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Credit Rating Dynamics and Competition

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Abstract. I analyze credit rating agencies and competition on a market with more than two agencies. Both investors and agencies react to each other's behavior. My model predicts cyclic dynamics in the base case: not only does the presence of trusting investors facilitate ratings inflation. In turn, ratings inflation also induces investors to be less trusting. A regulator can implement honest rating behavior by abolishing the "issuer pays" model or by a centralized monitoring of ratings quality. The effect of the entry or exit of an agency on ratings quality depends on the current number of rating agencies on the market.

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1. Introduction

How does the complex interaction between credit rating agencies (CRAs), issuers, and investors affect the quality and informativeness of credit ratings? CRAs are widely considered to have been a major factor within the development of the 2008 subprime mortgage crisis, accused of intentionally inflating ratings, i.e., giving good ratings to bad issues. Most recently, the U.S. Justice Department charged the largest CRA, Standard & Poor's (S&P), with fraud and demanded US$5-billion in restitution in early February 2013, see Mattingly (2013). In this paper, I aim to answer the question how honest rating behavior can be achieved. First, I analyze how investor behavior affects the behavior of CRAs and vice versa. Second, I make predictions about how this interaction is affected by the competitiveness of the market for credit ratings, as measured by the number of CRAs on the market. Third, I provide implications for policy makers how regulation can support honest rating behavior.

The U.S. market is characterized by a limited number of approved CRAs, so-called Nationally Recognized Statistical Rating Organizations (NRSRO). First there were only Moody's and S&P, and since approximately 1997, Fitch has been there as the third agency. In the meantime, seven more agencies have been approved, so there are now ten CRAs that are designated as NRSROs.[1] The three big CRAs have more than 90 percent of the market share, see Atkins (2008). In earlier years, the NRSRO designation possibly was a barrier of entry. However, the current situation with ten NRSROs on the market and seven of them not significantly improving their market shares suggests that further analysis is needed: within a theoretical framework that allows for ten or more agencies, it remains to be answered under which conditions players with negligible market shares can improve their positions.[2] It might be particularly interesting to determine the conditions under which a new rating agency that possibly has different ethical standards and business practices can successfully invade the market, even if it starts off with a tiny market share.

The current market for credit ratings is dominated by the "issuer pays" business model.[3] It was switched from an earlier "investor pays" model due to information drain and difficulties in collecting sufficient fees.[4] However, the "issuer pays" model has an inherent

[1] See http://www.sec.gov/answers/nrsro.htm, retrieved February 2, 2012.

[2] Jeon and Lovo (2011) suggest that apart from the approval as NRSRO, natural barriers of entry can be a critical factor and prevent new CRAs from being successful against the incumbent.

[3] In the present paper, I consider only the case of solicited credit ratings, i.e., that the issuer pays for receiving a rating. See, e.g., Bannier, Behr, and Güttler (2010) and Fulghieri, Strobl, and Xia (2012) for a discussion of unsolicited credit ratings, i.e., those provided by CRAs without receiving compensation from the market. For the U.S. market, Gan (2004) estimates that unsolicited ratings account for 22% of all new issue ratings between 1994 and 1998.

[4] There are some smaller CRAs on the market that apply the "investor pays" model. One of them, namely Egan-Jones Ratings Company, is even an NRSRO. A recent investigation by the U.S. Securities and Exchange Commission (SEC) against Egan-Jones shows that the "investor pays" model is not necessarily free of conflicts of interest either, see SEC (2012). The theoretical model by Stahl and Strausz (2011) suggests that the "issuer pays" business model might be superior to the "investor pays" model. They argue in a more general context that sellers (issuers) rather than buyers (investors) of an information-sensitive

conflict of interest: It can be profitable for the CRA to inflate ratings.[5] A related problem is ratings shopping: The issuer chooses to pay the fee only to a CRA that promises to give a favorable rating, and approaches a competitor otherwise. The question whether more competition can increase ratings quality is thus related to whether it helps to prevent ratings inflation, although it can give more opportunities for ratings shopping.

As two shortcomings of existing theoretical models, I identify that they only consider competition in duopoly and usually neglect the dynamic properties of the market. This gap that my paper aims to fill is well motivated by quoting Bolton, Freixas, and Shapiro (2012) (hereafter BFS), p.104: "It would be of interest (but beyond the scope of [their] paper) to explore these issues more systematically in a fully general dynamic game, possibly with an infinite horizon. There is currently no model of oligopolistic competition over an infinite horizon in the CRA literature; indeed, there are very few such models in the industrial organization literature for obvious reasons of tractability." Therefore I develop a tractable framework using Evolutionary Game Theory to analyze the interaction of CRAs over an infinite horizon in a competitive market with an arbitrary number of agencies. I model the CRAs' incentives to inform the investors honestly about the quality of investments, rather than to inflate ratings, as an interplay with investors' sophistication level. These characteristics of CRAs and investors are similarly modeled in BFS and other papers. As the main innovation on the modeling side, the methodology of Evolutionary Game Theory allows for an arbitrary number of market participants, as well as the analysis whether new behavioral traits successfully enter an established market.

As an important contribution, I do not only consider changes in CRA behavior, but I also allow the behavior of the investors to react on the characteristics of the CRA market they face. On the one hand, CRAs are more likely to inflate ratings for a high share of trusting investors, as the benefits from receiving more fees outweigh the possible reputation costs if caught inflating. On the other hand, investors will in turn change their behavior when they face a CRA market with a lot of ratings inflation. As sophisticated investors perform better than trusting investors on such a market, the latter will either start leaving the market or learn to be more sophisticated as well. In my model, sophisticated investors have to spend costs for the monitoring of investment and ratings quality, whereas trusting investors save these costs but suffer when they happen to buy bad investments.

First, I show that the interaction between the CRAs' and the investors' behavior can lead to different equilibria. Dependent on the parametrization, either honest or inflating CRAs dominate in the end, while the population of investors ends up being either trusting or sophisticated. There can even be cyclic dynamics with the distribution shares in both populations periodically increasing and decreasing over time.

good should pay for certification (ratings). While sellers want to signal quality, buyers have to inspect quality, the former being both socially more desirable and generating higher rents to the certifier (CRA).

[5] Throughout my paper I assume that CRAs have perfect knowledge about the actual quality of investments, and the only remaining issue is whether they truthfully report information to the investors. This assumption is questionable, especially in the light of recent underperformance when rating structured products. Among others, Pagano and Volpin (2010) investigate the interplay of ratings inflation and the failure of CRAs to provide accurate ratings, and Bar-Isaac and Shapiro (2012) discuss how ratings quality is related to analyst skills. Pagano and Volpin (2012) show that issuers may prefer (and CRAs produce) coarse and uninformative ratings even in the absence of ratings inflation and ratings shopping.

In a second step, I make additional assumptions about how the model's parameters could depend on the number of CRAs in the market. Thus, I can show how different market structures and outcomes result from changing the number of CRAs. Existing theoretical models can only distinguish between a monopolistic CRA and competition in duopoly. In contrast, I show that increasing competition can lead to significant changes in market structures and outcomes for any arbitrary current market size, for example when one new CRA enters a market currently consisting of two, three, or ten CRAs. I postulate that for example reputation costs increase with the number of CRAs on the market, as the investors' threat of punishing inflating CRAs becomes more effective. In this case there is a critical number of CRAs on the market, above which the reputation costs are high enough such that honest rating behavior pays off at least temporarily.

Finally, my analysis leads to the following two policy recommendations, without explicitly addressing the number of CRAs on the market: First, it is essential to find an alternative solution to the "issuer pays" model, and particularly to prevent that rating agencies can achieve higher revenues by issuing good ratings. Second, the monitoring of CRAs' performance and their possible punishment should rather be done (even more so) by a regulator or a central market authority, rather than individual investors. If at least one of these issues can be solved, then the market for credit ratings will function well in the sense that honest rating behavior is viable, independent of the size of the CRA market. Regarding the optimal number of CRAs on an oligopolistic market, I show that the entry or exit of an agency can change the equilibrium outcome and has an ambiguous effect on ratings quality. The direction of the effect depends on the current number of CRAs on the market.

Most researchers agree that ratings inflation is most severe for complex investment products as in structured finance, see e.g. Skreta and Veldkamp (2009) and BFS. On the other hand, Baghai, Servaes, and Tamayo (2013) show that CRAs have even become more conservative in assigning corporate credit ratings. In line with this, my paper should also be understood in the context of rating complex investment products rather than corporations.[6] Mathis, McAndrews, and Rochet (2009) analyze ratings inflation for a monopolistic CRA. They derive a reputation cycle similar to the cyclic dynamics in the base case of my model, however they exclude the possibility of the CRA to recover its reputation, once it is caught lying. In general, the existing literature comes to ambiguous conclusions on the relation between competition and ratings inflation.[7] Camanho, Deb, and Liu (2012) extend Mathis, McAndrews, and Rochet (2009) by competition effects. They find that competition results in greater ratings inflation. Similarly, Skreta and Veldkamp (2009) and BFS find that competition makes ratings shopping worse. On the contrary, Manso (2012) highlights that credit ratings can have feedback effects on the credit quality of issuers. He finds that increased competition between rating agencies can create downward pressure on ratings and tougher rating policies. Doherty, Kartasheva, and Phillips (2012) show both theoretically and empirically that the market entry of a new CRA can improve ratings quality and precision. Their story is that the entrant CRA can attract business from good issuers that have been pooled with worse quality issuers. By using a more

[6] I thank David Lando for pointing this out.

[7] Apart from competition effects, ratings inflation is influenced by other factors. For example, Opp, Opp, and Harris (2012) explain how rating-contingent regulation can contribute to ratings inflation. Stolper (2009) suggests that the problem might be solved by a proper regulatory approval scheme for CRAs.

precise rating scale, the entrant CRA allows the good issuers to receive higher prices for the investments they sell. Becker and Milbourn (2011) take the market entry of Fitch as a natural experiment to analyze the effect of increasing competition. Overall, they show a decrease in ratings quality. Bongaerts, Cremers, and Goetzmann (2012) suggest a different role of Fitch, namely being a tiebreaker if the other two big rating agencies, S&P and Moody's, disagree whether a bond issue has investment grade or high yield status. Assuming that a Fitch rating is solicited more often if the issuer expects it to break the tie towards investment grade, this endogeneity provides an alternative to the "ratings inflation" story, explaining why the observed Fitch ratings are higher on average. Similarly, Xia (2012) provides evidence in the opposite direction of Becker and Milbourn (2011). He shows that increased competition by the entry of investor-paid Egan-Jones Rating Company led to higher ratings quality for S&P. He, Qian, and Strahan (2011) examine whether rating agencies reward large issuers of mortgage-backed securities. After controlling for deal characteristics, they can analyze a situation in which small and large issuers differ only in the amount of possible future business. They find evidence for a positive bias of CRAs towards large issuers and thus for ratings inflation. Similarly, Mählmann (2011) shows that firms with longer rating agency relationships have better credit ratings, although they do not have lower default rates.

Hörner (2002) provides a general reputational theory in which competition can increase quality if the consumers' competitive choice makes loss of reputation a real threat. One of the first papers analyzing the trade-off between building up a long-term reputation and making higher short-term profits by misbehaving is by Klein and Leffler (1981). So in principle, if more competition should be a cure of ratings inflation, it would need to affect this trade-off towards the benefit of building up a long-term reputation. Related to Klein and Leffler (1981) and Hörner (2002), my hypothesis is that reputation costs are too low in a market with a small number of CRAs. The investors and issuers need to have a sufficient number of alternatives. Then the loss of reputation is a real threat that can induce honest rating behavior. The transition from monopoly to duopoly, or even to a market with three participants, still may not provide sufficient alternatives. This can explain that Becker and Milbourn (2011) do not find an increase in ratings quality following the market entry of Fitch. However, the change could come for an even larger number of CRAs, some of whom possibly offering true alternatives to the existing ones.

The paper is structured as follows: I introduce the modeling framework in Section 2.. In Section 3., I visualize and discuss the results for an arbitrary number of CRAs. Next, I discuss the effect of competition and explicitly focus on the number of CRAs in Section 4.. Then I provide empirical implications in Section 5.. In Section 6., I critically investigate limitations and present extensions of my model. Section 7. concludes the paper.

2. Model

The model is based on the methodology of Evolutionary Game Theory. More precisely, I build on the replicator dynamics as given in Taylor and Jonker (1978) and Taylor (1979), as well as Schuster, Sigmund, Hofbauer, and Wolff (1981), who derive additional results for evolutionary games between two populations. The economic setting of the model is given for the duopolistic case by BFS, from which I also use the notation as far as possible to ensure comparability.

2.1 SETUP

I consider a market on which issuers provide two types of investments. First, a good investment, which is present on the market with a share $\lambda \in [0, 1]$. It yields a payoff $1 + R > 1$ upon investment of 1, i.e., a net payoff of $R > 0$. Second, a bad investment, which is present on the market with a share $(1 - \lambda)$. It yields a payoff of zero, which can be interpreted as default, upon investment of 1. So the net payoff of the bad investment is (-1).

Apart from the issuers, there are two populations that interact with each other. First, I consider the population of investors (Inv). There is a share $\alpha \in [0, 1]$ of trusting investors (T), and a share $(1 - \alpha)$ of sophisticated investors (S). Second, I consider the population of rating agencies (CRAs). There is a share $\beta \in [0, 1]$ of honest CRAs (H), and a share $(1 - \beta)$ of inflating CRAs (I). The population space thus consists of all possible states (α, β) within the square (0,0), (1,0), (1,1), (0,1).

I concentrate on the interplay between investors and rating agencies, while taking the remainder of the market as exogenous. As discussed below, the issuers' ratings shopping behavior is also captured by my model, although the issuers are not modeled as active players. Also, I provide suggestions that a regulator should follow to change the boundary conditions of the market.

2.2 INTERACTIONS

For each interaction, there is a random investment from a given issuer to be rated, which is good with probability λ. Then the interaction takes place in a random pairing of one investor (type T or S) and one CRA (type H or I). The players cannot recognize each other's types. Depending on the players' types, they receive payoffs as derived in the following.

The CRA charges a fee $\Phi \geq 0$ from the issuer of the investment for giving the rating.[8] The fee is received only for a good rating. This assumption is common in the literature and can be interpreted as a reduced-form modeling of ratings shopping, as the issuer will then move on and hope to find another agency who promises to give him the good rating. Therefore one could argue that the issuer's behavior, namely the choice to accept and pay only for good ratings, is modeled here in a simple form. Effectively, an investment without rating is equivalent to an investment rated as bad in my model. I assume that CRAs can perfectly observe whether investments are good or bad. An honest CRA truthfully reports the type of the investment. Thus, if the investment is bad, it cannot sell the rating to the issuer and does not receive a fee. An inflating CRA, in contrast, always reports "good" and receives the fee.

[8] First, the fee is assumed exogenous and independent of the market conditions. In Section 4., I will allow the fee (and other parameters) to be a function of the number of CRAs on the market. In the Appendix, Section A1, I analyze the case in which the fees that issuers are willing to pay to the CRAs for a good rating depend on the fraction of trusting investors on the market. In my model, there are no costs for the CRAs to produce ratings. However, Φ can also be interpreted as the "net fee", i.e., what is remaining for the CRA after production costs, given that all CRAs face the same production costs. I discuss in the Appendix, Section A2 the case of different production costs for inflating and honest CRAs.

On the investor side, trusting investors cannot judge the investments. They buy all investments that are rated as good. In contrast, sophisticated investors spend a cost $C \geq 0$ to verify the CRA's work and evaluate the investments. If they meet an inflating CRA with a bad investment rated as good, they do not buy it, and at the same time they cause reputation costs $\rho \geq 0$ for the CRA. Here, I make a strong assumption that lying CRAs can immediately be recognized and punished. A more realistic assumption would be that such behavior can only be detected with some delay, if at all. However, the assumption is consistent with the previous assumption that investments turn out to be good or bad (without any uncertainty due to overlapping realizations of the outcomes) immediately after making the investment decision. A related critical assumption is that the sophisticated investors still spend monitoring costs to observe the CRAs' behavior, although they can perfectly verify the quality of the investments themselves. It can be motivated by assuming that only by the combination of the information they receive from the CRA and their own verification efforts, the sophisticated investors are able to make such a perfect judgment of the investments.

2.3 PAYOFFS

There are several possible economic explanations for the population shares changing over time, dependent on the realized payoffs. As a result of the realized payoffs, individuals switch their behavior towards another pure strategy, unsuccessful market participants leave the market, or new market entrants observe and imitate the most successful behavior.

2.3.1 Investors

Regarding the payoffs for the investors, I first consider the trusting investors. If they are meeting an honest CRA, they receive a good rating for a good investment, which occurs with probability λ. In this case they invest and receive a net payoff of R. If the investment is bad, the investors are warned, as the honest CRA refuses to give a good rating, so they do not invest and receive zero payoff. Together, the expected payoff is

$$V_{TH} = \lambda R. \tag{1}$$

Against an inflating CRA, they receive a net payoff of R for a good investment, which occurs with probability λ. However, if the investment is bad, which occurs with probability $(1 - \lambda)$, the CRA still gives a good rating. The trusting investors invest, and consequently receive a net payoff of (-1). Together, their expected payoff is

$$V_{TI} = \lambda R + (1 - \lambda)(-1). \tag{2}$$

The resulting expected payoff for trusting investors is

$$\Pi_T^{Inv} = \beta V_{TH} + (1 - \beta)V_{TI} = \lambda R - (1 - \beta)(1 - \lambda), \tag{3}$$

as they meet an honest (inflating) CRA with probabilities β and $1 - \beta$, respectively. Second, consider the sophisticated investors. If they are meeting an honest or inflating CRA,

Table I Investors' Payoffs. The investors' payoffs are summarized for each possible investor/CRA strategy pair, the investors being either trusting or sophisticated and the CRA being either honest or inflating. In addition, the last column shows the expected payoffs for either type of investor.

Investor / CRA	honest	inflating	expected
trusting	$V_{TH} = \lambda R$	$V_{TI} = \lambda R + (1-\lambda)(-1)$	$\Pi_T^{Inv} = \lambda R - (1-\beta)(1-\lambda)$
sophisticated	$V_{SH} = \lambda R - C$	$V_{SI} = \lambda R - C$	$\Pi_S^{Inv} = \lambda R - C$

they receive the same payoff,[9] namely

$$V_{SH} = V_{SI} = \lambda R - C. \tag{4}$$

In either case, they spend the cost C to verify the CRA's work. Thus, they manage to invest only in the good investments, which occur with probability λ. The resulting expected payoff for sophisticated investors is the same, namely

$$\Pi_S^{Inv} = \beta V_{SH} + (1-\beta) V_{SI} = \lambda R - C. \tag{5}$$

The resulting payoffs are summarized in Table I. The average payoff in the population of investors is

$$\bar{\Pi}^{Inv} = \alpha \Pi_T^{Inv} + (1-\alpha)\Pi_S^{Inv}. \tag{6}$$

2.3.2 Rating Agencies

Considering the rating agencies, I first state the payoffs for the honest CRAs. They give a good rating and receive the fee only if they observe a good investment, which occurs with probability λ.[10] On the other hand, they are never punished for inflating ratings. Their expected payoff against both trusting and sophisticated investors is therefore

$$X_{HT} = X_{HS} = \lambda \Phi. \tag{7}$$

Thus, the resulting expected payoff for honest CRAs is the same, namely

$$\Pi_H^{CRA} = \alpha X_{HT} + (1-\alpha) X_{HS} = \lambda \Phi. \tag{8}$$

Second, consider the inflating CRAs. If they are meeting a trusting investor, they receive

$$X_{IT} = \Phi. \tag{9}$$

As they always give good ratings, they are always paid by the issuers, regardless of the quality of the investment. Against a sophisticated investor, however, the inflating CRAs'

[9] As an alternative assumption, one can justify that $V_{SH} = \lambda(R - C)$, while still $V_{SI} = \lambda R - C$. The rationale is that within the model, an observed bad rating indicates an honest CRA, and therefore no monitoring costs have to be spent in that case. I thank Henri Pagès for suggesting this alternative. A motivation for my original modeling can be that there are more than two rating classes in practice, and then it cannot be inferred from observing a specific rating (unless it is the lowest one) whether the CRA is honest.

[10] As an alternative specification, one could assume that honest CRAs spend some production costs for rating the investment, while inflating CRAs do not. See the Appendix, Section A2 for a discussion of this case.

Table II CRAs' Payoffs. The CRAs' payoffs are summarized for each possible investor/CRA strategy pair, the investors being either trusting or sophisticated and the CRA being either honest or inflating. In addition, the last row shows the expected payoffs for either type of CRA.

Investor / CRA	honest	inflating
trusting	$X_{HT} = \lambda\Phi$	$X_{IT} = \Phi$
sophisticated	$X_{HS} = \lambda\Phi$	$X_{IS} = \Phi - (1-\lambda)\rho$
expected	$\Pi_H^{CRA} = \lambda\Phi$	$\Pi_I^{CRA} = \Phi - (1-\alpha)(1-\lambda)\rho$

expected payoff is

$$X_{IS} = \Phi - (1-\lambda)\rho. \tag{10}$$

While the issuer still pays them the fee regardless of the quality of the investment, they are punished whenever they rate a bad investment as good, which happens with probability $(1-\lambda)$, and meet a sophisticated investor. The resulting expected payoff for inflating CRAs is

$$\Pi_I^{CRA} = \alpha X_{IT} + (1-\alpha)X_{IS} = \Phi - (1-\alpha)(1-\lambda)\rho. \tag{11}$$

The payoffs are summarized in Table II. The average payoff in the population of CRAs is

$$\bar{\Pi}^{CRA} = \beta\Pi_H^{CRA} + (1-\beta)\Pi_I^{CRA}. \tag{12}$$

2.4 TWO-PLAYER GAME BETWEEN INVESTOR AND CRA

Before analyzing the evolutionary game between the two populations of investors and CRAs, I present the related two-player game between one investor and one CRA, with the payoffs given in Tables I and II. I show the game in the normal form, which is the combination of the two payoff matrices, in Table III. As usually done in two-player game theory, the first and second entry represent the payoffs for the investor (column player) and the CRA (row player), respectively.

Throughout the evolutionary dynamics, I will assume that for each interaction, there is a random draw of one investor and one CRA with built-in types out of their respective populations. In contrast, for the current section I assume that both the investor and the CRA each choose their optimal strategies. They can either choose a pure strategy, i.e., perform one of their two respective actions with certainty, or choose a mixed strategy, which is to randomize their actions. In the latter case, the investor chooses to be trusting with probability α, and the CRA chooses to be honest with probability β, respectively. The limit cases of choosing probabilities 0 or 1 reflect the pure strategies.

As standard in Game Theory, I check for the existence of Nash Equilibria, i.e., strategies that are the best responses to each other's choices. I restrict the attention to cases in which there are at least some bad investments on the market ($\lambda < 1$), and being caught lying causes positive reputation costs for the CRA ($\rho > 0$). Otherwise, there is either no role for rating investments, or no advantage of being an honest CRA in the model. Table IV summarizes the resulting possible cases.

For the interior case with $0 < C < 1-\lambda$ and $0 < \Phi < \rho$, there are no Nash equilibria in pure strategies. However, there is a Nash equilibrium in mixed strategies with the

Table III Two-Player Game in Normal Form. For each possible investor/CRA strategy pair, the investors being either trusting or sophisticated and the CRA being either honest or inflating, the table states the payoffs for the investor and CRA, respectively, as a pair separated by commas in each cell.

Investor / CRA	honest	inflating
trusting	λR, $\lambda\Phi$	$\lambda R - (1-\lambda)$, Φ
sophisticated	$\lambda R - C$, $\lambda\Phi$	$\lambda R - C$, $\Phi - (1-\lambda)\rho$

Table IV Nash Equilibria. Dependent on the parameters of the model, there are four possible Nash equilibria in pure strategies, and one Nash equilibrium in mixed strategies with (α^*, β^*) as in Equations (13) and (14) being the probabilities for being a trusting investor and an honest CRA, respectively. The Nash equilibria in pure strategies are labeled with the capital initials of the respective strategies, i.e., (T)rusting, (S)ophisticated, (H)onest, and (I)nflating. In brackets, I indicate the weakly dominated second Nash equilibrium, if applicable. The numbers indicate the sections in which the respective cases are treated for the evolutionary model.

Φ / C	$C = 0$	$0 < C < 1-\lambda$	$C \geq 1-\lambda$	α^*
$\Phi = 0$	S/H (T/H), 3.5	T/H, 3.4	T/H (T/I), 3.4	$\alpha^* = 1$
$0 < \Phi < \rho$	S/H, 3.5	(α^*, β^*), 3.1	T/I, 3.3	$0 < \alpha^* < 1$
$\Phi \geq \rho$	S/I (S/H), 3.2	S/I, 3.2	T/I (S/I), 3.3	$\alpha^* \leq 0$
β^*	$\beta^* = 1$	$0 < \beta^* < 1$	$\beta^* \leq 0$	

probabilities (α^*, β^*) for being a trusting investor and an honest CRA, respectively. This means that each player randomizes such that the other player is indifferent between the available strategies. More precisely, the investor chooses α such that

$$\Delta\Pi^{CRA} = \Pi_H^{CRA} - \Pi_I^{CRA} = 0 \Leftrightarrow \alpha = \alpha^* := 1 - \frac{\Phi}{\rho}. \tag{13}$$

Similarly, the CRA chooses β such that

$$\Delta\Pi^{Inv} = \Pi_T^{Inv} - \Pi_S^{Inv} = 0 \Leftrightarrow \beta = \beta^* := 1 - \frac{C}{1-\lambda}. \tag{14}$$

Moreover, as Table IV shows, there are four possible Nash equilibria in pure strategies:[11]

First, if $\Phi \geq \rho$ and $C < 1-\lambda$, the equilibrium is "sophisticated/inflating". For the CRA, the fees charged outweigh the possible reputation costs, and for the investor, the monitoring cost is low relative to the share of bad investments, so monitoring pays off. If in this first case, $\Phi = \rho$ and $C = 0$, then there is a second Nash equilibrium "sophisticated/honest", which is weakly dominated by "sophisticated/inflating". This means that it is equally good for the CRA to be honest or inflating given that the investor is sophisticated. However, if the investor chooses the off-equilibrium strategy to be trusting, then the CRA is better off being inflating.

Second, if $C \geq 1-\lambda$ and $\Phi > 0$, the equilibrium is "trusting/inflating". In this case, the investors do better not to monitor the investment quality, because there are relatively

[11] I thank Henri Pagès for suggesting a superior exposition of the Nash equilibria.

few bad investments on the market. Even with an inflating CRA and the investor buying all investments, the average return on the investment exceeds the monitoring costs. If in this second case, $\Phi = \rho$ and $C = 1 - \lambda$, then there is a second Nash equilibrium "sophisticated/inflating", which is weakly dominated by "trusting/inflating". This means that it is equally good for the investor to be trusting or sophisticated given that the CRA is inflating. However, if the CRA chooses the off-equilibrium strategy to be honest, then the investor is better off being trusting.

Third, if $\Phi = 0$ and $C > 0$, the equilibrium is "trusting/honest". Here, there is no fee differential for good and bad ratings, and thus no incentive for the CRA to inflate ratings. The investor, on the other hand, can encounter such a CRA in trusting mood and does not have to spend costs on monitoring quality. If in addition to $\Phi = 0$, it even holds that $C > 1 - \lambda$, then there is a second Nash equilibrium "trusting/inflating", which is weakly dominated by "trusting/honest". This means that it is equally good for the CRA to be honest or inflating given that the investor is trusting. However, if the investor chooses the off-equilibrium strategy to be sophisticated, then the CRA is better off being honest.

Fourth, if $C = 0$ and $\Phi < \rho$, the equilibrium is "sophisticated/honest". If the investor does not have to spend monitoring costs, it is the best choice to be sophisticated and thus deter inflating CRA behavior. If in addition to $C = 0$ it even holds that $\Phi = 0$, then there is a second Nash equilibrium "trusting/honest", which is weakly dominated by "sophisticated/honest". This means that it is equally good for the investor to be trusting or sophisticated given that the CRA is honest. However, if the CRA chooses the off-equilibrium strategy to be inflating, then the investor is better off being sophisticated.

As I will show in the following, the outcomes of the evolutionary dynamics are related to the outcomes of the two-player game presented in the current section. For the interior case with $0 < C < 1 - \lambda$ and $0 < \Phi < \rho$, I will show that the outcome is a cyclic dynamic behavior around a fixed point at (α^*, β^*). For the other cases, I show that the Nash equilibrium outcomes of the two-player game are also reached similarly as equilibria in the dynamic setting. The evolutionary game perspective allows to make predictions about the real-world situation in which not only a single investor and a single CRA, but populations of investors and CRAs interact over time. Although mathematically similar to the Nash equilibrium outcomes of the two-player game, only evolutionary dynamics allow the interpretation of interacting populations, rather than pure and mixed strategies of two single players. A two-player game does not describe the market well, and especially the interpretation of (α^*, β^*) as a fixed point of cyclic dynamics is completely different from a Nash equilibrium in mixed strategies. Also, the CRA payoff definitions are based on ratings shopping and thus only have a meaningful interpretation on a market with several CRAs. Moreover, I will discuss in Section 4. how the outcomes change if the model's parameters are dependent on the number of CRAs in the market.

2.5 EVOLUTIONARY GAME

From now on, I assume that there are interactions between a large number of individual investors and CRAs. For each interaction, there is a random draw of one investor and one CRA with built-in types out of their respective populations. The population shares change over time as a result of the payoffs achieved in the interactions. For such a situation, the corresponding replicator dynamics can be derived as in Taylor and Jonker (1978) and

Taylor (1979) to be[12]

$$\frac{\partial \alpha}{\partial t} = \alpha(\Pi_T^{Inv} - \bar{\Pi}^{Inv}) \quad \text{and} \quad \frac{\partial \beta}{\partial t} = \beta(\Pi_H^{CRA} - \bar{\Pi}^{CRA}). \tag{15}$$

This means that the growth rate $\frac{\partial \alpha}{\partial t}/\alpha$ of the trusting investors' population share equals the difference between the trusting investors' current payoff and the current average payoff in the investor population. If trusting investors perform better than average, their share is growing. Formerly sophisticated investors and new market entrants will adopt the successful behavior of being trusting. If trusting investors perform worse than average, their share is shrinking. The opposite holds for the share $(1-\alpha)$ of sophisticated investors, respectively, and an analogous mechanism is at work in the CRA population. Note that there is some stickiness in the behavior. The intuition behind is that it takes some time for the knowledge about the success of the respective strategy to reach all the market participants. The dynamics can be transformed into

$$\frac{\partial \alpha}{\partial t} = \alpha(1-\alpha)\underbrace{(\Pi_T^{Inv} - \Pi_S^{Inv})}_{\Delta\Pi^{Inv}} \quad \text{with} \quad \Delta\Pi^{Inv} = C - (1-\beta)(1-\lambda) \tag{16}$$

and

$$\frac{\partial \beta}{\partial t} = \beta(1-\beta)\underbrace{(\Pi_H^{CRA} - \Pi_I^{CRA})}_{\Delta\Pi^{CRA}}, \quad \text{with} \quad \Delta\Pi^{CRA} = (1-\lambda)((1-\alpha)\rho - \Phi). \tag{17}$$

The transformation allows the following interpretation: When trusting investors perform relatively better than sophisticated investors, then the trusting investors' share α in the investor population is increasing. Similarly, when honest CRAs perform relatively better than inflating CRAs, then the honest CRAs' share β in the CRA population is increasing. As there are only two strategies in each population, "better than average" is equivalent to "better than the other strategy".

My next step is to derive stationary regions. These refer to states in which there is no movement in either α or β direction, or neither. There is no movement in α direction, if $\frac{\partial \alpha}{\partial t} = 0$ in Equation (16). This is the case if either $\alpha = 0$ or $\alpha = 1$ (on the margins of the population space), or

$$\Delta\Pi^{Inv} = 0 \Leftrightarrow \beta = \beta^* := 1 - \frac{C}{1-\lambda}. \tag{18}$$

Similarly, there is no movement in β direction, if $\frac{\partial \beta}{\partial t} = 0$ in Equation (17). This is the case if either $\beta = 0$ or $\beta = 1$ (again, on the margins of the population space), or

$$\Delta\Pi^{CRA} = 0 \Leftrightarrow \alpha = \alpha^* := 1 - \frac{\Phi}{\rho}. \tag{19}$$

Note that the solutions for α^* and β^* are the same as those derived in Equations (13) and (14) as probabilities for the Nash equilibrium in mixed strategies. For states above or below the fixed lines, I observe from Equation (16) that $\Delta\Pi^{Inv}$ is increasing in β. Therefore, I have for $\beta > \beta^*$ $(<)$ a movement $\frac{\partial \alpha}{\partial t} \geq 0$ (≤ 0). Similarly, I observe from Equation (17) that $\Delta\Pi^{CRA}$ is decreasing in α. Likewise, I have for $\alpha > \alpha^*$ $(<)$ a movement $\frac{\partial \beta}{\partial t} \leq 0$ (≥ 0).

If there is no movement in either direction, then the corresponding state is a fixed point in the dynamics. From the required condition $\frac{\partial \alpha}{\partial t} = \frac{\partial \beta}{\partial t} = 0$, I derive the corners of

[12] See Appendix B for more details on the derivation.

the population space as fixed points, as well as the interior fixed point (α^*, β^*). For the existence of an interior fixed point, I need $\alpha^*, \beta^* \in (0, 1)$. From Equations (18) and (19), this means that $0 < 1 - \frac{C}{1-\lambda} < 1$ and $0 < 1 - \frac{\Phi}{\rho} < 1$, or reformulated,

$$0 < C < 1 - \lambda \tag{20}$$

and

$$0 < \Phi < \rho. \tag{21}$$

If the interior fixed point exists, then it is a center, as shown in the Appendix, Section C. This means that the orbits will spiral periodically around the fixed point, keeping their radius and speed constant.

2.6 INVESTOR WELFARE

As investor welfare, I define the current average payoff $\bar{\Pi}^{Inv}$ in the population of investors given by Equation (6), which can be rewritten as

$$\bar{\Pi}^{Inv} = \lambda R - [\underbrace{(1-\alpha)C}_{1.} + \underbrace{\alpha(1-\beta)(1-\lambda)}_{2.}]. \tag{22}$$

An intuitive interpretation is that welfare is created by investment into good projects, which generates a value of λR in expectation. Deductions occur for two reasons. First, there are the monitoring costs $(1-\alpha)C$ that sophisticated investors spend to avoid buying bad investments. Second, there are the losses $\alpha(1-\beta)(1-\lambda)$ occuring upon the default of bad investments bought by trusting investors and rated as good by inflating CRAs. The usual first-order condition for a maximum with respect to the share of trusting investors α is

$$\frac{\partial \bar{\Pi}^{Inv}}{\partial \alpha} = C - (1-\beta)(1-\lambda) = 0, \tag{23}$$

whereas $\frac{\partial^2 \bar{\Pi}^{Inv}}{\partial \alpha^2} = 0$ shows that further analysis is needed. Note that $\frac{\partial \bar{\Pi}^{Inv}}{\partial \alpha} = \Delta\Pi^{Inv}$ from Equation (16), therefore solving Equation (23) for β yields the same solution $\beta^* = 1 - \frac{C}{1-\lambda}$ as in Equation (18).

Given that there is a share β^* of honest CRAs on the market, the corresponding investor welfare is $\bar{\Pi}^{Inv}(\beta^*) = \lambda R - C$. Furthermore, Equation (23) says that for β higher (lower) than β^*, the investor welfare is increasing (decreasing) in α, with the boundaries $\alpha = 1$ $(= 0)$ being the solutions maximizing investor welfare at values of $\lambda R - (1-\beta)(1-\lambda)$ and $\lambda R - C$, respectively. Comparing to the dynamics from Equation (16), note that whenever $\beta > \beta^*$ induces an increase of trusting investors in the market over time, then investor welfare also increases with the share of trusting investors. However, it is important to note that the behavior of investors and CRAs is mutually dependent. While the investor behavior is changing over time, the CRA behavior is as well. Particularly, in the base case with an interior fixed point and cyclic dynamics, the boundary solutions $\alpha = 1$ $(= 0)$ that maximize investor welfare will generally not be reached. Therefore I will look at the joint dynamic evolution of population shares in the following section.

Table V Base Case Parameter Values. The table states the numerical values chosen for each of the parameters in the base case of the model.

share of good investments	$\lambda = 0.5$
net payoff upon investment	$R = 0.5$
verification cost	$C = 0.2$
fee charged by CRA	$\Phi = 1$
reputation cost	$\rho = 1.4$

3. Results for an Arbitrary Number of CRAs

The equilibrium resulting from the interaction of an arbitrary number of CRAs and investors depends on the parametrization of the model. More precisely, it depends on whether an interior fixed point (α^*, β^*) exists, which is the case whenever Equations (20) and (21) are satisfied. If (α^*, β^*) lies outside the population space, it still makes a difference on which side it lies. This results in several interesting cases, which will be analyzed in the following sections.

3.1 BASE CASE: INTERIOR FIXED POINT AND CYCLES

As base case I define the situation with one interior fixed point at (α^*, β^*), i.e., $\alpha^*, \beta^* \in (0, 1)$. To satisfy Equations (20) and (21) derived in the previous section, I choose the share of good investments as $\lambda = 0.5$, the verification cost borne by sophisticated investors as $C = 0.2$, the fee charged by the CRA to the issuer for a good rating as $\Phi = 1$, and the CRA's reputation cost if caught lying as $\rho = 1.4$. The net payoff upon investment is chosen as $R = 0.5$. Note that the latter affects neither the location of (α^*, β^*) nor the dynamics in Equations (16) and (17). These are driven only by the differences in payoffs between strategies, and the investor payoffs can all be shifted by λR to eliminate R. The parameter values are summarized in Table V. According to Equations (18) and (19), the resulting interior fixed point is located at $(\alpha^*, \beta^*) = (0.29, 0.6)$. Note that the qualitative properties of the model depend only on whether α^* and β^* lie in the interior of the population space. Therefore the choice of parameters serves purely illustrative purposes.

The resulting dynamics are visualized in Figure 1. The arrows indicate the vectors $(\frac{\partial \alpha}{\partial t}, \frac{\partial \beta}{\partial t})$, i.e., the direction and speed of development of the shares in each population for given current shares of trusting investors (α) and honest CRAs (β). Also shown are the fixed lines on the margins of the population space, and the two interior fixed lines.

The latter separate four different regimes. Dependent on the current investor sophistication level, either the honest or inflating CRAs are more successful. For example, start in the middle of the population space, at (0.5,0.5), which, for the given parametrization, lies in the lower right quadrant of the population space. Then the share of trusting investors (α) is higher than on the fixed line (α^*). As a consequence, the inflating CRAs are the more successful ones, and their share is growing. Also, the current state has a level of honest CRAs (β) that is below the fixed line (β^*). Therefore the trusting investors often happen to put their money in bad investments. The sophisticated investors are better off in such a situation and can improve their market share. As the arrows indicate, the combined effect is that the market moves towards less honest (and more inflating) CRAs and less trusting (and more sophisticated) investors.

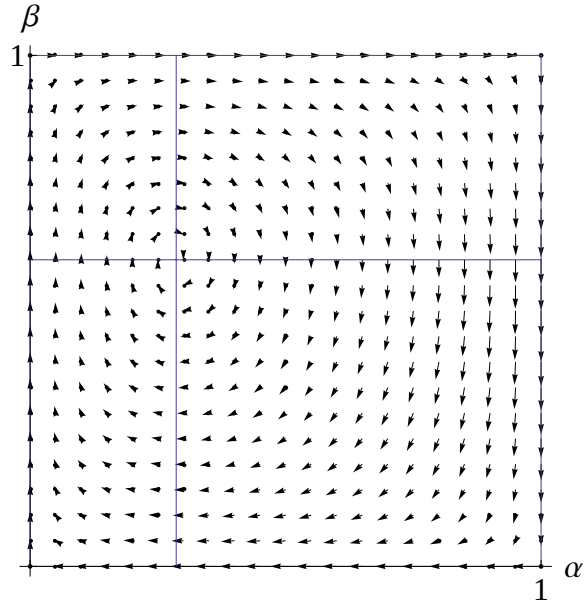


Fig. 1. Base case: interior fixed point and cycles.
The figure shows the dynamic evolution of the population shares, with α representing the share of trusting (rather than sophisticated) investors and β representing the share of honest (rather than inflating) CRAs. It is displayed as a vector field for $\alpha^*, \beta^* \in (0, 1)$. The fixed lines α^* and β^* are indicated by vertical and horizontal lines, respectively. Parameter values correspond to the base case parametrization given in Table V.

Once the fixed line at α^* is crossed, the lower left quadrant of the population space is entered. Now the share of trusting investors is still shrinking, because of the high market share of inflating CRAs. However, now there are enough sophisticated investors in the market to make reputation costs more important for the CRAs than rating fees. Therefore the honest CRAs make more profit now, and they consequently improve their market share.

When crossing the fixed line at β^*, yet another regime is reached in the upper left quadrant of the population space. Now the share of honest CRAs is high enough that it does not pay off anymore for the individual investor to invest in the monitoring of the CRAs. Therefore the trusting investors now perform better than the sophisticated ones and gain in market share. Still, there are enough sophisticated investors in the market to make the honest CRAs better off than the inflating ones, and thus the honest CRAs' market share is further growing.

In the next regime transition, the fixed line at α^* is crossed again, and the upper right quadrant of the population space is entered. Here, the trusting investors still become more numerous. Due to the low share of the inflating CRAs, it is still safe to buy all investments rated as good, rather than investing in monitoring. However, the trusting investors have already become such a big group that again it is beneficial for the CRAs to inflate ratings, as they can do so with little risk of being caught. Therefore the inflating CRAs gain market

share on expense of the honest ones. Finally, the last transition over the fixed line at β^* leads again into the lower right quadrant of the population space, where I started the investigation.

As previously derived, the fixed point (α^*, β^*) is a center in the dynamics. This means that the clockwise cycles of movement in the population that are displayed in Figure 1 will spiral periodically around the fixed point, rather than move towards or away from it. These cycles of movement are consistent with evidence of both CRAs' and investors' behavior varying over the business cycle. Related are the following two theoretical predictions: Bar-Isaac and Shapiro (2012) find that ratings accuracy is counter-cyclical. BFS predict that "ratings inflation is more likely in boom times when investors have lower incentives to perform due diligence, as the ex ante quality of investments is then higher." In the language of my model, this corresponds to a higher share of inflating CRAs for a lower share of sophisticated investors in the market. The effect of the ex-ante investment quality will be analyzed in Section 3.3.

The conclusion for the base case depends on the current regime of the market. It pays off temporarily for CRAs to be honest, but only if there are enough sophisticated investors in the market, who make reputation loss a real threat. Otherwise, ratings inflation is the best strategy for the CRAs. My result for the base case is consistent with other theories, e.g., Skreta and Veldkamp (2009) and BFS, who suggest that ratings inflation is most severe for complex investment products, i.e., products exceeding the investor sophistication level, and for more trusting investors. However, in contrast to the existing theories I emphasize that even for the base case of my model, "strategic honesty" can pay off at least temporarily. Investor welfare in the base case is varying over time according to Equation (22), driven by the changing population shares. For $\beta = \beta^*$ as in the center of the dynamics, the corresponding investor welfare is $\bar{\Pi}^{Inv}(\beta^*) = \lambda R - C$.

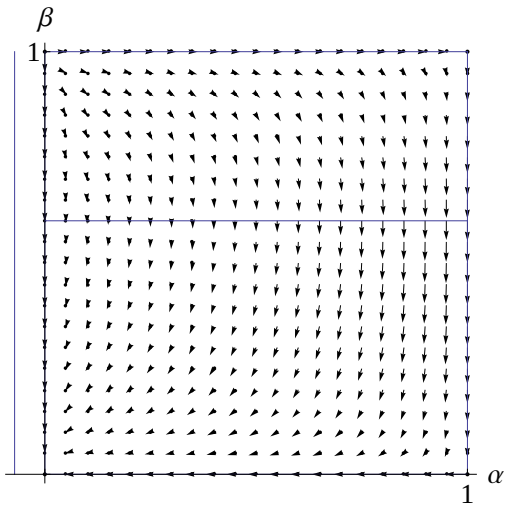
Apart from the base case, I highlight several other cases in the following. These occur for parameter constellations in which one or both of the coordinates (α^*, β^*) lie outside of the population space. Interestingly, very different outcomes will be reached for these cases, including stable equilibria in which the competitive market for credit ratings will be served exclusively by honest CRAs. The analysis of these cases will also allow policy recommendations on how to induce honest rating behavior.

3.2 SOPHISTICATED INVESTORS AND INFLATING CRAS

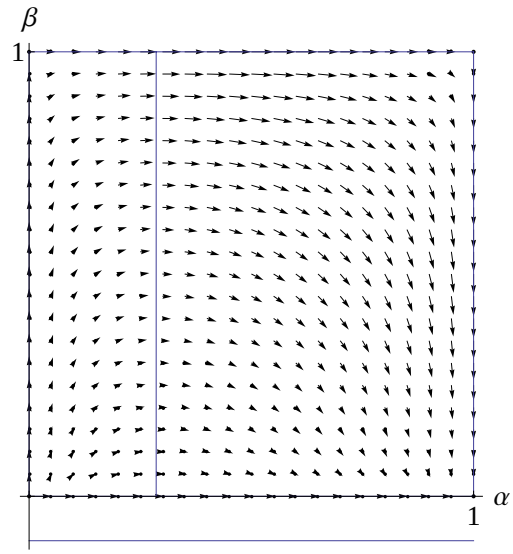
I begin with describing the equilibrium that occurs if $\Phi > \rho$. This means that the inflating CRA, if caught, faces reputation costs below the fees earned. Then it follows from Equation (19) that $\alpha^* < 0$. The dynamics are visualized in Figure 2(a) given that $0 < C < 1 - \lambda$ (the resulting equilibrium is the same for $C = 0$).

The resulting equilibrium is that the honest CRAs die out, and trusting investors die out as well. The corresponding investor welfare according to Equation (22) is $V_{SH} = \lambda R - C$. From an investor point of view, it pays off to collect information oneself instead of listening to the CRAs. The sophisticated investors can thus avoid to buy bad investments. Still, the issuers pay fees to the rating agencies. As these fees are higher than the reputation costs, it is still worth for the rating agencies to produce useless information and get paid for it.

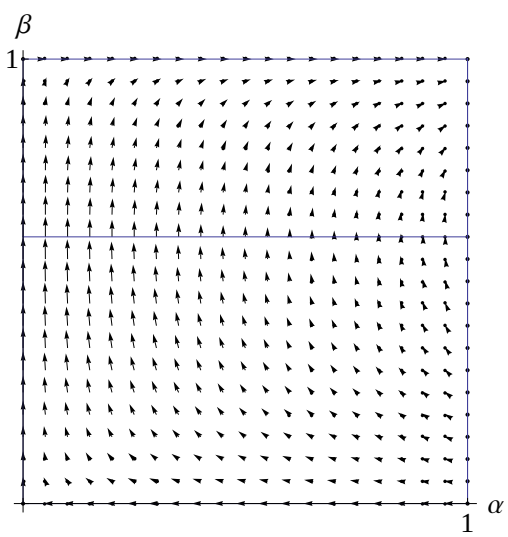
A possible critique is that even if there are no trusting investors on the market ($\alpha = 0$), the CRAs still receive fees from the issuers. Instead, the issuers might want to make sure



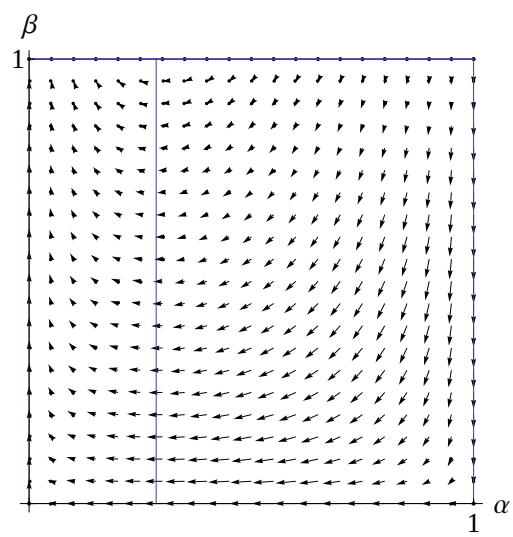
(a) Sophisticated investors and inflating CRAs, $\Phi = 1.5$



(b) Trusting investors and inflating CRAs, $C = 0.55$



(c) Trusting investors and honest CRAs, $\Phi = 0$



(d) Sophisticated investors and honest CRAs, $C = 0$

Fig. 2. Equilibrium outcomes for non-base parametrizations.
The four figures show the dynamic evolution of the population shares, with α representing the share of trusting (rather than sophisticated) investors and β representing the share of honest (rather than inflating) CRAs. The fixed lines α^* and β^* are indicated by vertical and horizontal lines, respectively. Different from the base case parametrization given in Table V, each figure shows the effect of changing one of the parameters, each leading to a different equilibrium (both as given in the captions).

that they do not pay for the inflated ratings delivered to the sophisticated investors, who are able to verify the work of the CRAs and the quality of the investments. I tackle this critique in the Appendix, Section A1. There I assume that the CRAs only receive fees if they meet trusting investors and report a good rating to them.

3.3 TRUSTING INVESTORS AND INFLATING CRAS

Given there are costs to monitor the behavior of the CRAs, but the final outcome is that honest CRAs die out anyway, one could think that it would be more efficient to have only trusting investors remaining, so there is no waste of money for monitoring the CRAs. Indeed, such an outcome is reached if $C > 1 - \lambda$, meaning that the monitoring costs are high relative to the share of bad investments. Then it follows from Equation (18) that $\beta^* < 0$. The dynamics are visualized in Figure 2(b) given that $0 < \Phi < \rho$ (the resulting equilibrium is the same for $\Phi > \rho$).

In the resulting equilibrium, sophisticated investors die out, because monitoring does not pay off. This is a situation similar to what BFS associate with boom times. If there is a high ex-ante quality of investments on the market, then trusting investors perform better than sophisticated ones. The corresponding investor welfare according to Equation (22) is $V_{TI} = \lambda R - (1 - \lambda)$, which is actually a better outcome than the previous section's $\lambda R - C$, given that the current section deals with the case $C > 1 - \lambda$.

3.4 TRUSTING INVESTORS AND HONEST CRAS

Now I present an outcome that leads to a stable equilibrium in which CRAs are honest and provide the best possible information, whereas investors are trusting and do not have to verify the work of the CRAs.[13] Such an outcome is reached when $\Phi = 0$, which means that the CRA does not receive any fees. Then it follows from Equation (19) that $\alpha^* = 1$. Equivalently, the same outcome is reached in a variant of the model in which the CRA always receives the same fee, no matter whether it assigns a good or bad rating.[14] The dynamics are visualized in Figure 2(c) given that $0 < C < 1 - \lambda$ (the resulting equilibrium is similar for $C > 1 - \lambda$). The corresponding investor welfare according to Equation (22) is $V_{TH} = \lambda R$, which is indeed the highest achievable value in my model.

The result confirms the intuition that the "issuer pays" model is inherently wrong, if ratings shopping is possible and the CRAs effectively only receive fees for good ratings. My model suggests that the threat of sophisticated investors monitoring and possibly punishing the CRAs is effective. In the final outcome, however, no resources have to be spent on monitoring, and therefore all investors become trusting, because there will be no inflating CRAs in the market, as long as the CRAs' income is not driven by whether

[13] As a remark, it is possible that the $\alpha = 1$ line is reached before $\beta = 1$. This means that the sophisticated investors have already died out, while there are still inflating CRAs in the market. Then there is no further change, as either type of CRA shows the same performance.

[14] The equivalence can be shown as follows: On the one hand, for $\Phi = 0$ it follows from Equations (8) and (11), respectively, that $\Pi_H^{CRA} = 0$ and $\Pi_I^{CRA} = -(1 - \alpha)(1 - \lambda)\rho$. On the other hand, if the honest CRA receives Φ also for assigning bad ratings, then $\Pi_H^{CRA} = \Phi$, while Π_I^{CRA} remains as in Equation (11). In either case, the resulting dynamics in Equation (17) are the same, as $\Delta\Pi^{CRA} = \Pi_H^{CRA} - \Pi_I^{CRA} = (1 - \lambda)(1 - \alpha)\rho$.

they issue good or bad ratings. So the conclusion is, similar to Camanho, Deb, and Liu (2012), that the main problem in the market for ratings and the first thing to critically investigate is the "issuer pays" model. Also BFS come to the conclusion that "up-front fees (as in the Cuomo plan) accompanied by automatic disclosure of ratings and oversight of analytical standards will minimize distortions from conflicts of interest and shopping."

3.5 SOPHISTICATED INVESTORS AND HONEST CRAS

Finally, consider the case $C = 0$, i.e., sophisticated investors can monitor the CRAs at zero cost. Then it follows from Equation (18) that $\beta^* = 1$. The dynamics are visualized in Figure 2(d) given that $0 < \Phi < \rho$ (the resulting equilibrium is similar for $\Phi = 0$).

The resulting equilibrium is that trusting investors die out, and so do inflating CRAs.[15] Independent of the initial population shares, the sophisticated investors are the more successful ones and therefore survive, whereas the inflating CRAs will finally be driven out of the market and all CRAs will be honest. The lesson could be that a regulator or a central market authority, instead of individual investors, should take over the burden of monitoring the CRAs' performance. After all, the sophisticated investors bear the cost of monitoring individually, but the trusting investors can free-ride on the benefits of these monitoring efforts. The corresponding investor welfare according to Equation (22) is $V_{SH} = \lambda R - C$. As the present section considers the case $C = 0$, the investor welfare is actually again at the highest achievable value in my model, λR, just as in the previous section.

4. Effects of Competition and Number of CRAs

The framework of Evolutionary Game Theory does not require to explicitly specify the number of market participants. It allows for an arbitrary number of participants in the populations of both investors and CRAs. I see it as a reasonable assumption to regard the number of investors as uncountably large. In the following, the focus will therefore be on the number of CRAs and their effect on the outcome of the game. The channel is that I allow the model's parameters, in particular the verification cost C_N, the fee Φ_N charged by the CRA, and the reputation cost ρ_N, to be dependent on the number of CRAs N, as indicated by the subscript. Throughout the section, I assume that the share of honest CRAs β is not affected by a change in N. Otherwise, the market entry of one CRA would affect both the parameters C_N, ρ_N, and Φ_N, and thus the coordinates of the interior fixed point (α^*, β^*), and also the location of the current state (α, β). This simplification can be justified by the fact that the latter does not affect the qualitative properties of the equilibrium outcome.

4.1 EFFECT OF NUMBER OF CRAS ON MODEL PARAMETERS

The following specifications should be seen as examples for the effect of varying the number of CRAs N on the model parameters (see Section 6. for a critical discussion). I hypothesize

[15] Note that similar to the previous section, there are some stable states on the $\beta = 1$ line, meaning that once the inflating CRAs have died out, a remaining share of trusting investors can survive on the market, as they perform just as good as the sophisticated investors in that case.

that the verification cost C_N and the reputation cost ρ_N should be increasing in the number of CRAs N, while the fee Φ_N charged by the CRA should be decreasing. The motivation is that it takes more effort, i.e., higher verification costs, for the sophisticated investors to monitor the CRAs' work, if they have to cover a market with more participants.[16] On the other hand, such a market provides more alternatives, therefore the CRAs' risk of losing business to competitors and thus their reputation costs are higher with more competitors. While I assume that a monopolistic CRA can charge the highest fees for its services, increasing competition drives down the fees. As a simple specification of the three functions that satisfies the mentioned hypotheses, I choose

$$C_N = C_1 \cdot N \tag{24}$$

given a verification cost $C_1 > 0$ for the monopolistic CRA. Moreover, I choose

$$\rho_N = \rho_2(N-1) \tag{25}$$

given a reputation cost $\rho_2 > 0$ for the CRAs in duopoly. This implies zero reputation cost for the monopolistic CRA. The motivation is that for example for regulatory reasons, the issuers need at least one rating in any case. Therefore there is no outside option, even if the CRA is known to be cheating.[17] Finally, I choose

$$\Phi_N = \frac{\Phi_1}{N} \tag{26}$$

given a fee $\Phi_1 > 0$ charged by the monopolistic CRA. The fee is thus approaching zero for a perfectly competitive market with high N.[18]

4.2 EFFECT ON EQUILIBRIUM OUTCOME

I start with rewriting the fixed lines given by Equations (18) and (19) as

$$\beta^* = 1 - \frac{C_N}{1-\lambda} \quad \text{and} \quad \alpha^* = 1 - \frac{\Phi_N}{\rho_N}, \tag{27}$$

respectively, to account for the dependence on the number of CRAs N. Similarly, the conditions for an interior fixed point, i.e., Equations (20) and (21), become

$$0 < C_N < 1-\lambda \quad \text{and} \quad 0 < \Phi_N < \rho_N, \tag{28}$$

respectively. Recalling that C_N should be increasing in N, it is possible (given that $C_1 < 1-\lambda$) that there is a switch from the base case towards the case with $\beta^* < 0$, resulting in trusting investors and inflating CRAs, as described in Section 3.3. According to Equation (24), this happens when

$$N > \frac{1-\lambda}{C_1}. \tag{29}$$

[16] One could also argue that the sophisticated investors' effort is mainly dependent on the investments they evaluate rather than the number of CRAs. In that case, one should choose C independent of N, and for example relate it to the investment quality λ instead.
[17] If the issuers need more than one rating for regulatory reasons, then the reputation cost will remain zero as long as there are no more CRAs than ratings needed.
[18] An alternative specification would be $\Phi_N = (\Phi_1 - MC)/N + MC$, ensuring that on a perfectly competitive market the fees charged still equal the CRAs' marginal costs MC.

This illustrates that the switch does not necessarily happen when switching from monopoly to duopoly or from duopoly to a market with three CRAs. Depending on the magnitude of the verification cost relative to the average quality of investments in the market, one market structure could be prevailing even for a market of several CRAs, and then the market entry of one more CRA could suddenly cause a switch in the market structure. Here I see a relevant contribution to the existing literature, and I point out that it is not enough to compare monopolistic and duopolistic markets to answer the general question what the effect of more competition is.

Now I analyze the behavior for perfect competition, i.e., when the number of CRAs N approaches infinity. From Equations (24), (25), and (26), I have

$$C_N \to \infty, \quad \rho_N \to \infty, \quad \text{and} \quad \Phi_N \to 0 \quad \text{for} \quad N \to \infty. \tag{30}$$

In this case, $\alpha^* \to 1$ and $\beta^* < 0$, which corresponds to the case resulting in trusting investors and honest CRAs, as described in Section 3.4.

For a finite number of CRAs, I focus on the assumptions that ρ_N should be increasing and Φ_N should be decreasing in N. Thus, for large N, I have $\Phi_N < \rho_N$, which corresponds to the base case. But if $\Phi_1 > \rho_1$, which is the case for my assumption of $\Phi_1 > 0$ and $\rho_1 = 0$, there will be a switch of the market structure at some specific N. For the monopolistic case, the market structure results in sophisticated investors and an inflating CRA, as described in Section 3.2.[19] The critical N, at which the market switches to the base case, satisfies

$$\Phi_N < \rho_N \Leftrightarrow \frac{\Phi_1}{N} < \rho_2(N-1) \Leftrightarrow N > \frac{1}{2}(1 + \sqrt{1 + 4\frac{\Phi_1}{\rho_2}}) > 1. \tag{31}$$

Again, the switch in market structure does not necessarily happen when switching from monopoly to duopoly. Depending on the relation between reputation cost and fees, it could be that monopoly and duopoly show the same behavior, i.e., do not leave room for honest CRAs, and the switch happens when a third or fourth CRA enters the market.

Summarizing, there are the following possible transitions dependent on the number of CRAs N: For the monopolistic case ($N = 1$) with zero reputation costs, I have the market structure resulting in sophisticated investors and an inflating CRA. This can remain the same for small numbers of CRAs, e.g., in duopoly. Then, when $N > \frac{1}{2}(1 + \sqrt{1 + 4\frac{\Phi_1}{\rho_2}})$, the reputation costs exceed the fees ($\Phi_N < \rho_N$) and there is a switch to the base case. For even higher N, it becomes too expensive for sophisticated investors to monitor the CRAs, relative to the risk of receiving a bad investment once in a while ($C_N > 1 - \lambda$). Therefore there is a switch from the base case towards the case with trusting investors and inflating CRAs, when $N > \frac{1-\lambda}{C_1}$.[20] Finally, for $N \to \infty$, the fees become minimal and reputation costs are huge. Thus there are no more incentives for ratings inflation, and the

[19] Note that my specification $\rho_1 = 0$ leads to a special situation for the monopolistic CRA. The solution for α^* from Equation (19) is not valid. Instead, Equation (17) yields $\Delta\Pi^{CRA}(\rho = 0) = -(1 - \lambda)\Phi$, which is negative for all α, if $\Phi > 0$, and zero for all α, if $\Phi = 0$. Thus, for positive fees the honest CRAs die out. The case $\Phi = \rho = 0$ is not reached in my definitions above; in the monopolistic situation, which is the only case with $\rho = 0$, I assume that the monopolistic CRA still receives positive fees.

[20] If $\frac{1-\lambda}{C_1} < \frac{1}{2}(1 + \sqrt{1 + 4\frac{\Phi_1}{\rho_2}})$, then the intermediate area with the base case disappears. There is a switch from the case with sophisticated investors and inflating CRAs towards

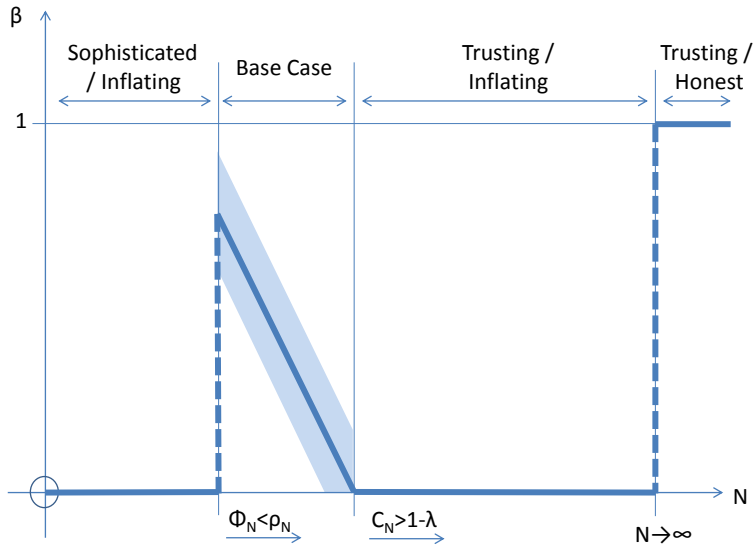


Fig. 3. CRA Honesty and Number of CRAs.
The figure visualizes the share of honest CRAs β as a function of the number of CRAs N, i.e., the possible transitions between the equilibrium outcomes for different degrees of competition on the CRA market.

case resulting in trusting investors and honest CRAs is reached. Figure 3 visualizes these possible transitions and the respective share β of honest CRAs for different degrees of competition on the CRA market.

4.3 NUMERICAL EXAMPLE

As a simple numerical example, consider the base case parameters given in Table V as describing the current market situation with $N = 10$ approved rating agencies (NRSROs). That means that $C_{10} = 0.2$, $\Phi_{10} = 1$, and $\rho_{10} = 1.4$. The share of good investments is given as $\lambda = 0.5$ independent of N. From Equations (24), (26), and (25), respectively, it follows that $C_1 = 0.02$, $\Phi_1 = 10$, and $\rho_2 = 0.156$. In turn, it is derived using Equation (29) that there will be a switch from the base case towards the case with trusting investors

the case with trusting investors and inflating CRAs, when $N > \frac{1-\lambda}{C_1}$, and then a further switch between the two subcases of Section 3.3, with the same resulting outcome, when $N > \frac{1}{2}(1 + \sqrt{1 + 4\frac{\Phi_1}{\rho_2}})$.

and inflating CRAs, when a critical N of 25 is exceeded. Similarly, according to Equation (31), the previous switch from the case with sophisticated investors and inflating CRAs to the base case has occurred when exceeding a critical N of 8.533. This could correspond to the recent approval of ten instead of three NRSROs, which should have led to a situation in which honest rating behavior pays off at least temporarily, while ratings inflation and loss of trust in the rating business were the characteristics of a market with only three approved agencies.

Alternatively, assume that the base case parameters reflected the situation when there were only three NRSROs. This corresponds to $C_3 = 0.2$, $\Phi_3 = 1$, and $\rho_3 = 1.4$. Consequently, it follows that $C_1 = 0.067$, $\Phi_1 = 3$, and $\rho_2 = 0.7$. Then, the previous switch from the case with sophisticated investors and inflating CRAs to the base case has occurred when exceeding a critical N of 2.63 – in the real world corresponding to the market entry of Fitch. Similarly, the switch from the base case towards the case with trusting investors and inflating CRAs is calculated to happen when a critical N of 7.5 is exceeded. This would mean that after approving ten instead of three rating agencies, it should indeed be observable that the market evolves from the cycles in the base case towards a stable equilibrium with trusting investors and inflating CRAs.

While the numbers and market structure transitions are not calibrated rigorously to actual data, the numerical example illustrates well that significant changes can happen beyond the step from a monopoly to a duopoly.

5. Empirical Implications

I divide the empirical implications into three parts. First, I discuss the dynamics predicted for the interaction between CRAs and investors, regarding the market participants' behavior and population shares over time. Second, I summarize the effect that a changing number of CRAs and thus competition should have on the equilibrium outcome. Finally, I present policy recommendations resulting from my model.

5.1 EVOLUTIONARY DYNAMICS AND VIABILITY OF NEW TYPES

For the base case setting of Section 3.1, my model predicts cyclic dynamics of market shares. First, it allows the prediction made by BFS: for a high share of trusting investors, CRAs are more likely to inflate ratings. However, in BFS the share of trusting investors α is exogenously given and constant over time. In contrast, my model allows for a second effect in the opposite direction: I predict that facing a CRA market with a lot of ratings inflation, the investors will in turn change their behavior. As sophisticated investors perform better than trusting investors on such a market, the latter will either start leaving the market or learn to be more sophisticated as well. My theory is also in line with the predictions by Skreta and Veldkamp (2009) and BFS that ratings inflation is most severe for complex investment products, i.e., products exceeding the investor sophistication level. To sum up, the dynamics in the base case do not only explain the CRAs' behavior as a result of the investors' sophistication level, but also the investors' behavior as a function of the CRAs' ratings quality. The result is the cyclic dynamics of the base case.

The ex ante quality of investments λ is a constant parameter in my model. However, considering different levels of λ allows the following prediction: The situation with low ex ante investment quality corresponds to the base case, in which it pays off at least

temporarily to be a sophisticated investor and to spend some monitoring effort. In Section 3.3, I show the situation for a high enough ex ante investment quality, which BFS associate with boom times; then the trusting investors perform better, as the sophisticated investors' monitoring does not pay off. Similarly, Bar-Isaac and Shapiro (2012) find that CRAs work less accurate in boom times, which corresponds well with the lower risk of being caught lying. It might be questionable whether there is indeed a higher ex ante investment quality in boom times, rather than merely more optimism into products of possibly lower quality. Still, the association of trusting investors with boom times seems universally valid, and thus also the dynamic bidirectional relation between investor and CRA types over the business cycle, as predicted by my model.

Apart from the base case, my analysis provides solutions in which the equilibrium outcomes lie on the boundaries and in the corners of the population space. The equilibria are stable, which means the following: assume that, without being explained by the model, one participant of a new type enters the market that otherwise consists purely of the other type. Then, such a new type is not viable: my model predicts that the new type will not be successful, but be driven out of the market. As an example, consider the case with low reputation costs relative to the fees earned, illustrated in Section 3.2. Here, the market's equilibrium outcome consists of sophisticated investors and inflating CRAs. My model predicts that if one trusting investor or one honest CRA enter the market, they will be driven out of the market again. Only if, by the entry of the new CRA, the resulting reputation costs faced by inflating CRAs exceed the critical level, then the whole market can switch into the cyclic base case, in which both trusting investors and honest CRAs are persistently present. Therefore the following predictions are derived from explicitly analyzing the number of CRAs.

5.2 COMPETITION AND NUMBER OF CRAS

Section 4. allows the prediction that a perfectly competitive CRA market can prevent incentives for ratings inflation and is thus beneficial for ratings quality. For small numbers of CRAs, e.g., monopoly, duopoly, or maybe a market with three or four agencies, I predict a market structure resulting in sophisticated investors and inflating CRAs. For intermediate CRA market sizes, one might observe a cyclic change of market shares as described in my base case. If the CRA market is even larger, trusting investors will dominate, but CRAs still behave inflating.

Summarizing, I predict that the effect of an increasingly competitive CRA market on the degree of honest rating behavior has to be considered very carefully. While a monopolistic or duopolistic market can be populated exclusively by inflating CRAs, the first change of market structure results in a role for honest CRAs as a result of increasing the number of CRAs. However, the second transition shows that further increasing competition can lead to the honest CRAs again being eliminated from the market.

Only for a perfectly competitive CRA market, on which ratings inflation creates only little fees, but prohibitively high reputation costs, I predict both trusting investors and honest CRAs to prevail in the long run. Although there are currently more than two CRAs with NRSRO status on the market, it is unlikely that the business will ever show the characteristics of perfect competition and uncountably many CRAs. In the next section, I therefore also provide policy recommendations that can implement honest rating behavior even on a market with limited competition.

On the empirical side, it will be interesting to examine, similar to Becker and Milbourn (2011) with the market entry of Fitch, how the market is changing by the designation of additional agencies as NRSRO. The existing evidence is mixed: Becker and Milbourn (2011) argue that increasing competition after the market entry of Fitch leads to a decrease in ratings quality. In contrast, Bongaerts, Cremers, and Goetzmann (2012) point out the role of Fitch as a tiebreaker, thus endogenously being chosen by the better issuers. Similarly, Doherty, Kartasheva, and Phillips (2012) show that the market entry of a new CRA can improve ratings quality and precision, if it helps good issuers to separate themselves from bad ones. My model suggests that the effect of a new CRA entering the market depends on whether this event affects the CRAs' trade-off between current fees and future reputation positively or negatively.

5.3 POLICY RECOMMENDATIONS

The predictions for successful regulatory measures, without explicitly addressing the number of CRAs on the market, are based on the analysis of the desirable cases in Section 3., i.e., those in which the honest CRAs survive in the long run. Based on Section 3.4, I predict that it is essential to find an alternative solution to the current version of the "issuer pays" model, particularly to prevent that rating agencies can achieve higher revenues by issuing good ratings. This is in line with most other recent studies and also with the Cuomo plan aiming to enforce up-front payment for ratings.[21] Moreover, Roland Berger's recent proposal for a European rating agency involves an "investor pays" model, see Reuters (2012). In the language of my model, if the market entry of the European rating agency changes the whole market to use the "investor pays" model, this will correspond to the outcome of Section 3.4. So indeed, Roland Berger's proposal is a candidate solution for establishing an honest CRA market. Another alternative to the "issuer pays" model is the "platform pays" model suggested by Mathis, McAndrews, and Rochet (2009). However, it is still a question for further research and debate as well as subject to real-world test, whether it will be possible to implement an alternative to the "issuer pays" model.

Based on Section 3.5, I recommend that the monitoring of CRAs' performance and their possible punishment should rather be done (even more so) by a regulator or a central market authority, rather than individual investors. BFS suggest on the one hand the oversight of minimum analytical standards by regulators, and on the other hand the enhancement of the market's ability to punish CRAs and thus increase the reputation cost. In contrast, I predict that the market participants are not likely to fulfill their part of control and punishment, as there is the incentive for trusting investors to free-ride on the efforts of sophisticated investors. Therefore I suggest that regulators should fully take over the task of monitoring, the costs of which could be covered by fees that all market participants have to pay to the regulating authorities. As far as CRAs can be found guilty of fraud, they can also be held legally liable as in the recent attempt by the U.S. Justice Department, see the first paragraph of this paper and Mattingly (2013). Instead of direct punishment, like for example the denial of approval as suggested by Stolper (2009), one could also think of the regulator "rating the raters"[22], i.e., giving the CRAs grades based on their past performance, as a guideline for the market participants.

[21] See BFS, p. 105 for a discussion of the Cuomo plan.
[22] Credits for this formulation go to the title of Mathis, McAndrews, and Rochet (2009).

If at least one of my suggested regulatory measures can be implemented, then I predict that the market for credit ratings will function well in the sense that honest rating behavior is viable, independent of the number of CRAs on the market.

6. Limitations and Extensions

I develop a framework for modeling the market for credit ratings with competition between more than two CRAs. The aim of my model is to examine whether more competition is favorable. To achieve this goal, I see two possible approaches. The first is via simulation and numerical solution of a multiple-agent model. Such an approach is expected to become computationally more and more expensive with the number of players (here, CRAs). The nature of the approach precludes parameter-free, analytical results. The second approach, which I follow in the present paper, is based on Evolutionary Game Theory. Advantages of the second approach are the tractable analysis and analytical solutions. Also, the approach does not specify the number of players. Therefore it can handle an arbitrary number of credit rating agencies. Furthermore, I allow for a true interplay between investors and agencies, i.e., both investors and agencies react to each other's behavior.

A disadvantage of the approach is that the model flexibility is limited, compared to a simulation approach. For example, the model assumes a random pairing between investors and CRAs. Moreover, it is not possible to remember and update the individual CRA's reputation over time. The only state variables are the population shares. Regarding the change of population shares over time, my modeling framework does not distinguish between market entry/exit and adoption of other agents' behavior.[23] Still, since the market share of each type of CRA is related to its reputation, this share can also be seen as a proxy for the CRA's current reputation in the model. After all, a CRA can draw a big part of its reputation from the fact that many market participants observe each other believing in the CRA's judgments, even though it is not necessarily justified by superior past performance. The fact that investors cannot deliberately choose one specific CRA is outweighed by the opportunity of instantly imposing reputation costs on misbehaving CRAs. The history of obvious misjudgments, for example during the 2008 subprime mortgage crisis, and the lack of long-term consequences for the CRAs' reputation, might also suggest that the investors have short memory. Therefore, instant reputation costs might be more realistic than the investors ignoring the inflating CRAs' reports in all future periods, as assumed e.g. by BFS. My model provides useful implications that are similar to those expected for a model with a more complex treatment of reputation effects, which supports that the assumptions made are justifiable.

A theoretical limitation is that my approach assumes two populations of infinite size, whereas the CRA population is clearly rather small in reality. Fogel, Andrews, and Fogel (1998) show that the simulation outcomes for small populations can differ from the analytical solution of an infinite-population size evolutionary game.[24] Here it is comforting

[23] Within my basic model, a new type, for example an honest CRA, does not appear on a market that has so far been served by 100% inflating CRAs. However, I can analyze whether an entrant with a small, but positive market share will be able to attract more and more market share over time or will be driven back out of the market. In Section 4., I analyze the explicit effects of the number of CRAs.

[24] I thank Olivier Toutain for pointing this out.

that as shown in Section 2.4, the equilibrium solutions for all cases but the base case can similarly be achieved in a two-player game between one investor and one CRA. Moreover, my solutions do not have multiple basins of attraction, i.e., the equilibrium reached is independent of the starting point of the dynamics. Therefore, even large perturbations of the current state will still lead to the same final outcome.

Another possible critique of my approach is that the model's parameters, particularly the fees charged by CRAs, the reputation cost faced by a CRA caught inflating, and the monitoring costs borne by the sophisticated investors, are not derived from an equilibrium model. My model is flexible enough, though, to define these parameters as functions of explanatory variables in a preceding model. One example for such an explanatory variable is the number of CRAs in the market, an approach that is pursued in Section 4.. Any such parametrization results in one of the cases described in Section 3., as long as the parameters stay constant over time. Also, it is possible to define the parameters as functions of the two state variables, i.e., the population shares. This can possibly change also the dynamic properties of the model into situations that are not described in Section 3.. One example is that the fees that issuers are willing to pay to the CRAs for a good rating might reasonably be argued to depend on the fraction of trusting investors on the market, and thus the issuers' expected benefits from receiving a good rating. I analyze such a case in the Appendix, Section A1. One could also justify that the reputation cost faced by a CRA caught inflating depends on the fraction of honest CRAs on the market, as the latter increase the choice of desirable alternatives for investors and issuers. The implementation of the many alternative specifications is quite straightforward, but will not be discussed in further detail. Also, the calibration of the model parameters to real-world data and an empirical test of the predictions are left to further research.

7. Conclusion

Recent theoretical work on the credit rating industry considers competition in duopoly, although the U.S. market consists of three major players, and even ten agencies that are designated as Nationally Recognized Statistical Rating Organizations. Therefore I develop a framework using Evolutionary Game Theory to analyze the interaction of credit rating agencies on a market with more than two agencies. The focus of interest is how honest rating behavior can be achieved, i.e. how it can be avoided that agencies give good ratings to bad issues. From the analysis I draw the following three conclusions:

First, the presence of trusting investors facilitates ratings inflation. In turn, ratings inflation also induces investors to be less trusting. The interplay of the two effects can lead to cyclic dynamics in the distribution of both CRA and investor types.

Second, it crucially depends on the current number of CRAs on the market whether increasing competition will improve or decrease the degree of honest rating behavior. The effects beyond duopoly can be significantly different from what is learned by comparing monopoly and duopoly.

Finally, a market with solely honest rating agencies can be achieved in the hypothetical case of perfect competition. Alternatively, it can be implemented by regulatory measures such as abolishing the current "issuer pays" model or by a centralized monitoring of ratings quality.

Appendix A Alternative Specifications of Payoffs

A1 FEES DEPENDENT ON SHARE OF TRUSTING INVESTORS

I tackle the critique that so far, even if there are no trusting investors on the market ($\alpha = 0$), CRAs still receive fees from issuers. From now on, I assume that CRAs only receive fees if they meet trusting investors and report a good rating to them. In that way, issuers make sure that they do not pay for the worthless information delivered to the sophisticated investors, who are able to verify the work of the CRAs and the quality of the investments. So in my alternative specification, issuers effectively become an active third population in the game and negotiate fees dependent on the expected benefits. The payoffs to the investors are not affected by my alternative specification of payoffs.

First, I state the new payoffs for the honest CRAs. They still only give a good rating if they observe a good investment, which occurs with probability λ. Whether they receive their fee depends on the type of investor they meet. If it is a trusting investor, they do, so

$$X_{HT} = \lambda\Phi. \tag{32}$$

However, if they meet a sophisticated investor, they do not receive their fee, and

$$X_{HS} = 0. \tag{33}$$

Thus, the resulting expected payoff for honest CRAs is

$$\Pi_H^{CRA} = \alpha X_{HT} + (1-\alpha)X_{HS} = \alpha\lambda\Phi. \tag{34}$$

It might be hard to motivate that fees paid by the issuers can be conditioned on the individual investors that the CRAs meet, i.e., that $X_{HT} \neq X_{HS}$. An alternative interpretation is that the fee Π_H^{CRA} is paid by the issuers according to their expectation about how many trusting investors the CRAs will meet on average. The dynamics only require Π_H^{CRA}, not the individual payoffs. So an equivalent specification would be that the CRAs receive Π_H^{CRA} in each interaction, regardless of the type of investor they meet.

Second, consider the inflating CRAs. If they meet a trusting investor, they receive

$$X_{IT} = \Phi, \tag{35}$$

as in the original specification. As they always give good ratings, they are always paid by the issuers, regardless of the quality of the investment. Against a sophisticated investor, however, their expected payoff is

$$X_{IS} = -(1-\lambda)\rho. \tag{36}$$

Here the issuer does not pay the fee, as the sophisticated investors can judge the investment quality themselves. Still, the CRAs are punished whenever they rate a bad investment as good, which happens with probability $(1-\lambda)$. The resulting expected payoff for inflating CRAs is

$$\Pi_I^{CRA} = \alpha X_{IT} + (1-\alpha)X_{IS} = \alpha\Phi - (1-\alpha)(1-\lambda)\rho. \tag{37}$$

Similar to the specification for the honest CRAs, one might alternatively interpret the resulting expected payoff Π_I^{CRA} in the following way: the CRAs receive the fee $\alpha\Phi$ for each interaction, independent of the type of investor they meet. The issuers adjust the fee by the expected usefulness of the rating, i.e., the likelihood α that it reaches a trusting investor.

Table VI CRAs' Payoffs, Alternative Specification.

Investor / CRA	honest	inflating
trusting	$X_{HT} = \lambda\Phi$	$X_{IT} = \Phi$
sophisticated	$X_{HS} = 0$	$X_{IS} = -(1-\lambda)\rho$
expected	$\Pi_H^{CRA} = \alpha\lambda\Phi$	$\Pi_I^{CRA} = \alpha\Phi - (1-\alpha)(1-\lambda)\rho$

On top of that, the CRAs face the cost $(1-\lambda)\rho$ whenever they meet a sophisticated investor.

The payoffs are summarized in Table VI. As in the original specification, the average payoff in the population of CRAs is defined as

$$\bar{\Pi}^{CRA} = \beta\Pi_H^{CRA} + (1-\beta)\Pi_I^{CRA}. \tag{38}$$

For the evolutionary dynamics as in Section 2.5, I need

$$\Delta\Pi^{CRA} = \Pi_H^{CRA} - \Pi_I^{CRA} = (1-\lambda)((1-\alpha)\rho - \alpha\Phi). \tag{39}$$

The corresponding value $\Delta\Pi^{Inv}$ for the investors remains unchanged. Likewise, for the fixed lines as in Section 2.5 it holds that β^* remains unchanged. However, the second fixed line becomes

$$\Delta\Pi^{CRA} = 0 \Leftrightarrow \alpha = \alpha^* := \frac{\rho}{\rho+\Phi}. \tag{40}$$

For an interior fixed point, the conditions are now $0 < C < 1-\lambda$ as in Equation (20), and from $0 < \frac{\rho}{\rho+\Phi} < 1$, I have

$$0 < \Phi \quad \text{and} \quad 0 < \rho. \tag{41}$$

For the extreme cases $\Phi = 0$ and $\rho = 0$, I have $\alpha^* = 1$ and $\alpha^* = 0$, respectively. For the properties of the fixed point, I derive the corresponding matrices as in Section 2.5. The investor payoff matrices $\bar{A}$ and A remain unchanged. The CRA's payoffs from Table VI become

$$\bar{B} = \begin{pmatrix} \lambda\Phi & 0 \\ \Phi & -(1-\lambda)\rho \end{pmatrix}. \tag{42}$$

Transformation leads to

$$B = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix} = \begin{pmatrix} 0 & (1-\lambda)\rho \\ (1-\lambda)\Phi & 0 \end{pmatrix}. \tag{43}$$

As in Section 2.5, I can verify that $\alpha^* = \frac{b_{12}}{b_{12}+b_{21}}$. Next I test whether an interior fixed point in my model, under the alternative specification, is a saddle or a center. Since

$$a_{12}b_{12} = (\underbrace{C-(1-\lambda)}_{<0 \text{ see Equation (20)}})(1-\lambda)\rho < 0 \tag{44}$$

holds, it is a center according to Schuster, Sigmund, Hofbauer, and Wolff (1981).

Next, I repeat the analysis of Section 4. for the alternative specification of payoffs. The fixed lines as functions of the number of CRAs N are now

$$\beta^* = 1 - \frac{C_N}{1-\lambda} \quad \text{and} \quad \alpha^* = \frac{\rho_N}{\rho_N + \Phi_N}, \tag{45}$$

respectively. While β^* remains unchanged, α^* reflects the changes derived earlier in the present section. Similarly, the conditions for an interior fixed point become

$$0 < C_N < 1 - \lambda, \quad 0 < \Phi_N, \quad \text{and} \quad 0 < \rho_N, \tag{46}$$

respectively. I use the definitions introduced in Section 4. for the verification cost C_N, the fee Φ_N charged by the CRA, and the reputation cost ρ_N. Then in monopoly, $\rho_1 = 0$ and thus $\alpha^* = 0$. If the monopolistic CRA can choose its strategy, it will be inflating. Depending on the relation between the verification cost and the average quality of investments on the market, the sophisticated investors will be the only ones to survive for $C_1 < 1 - \lambda$, and the trusting ones for $C_1 > 1 - \lambda$, respectively. The expected payoff for the CRA is $\Pi_I^{CRA}(\rho = 0) = \alpha\Phi$. Thus, the monopolistic CRA can either not collect any fees, if $\alpha = 0$, and the issuers know there are no trusting investors on the market, or the CRA collects the full fee, if the investment quality is so good that only trusting investors remain in the market, who are happy to use the CRA's services despite their bad quality. For perfect competition ($N \to \infty$), I have $\Phi_N \to 0$ and thus $\alpha^* \to 1$. In duopoly and oligopoly with any finite number of CRAs, $\alpha^* \in (0, 1)$ will be an interior solution.

A2 PRODUCTION COSTS FOR CREDIT RATINGS

In my model, there are no costs for the CRAs to produce ratings. However, Φ can also be interpreted as the "net fee", i.e., what is remaining for the CRA after production costs. The results will not be changed as long as honest and inflating CRAs face the same production costs.

In contrast, assume that honest CRAs have to spend some production costs or effort (call it ϵ) for rating the investment, while inflating CRAs do not. Then, the payoffs for honest CRAs change to

$$\Pi_H^{CRA} = X_{HT} = X_{HS} = \lambda\Phi - \epsilon, \tag{47}$$

while the payoffs for inflating CRAs remain unchanged. Then the new α^* in Equation (13) becomes

$$\alpha^* = 1 - \frac{\Phi}{\rho} - \frac{\epsilon}{(1-\lambda)\rho}, \tag{48}$$

and the threshold level of $\alpha^* = 0$ in Section 3.2 is reached for

$$\Phi = \rho - \frac{\epsilon}{1-\lambda}. \tag{49}$$

This means that for given levels of ϵ and λ, there is still a critical level of Φ relative to ρ that has to be exceeded, such that the regime switches from the base case to the case with sophisticated investors and inflating CRAs in equilibrium.

The case of $\alpha^* = 1$ in Section 3.4 is only reached for $\Phi = \epsilon = 0$. This means that in order to reach the case with trusting investors and honest CRAs in equilibrium, it is needed both zero fees or a zero fee differential between issuing good and bad ratings, and also zero production costs or a zero production cost differential between issuing good and bad ratings.

Appendix B Derivation of the Replicator Dynamics

The derivation of Equation (15) follows Taylor and Jonker (1978) for one population and Taylor (1979) for the extension to two populations.

For one population, Taylor and Jonker (1978) define as n_i the number of i-strategists in the population. Let I be the number of available strategies (in my paper, $I = 2$). Then $N = \sum_{i=1}^{I} n_i$ is the total number of individuals. Defining furthermore $s_i = n_i/N$ as the proportion of i-strategists, the state of the population is $s = (s_1, \ldots, s_I)$. With r_i being the current growth rate of n_i, it follows for the evolution that $\frac{\partial n_i}{\partial t} = r_i n_i$. Moreover, with $\bar{r} = \sum_{i=1}^{I} s_i r_i$ being the average growth rate, it follows that $\frac{\partial N}{\partial t} = \bar{r} N$. Differentiating $s_i = n_i/N$ leads to

$$\frac{\partial s_i}{\partial t} = s_i(r_i - \bar{r}). \tag{50}$$

Now the assumption is that the fitness of strategy i is an estimate of the growth rate r_i. In my model, the fitness of strategy i corresponds to the expected payoff of the respective strategy (e.g., Π_T^{Inv} for the expected payoff of the trusting investors), and the average growth rate corresponds to the average payoff in the population (e.g., $\bar{\Pi}^{Inv}$ for the average payoff in the investor population). Then the growth of the share of trusting investors α follows

$$\frac{\partial \alpha}{\partial t} = \alpha(\Pi_T^{Inv} - \bar{\Pi}^{Inv}), \tag{51}$$

as given in Equation (15). As there are only two strategies for the investors, the dynamics for the share $(1 - \alpha)$ of the second strategy follow implicitly. An analogous formula is given in Equation (15) for the CRA population. As Taylor (1979) shows, the solutions are also valid for two interacting populations. In addition, Weibull (1997) introduces background birth and death rates in the populations, and he shows that they will not affect the Taylor and Jonker (1978) dynamics.

Appendix C Properties of the Interior Fixed Point

To analyze the properties of the interior fixed point, I follow the method in Schuster, Sigmund, Hofbauer, and Wolff (1981). For that purpose, I first write the payoffs for investors and CRAs as matrices $\bar{A}$ and $\bar{B}$, respectively. From Table I, I have

$$\bar{A} = \begin{pmatrix} \lambda R & \lambda R - (1-\lambda) \\ \lambda R - C & \lambda R - C \end{pmatrix} \tag{52}$$

for the investor payoffs. Schuster, Sigmund, Hofbauer, and Wolff (1981) first transform the payoff matrix into one with zeros on the diagonal by subtracting a constant from each column, which does not affect the dynamics. In my case, this leads to

$$A = \begin{pmatrix} 0 & a_{12} \\ a_{21} & 0 \end{pmatrix} = \begin{pmatrix} 0 & C - (1-\lambda) \\ -C & 0 \end{pmatrix}. \tag{53}$$

Similarly, I write the CRA's payoffs from Table II as

$$\bar{B} = \begin{pmatrix} \lambda\Phi & \lambda\Phi \\ \Phi & \lambda\Phi + (1-\lambda)(\Phi - \rho) \end{pmatrix}. \tag{54}$$

The matrix results from transposing Table II, as it should represent the population that receives the payoffs as the column player. Again, transformation leads to

$$B = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix} = \begin{pmatrix} 0 & (1-\lambda)(\rho-\Phi) \\ (1-\lambda)\Phi & 0 \end{pmatrix}. \tag{55}$$

From these matrices, Schuster, Sigmund, Hofbauer, and Wolff (1981) derive the interior fixed point as

$$(\alpha^*, \beta^*) = (\frac{b_{12}}{b_{12}+b_{21}}, \frac{a_{12}}{a_{12}+a_{21}}), \tag{56}$$

which corresponds to the results in Equations (18) and (19) above. If the fixed point is indeed in the interior of the population space, Schuster, Sigmund, Hofbauer, and Wolff (1981) derive that it can either be a saddle or a center. If $a_{12}b_{12} > 0$, it is a saddle. In contrast, if $a_{12}b_{12} < 0$, it is a center. For an interior fixed point in my model,

$$a_{12}b_{12} = (\underbrace{C-(1-\lambda)}_{<0 \text{ see Equation (20)}})(1-\lambda)\underbrace{(\rho-\Phi)}_{>0 \text{ see Equation (21)}} < 0 \tag{57}$$

holds, i.e., it is a center.

References

Atkins, Paul S., 2008, Speech by sec commissioner: Remarks to the institute of international bankers, http://www.sec.gov/news/speech/2008/spch030308psa.htm, retrieved November 17, 2011.

Baghai, Ramin, Henri Servaes, and Ane Tamayo, 2013, Have rating agencies become more conservative? implications for capital structure and debt pricing, Working Paper.

Bannier, Christina E., Patrick Behr, and Andre Güttler, 2010, Rating opaque borrowers: why are unsolicited ratings lower?, *Review of Finance* 14, 263–294.

Bar-Isaac, Heski, and Joel Shapiro, 2012, Ratings quality over the business cycle, Journal of Financial Economics, forthcoming, http://dx.doi.org/10.1016/j.jfineco.2012.11.004.

Becker, Bo, and Todd Milbourn, 2011, How did increased competition affect credit ratings?, *Journal of Financial Economics* 101, 493–514.

Bolton, Patrick, Xavier Freixas, and Joel Shapiro, 2012, The credit ratings game, *Journal of Finance* 67, 85–112.

Bongaerts, Dion, Martijn Cremers, and William N. Goetzmann, 2012, Tiebreaker: Certification and multiple credit ratings, *Journal of Finance* 67, 113–152.

Camanho, Nelson, Pragyan Deb, and Zijun Liu, 2012, Credit rating and competition, Working Paper.

Doherty, Neil A., Anastasia Kartasheva, and Richard D. Phillips, 2012, Information effect of entry into credit ratings market: The case of insurers' ratings, *Journal of Financial Economics* 106, 308–330.

Fogel, Gary B., Peter C. Andrews, and David B. Fogel, 1998, On the instability of evolutionary stable strategies in small populations, *Ecological Modelling* 109, 283–294.

Fulghieri, Paolo, Günter Strobl, and Han Xia, 2012, The economics of solicited and unsolicited credit ratings, Working Paper.

Gan, Yingjin Hila, 2004, Why do firms pay for bond ratings when they can get them for free?, Working Paper, University of Pennsylvania.

He, Jie, Jun Qian, and Philip E. Strahan, 2011, Credit ratings and the evolution of the mortgage-backed securities market, *American Economic Review: Papers & Proceedings* 101, 131–135.
Hörner, Johannes, 2002, Reputation and competition, *American Economic Review* 92, 644–663.
Jeon, Doh-Shin, and Stefano Lovo, 2011, Reputation as an entry barrier in the credit rating industry, Working Paper, Toulouse/HEC.
Klein, Benjamin, and Keith B. Leffler, 1981, The role of market forces in assuring contractual performance, *Journal of Political Economy* 89, 615–641.
Manso, Gustavo, 2012, Feedback effects of credit ratings, Journal of Financial Economics, forthcoming.
Mathis, Jerome, Jamie McAndrews, and Jean-Charles Rochet, 2009, Rating the raters: Are reputational concerns powerful enough to discipline rating agencies?, *Journal of Monetary Economics* 56, 657–674.
Mattingly, Phil, 2013, Obama's $5-billion s&p lawsuit a culmination of four years of investigation, http://business.financialpost.com/2013/02/12/obamas-5-billion-sp-lawsuit-a-culmination-of-four-years-of-investigation/, retrieved February 14, 2013.
Mählmann, Thomas, 2011, Is there a relationship benefit in credit ratings?, *Review of Finance* 15, 475–510.
Opp, Christian C., Marcus M. Opp, and Milton Harris, 2012, Rating agencies in the face of regulation, Journal of Financial Economics, forthcoming, http://dx.doi.org/10.1016/j.jfineco.2012.10.011.
Pagano, Marco, and Paolo Volpin, 2010, Credit ratings failures and policy options, *Economic Policy* 25, 401–431.
——— , 2012, Securitization, disclosure and liquidity, *Review of Financial Studies* 25, 2417–2453.
Reuters, 2012, European rating agency to rival s&p, moody's in 2012, http://www.reuters.com/article/2012/01/21/berger-ratingagency-idUSL6E8CL08S20120121, retrieved April 19, 2012.
Schuster, Peter, Karl Sigmund, Josef Hofbauer, and Robert Wolff, 1981, Selfregulation of behaviour in animal societies. part 2: Games between two populations without selfinteraction, *Biological Cybernetics* 40, 9–15.
SEC, 2012, Egan-jones ratings co. and sean egan charged with making material misrepresentations to sec, http://sec.gov/news/press/2012/2012-75.htm, retrieved August 13, 2012.
Skreta, Vasiliki, and Laura Veldkamp, 2009, Ratings shopping and asset complexity: A theory of ratings inflation, *Journal of Monetary Economics* 56, 678–695.
Stahl, Konrad O, and Roland Strausz, 2011, Who should pay for certification?, CEPR Discussion Paper No. 8190.
Stolper, Anno, 2009, Regulation of credit rating agencies, *Journal of Banking & Finance* 33, 1266–1273.
Taylor, Peter D., 1979, Evolutionarily stable strategies with two types of player, *Journal of Applied Probability* 16, 76–83.
——— , and Leo B. Jonker, 1978, Evolutionarily stable strategies and game dynamics, *Mathematical Biosciences* 40, 145–156.
Weibull, Jorgen W., 1997, *Evolutionary Game Theory* (MIT Press: Cambridge, MA).
Xia, Han, 2012, Can competition improve the information quality of credit ratings?, Working Paper, http://ssrn.com/abstract=1981516.