monetary policy steers interest rates in the economy by influencing banks’ costs of refinancing, which is one of the reasons \( i_{cb} \) is referred to as the benchmark rate. Another reason is that \( i_{cb} \) reflects the opportunity costs of holding money, which are high when investment opportunities are promising. Empirically, the spread between the secured rate and the expected return on near-money assets, \( i_t^s - \phi_t^b \), co-moves with the unsecured rate in the United States (Nagel, 2015). Extending the finding of Nagel (2015), Equation (14) suggests that the spread includes the term \( \theta i_t^s \), which is time-varying and increasing in bank leverage, i.e., credit risk.

### 2.2. Asset Market

We derive the optimal values for \( x_t \) and \( y_t \) by backward induction, starting with optimal asset holdings at time \( t = 2 \).
Time 2. Let $\Gamma$ be a customer’s value function, then his optimization problem at $t = 2$ becomes

$$\Gamma_2(W_k^2, p_2, \nu_2) = \max_{y_k^2} -E_2[e^{-\gamma W_k^2}].$$

(16)

Knowing that assets pay off at date 3, the solution to this problem is

$$y_{jk}^2 = \nu_{jk}^2 - p_{jk}^2 \gamma (\sigma_{jk}^2) - z_{jk}^2.$$  

(17)

Since all customers are in the market at time 2, the aggregate endowment shock is $Z_2 := \sum_k y_{jk}^2 = 0$ and equilibrium prices equal fundamental values, $p_2 = \nu_2$. The customer’s value function is $\Gamma_2(W_k^2, p_2 = \nu_2, \nu_2) = -e^{-\gamma W_k^2}$. As markets must clear, borrowers’ demand is zero and the their value function is $\Psi_2(W_2, p_2 = \nu_2, \nu_2) = W_2$.

Time 0 and 1. If all customers arrive at time 0, $Z_t = 0$ for all $t$. Hence, we concentrate on the case with potential trade at times $\tau = 0, 1$ when $Z_\tau > 0$. Customers $k = 0, 1$ optimally choose $y_{jk}^1$ according to

$$y_{jk}^1 = \nu_{jk}^1 - p_{jk}^1 \gamma (\sigma_{jk}^2) - z_{jk}^1,$$  

(18)

and the aggregate endowment shock is $Z_1$. Note that Equation (18) resembles Equation (17), because prices at time 2 are equal to fundamental values. At time 0, customer $k = 0$ enters the market and maximizes his expected wealth at time 1, $E_0[\Gamma_1(W_{1}^k, p_1, \nu_1)]$, subject to the value function

$$\Gamma_1(W_{1}^k, p_1, \nu_1) = - \exp \left\{ -\gamma \left[ W_{1}^k + \sum_j (\nu_{1j}^1 - p_{1j}^1)^2 \right] \right\}.$$  

(19)

Borrowers maximize their expected wealth as given by Equation (13). The corresponding value function at time 1 is

$$\Psi_1(W_1, p_1, \nu_1, c_1) = \max_{x_1^s, x_1^u} E_1[\Psi_2(W_2, p_2 = \nu_2, \nu_2)].$$  

(20)

Maximizing expected wealth with respect to $x_1^s$, we obtain $\phi_{1j}^1 - c_{1j}^{s,a}$. This implies a corner solution in the secured market, i.e., borrowers invest in $\max_j \{ \phi_{1j}^1 - c_{1j}^{s,a} \}$ until the capital constraint binds,

$$x_1^s = \frac{1}{m_1} W_1.$$  

(21)
Fully leveraging $W_1$ in the secured market is optimal as neither holding capital nor directly purchasing asset provides a higher expected net return. If the maximum spread $\phi_j^j - c_1^j$ is equal to zero, borrowers are indifferent between any position up to the constraint (and Equation (21) holds with “≤”).

Next, we differentiate Equation (12) with respect to $x_1^u$ and denote $\phi_1 = \max_j \{\phi_j^j\}$:

$$x_1^u = \frac{\phi_1 - i_1^u}{\partial \mu \partial L_1} W_1.$$  \hspace{1cm} (22)

From Equation (22), a larger net return increases demand for unsecured borrowing, while higher leverage reduces it. In other words, unsecured funding is optimal when the marginal return of investing in an additional asset equals marginal funding costs, i.e.,

$$\phi_1 - i_1^u = \frac{M_1^u \partial \mu}{W_1 \partial L_1}.$$  \hspace{1cm} (23)

Equation (23) establishes an endogenous financial constraint for unsecured funding. That is, if $M_1 < D_0$, a borrower’s leverage is constrained by marginal profitability and leverage is optimal when Equation (23) holds. Conversely, leverage is higher when assets yield higher returns, providing a link to macroeconomic conditions and pro-cyclicality of bank leverage (Adrian and Shin, 2010). If $M_1$ exceeded $D_0$, the lending volume would be limited to $D_0$, and Equation (23) would hold with “≥”. Alternatively, credit rationing could also occur if $D_1$ was subject to, e.g., contagion risk of deposit withdrawals at time 1 (e.g., Diamond and Dybvig, 1983; Shin, 2009). Instead, we let credit be rationed endogenously by lenders charging a higher risk premium $\mu$ in anticipation of borrowers’ default probability, which constrains leverage by expected asset returns and capital $W_1$ (e.g., Stiglitz and Weiss, 1981; Acharya and Viswanathan, 2011).

Finally, the shadow costs of capital, denoted by $\varphi_1^u$ for the unsecured market, are the net return on a borrower’s wealth:

$$\varphi_1^u = \phi_1 - i_1^u.$$  \hspace{1cm} (24)

When Equation (23) holds, the funding constraint is binding and the price for relaxing the constraint is given by the shadow cost. Similarly, the shadow costs of capital in the secured market are given by $\varphi_1^s = \phi_1 - c_1^s$, and reflect the net return on a borrower’s wealth when the capital constraint becomes binding. The value function at time 1 is given by $\Psi_1(W_1, p_1, \nu_1, c_1) =$
At time 0, each borrower maximizes \( E_0[\Psi_1(W_1, p_1, \nu_1, c_1)] \), subject to the financial constraints. Since banks in our model have no disutility from negative wealth, the analysis is similar to time 1.

In sum, equilibrium interest rates are given by Equation (15) and borrowers’ positions as shown by Equations (21) and (22). We discuss the properties of asset prices at times 0 and 1 and the implications for money market fragility in the next section.

3. Money Market Fragility

In line with Brunnermeier and Pedersen (2009), markets are fragile when the equilibrium price \( p_1(\eta_1, \Theta_1) \) cannot be chosen to be continuous in the exogenous shocks \( \eta_1 \) and \( \Theta_1 \). Fragility occurs when a small fundamental shock \( \Theta_1 < 0 \) is followed by a (much) larger drop in the market price, i.e., \( \Delta p_1 < \Delta \nu_1 \). In this section, we show that fragility can arise due to funding constraints, even if borrowers have access to both the secured and unsecured money market. We show our main results for a single asset \( J = 1 \) at time 1.\(^{10}\) From the equilibrium derivations, we know that all the trades are reversed at time 2 when the complementary customer enters the market, and so we focus on the dynamics of funding liquidity at time 1 conditional on the borrower’s security holdings at time 0.

Only if borrowers are constraint in both funding markets, equilibrium prices can be fragile and subject to market illiquidity. Under the assumption that borrowers do not have access to the unsecured money market, funding liquidity in the secured market has been shown to be fragile in Brunnermeier and Pedersen (2009). More specifically, excess demand \( x_1 + \sum_{k=0}^{1} y_k \geq 0 \) can be decreasing for lower prices due to binding funding constraints, whereas a monotonic demand function has higher demand (i.e., increasing excess demand) when prices decrease. What happens if borrowers can also borrow in the unsecured market? Lemma 2 shows that liquidity in the unsecured market can also be fragile when considering that funding source in isolation.

**Lemma 2.** There exists \( x^u \) such that the market is fragile if borrowers’ position at time 0 is larger than \( x^u \), i.e., \( x_0^u > x^u \) and customers’ demand shock is in the same direction, i.e., \( Z_1 > 0 \).

We illustrate the non-monotonic shape of the demand curve in the unsecured market in the left panel of Figure 2. As shown by the dashed curve, if borrowers have no previous position and

\(^{10}\)We discuss portfolio dynamics in Section 4.3.
$x_0^u = 0$, demand increases with lower prices as $\frac{\partial (\phi_1 - i_1^u)}{\partial p_1} < 0$. Intuitively, marginal returns not only increase due to higher $\phi_1$, but also because $i_1^u$ decreases with $p_1$. However, the funding constraint binds when lower market prices decrease marked-to-market capital $W_1$ and thereby increase leverage for existing positions $x_0^u > 0$. Since higher leverage increases default probability $\theta(L_1)$ and risk premium $\mu$, lenders increase interest rate $i_1^u$ for unsecured borrowing, which reduces the marginal return and thus $x_1^u$. Although lower prices raise the expected return and demand for the asset, higher interest rates in combination with customers’ selling pressure ($Z_1 > 0$) can prevent the borrower from investing into the asset until marginal returns equal marginal costs again (Equation (22)). As shown in Figure 2, the demand function becomes decreasing as $p_1$ decreases due to higher $i_1^u$, and increases again only for high enough return $\phi_1$. Hence, binding funding constraints can lead to a large, discontinuous drop in the market price $p_1$ when the borrower has a sufficiently large stake in the asset.

![Figure 2. Equilibrium](image)

This figure shows the equilibrium price at time 1 for which the market clears, $x_1 = -y_1$. The left panel shows the equilibrium for demand $x_1^u$ in the unsecured market as a function of the previous position at time 0. The dashed line represents a liquid equilibrium with position $x_0^u = 0$, and the dark line illustrates an illiquid equilibrium for $x_0^u > 0$, which has a decreasing part in $p_1$ as shown in Lemma 2. The right panel shows an illiquid equilibrium for both secured and unsecured funding for a borrower with $x_0 > 0$, where the dashed line represents the previous (liquid) equilibrium at time 0.

After establishing fragility in the unsecured market, we combine this result with the secured market and analyze the case in which borrowers have access to both funding sources simultaneously. The right panel of Figure 2 illustrates the equilibrium market price for a borrower’s total demand $x_1$ when $x_0 > 0$ and funding constraints bind. In addition to $x_1^u$, funding demand in the secured
market, $x^*_t$, as given by Equation (21), decreases with lower prices, too, due to higher margins and a reduction in capital. In both funding markets, a shock to the fundamental value of an asset, $\Theta_1 < 0$, turns a former liquid equilibrium (as depicted by the dashed line) into an illiquid equilibrium with $\Lambda_1 < 0$ when funding liquidity is fragile. As summarized in Proposition 1, the result are mutually reinforcing liquidity spirals across money markets, exacerbating funding problems and causing common money market fragility.

**Proposition 1.** When funding constraints are slack, the price change is equal to the change in the fundamental value, $\Delta p_1 = \Delta \nu_1$, and the market is in a liquid equilibrium. In a stable illiquid equilibrium with selling pressure from customers, $Z_1 > 0$, and exposure $x_0 > 0$, the price sensitivity to a fundamental shock $\Theta_1 < 0$ is

(i) in the secured money market

$$
\frac{\partial p_1}{\partial \Theta_1} = m_1 \left( \frac{2\gamma \sigma_2^2 - 4\gamma \sigma_2 (\nu_1 - p_1) \frac{\partial \sigma_2}{\partial \nu_1}}{(\gamma (\sigma_2)^2)^2} \right) - \frac{\partial m_1}{\partial \Theta_1} x^*_1 \partial p_1 \left( \frac{2}{\gamma (\sigma_2)^2} \right) - \frac{\partial W_1}{\partial p_1}.
$$

(ii) and in the unsecured money market

$$
\frac{\partial p_1}{\partial \Theta_1} = -\frac{4\gamma \sigma_2 (\nu_1 - p_1) \frac{\partial \sigma_2}{\partial \nu_1} - 2\gamma \sigma_2^2}{(\gamma (\sigma_2)^2)^2} - \frac{W_1 \frac{\partial W_1}{\partial p_1}}{\partial \mu_1^1 (p_1)^2} - \left( \frac{\partial \phi_1}{\partial p_1} \right)^2 \left( \frac{\partial \mu_1^1}{\partial \mu_1^1} \right)^2 \left( \frac{\partial \mu_1^1}{\partial p_1} \right)^2.
$$

(iii) There exist two mutually reinforcing liquidity spirals for the unsecured money market, namely, an interest rate spiral, $\frac{\partial \mu_1}{\partial p_1} < 0$, and a loss spiral, $\frac{\partial W_1}{\partial p_1} > 0$.

(iv) **(Money Market Fragility)** Margin and loss spirals for the secured market, $\frac{\partial m_1}{\partial p_1} < 0$ and $\frac{\partial W_1}{\partial p_1} > 0$, are integrated with liquidity spirals in the unsecured market by mutually exerting downward pressure on prices and deteriorating borrowers’ capital.

This result is intuitive as funding problems arise from a reduction in the borrower’s capital, both directly and indirectly. The direct effect comes from the loss spirals due to the deteriorating effect of lower prices on mark-to-market capital. In turn, this reduction causes two indirect, market-specific effects, namely the margin and interest rate spirals. Margins increase in response.
to lower prices as shown in Lemma 1 and force the borrower to sell assets as the capital constraint becomes binding. Additionally, less capital reinforces selling pressure as now fewer of the higher margins can be paid. In the unsecured market, less capital and lower prices increase interest rates due to higher leverage. This intensifies asset sales further, reinforces loss and margin spirals, and ultimately leads to fragility of common money market liquidity. Overall, Proposition 1 shows that all three sources of immediate funding for financial institutions, namely borrowing in the secured market, borrowing in the unsecured money market, and selling assets are interconnected and may become illiquid at the same time.

4. Secured and Unsecured Funding Liquidity

Having established that common money market liquidity can be fragile, we want to analyze in more detail the relation between secured and unsecured funding in equilibrium. That is, we are interested in the relative funding dynamics after a negative price shock and how the time-0 position $x_0$ and leverage $L_0$, affect these dynamics.

4.1. Liquidity Dynamics

Due to liquidity spirals, a drop in the market price $p_1$ affects the borrower’s security positions $x_1^s$ and $x_1^u$ as shown by the demand curves in Figure 2. Formally, the price sensitivity of demand in the secured market is given by

$$\frac{\partial x_1^s}{\partial p_1} = \frac{m_1 \frac{\partial W_1}{\partial p_1} - W_1 \frac{\partial m_1}{\partial p_1}}{m_1^2} > 0.$$  \hspace{1cm} (27)

The concave shape of the demand curve indicates that as prices decrease and the capital constraint becomes binding, the margin and loss spirals force the borrower to deleverage. Through the increase in haircuts, the secured funding volume $M_1^s$ decreases. Since $L_1 = L_1^s + L_1^u$, a price shock not only affects secured leverage $L_1^s$, but also total leverage by

$$\left. \frac{\partial L_1^s}{\partial p_1} \right|_{\Theta_1 < 0} = \frac{m_1 - p_1 \frac{\partial m_1}{\partial p_1}}{m_1^2} > 0.$$  \hspace{1cm} (28)
As a fundamental shock increases the margin initially, secured leverage decreases since \( L_s^1 = h_1^{-1} \). In an illiquid equilibrium, margins increase further, and the borrower is forced to deleverage.

Due to the non-monotonicity of the demand curve in the unsecured market, the equilibrium effect of a price shock on \( x_u^1 \) can be positive or negative. However, since unsecured leverage \( L_u^1 \) is given by

\[
L_u^1 = \frac{\phi_u^1 - i_u^1}{\mu'(L_1)},
\]

unsecured leverage unambiguously increases after a shock. That is, if total leverage has decreased in equilibrium, the risk premium and interest rate \( i_u^1 \) have done so as well, and \( L_u^1 \) has increased less than \( L_s^1 \) has decreased in total. Conversely, if \( L_1 \) has increased, unsecured leverage must have increased as well. Thus, in any case, the numerator of Equation (29), which represents the shadow cost of capital in the unsecured market \( \phi_u^1 \), increases. As secured and unsecured leverage represent the relative share of total assets funded in each market, the opposing effect on \( L_s^1 \) and \( L_u^1 \) implies a reallocation of funding from the secured to the unsecured money market after a shock.

**Proposition 2.** In equilibrium, a fundamental shock leads to a re-allocation of money market liquidity from secured to unsecured funding. While higher margins reduce the funding volume in the secured money market, they

(i) *(Substitution)* relax the funding constraint in the unsecured money market through lower secured leverage, such that

\[
\Delta \left| \frac{M_u^1}{M_1} \right|_{\Theta_1 < 0} > 0 \quad \text{and} \quad \Delta \left| \frac{M_s^1}{M_1} \right|_{\Theta_1 < 0} < 0.
\]

(ii) *(Commonality)* Funding illiquidity, as measured by the borrowers’ shadow costs of capital, \( \phi_s^1 \) and \( \phi_u^1 \), co-moves, i.e.,

\[
\text{Cov}_0(\phi_s^1, \phi_u^1) > 0,
\]

implying common illiquidity if shadow costs increase, and perfect substitution from secured to unsecured funding if shadow costs decrease.

The economic interpretation for Proposition 2 follows from the shadow costs of capital. After a shock, haircuts increase the funding costs in the secured market, and higher leverage increases \( i_u^1 \) in the unsecured market. If the funding constraint in the unsecured market is binding, the market
price must fall as shown in Proposition 1. This leads to a (non-linear) increase in $\phi_1$, so that a higher shadow cost $\varphi_1^u$ reflects funding illiquidity, co-moving with funding illiquidity in the secured market. As borrowers reduce positions, the shadow costs $\varphi_1^s$ and $\varphi_1^u$ jointly increase, indicating that liquidity spirals in the money market are mutually reinforcing as shown in Proposition 1. However, as borrower sell assets, the allocation of funding liquidity changes towards more unsecured funding. Intuitively, this happens because on the one hand, the funding constraint in the secured market is exogenously imposed by capital $W_1$, whereas the funding constraint in the unsecured market is more flexible and endogenous to marginal returns. On the other hand, lenders in the secured market know the collateral risk, so that the borrower partly replaces the reduction in secured liquidity by funding a larger share of the security in the unsecured market. If the leverage constraint is slack, there is perfect substitution of funding liquidity in the sense that the reduction of secured funding after an asset shock is compensated by an equivalent increase in unsecured funding, and shadow costs decrease.

In fact, the commonality of funding liquidity, as given by the shadow costs of capital in Proposition 2, has important implications for the extent of fragility, which we analyze in the next section.

### 4.2. Extent of Fragility

Fragility at time 1 arises from a large enough previous position $x_0$, leading to capital losses and non-monotonicity in the borrower’s demand function. This happens because deleveraging of assets lead to drops in asset prices until $\varphi_1^u = \phi_1 - i_1^u$ is sufficiently large to allow the borrower to raise enough funds to clear the market. Since more $x_0$ increases $i_0^u$ through higher leverage, the borrower’s ability to substitute funding liquidity at time 1 depends on his shadow costs of capital at time 0.

**Lemma 3.** The funding constraint in the unsecured money market at time 1 is slack for $\varphi_0^u \geq \varphi_0^s$, and binding for $\varphi_0^u < \varphi_0^s$.

Intuitively, Lemma 3 implies that large holdings $x_0$ imply sufficient access to funding liquidity. This means, shadow costs in the unsecured market are relatively small due to a high interest rate $i_0^u$, and shadow costs of capital in the secured market, $\varphi_0^s = \phi_0 - c_0^s$, are relatively large for low haircuts and/or high returns. Since borrowers encumber all their capital in the secured market at time 0, $\varphi_0^u < \varphi_0^s$ lead to funding problems at time 1 if a borrower faces initial losses. As the capital
constraint becomes binding after a shock, low shadow costs in the unsecured market at time 0 exert selling pressure at time 1, because the borrowers’ leverage constraint becomes binding from too high leverage \( L_0 \). As shown in Proposition 2, this leads to asset sales until prices are low enough and \( \varphi_0^u \) sufficiently high to clear the market. In contrast, \( \varphi_0^u \geq \varphi_0^s \) allow the borrower to substitute funding liquidity by raising the required amount of unsecured debt to compensate for the reduction in secured funding from an increase in haircuts. Since \( \varphi_0^u \) is linked to the marginal funding costs in the unsecured market, as given by Equation (23), Proposition 3 relates the potential for liquidity substitution to the shadow costs of capital.

**Proposition 3.** Common funding liquidity at time 1 depends on the relative shadow costs of capital at time 0:

(i) **(Shadow illiquidity)** The shadow costs of capital in the secured market represent the shadow marginal funding costs in the unsecured market, and constitute shadow illiquidity if

\[
\frac{\partial i_0^u}{\partial L_0} L_0^u \leq \varphi_0^u < \varphi_0^s.
\] (31)

In contrast, for \( \varphi_0^s \leq \frac{\partial i_0^u}{\partial L_0} L_0^u \), the market at time 1 is liquid and \( p_1 = \nu_1 \).

(ii) **(Liquidity dry-up)** When there is shadow illiquidity at time 0, leverage \( L_0 \) is so high that it impairs the substitution of funding liquidity at time 1 in case of a fundamental shock, \( \Theta_1 < 0 \), and selling pressure in the market, \( Z_1 > 0 \). In extremis, the unsecured market is “frozen” and borrowers default.

Shadow illiquidity exists if the shadow costs of capital in the secured market exceed the marginal funding costs in the unsecured market. This is intuitive, because the shadow costs of capital represent the expected loss in case the constraint becomes binding. That is, when the borrower faces initial losses at time 1 or the haircuts increase, the capital constraint becomes binding and positions funded in the secured market must be reduced. Consequently, the borrower shifts these assets to the unsecured market and obtains funding if the leverage constraint is slack. Hence, \( \varphi_0^s \) represents the marginal return required to substitute secured funding liquidity in the unsecured market. In other words, if Equation (31) holds, there exists shadow illiquidity at time 0, because the marginal funding costs in the unsecured market will exceed the marginal return in case the
capital constraint becomes binding at time 1. Clearly, it is shadow illiquidity since there is no problem if time-1 fundamentals remain solid.

Moreover, shadow illiquidity implies that leverage $L_0$ is so high as to constitute a low spread $\varphi_0^u$, which impairs substitution of liquidity at time 1. This happens because high initial leverage increases further after capital losses and induces assets sales until the market price drop creates sufficient marginal return for borrowers to stop reducing positions. As shown in Figure 3, $\Delta x_1 < 0$ is high (in absolute value) the higher was initial leverage $L_0$, forcing borrowers to deleverage until they can raise unsecured funding. In turn, $\Delta x_1 > 0$ increases if the leverage constraint is slack at time 1, and borrowers take advantage of prosperous investment opportunities by access to unsecured funds.

![Figure 3. Leverage Cycle](image)

The solid line is drawn from the change in $\Delta x_1$ as a function of the relation between the shadow costs of capital at time 0.

In fact, Figure 3 relates to the leverage cycle, which describes that excess leverage is followed by excessive deleveraging (Geanakoplos, 2010). Similarly, $\Delta x_1 > 0$ refers to an increase in leverage, whereas $\Delta x_1 < 0$ applies to deleveraging. When leverage is low, borrowers raise additional debt to benefit from profitable opportunities and deleverage in a crisis when markets are illiquid. For very high leverage, corresponding deleveraging is due to illiquidity in the unsecured market, which can be considered stressed until volumes have decreased sufficiently (Afonso, Kovner, and Schoar, 2011). If borrowers cannot raise unsecured debt to fund their assets at affordable costs, the unsecured market is “frozen”, i.e., closed for borrowers with too high counterparty credit risk.

Our results show that fragility arises because high leverage impairs the flexible re-allocation of
liquidity from secured to unsecured funding after a fundamental shock. In the next section, we analyze the relation between secured and unsecured funding liquidity when borrowers anticipate liquidity risk, i.e., we analyze price dynamics at time 0.

4.3. Liquidity Risk

Since the market is perfectly liquid at time 2, borrowers face no liquidity risk at time 1. Hence, we analyze borrowers’ portfolio decision at time 0, when they build expectations about future illiquidity. This allows us to determine the role of liquidity risk for the use of funding markets, and the impact on time-0 shadow costs of capital.

At time 0, borrowers maximize their expected wealth \( E_0[W_1(1 + \varphi_1)] \), which depends on time-1 illiquidity, measured by the weighted shadow costs of capital, \( \varphi_1 \).\(^{11}\) Solving for \( x_0 \), we get a time-0 price

\[
p_0 = \frac{E_0[p_1]}{1 + c_0} + \frac{Cov_0[p_1, \varphi_1]}{(1 + c_0)E_0[1 + \varphi_1]},
\]

where the covariance term captures the notion of liquidity risk (Brunnermeier and Pedersen, 2009). Accordingly, negative covariance means the market price decreases when funding illiquidity is high, exposing the borrower to potential fragility. Thus, the higher liquidity risk, the lower is the time-1 market price, constituting a liquidity risk premium. Moreover, as \( p_0 \) decreases, volatility and margins increase, and unsecured demand increases as can be seen from Figure 2. Proposition 4 highlights how liquidity risk affects money market funding liquidity.

**Proposition 4.** Liquidity risk affects money market funding liquidity as follows:

(i) (Flight-to-safety) Securities with low liquidity risk, i.e., higher time-0 price \( p_0 \), are funded more in the secured money market,

\[
\frac{\partial M_0^s}{\partial p_0} > 0,
\]

due to lower funding costs \( c_0^s \), and higher secured leverage.

(ii) (Risk-seeking) Securities with high liquidity risk, i.e., \( Cov_0(p_1, \varphi_1) < 0 \), are funded more in the unsecured money market. As \( p_0 \) decreases,

\[
\frac{\partial M_0^u}{\partial p_0} < 0,
\]

\(^{11}\)See the appendix on Proposition 4.
unsecured funding increases due to higher expected marginal returns and unsecured leverage.

These cross-sectional findings relate to aggregate patterns observed in money markets before and during the recent crisis. As a "flight to quality" or "flight to liquidity" set amidst the subprime crisis, borrowers shifted their portfolio holdings to safer, less volatile assets. This change on the asset side corresponds to a shift to secured funding on the liability side, which has been shown empirically for the European money market (Mancini, Ranaldo, and Wrampelmeyer, 2015). The reason is that securities that are expected to be little or unaffected by common funding illiquidity in the next period carry a low return and low haircuts. This implies that secured funding costs are low and secured leverage high, which crowds out unsecured funding volumes and appears as a "flight to secured funding".

In contrast, a portfolio of securities carrying a liquidity risk premium is funded predominantly in the unsecured market due to higher expected returns and haircuts. Hence, a lower time-0 price causes volatility and higher margins, so that borrowers engaging in risk-seeking fund their portfolio in the unsecured market. Related to this pattern, Acharya and Steffen (2015) provide empirical support for risk-seeking behavior during the recent European sovereign debt crisis. Banks invested in riskier government bonds, expecting prices to go up as common funding conditions would improve. As these assets carry high haircuts or are even ineligible as collateral in the secured market, risk-seeking behavior corresponds to an increase in unsecured funding.\(^\text{12}\)

Proposition 4 also relates to our previous findings. The substitution effect holds even before markets are fragile, and access to funding liquidity increases leverage. Importantly, also a shift to safe assets increases the shadow cost differential, \(\varphi^u_0 < \varphi^s_0\), and makes markets more susceptible to future shocks. This is consistent with actual developments in financial markets. For example, after banks increased their holdings of a priori safe (European) government bonds with low volatility following the 2007 to 2009 financial crisis, markets became fragile again when these assets were shocked. As derived in this section, the vulnerability of borrowers stems from high leverage, which impairs the substitution of funding liquidity and leads to fragility. In the next section, we turn to the social optimum to assess monetary and regulatory policy measures with regard to their effectiveness in preventing fragility.

\(^{12}\)We discuss the impact of central bank monetary policy on funding conditions in the next section.
5. Social Optimum and Central Bank Monetary Policy

The social optimum is characterized by the optimal demand, $x_t^*$, and leverage ratio, $L_t^*$, which determines the social costs.

5.1. The Social Equilibrium

We consider an unconstrained social planner that maximizes total welfare. From all agents’ utility functions, the optimal demand is $x_t^* = Z_t$ (Gromb and Vayanos, 2002). The social costs differ from the private funding costs by the externality of leverage on the borrower’s funding constraint in the unsecured market. Hence, the planner derives the socially optimal leverage ratio by maximizing the borrower’s expected wealth for $x_t^*$ and $x_t^u$, internalizing that $i_t^u(L_t)$ is a function of total leverage. For the first-order condition with respect to $x_t^s$, we have

$$\frac{\partial E_t[W_{t+1}]}{\partial x_t^s} = (\phi_t - c_t^s) - x_t^u \frac{\partial \mu}{\partial L_t} \frac{\partial L_t}{\partial x_t^s}. \quad (33)$$

The first term of Equation (33) is identical to the borrower’s optimization and yields the net return of an asset funded in the secured market. The second term is neglected by the borrower and adds to the funding costs in the secured market by an amount equal to the shadow costs in the unsecured market as shown in Equation (22). Thus, social costs amount to the sum of $c_t^s$ and $\phi_t^u$.

The planner’s optimal choice of $x_t^u$ is equivalent to the borrower’s solution. Thus, we arrive at the socially optimal leverage ratio by inserting $x_t^u$ from Equation (22) into Equation (33).

Proposition 5. The social optimum is determined by the following conditions:

(i) **(Liquidity)** Market and funding liquidity is optimal when $x_t^* = Z_t$.

(ii) **(Leverage)** The optimal leverage ratio $L_t^*$ is determined by equal shadow costs of capital,

$$\phi_t^u = \phi_t^s, \quad (34)$$

implying equal funding costs, i.e., $L_t = L_t^*$ when $c_t^u(L_t^*) = c_t^s$.

The social equilibrium is characterized by (i) perfect liquidity, and (ii) sufficient capital such that $L_t = L_t^*$. As shown for the market equilibrium, the relation of shadow costs indicates whether
liquidity can be substituted in the money market. If Equation (34) holds with “<”, borrowers have excess leverage, i.e., \( c_i^u > c_i^s \), which exerts negative externalities on customers due to illiquidity, and on lenders if borrowers go bankrupt. Moreover, the equality of funding costs indicates that the socially optimal leverage ratio is time-varying, i.e., lower when margins are low and higher when volatility is high.

When funding constraints bind, the market allocation fails to achieve the social optimum, which calls for policy intervention. In the next section, we analyze how central bank monetary policy affects the market allocation, and evaluate its effectiveness in preventing fragility by comparing with the social optimum.

5.2. Central Bank Monetary Policy

The central bank in our model operates as the lender of last resort (Bagehot, 1873), and sets interest rate \( i_{cb} \) and haircut \( h_{cb} \) for collateralized lending/refinancing. We consider lending via a standing facility through which the central bank offers liquidity for a fixed rate \( i_{cb} \). To comply with its role as lender of last resort, we assume that at time 0 the central bank adds an additional buffer to the \( \pi \)-value-at-risk and borrowers face lower costs \( c_0^s < c_{cb} \) for interbank liquidity.\(^{13}\)

To ease funding conditions, central banks usually intervene by conventional interest rate policy and reduce \( i_{cb} \), which affects money market rates according to Equation (15). The reduction in \( i_{1u} \) loosens the funding constraint in the unsecured market and increases demand as \( \frac{\partial x_{1u}}{\partial i_{1u}} < 0 \).

In contrast, unconventional measures are directly targeted at distressed assets, and include haircut policy and asset purchase programs. Haircut policy is conducted to provide funding liquidity for lower margins than in the private market, which is particularly effective when assets have become ineligible as collateral or highly capital-intense in the secured market. Since costs of secured funding depend positively on the haircut, haircut policy essentially represents a subsidy vis-à-vis the interbank money market (Drechsler et al., 2013). In contrast, outright asset purchases provide market liquidity, as the central bank directly intervenes in the open market with position \( x_{cb}^1 \). Both haircut policy and asset purchases are associated with financial risk, as central banks provide liquidity against assets or collateral from private agents without knowing the fundamental value.

To be socially efficient, monetary policy interventions must meet the liquidity and leverage

\(^{13}\)Alternatively, the central bank haircut is simply \( h_{cb} = 1 \), and assets are ineligible.
condition of Proposition 5. Trivially, all policy measures increase demand and prevent fragility if $x^*_1 = Z_1$. Through asset purchases, the central bank reduces supply by $Z_1 - x^{cb}_1$, which alleviates downward price pressure. For interest rate and haircut policy, borrowers’ equilibrium demand for secured and unsecured funding increases as given by Equations (21) and (22). Most importantly, the optimal liquidity condition assures the prevention of present fragility, while the optimal leverage condition aims to prevent future fragility.

**Proposition 6.** After a fundamental shock, conventional and unconventional monetary policy can prevent present fragility and restore liquidity such that $x_1 = x^*_1$. However, monetary policy measures differ in their effectiveness in achieving the socially optimal leverage ratio and preventing future fragility:

(i) **(Interest rate policy)** Conventional policy decreases money market interest rates and retains excess leverage at $L_1 > L^*_1$. Lower interest rates facilitate the substitution from secured to unsecured funding and the shadow cost differential $\varphi^u_1 < \varphi^s_1$ remains.

(ii) **(Haircut policy)** Haircut policy retains excess leverage at $L_1 > L^*_1$. If $h_{cb} < h_0$, haircut policy leads to substitution from unsecured to secured funding and $\varphi^u_1 < \varphi^s_1$ increases.

(iii) **(Security purchases)** An asset purchase program can reach $L_1 = L^*_1$ by allowing borrowers to sell illiquid securities to the central bank, such that $\varphi^u_1 = \varphi^s_1$.

During the recent crises, central banks around the world have conducted conventional as well as unconventional monetary policy. Besides decreasing interest rates close to the zero lower bound, most central banks have provided emergency lending facilities for financial institutions to obtain funding liquidity at haircuts lower than in the market, where many distressed assets have even become ineligible as collateral. Moreover, the Federal Reserve intervened in the secondary market by purchasing distressed securities as well as liquid “near-money” bonds. For the latter, the effect was similar to a further reduction in money market interest rates, which can be seen from Equation (14). As the central bank buys bonds $b$, the expected return $\phi^b_1$ decreases due to higher demand and prices, which pushes interbank rates further down. On the other hand, purchase programs of distressed assets in the secondary market allowed banks to shift off illiquid securities to the central bank.
As shown in Proposition 6, interest rate and haircut policies provide funding liquidity at conditions that prevent borrowers from selling these assets as fragility arises. If haircuts and interest rates reduce after prices have started to decrease, borrowers maintain funding the shocked asset at lower funding costs, especially in the unsecured market, or by means of lower margin requirements for secured funding at the central bank. However, the adverse consequence of such liquidity provision is that borrowers hold on to their excess leverage, which is shown by $\varphi_1^u < \varphi_1^s$ (Acharya and Tuckman, 2014). As shadow costs are lower in the unsecured market, a future shock to a borrower’s capital causes unsecured liquidity to dry up until prices decrease sufficiently and marginal returns increase. As shown by the leverage cycle, excess liquidity is followed by excess illiquidity, the higher a borrower’s leverage. Moreover, if haircuts are set lower than they were before the shock, borrowers replace unsecured liquidity by secured liquidity, which increases the shadow cost differential and potential illiquidity in the future.

In contrast, an asset purchase program allows borrowers to get rid of illiquid leverage and reduce their exposure to risky securities. Consequently, the shadow costs in the unsecured market increase due to the reduction in leverage, improving borrowers’ resilience to future shocks. If liquidity can be substituted in the money market without the need to deleverage assets, future fragility is absent. Nevertheless, slack funding constraints incentivize borrowers to lever up in other assets, which raises the need for regulatory requirements to prevent future fragility before a shock occurs.

5.3. Regulatory Policy

In the previous subsection, we show that central bank intervention may have adverse effects on banks’ future resilience. Therefore, it is important to understand what policy makers can do to make banks less vulnerable and prevent future fragility.

We discuss three important policy measures, namely enforcing maximum leverage ratios, capital buffers, and liquidity coverage ratios in the context of our model. These measures are part of the Basel III and Dodd-Frank regulatory frameworks and strive to improve banks’ resilience to sudden changes in asset values.
5.3.1 Leverage Ratio

Excessive leverage has been identified as one of the key triggers of the recent crises, which led regulators to cap bank leverage at a maximum ratio. A maximum leverage ratio is only effective when it becomes binding, such that it inhibits banks from levering up more. It is socially optimal when it prevents excessive leverage, i.e., if it enforces $L \leq L^*$. An important outcome from our model is that funding liquidity follows pro-cyclical patterns. This implies that a dynamic rather static leverage ratios are more effective in preventing future fragility. Leverage is time-varying and depends on liquidity, meaning that in times of low haircuts and profitable investment opportunities, access to unsecured funding liquidity leads to $\varphi^u_1 < \varphi^s_1$. In those times, it is socially optimal to have a binding leverage ratio, which enhances financial stability by creating slack funding constraints. As leverage increases after an asset shock, a constant maximum leverage cap inhibits banks from efficiently substituting liquidity, which enforces asset sales and illiquidity. Moreover, from Proposition 4, banks unexposed to distressed assets are potential providers of market liquidity in times of stress, and may be inhibited from buying assets as $L < L^*$. Thus, our model suggests that a dynamic maximum leverage ratio, which takes into account economic cycles would be preferrable over a static cap on bank leverage.

5.3.2 Capital Buffers

According to the current regulatory framework, banks are required to hold capital as a percentage of their risk-weighted assets, plus an additional countercyclical capital buffer in good times, if required by national regulators. As holding capital is costly, capital buffers effectively increase banks’ funding costs by an increasing function of the assets’ riskiness. As long as risk weights actually mirror the riskiness of asset, this buffer disincentivizes the build-up of risky portfolios and encourages safer investments. In our model, a buffer restraints borrowers from fully encumbering their capital $W$ for margins in the secured market, and makes the capital constraint slack. However, by holding more capital for riskier assets, such a buffer actually fosters a sort of regulatory arbitrage by incentivizing banks to invest in low-margin assets with very low risk weights. As the capital buffer is low for these assets, leverage may become excessive, which increases the vulnerability of banks to an asset shock and makes the extent of future fragility more severe. Moreover, being based on risk weights, the countercyclical buffer inherits all their drawbacks, e.g., zero buffer for
sovereign bonds.

According to Equation (33), the social funding costs in the secured market differ from the private costs by the negative externality on the funding constraint in the unsecured market. In particular, the externality increases with lower haircuts, as higher (secured) leverage implies more fragility in case of a shock. However, such externality is difficult to estimate for regulators. Yet, a similar effect is achieved by requiring banks to hold a countercyclical capital buffer linked to haircuts. In calm periods, when haircuts are low, $x^s$ is high and $\frac{\partial \mu}{\partial L} \frac{\partial L}{\partial x^s}$ increases with lower margins. By holding more capital for high-leverage assets, borrowers internalize the negative externality of leverage on the inability to substitute secured funding in times of increasing volatility and margins.

Thus, our model suggests to link the buffer to haircuts instead of risk weights, which could be implemented having banks report their portfolio holdings to regulation authorities or disclose during stress tests. That way, and assuming that haircuts accurately reflect the security risk, risk weights would consistently be adjusted for market risk and individual holdings. Risk weights essentially become an inverse function of volatility, while regulators determine the size of the buffer.

5.3.3 Liquidity Coverage Ratio

New regulations also comprise a liquidity coverage ratio, imposing banks to hold high-quality liquid assets (as a fraction of short-term expected liabilities) to better cope with sudden liquidity needs. In essence, these assets are similar to near-money bonds in our model, as they are required to be readily convertible into cash and eligible for central bank refinancing. In case of an asset shock, these securities can be sold or pledged to repay short-term loans, essentially functioning like a capital buffer without being conditioned on risk weights. As these assets earn a lower return, they reduce borrowers’ profits (Equation (15)). However, due to flight-to-safety or central bank asset purchase programs, their demand and prices increase in bad times, thus providing a hedge to riskier securities.

In contrast to a capital buffer, a liquidity coverage ratio provides no incentive for regulatory arbitrage, but it also does not incentivize banks to implement a socially optimal leverage ratio. In the context of money market fragility, if the socially optimal leverage cannot be determined by regulators, a liquidity coverage ratio in combination with a flexible capital buffer inversely related to haircuts most effectively counteracts future fragility and makes banks more resilient to adverse market situations.
6. Empirical Application

As a final step, we perform a simple empirical exercise to assess the main mechanisms of our model. To that end, we take the recent European sovereign debt crisis as a real-world example of an asset shock, including significant losses in value of government bonds of Greece, Ireland, Italy, Portugal, and Spain (GIIPS). We analyze changes in banks’ funding structure following the shock and highlight the role of margins.

6.1. Data

A complete empirical analysis of our theoretical results would require bank-specific data on assets and liabilities, money market and central bank borrowing volume, margins, and interest rates. These data are not publicly available, unfortunately. Thus, for our empirical investigation, we combine data from the richest sources available to construct proxies for the various quantities of interest.

First, we use European government bond holdings from stress test data published by the European Banking Association (EBA) since March 2010. From this list, we take all banks that participated in the stress tests for March and December 2010 as well as December 2011. We denote total bond holdings of bank \( j \) in year \( t \) by \( B_{j,t} \).\(^{14}\) Second, for these banks we collect yearly balance sheet data on money market funding volumes, including secured and unsecured borrowing and lending for 2009 through 2011. We compute each bank’s yearly secured (unsecured) net borrowing volume as the difference between the secured (unsecured) borrowing and lending volumes. To comply with our theoretical analysis, we consider net borrowers in the money market, for which the sum of secured and unsecured net borrowing is positive. For each bank, we compute the share of unsecured funding, denoted by \( S_{j,t} \), as the ratio of unsecured net borrowing over total money market net borrowing. Third, we construct a proxy for the average margin for banks’ bond portfolio. We obtain margin span parameters published by LCH.Clearnet, a major clearing house and provider of risk and collateral management services, as a measure of country-specific margins for a range of government bonds, including Germany, France, Italy, Spain, Netherlands, Belgium,

\(^{14}\)We use the March 2010 EBA stress test bond holdings for end-of-year 2009, as stress tests were first conducted in March 2010.
and Greece (until end of 2010). We construct bank-specific margins by the average of margin parameters weighted by each bank’s exposure to each country from the EBA stress test data. That is, we multiply each margin with a bank’s position in that bond and divide by the total position of that portfolio. We denote the weighted average margin by $m_{j,t}$. Lastly, we obtain data on central bank borrowing from Bruegel, which include a breakdown of Eurosystem liquidity across national European central banks. Countries not considered by Bruegel include non-Euro countries such as U.K. and Sweden for which we collect data manually from the respective national central banks and convert them into Euro. As a proxy for banks’ individual central bank exposure we use the share of their GIIPS holdings relative to all their national peers’ GIIPS holdings, taking the full list of banks participating in the stress tests. To measure the reliance on central bank funding, we divide each bank’s borrowing volume from the central bank by the banks total assets. We denote this variable by $CB_{j,t}$. Merging the different data sources results in a total sample of 26 banks.

6.2. Regression Analysis

To empirically test the main mechanisms of our model, we perform cross-sectional least squares regressions for changes in variables between two time periods, namely 2009 to 2010 and 2009 to 2011. We use 2009 as the pre-European-sovereign-debt-crisis date and 2010/2011 as time of stress in GIIPS bonds. First, we investigate the relation between margins and the share of unsecured funding. According to the model, we expect a positive relation, as higher margins lead to a substitution from secured to unsecured funding, and thus an increase in $S_{j,t}$. We control for banks’ reliance on central bank by including the change in $CB_{j,t}$ from 2009 to 2010. In sum, we estimate the following regression:

\[ \Delta S_{j,10} = \beta_0 + \beta_1 \Delta m_{j,10} + \beta_2 \Delta CB_{j,10} + \epsilon_j. \]  

\[ \text{(35)} \]
Second, we investigate the relation between banks’ deleveraging from 2009 to 2010 and their pre-crisis leverage as well as the change in their reliance on central bank funding. According to our model, banks with higher initial leverage should be affected more by the shock and thus have to deleverage more and rely more on central bank funding. We estimate the following regression model:

$$\Delta B_{j, 10} = \beta_0 + \beta_1 \Delta CB_{j, 10} + \beta_2 L_{j, 09} + \epsilon_j.$$  (36)

Table 1 shows the regression results. Despite the limited number of observations and the simplicity of the empirical exercise, the results support the predictions of our theoretical model. An increase of margins and a decrease of reliance on central bank liquidity is associated with an increase in the share of unsecured borrowing. Moreover, higher leverage in 2009 and increases in central bank borrowing are associated with a larger deleveraging.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta^0_{09}$ Share</th>
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<th>$\Delta^0_{09}$ Bonds</th>
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</tr>
</thead>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\Delta m$</td>
<td>0.081**</td>
<td></td>
<td>0.079*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.038)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta CB$</td>
<td>-0.777</td>
<td>-0.553</td>
<td>145.357*</td>
<td>145.282**</td>
</tr>
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<td></td>
<td>(1.083)</td>
<td>(1.019)</td>
<td>(76.176)</td>
<td>(63.002)</td>
</tr>
<tr>
<td>$L_{09}$</td>
<td>-1.312***</td>
<td></td>
<td>-1.291***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.386)</td>
<td></td>
<td>(0.363)</td>
<td></td>
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<td>-0.030</td>
<td>-0.101</td>
<td>10.745</td>
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<tr>
<td></td>
<td>(0.063)</td>
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<td>(0.067)</td>
<td>(9.747)</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>9.887</td>
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<td></td>
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<td></td>
<td>(9.112)</td>
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<td>$R^2$</td>
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<td>0.02</td>
<td>0.18</td>
<td>0.32</td>
</tr>
<tr>
<td>Obs.</td>
<td>26</td>
<td>25</td>
<td>25</td>
<td>26</td>
</tr>
</tbody>
</table>

7. Conclusion

This paper provides the first comprehensive theoretical model for money markets. It offers a unified framework to analyze the relation between short-term secured and unsecured money market.
funding liquidity, and the interaction with assets’ market liquidity. Additionally, a central bank affects money markets with conventional and unconventional monetary policies. Our model shows that liquidity spirals also exist in the unsecured money market, mutually reinforcing with liquidity spirals in the secured market by exerting downward pressure on prices and deteriorating borrowers’ capital. Moreover, we explain the cross-sectional differences and time-series variation in funding shares as well as the dynamics of liquidity substitution and how the extent of money market fragility depends on bank leverage.

We derive the optimal leverage ratio from a social planner’s point of view and analyze central bank and regulatory policy. We show that monetary policies can prevent present fragility and restore liquidity, but with potentially adverse effects on leverage and future fragility. A relaxation of interest-rate (haircut) policy facilitates the substitution from secured (unsecured) to unsecured (secured) funding. However, both interest-rate and haircut policies are ineffective in reducing excess leverage. In contrast, an asset purchase program allows borrowers to reduce leverage and risky securities. Nevertheless, the resulting slackness of the borrower’s funding constraints incentivizes to lever up in other assets.

On the regulatory side, our model suggests that countercyclical maximum leverage ratios and capital buffers enhance banks’ resilience to future shocks more adequately than static measures. A capital buffer restrains borrowers from fully encumbering their capital for margins to obtain secured funding, and relaxes the capital constraint. However, it is not sufficient on its own, because it can induce banks to invest in low-margin assets, resulting in excess leverage and in larger vulnerability of banks to an asset shock. Thus, an important policy implication from our analysis is that a combination of a countercyclical leverage ratio, preventing excess leverage, and a capital buffer inversely linked to haircuts, easing funding strains in the secured market, counteract future fragility and make banks more resilient to adverse market situations.
References


Bagehot, W., 1873. Lombard Street: A Description of the Money Market.


36
Appendix A: Proofs

Proof of Lemma 2. Since $Z_1 > 0$, it follows that $p_1 \leq \nu_1$ and $x_1^u \geq 0$. From the market clearing condition, we have that $x_1^u = -\sum_{k=0}^{1} y_k^1$. From Equation (22), we get

$$\frac{\phi_1 - i_1^u}{\mu'(L_1)p_1} W_1 \geq Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2}. \quad (37)$$

We prove non-monotonicity in the borrower’s demand function for an exogenous shock $\eta_1 < 0$. As shown in Brunnermeier and Pedersen (2009) for the secured market, fragility arises if a borrower’s funding need is higher when the price decreases further below the fundamental value. Therefore, we re-arrange Equation (37) for $\eta_1$,

$$G(p_1) := \frac{Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2}}{\frac{\phi_1 - i_1^u}{\mu'(L_1)p_1}} - p_1 x_0^u - b_0 \leq \eta_1, \quad (38)$$

where $G(p_1)$ intuitively measures the borrower’s funding need in equilibrium as a function of the market price $p_1$. Thus, fragility occurs if $G'(p_1)$ can be negative:

$$G'(p_1) := \frac{x_0^u}{W_1 \gamma(\sigma_2)^2} - \left( \frac{Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2}}{\frac{\phi_1 - i_1^u}{\mu'(L_1)p_1}} \right) \left[ \frac{\mu'(L_1)p_1 \frac{\partial(\phi_1 - i_1^u)}{\partial p_1} - (\phi_1 - i_1^u)\mu'(L_1)}{(\mu'(L_1)p_1) \gamma(\sigma_2)^2} \right] - x_0^u. \quad (39)$$

For the individual terms, we have $\frac{\partial \phi_1}{\partial p_1} < 0$ and $\frac{\partial i_1^u}{\partial p_1} = \frac{\partial L_1}{\partial L_1} \frac{\partial L_1}{\partial p_1} < 0$, because $\frac{\partial L_1}{\partial p_1} < 0$ since $A_0 > W_0$. Since Equation (39) is decreasing in all terms containing $x_0$, there exists a position $x_0^u$, such that for $x_0 > x_0^u$, the whole expression is decreasing in $p_1$, and $G'(p_1) < 0$.

Proof of Proposition 1. We show the price sensitivity to a fundamental shock $\Theta_1 < 0$ when funding constraints bind. As in Brunnermeier and Pedersen (2009), using the implicit function theorem to compute the derivatives at the equilibrium position, for the secured market we have

$$m_1 \left( Z_1 - \frac{2(\nu_1 - p_1)}{\gamma(\sigma_2)^2} \right) = b_0 + x_0^s p_1 + \eta_1. \quad (40)$$
Among all parameters, margin $m_1$, volatility $\sigma_2$, and the fundamental value $\nu_1$ are functions of the shock $\Theta_1$. In an illiquid equilibrium, the denominator of Equation (25) is positive. Thus, liquidity spirals occur because $\frac{\partial m_1}{\partial p_1} < 0$ and $\frac{\partial m_1}{\partial \Theta_1} < 0$, representing the margin spiral, and $\frac{\partial W_1}{\partial p_1} > 0$ for $x_0 > 0$ showing the loss spiral.

The equilibrium position in the unsecured market is as given by Equation (37), and $\sigma_2$, $\nu_1$, and $\phi_1$ are functions of $\Theta_1$. As above, the whole term is positive in an illiquid equilibrium, and so the denominator of Equation (26) is negative. Due to the interest rate spiral, $\frac{\partial i_u}{\partial p_1} < 0$, and loss spiral, $\frac{\partial W_1}{\partial p_1} > 0$ for $x_0 > 0$, the value of the denominator increases, i.e., becomes less negative, and liquidity spirals exert a negative effect on the price.

**Proof of Proposition 2.** Part (i) of this proposition follows directly from the way leverage is computed. For the same capital $W_1$, $L_1 = L_1^s + L_1^u$ is additive, and the only variable by which secured and unsecured leverage differ is the respective position $x_1^s$ and $x_1^u$. Since $L_1^s$ decreases and $L_1^u$ increases, the share $\Delta \frac{x_1^u}{x_1^s}$ increases, provided that $\Theta_1 < 0$. By the definition of funding volumes, this refers to an increase in $M_1^u$ relative to $M_1^s$.

For part (ii) of Proposition 2, the covariance is positive since both shadow costs $\varphi_1^s$ and $\varphi_1^u$ depend on the same random variable $p_1$. When constraints bind and the price decreases, shadow costs increase due to the increase in $\phi_1$. In a liquid equilibrium, constraints are slack and the funding costs increase from a small reduction in $p_1$, such that shadow costs decrease jointly.

**Proof of Lemma 3.** Since $Z_1 \geq 0$, borrowers’ funding constraints are slack when $dx_1 \geq 0$, which happens if wealth $dW_1 \geq 0$. The intertemporal time-1 budget constraint is slack for

$$W_0 + x_0 \phi_0 \geq x_0^c c_0^s + x_0^u c_0^u. \quad (41)$$

In the time-0 equilibrium, we have $dW_0 = 0$, and $dx_0 = 0$ implies that $dx_0^s = -dx_0^u$, i.e., less secured funding is perfectly substituted by more unsecured funding, and

$$\phi_0 - c_0^u \geq \phi_0 - c_0^s. \quad (42)$$
If the shadow costs in the unsecured market are larger (or equal) than the shadow costs in the secured market, a binding capital constraint in response to a shock at time 1 allows the borrower to fund \( dx_1 \geq 0 \) in the unsecured market due to a sufficiently large marginal return. If Equation (42) holds with “\(<\)”, then a (small) capital loss leads to binding funding constraints in both markets and \( dx_1 < 0 \).

**Proof of Proposition 3.** Part (i) of Proposition 3 follows directly from Lemma 3. Since \( c^u_0 = i^u_0(L_0) \) and for any given expected return \( \phi_0 \), the spread \( \phi_0 - i^u_0 \) is smaller the larger is \( L_0 \). From Lemma 2, for \( x_0 > x_0 \), there is fragility at time 1 if \( \Theta_1 < 0 \) and \( Z_1 > 0 \). Consequently, funding illiquidity, as measured by the shadow costs, increases until the market clears. Since \( p_1 \) decreases from \( dx_1 < 0 \), the price drop is larger the higher was \( L_0 \). If \( dx_1 < 0 \) is so large that \( p_1 \leq \bar{p}_1 \), where \( \bar{p}_1 \) is such that \( x_0(p_0 - \bar{p}_1) \geq W_0 \), funding in the unsecured market is unavailable for a borrower at an interest rate that allows borrowing sufficient funds to clear the market at a price \( p_1 > \bar{p}_1 \) (part (ii)).

**Proof of Proposition 4.** We show this proposition for any asset \( j \in J \). Since there is commonality in funding liquidity, \( Cov_0(\varphi^s_1, \varphi^u_1) > 0 \), common money market illiquidity is the volume-weighted average of the shadow costs, i.e., \( \varphi_1 = \frac{1}{x_1}(x^s_1\varphi^s_1 + x^u_1\varphi^u_1) \). Common funding costs \( c_0 \) at time 0 are derived similarly. Hence, the borrower at time 0 solves \( E_0[W_1(1 + \varphi_1)] \) for \( x_0 \) and gets

\[
E_0 [(p_1 - p_0(1 + c_0))(1 + \varphi_1)] = 0. \tag{43}
\]

Since covariance is bilinear, rearranging yields a time-0 price of

\[
p_0 = \frac{E_0[p_1(1 + \varphi_1)]}{(1 + c_0)E_0[1 + \varphi_1]} = \frac{E_0[p_1]}{1 + c_0} + \frac{Cov_0[p_1, \varphi_1]}{(1 + c_0)E_0[1 + \varphi_1]]. \tag{44}
\]

Accordingly, the time-0 price is lower the higher are funding costs \( c_0 \), and if an asset has liquidity risk, \( Cov_0[p_1, \varphi_1] < 0 \).
Proof of Proposition 6. We show the results of Proposition 6 jointly for all parts (i) to (iii). First, a reduction in $i_{cb}$ reduces money market interest rates according to Equation (14), which relaxes the borrower’s leverage constraint and, ceteris paribus, increases unsecured funding,

$$\frac{\partial M_i^u}{\partial i_{cb}} = \frac{-W_1}{(1 - \theta)\mu'(L_1)} < 0.$$  \hfill (45)

Second, haircut policy relaxes the capital constraint and provides secured funding liquidity from the central bank. We have

$$\frac{\partial M_i^s}{\partial h_{cb}} = \frac{-W_1}{h_{cb}^2} < 0.$$  \hfill (46)

Third, asset purchases $x^{cb}_1 > 0$ decrease the supply of securities to $-(Z_1 - x^{cb}_1)$, which leads to market clearing at a higher equilibrium price. Since interest and haircut policy simply reverse the downward pressure on the price, leverage $L_1$ and thus shadow costs remain unchanged. Finally, asset purchases reduce borrowers’ total position $x_1$ due to positive demand $x^{cb}_1$, and $L_1$ decreases.
Appendix B: Additional Tables

Table A.1
Summary Statistics

This table provides a list of all banks and parameters included in the regression analysis. Descriptive statistics of the parameters are given below. All data are rounded and denominated in Euro.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Country</th>
<th>∆Share</th>
<th>∆m</th>
<th>∆CB</th>
<th>L09</th>
<th>∆Bonds</th>
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<td>IT</td>
<td>-0.048</td>
<td>0.203</td>
<td>0.016</td>
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Mean        | -0.031 | 0.889 | 0.005 | 23.317| -19.846|
Median      | -0.014 | 0.410 | -0.002| 21.923| -12.823|
Std. dev.   | 0.291  | 1.502 | 0.056 | 9.858 | 22.695 |
Min         | -0.522 | -0.220| -0.063| 10.900| -83.138|
Max         | 0.637  | 7.184 | 0.239 | 48.184| 13.527 |
This table shows the results of regressing changes in banks’ funding shares (Columns (1) to (3)) and changes in bond holdings (Columns (4) to (6)) on explanatory variables derived from our model. Banks for which central bank figures are unavailable include ING Groep NV and Dexia SA. Standard errors are shown in parentheses. The stars *, **, and *** indicate statistical significance at the 1%, 5%, and 10% level, respectively.

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<td>0.009</td>
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\(R^2\)