Stress Testing and Bank Lending*

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Abstract

Bank stress tests are a major form of regulatory oversight implemented in the wake of the financial crisis. Banks respond to the strictness of the tests by changing their lending behavior. As regulators care about bank lending, this affects the design of the tests and creates a feedback loop. We analyze a model of this feedback effect when the regulator trades off the costs of reduced credit to the real economy versus the benefits of preventing default. Stress tests may be (1) lenient, in order to encourage lending in the future, or (2) tough, in order to reduce the risk of costly bank defaults. Reputational concerns on the part of the regulator drive how tough/lenient the test is. Due to a strategic complementarity between the regulator's stress testing strategy and the bank's lending decision, there may be multiple equilibria. We find that in some situations, surplus would be higher without stress testing and regulators may strategically delay stress tests.

Keywords: Bank regulation, stress tests, bank lending

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1 Introduction

Stress tests are a new policy tool for bank regulators that were first used in the recent financial crisis and have become regular exercises subsequent to the crisis. They are assessments of a bank's ability to withstand adverse shocks, and are generally accompanied by requirements intended to boost the capital of those banks who have been found to be at risk.

Naturally, bank behavior reacts to stress testing exercises. Acharya, Berger, and Roman (forthcoming) find that all banks that underwent the U.S. SCAP and CCAR tests reduced their risk by raising loan spreads and decreasing their commercial real estate credit and credit card loans activity.[1]

Regulators should take into account the reaction of banks when conducting the tests. One might posit that if regulators want to boost lending, they might make stress tests more lenient. Indeed, in the case of bank ratings, Agarwal et al. (2014) show that state level banking regulators rate banks more leniently than federal regulators due to concerns over the local economy and this may lead to more bank failures.

In this paper, we study the feedback effect between stress testing and bank lending. Banks may take excess risk or not lend enough to the real economy. Regulators react with either a lenient or tough approach. We demonstrate that this behavior may be self-fulfilling and result in coordination failures. A regulator may prefer to conduct an uninformative stress test or to strategically delay the test.

We examine a model in which there are two sequential stress testing exercises. For simplicity, there is one bank that is tested in each exercise. Each period, the bank chooses how much effort to put in to originate a risky loan, with their outside option being to invest in a risk free asset. The regulator can observe the quality of the risky loan, and may require the bank to raise capital (which we denote as "failing" the stress test) or not. Therefore, stress tests in the model are about gathering information and boosting capital, rather than relaying information to the market. This is in line with the annual exercises during non-crisis times, where runs are an unlikely response to stress test results.[2]

In the model, there is uncertainty about whether a regulator is lenient. A lenient regulator conducts uninformative stress test exercises.[3] A strategic regulator who may fail a bank trades off the cost of foregone credit[4] with the benefit of reducing costly default. After the first stress test result, the bank updates its beliefs about the preferences of the regulator, tries to originate a loan, and undergoes a stress test. Thus, the regulator's first stress test serve two roles: to possibly boost capital for the bank in the first period and

[1] Connolly (2017) and Calem, Correa, and Lee (2017) have similar findings.

[2] This process also resembles banking supervision (for a discussion of banking supervision, see Eisenbach et. al, 2017).

[3] In the text, we demonstrate that the results can be qualitatively similar if the uncertainty was about whether the regulator were tough (failed all banks).

[4] Increased credit is positively associated with economic growth and income for the poor (both across countries and U.S. states, see Levine, 2011). Moskowitz & Garmaise (2006) provide causal evidence the social effects of credit allocation such as reduced crime.

to signal the regulator's willingness to force the bank to raise capital in the second period.

Banks may take too much or too little risk from the regulator's point of view. On the one hand, the bank will take too much risk due to its limited downside. On the other hand, the bank may take too little risk when potential capital raising requirements extract rents from the bank's owners.[5] The banks' choices affect the leniency of the stress test and the leniency of the stress test affects the banks' choices.

The regulator faces a natural trade-off in conducting the first stress test:

First, the regulator may want to build a reputation for being lenient, which can increase lending by the bank in the second period. Since lenient regulators prefer not to require banks to raise capital, there is an equilibrium where the regulator builds the perception that it is lenient by passing the banks that should fail in the first stress test. This is reminiscent of EU's 2016 stress test, where the pass/fail grading scheme was eliminated and only one bank was found to be undercapitalized.[6]

Second, the regulator may want to build a reputation for being tough, which can prevent future excess risk-taking. This leads to an equilibrium where it is tough in its first stress test: it fails banks that should pass. Post-crisis, the U.S. has routinely been criticized for being too strict: its very adverse scenarios, not providing the model to banks, accompanying asset quality reviews, and qualitative reviews all feed into a stringent test.

Finally, there is one more type of equilibrium, where the regulator doesn't have reputation concerns and acts in accordance with its information.

Intriguingly, these equilibria can coexist, leading to a natural coordination failure. This occurs due to a strategic complementarity between the strategic regulator's choice of leniency in the first stress test and the bank's second period risk choice. The less likely the strategic regulator is to pass the bank in the first period, the more risk the bank in the second period takes when it observes a pass (as it believes the regulator is the lenient type). This prompts the strategic regulator to be even tougher, and leads to a self-fulfilling equilibrium. This implies that the presence of stress tests may introduce fragility in the form of excess default or reduced lending to the real economy. The inefficiency in equilibrium choices may be sufficiently large that a regulator may prefer not to conduct stress tests or strategically delay them.

This naturally raises the issue of how a particular equilibrium may be chosen. One way might be if regulators can commit ex-ante to how to use information provided to them (as in games of Bayesian Persuasion). In practice, this might mean announcing stress test scenarios in advance or allowing banks to develop their own scenarios. The regulator might also take costly actions to commit by auditing bank data

[5] Thakor (1996) provides evidence that the adoption of risk-based capital requirements under Basel I and the passage of FDICIA in 1991 led to banks substituting risky lending with Treasury investments potentially prolonging the economic downturn.

[6] That bank, Monte dei Paschi di Siena, had already failed the 2014 stress test and was well known by the market to be in distress.

(e.g. asset quality reviews).

In the model, uncertainty about the regulator's preferences plays a key role. Given that (i) increased lending may come with risk to the economy and (ii) bank distress may have systemic consequences, there is ample motivation to keep this information/intention private. This uncertainty may also arise from the political process. Decisionmaking may be opaque, bureaucratic, or tied up in legislative bargaining.[7] Meanwhile, governments with a mandate to stimulate the economy may respond to lobbying from various interest groups or upcoming elections.[8]

There is little direct evidence of regulators behaving strategically during disclosure exercises, but much indirect evidence. The variance in stress test results to date seem to support the idea of regulatory discretion.[9] Beyond Agarwal et. al. (2014) cited above, Bird et al. (2015) show U.S. stress tests were lenient towards large banks and strict with poorly capitalized banks, affecting bank equity issuance and payout policy. The recent Libor scandal revealed that Paul Tucker, deputy governor of the Bank of England, made a statement to Barclays' CEO that was interpreted as a suggestion that the bank lower its Libor submissions.[10] Hoshi and Kashyap (2010) and Skinner (2008) discuss accounting rule changes that the government of Japan used to improve the appearance of its financial institutions during the country's crisis.[11]

Theoretical Literature

There are a few papers on reputation management by a regulator. Morrison and White (2013) argue that a regulator may choose to forbear when it knows that a bank is in distress, because liquidating the bank may lead to a poor reputation about the ability of the regulator to screen and trigger contagion in the banking system. Boot and Thakor (1993) also find that bank closure policy may be inefficient due to reputation management by the regulator, but this is due to the regulator being self-interested rather than being worried about social welfare consequences as in Morrison and White (2013). Shapiro and Skeie (2015) show that a regulator may use bailouts to stave off depositor runs and forbearance to stave off excess risk taking by banks. They define the type of the regulator as the regulator's cost of funding. We also have potential contagion through reputation as in these papers, but model the bank's lending decision and how it interacts with the choice of the regulator to force banks to raise capital. Furthermore, we define the regulator's type as whether it lenient (uninformative) or strategic.

[7] Shapiro and Skeie (2015) provide examples of related uncertainty around bailouts during the financial crisis.

[8] Thakor (2014) discusses the political economy of banking.

[9] The 2009 U.S. SCAP was widely perceived as a success (Goldstein and Sapra, 2014), with subsequent U.S. tests retaining credibility. European stress tests have varied in perceived quality (Schuermann, 2014) with the early versions so unsuccessful that Ireland and Spain hired independent private firms to conduct stress tests on their banks.

[10] The CEO of Barclays wrote notes at the time on his conversation with Tucker, who reportedly said, "It did not always need to be the case that [Barclays] appeared as high as [Barclays has] recently." This quote and a report on what happened appear in the *Financial Times* (B. Masters, G. Parker, and K. Burgess, Diamond Lets Loose Over Libor, *Financial Times*, July 3, 2012).

[11] Nevertheless, stress tests do contain significant information that is valued by markets (Flannery, Hirtle, and Kovner (2017) demonstrate this and survey recent evidence).

There are several recent theoretical papers on regulatory disclosure and stress tests. Goldstein and Sapra (2014) survey the stress test and disclosure literature to describe the costs and benefits of information provision. Prescott (2008) argues that more information disclosure by a bank regulator decreases the amount of information that the regulator can gather on banks. Bouvard, Chaigneau, and de Motta (2015) show that transparency is better in bad times and opacity is better in good times. Goldstein and Leitner (2018) find a similar result in a very different model where the regulator is concerned about risk sharing (the Hirshleifer effect) between banks. Williams (2017) looks at bank portfolio choice and liquidity in this context. Orlov, Zryumov, and Skrzypacz (2018) show that the optimal stress test will test banks sequentially. Faria-e-Castro, Martinez, and Philippon (2016) demonstrate that stress tests will be more informative when the regulator has a strong fiscal position (to stop runs). In contrast to these papers, in our model *reputational incentives* drive the regulator's choices. In addition, we incorporate capital requirements as a key element of stress testing and focus on banks' endogenous choice of risk. We also don't allow the regulator to commit to a disclosure rule (as all of the papers except for Prescott (2008) and Bouvard, Chaigneau, and de Motta (2015) do).

Our paper identifies the regulator's reputation concern as a source of fragility in the banking sector. In a different context, Ordonez (2013, 2017) show banks' reputation concerns, which provides discipline to keep banks from taking excessive risk, can lead to fragility and a crisis of confidence in the market.

2 The model

We consider a model with five types of risk-neutral agents: the regulator, the bank, borrowers, existing debtholders and a capital provider. The model has two periods $t \in \{1, 2\}$ and the regulator conducts a stress test for the bank in each period. We assume that the regulator has a discount factor $\delta \geq 0$ for the payoffs from the second period bank, where δ may be larger than 1 (as, e.g., in Laffont and Tirole, 1993). The discount factor captures the relative importance of the future of the banking sector for the regulator. For simplicity, we do not allow for discounting within a period.

2.1 Banks and rollovers

In each period t, where $t = \{1, 2\}$, there are four stages:

1. The bank exerts effort to originate a risky loan. If it does not find an appropriate loan, it invests in the safe asset;

2. A risky loan may be good or bad quality. The regulator conducts the stress test by privately observing

the quality of the bank's asset, and decides whether to pass or to fail the bank. In the case of failure, the regulator requires the bank to raise capital;

3. The bank rolls over or liquidates its maturing debt;

4. The payoff of the bank realizes.

Before the start of the game, the bank has raised one unit of debt (or uninsured deposits). At stage 1, the bank has access to a safe investment opportunity which returns $R_0 > 1$ at stage 4. With probability e_t, the bank identifies an appropriate risky loan at stage 1. Originating this loan incurs a quadratic screening cost of $\frac{1}{2}ke_t^2$. In order to focus on the reputation building incentives of the regulator when conducting the stress test in the first period, we make the simplifying assumption that in the first period, the bank has the opportunity to extend a risky loan with fixed probability e_1.[12] In the second period, we assume that the bank can choose its costly loan origination effort to improve the chance that it identifies a risky loan.

If the bank extends a risky loan, the loan quality q_t can be good (g) or bad (b), where the prior probability that the loan is good is denoted by α. The loan can be liquidated at stage 3 to generate 1. If not liquidated, a good loan ($q_t = g$) repays R with probability 1 at stage 4, whereas a bad loan ($q_t = b$) repays R with probability $1 - d$ and 0 otherwise at stage 4. We make the following assumption about the loan returns.

Assumption 1. *(i)* $\alpha R > R_0$*; (ii)* $(1 - d)R > 1$.

Part (i) of Assumption 1 ensures that the bank extends a risky loan whenever the opportunity arises.[13] This is because the bank's owners receive at least αR from the risky loan, which repays R with certainty if it is good (with probability α). If the loan is bad (with probability $1 - \alpha$), the bank's owners may not receive the value of the loan because it may have to raise capital (as required by the regulator during a stress test, see Section 2.2), which incurs dilution costs. Assumption 1 (i) therefore provides a sufficient condition. Part (ii) of Assumption 1 implies that liquidation is inefficient at stage 3.

We make the following parameter restriction on the marginal cost of effort k to ensure a unique interior solution to the bank's effort problem in equilibrium.

Assumption 2. $k > [\alpha + (1 - \alpha)(1 - d)]R - R_0$.

We assume that it is observable to all market participants whether the bank has made a safe investment or a risky one. As we will discuss below, the regulator will privately learn about the credit quality of the bank's risky investment.

[12] Allowing endogenous loan origination effort by the first bank does not alter the reputation building incentives we demonstrate in Section 4.

[13] This claim is formally shown in the proof of Proposition 1.

The bank's debt matures at stage 3 and has an exogenous promised repayment of 1.[14] Since the bank's asset only matures at stage 4, the bank must rollover its debt at stage 4. Based on whether the bank has invested in a risky loan at stage 1 and the information revealed by the stress test results at stage 2, the bank rolls over its debt with an endogenously determined promised repayment at stage 4 of $\tilde{R} \in [1, R]$. For simplicity, we assume that the bank has all the bargaining power, so that $\tilde{R}$ is such that the expected payoff to the debtholders is equal to 1. Assumption 1 implies that the bank is always solvent, and thus refinancing is always feasible and efficient at stage 3.

2.2 The regulator and stress testing

The regulator conducts the stress test by observing the quality q_t of the bank's risky loan at stage 2, and then decides whether to require the bank to raise capital. We will henceforth refer to the regulatory action of requiring the bank to raise capital as "failing", and not requiring the bank to raise capital as "passing". The regulator's objective function is to maximize the expected value of the bank minus the social cost of bank defaults and recapitalizations (which we detail below).

The stress test in the model therefore is not about conveying information to the market about the health of the banks. The test provides the regulator with information on the bank's health, which the regulator uses by requiring recapitalizations. Nevertheless, the stress test accompanied by the recapitalizations[15] does convey information to the market. This information is about the private information of the regulator (defined below). In the model, the bank in the second period reacts to this information, forming the basis of the reputation mechanism.

If a bank fails the stress test, we assume that the bank is required to raise 1 unit of capital kept in costless storage with zero net return so that the bank with the risky loan will not default at stage 4 even if its borrower does not repay.[16] There is a capital provider who can fund the bank. We assume that the capital provider has some bargaining power due to the scarcity of capital, enabling it to capture a fraction β of the expected surplus of the bank. Raising capital thus results in a (private) dilution cost for the bank's owners. The banking literature generally views equity capital raising as costly for banks (for a discussion see Diamond (2017)). We model this cost as dilution due to bargaining power from capital providers, which fits our scenario of a public requirement by a regulator, though other mechanisms that impose a cost on the bank when trying to shore up capital are possible.[17]

[14] While the promised repayment can be greater than 1, the renegotiation-proof repayment is equal to 1 when the bank has all the bargaining power. This is because the liquidation value of the debt is equal to 1.

[15] The stress test results themselves are cheap talk in the model, but the recapitalizations incur costs (and benefits) for the regulator, making costly signals possible.

[16] We assume capital earns zero net return for simplicity. The results do not change if capital is reinvested in the safe investment with a return $R_0 > 1$.

[17] For example, the bank may be forced to sell assets at fire-sale prices. This is a loss in value for the bank. And those who

The regulator can be of two types, the strategic type or the lenient type. The lenient type is behavioral and always passes the bank, whereas the strategic type trades off the social benefits and costs associated with capital when deciding whether to fail a bank, as detailed just below. The regulator knows its own type, but during the stress testing of bank t (where $t = \{1, 2\}$), the market (the existing debtholders of bank t, capital providers and the owners of bank t) is uncertain about the regulator's type. The market has an ex ante belief that, with probability $1 - z_t$, the regulator is strategic. With probability z_t, the regulator is believed to be a lenient type. In our model, z_1 is the probability that nature determines that the regulator is a lenient type. The term z_2 is the updated belief held by the market that the regulator is a lenient type after bank 1's stress test.

The social benefit of capital is to eliminate the cost to society of a bank default at stage 4. Specifically, if a bank operates without recapitalization and the borrower repays 0 at stage 4, the bank defaults and a social cost to society D is incurred. The cost of bank default may represent the loss of value from future intermediation the bank may perform, the cost to resolve the bank, or the cost of contagion.

The strategic regulator has a social cost of capital of C from recapitalization. The social cost of capital here reflects the opportunity cost of the capital used, i.e. the positive externality generated by alternative projects the capital providers could invest in.[18] Note that the social cost of capital is often thought of as the positive value lost from the bank not investing in other projects, not from the capital provider's foregone investments. Nevertheless, stress tests have generally been accompanied by dividend restrictions, compensation restrictions, and recommendations to seek outside funding. Plantin (2014) similarly models the forgone cost of outside investment opportunities due to equity capital raising. Stein's (2012) patient investors are like our capital provider in that they forgo investments to provide capital to the bank, albeit in return for assets.

Therefore when a bank is recapitalized, the social cost is incurred, but the expected social benefit depends on expectations about the likelihood that the bank's risky loan produces a low return. We make the following assumption about the relative social costs and benefits of capital.

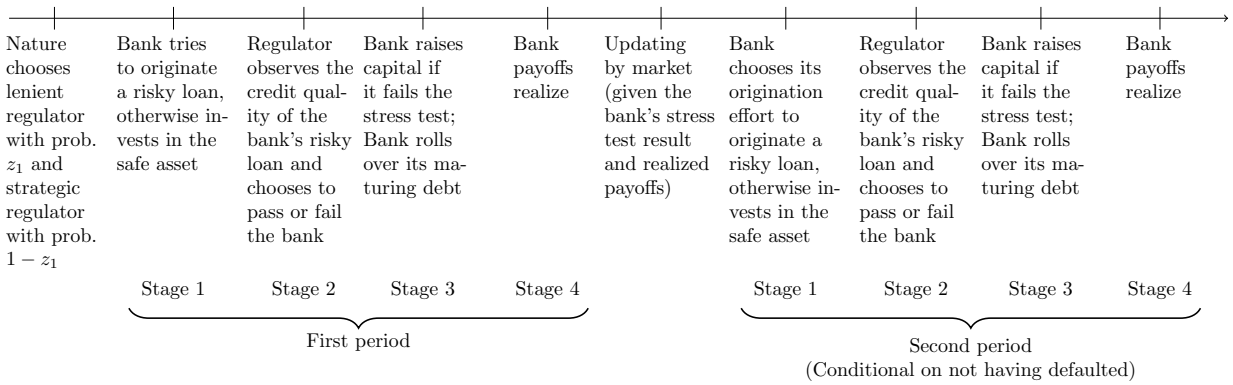
Assumption 3. *(i)* $dD > C > 0$*; (ii)* $[\alpha + (1-\alpha)(1-d)]\, R - R_0 > (1-\alpha)dD$.

Part (i) of this assumption states that the strategic regulator finds that it is beneficial to recapitalize a bank whose risky loan is known to be bad. This puts the strategic regulator in conflict with the lenient regulator and drives reputational incentives. Part (ii) of this assumption states that the risky loan produces a higher expected value than the risk free investment even after taking into account the social cost associated

are purchasing the assets are distorting their investment decisions, as in our model. Hanson, Kashyap, and Stein (2011) discuss this effect and review the literature on fire sales.

[18] As we note in the introduction, increased credit has been found to promote economic growth and increase income for the poor.

Figure 1: Time line of events



with a potential bank default. This assumption ensures that it is socially desirable that the bank invests in the risky asset with positive probability.

The lenient type, on the other hand, always passes the bank. The stress test conducted by the lenient type regulator is therefore uninformative about the bank's assets. Agents may view this type as not conducting "serious" stress test exercises. The behavior of the lenient type regulator can be microfounded by a high social cost of capital relative to the social benefit of capital.[19] In subsection 6.2, we demonstrate that our qualitative results still may hold if we replace the behavioral lenient type with a behavioral tough type who always fails banks and recapitalizes them.

2.3 Summary of timing

The regulator conducts stress testing of the bank in first period, and then in the second period if the bank has not defaulted in the first period. If the bank defaults in the first period, the bank is closed down and does not continue into the second period. At the beginning of the second period, the beliefs about the type of the regulator will be updated depending on the result of bank's stress test and the realized payoff of the bank in the first period. The timing is illustrated in Figure 1.

We assume that the probabilities that the risky loan opportunity is good in the second period is independent of whether the risky loan opportunity is good in the first period, and that the type of the regulator is independent from the quality of the banks' risky loans. Furthermore, the regulator's type remains the same in both periods.

We use the equilibrium concept of Perfect Bayesian equilibrium.

[19] Specifically, if the lenient type regulator faces a high social cost of capital $\bar{C}$ and/or a low cost of bank default $\underline{D}$, then passing the bank with certainty is indeed the unique equilibrium strategy if $\bar{C} \geq d\underline{D} + \delta\frac{1}{2}k$, i.e. that the high cost of capital for the lenient regulator exceeds the maximum benefit of capital over two banks. This includes a direct benefit at the first bank by the eliminating the expected cost of a bank default dD and a potential indirect benefit due to a reduced screening cost for the second bank, if failing the first bank increases the market's perception that the regulator has a high cost of capital. The maximum benefit at the second bank due to a reduced screening cost is no greater than $\frac{1}{2}k$.

3 Stress testing in the second period

We begin the analysis of the model by using backward induction, and characterize the equilibrium in the second period. We first characterize the strategic regulator's stress test strategy at stage 2, taking as given the bank's loan origination effort in stage 0 and investment decision in stage 1. We then analyze the bank's investment decision and screening effort.

If the bank makes a safe investment at stage 1, it is clear that the bank will not default and therefore requires no capital at stage 2. The bank will be able to roll over its maturing debt at stage 3 with a promised repayment of $\tilde{R} = 1$. In this section, we will focus on describing the equilibrium stress test outcome given that the bank extends a risky loan at stage 1 to a good borrower.

Because the game ends after the stress test in the second period, the regulator has no reputational incentives. The stress test strategy of the strategic regulator at stage 2 depends on the quality of the bank's risky loan $q_2 \in \{g, b\}$. Specifically, the strategic regulator passes the bank only if the loan is good, as implied by Assumption 3. Table 1 depicts the regulator's equilibrium stress testing strategy.

Table 1: Equilibrium stress testing in the second period

	Strategic regulator	Lenient regulator
$q_2 = g$	Pass	Pass
$q_2 = b$	Fail	Pass

At stage 3, given the stress test result, the bank raises capital if it fails the stress test, and rolls over its maturing debt. Let α_2^p and α_2^f denote the market's posterior belief that the bank's risky loan is good, given that the bank passes (henceforth denoted by superscript p) and fails (henceforth denoted by superscript f) the stress test, respectively. Given the regulator's stress testing strategy depicted in Table 1, the market's posterior beliefs are given by

$$\alpha_2^p = \frac{\alpha}{1 - (1 - z_2)(1 - \alpha)}, \qquad \alpha_2^f = 0$$

The promised repayment when the bank rolls over its maturing debt at stage 3 depends on the stress test results and the market's posterior belief about the bank's type. First, only a bank with a bad risky loan fails the stress test. It is then required by the regulator to raise 1 unit of capital at stage 3. The capital allows the bank to roll over its maturing debt with a promised repayment of $\tilde{R}_2^f = 1$. Because of the capital providers' bargaining power, the bank's expected payoff captures a fraction $1 - \beta$ of the surplus in this case and is given by $(1 - \beta)[(1 - d)R - 1]$. Second, if a bank passes the stress test, it rolls over its maturing debt at stage 3 by promising a repayment $\tilde{R}_2^p(\alpha_2^p)$, such that

$$[\alpha_2^p + (1 - \alpha_2^p)(1 - d)]\, \tilde{R}_2^p(\alpha_2^p) = 1 \tag{1}$$

We can now analyze the bank's screening effort at stage 0. At stage 0, the bank takes as given the rollover repayments $\tilde{R}_2^p$ and $\tilde{R}_2^f$ in case it passes and fails the stress test. The bank chooses its screening effort to maximize the expected payoff to its owners net of effort cost. That is,

$$\max_{e_2} \quad \underbrace{e_2 \left(\overbrace{[\alpha + (1-\alpha) z_2 (1-d)] \left(R - \tilde{R}_2^p \right)}^{\text{pass}} + \overbrace{(1-\alpha)(1-z_2)(1-\beta)[(1-d)R - \tilde{R}_2^f]}^{\text{fail}} \right)}_{\text{risky loan}} + \underbrace{(1-e_2)(R_0 - 1)}_{\text{safe asset}} - \underbrace{\frac{1}{2} k e_2^2}_{\text{effort cost}} \tag{2}$$

The first term of Eq. 2 represents the expected payoff to the bank's owners when it originates a risky loan, with probability e_2. Subsequently, if the bank passes the stress test, it rolls over its debt with a promised repayment of $\tilde{R}_2^p$. The bank's owners thus receives the residual payoff $R - \tilde{R}_2^p$ if the loan is good (with probability α) or if the loan is bad but repays R when facing a lenient type regulator (with probability $(1-\alpha) z_2 (1-d)$). If the loan is bad and the regulator is the strategic type, the bank fails the stress test. In this case, the expected payoff to the owners of the bank is given by $(1-\beta)[(1-d)R - 1]$. The second term of Eq. 2 represents the expected payoff to the bank's owners when it invests in the safe asset (with probability $1 - e_2$). In this case, the expected payoff to the owners of the bank is $(R_0 - 1)$. Finally, the bank incurs the cost of loan origination effort.

Let us now characterize the equilibrium in the second period in the following proposition.

Proposition 1. *In the second period, there exists a unique equilibrium in which the bank expends loan origination effort $e_2^*(z_2)$ at stage 1. If the bank extends a risky loan, at stage 2, the lenient type regulator passes the bank with certainty and the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

The bank's loan origination effort in equilibrium $e_2^*(z_2)$ is increasing in the market's belief that the regulator is of the lenient type, z_2, where $e_2^*(z_2)$ is given by

$$e^*(z_2) = \frac{1}{k} \left(\underbrace{[\alpha + (1-\alpha)(1-d)]R - R_0}_{\textit{NPV effect}} - \underbrace{(1-\alpha)(1-z_2)\beta[(1-d)R - 1]}_{\textit{dilution cost}} \right) \tag{3}$$

The expected marginal gain from extending a risky loan has two components. On the one hand, the bank benefits from extending a risky loan because it produces a higher expected return than the safe investment (*NPV effect*). On the other hand, the bank faces a dilution cost whenever it is required to raise capital, because the capital providers require a higher rate of return (*dilution cost*). Since the bank only faces the

possibility of failing the stress test and thus having to raise capital if it extends a risky loan, the bank's expected marginal gain from extending a risky loan also reflects the potential dilution cost of recapitalization.

Importantly, the regulator's reputation of being the lenient type, z_2 increases the bank's loan origination effort in the second period in equilibrium, $e_2^*(z_2)$. This is because the lenient type regulator does not require the bank to raise capital even if the bank's risky loan is bad (Table 1). As a result, the bank's incentive to originate a risky loan is greater when the bank expects that the regulator is of the lenient type, because the bank's owners receive the higher expected return from the risky loan without having to incur a dilution cost due to recapitalization.

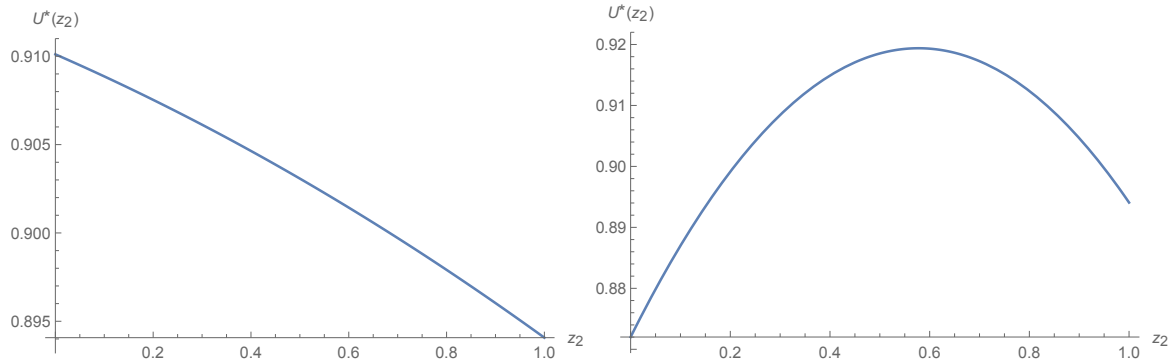
Since the regulator's reputation z_2 determines the bank's screening incentives in equilibrium, we now turn to understanding how the regulator's reputation affects its surplus (i.e. welfare). Let $U(e_2)$ denote the strategic regulator's expected surplus from the bank in the second period, given the bank's screening effort e_2. We can express the expected surplus as follows:

$$U(e_2) = e_2\left([\alpha + (1-\alpha)(1-d)]\,R - 1 - (1-\alpha)C\right) + (1-e_2)(R_0 - 1) - \frac{1}{2}ke_2^2$$

In the above expression, the first term represents the expected surplus when the bank extends a risky loan, the second term represents the surplus when the bank invests in the safe asset, and the last term represents the cost of the bank's screening effort. Notice that, when the bank extends a risky loan, the strategic regulator internalizes the social cost of a potential bank default, which is equal to C, the social cost of bank recapitalization.

Crucially, because the regulator internalizes the social cost of a bank default, whereas the bank's does not, the bank's private choice of screening effort characterized in Proposition 1 generally differs from the socially optimal level. Moreover, the wedge between the two depends on the regulator's reputation, z_2. If the bank expects the regulator to be strategic (z_2 low), it expects a tough stress test and a high probability of having to raise capital when it extends a risky loan. In this case, the bank will be too conservative, and expend too little loan origination effort, resulting in too little lending. If the bank perceives the regulator to be of the lenient type (z_2 high), however, it expects a lenient stress test and a low probability of having to raise capital even if it extends a risky loan. In this case, the bank does not internalize the social cost of its risky investment and exerts too much screening effort, resulting in excessive risk taking. This implies that the expected surplus of the strategic regulator will be increasing in the regulator's reputation z_2 for z_2 low, and decreasing for z_2 high. In particular, this leads to a strictly interior level of reputation $\hat{z}$ that maximizes the expected surplus of the strategic regulator. This result is formally stated in the following proposition and depicted in Figure 2. Let $U^*(z_2) = U(e_2^*(z_2))$ denote the expected surplus from the bank in the second

Figure 2: The expected surplus from the bank in the second period in equilibrium, given a strategic regulator. The parameter values used in this plot are $\alpha = 0.4$, $R = 4$, $R_0 = 1.3$, $d = 0.01$, $D = 100$, $C = 0.75$ and $k = 4$. The left panel represents the case where $\beta = 0.1 < \underline{\beta} = 0.253$, and the right panel represents the case where $\beta = 0.6 > \underline{\beta}$.



period for the strategic regulator in equilibrium.

Proposition 2. *The expected surplus from the bank in the second period for the strategic regulator in equilibrium, $U^*(z_2)$, is strictly increasing in the probability that the regulator is of the lenient type z_2 for $z_2 < \hat{z}$, and decreasing in z_2 for $z_2 \geq \hat{z}$. $\hat{z} \in [0, 1)$ is defined by*

$$\hat{z} = \begin{cases} 0, & \text{if } \beta \leq \underline{\beta} \\ \frac{\beta[(1-d)R-1]-C}{\beta[(1-d)R-1]}, & \text{if } \beta > \underline{\beta} \end{cases} \tag{4}$$

where $\underline{\beta}$ is defined by

$$\underline{\beta}\,[(1-d)R-1] = C \tag{5}$$

Recall that β reflects the surplus extracted from the bank by the capital provider. The cutoff $\underline{\beta}$ defines the situation where the bank's private cost of capital for the bank is equal to the social cost of capital for the strategic regulator. Both the private cost of capital and the social cost of capital reflect the opportunity cost of foregone loans by the capital provider. The private cost of capital may additionally reflect the bargaining power of the bank vis-a-vis capital providers or a monitoring/information premium demanded by the capital provider. The social cost of capital may additionally reflect the positive externalities forgone by the capital provider channeling its funds to the bank. Therefore, depending on these measures, the private cost of capital may be higher or lower than the social cost of capital.

Proposition 2 thus states that, the expected surplus of the bank in the second period in equilibrium is hump-shaped in its reputation of being the lenient type if the social cost of capital is lower than the private

cost of capital, for reasons discussed above. However, if the social cost of capital is greater than the private cost of capital, then the expected surplus from the bank in the second period in equilibrium for the strategic regulator is always decreasing in its reputation of being the lenient type. This is because when the private cost of capital is low, the bank always tends to over-invest the risky loans. Therefore the more the bank expects that the regulator is of the lenient type who does not require the bank to raise capital, the more risk the bank takes and the lower is the expected surplus.

4 Stress testing in the first period

In this section, we analyze the equilibrium stress testing strategy of the regulator for the bank in the first period, given the equilibrium in the second period.

Let us consider the incentives of the strategic regulator to pass the bank in the first period. Let G_{q_1} denote the net gain of passing the bank relative to failing the bank, given the quality of the bank's risky loan $q_1 \in \{g, b\}$.

$$\begin{aligned} G_g &= \underbrace{C}_{\text{bank 1 surplus effect}} + \underbrace{\delta\left[U^*(z_2^R) - U^*(z_2^f)\right]}_{\text{reputation effect}} \\ G_b &= \underbrace{(C - dD)}_{\text{bank 1 surplus effect}} + \underbrace{\delta\left[(1-d)U^*(z_2^R) - U^*(z_2^f)\right]}_{\text{reputation effect}} \end{aligned} \tag{6}$$

where the first term represents the net gain in terms of the expected surplus from the bank in the first period, and the second term represents the reputation concern in terms of the expected surplus from the bank in the second period. The term z_2^R is the posterior belief held by the market about the probability that the regulator is of the lenient type, given that the bank passes the stress test in the first period and realizes a payoff of R. If the bank passes the stress test in the first period and realizes a payoff of 0 instead, the bank defaults and does not continue to the second period, producing an expected surplus of 0. The term z_2^f is the posterior belief held by the market about the probability that the regulator is the lenient type, given that the bank fails the stress test in the first period.[20] Since the bank recapitalizes after failing the stress test, the bank does not default in the first period and thus continues to the second period.

We first show that there is an equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period.

Proposition 3. *The equilibrium stress testing strategy of the strategic regulator in the second period described*

[20] Note that since only a strategic regulator fails the bank in equilibrium, the posterior belief $z_2^f = 0$ and does not depend on the realized payoff of the bank in the first period.

in Proposition 1 is an equilibrium strategy in the first period. That is, in the first period, if the bank extends a risky loan, at stage 2, the strategic regulator passes the bank if and only if the risky loan is good.

There exist thresholds $\bar{\delta}_g(z_1)$ and $\bar{\delta}_b(z_1)$, such that the necessary and sufficient conditions for this equilibrium to exist is $\delta < \min\{\bar{\delta}_g(z_1), \bar{\delta}_b(z_1)\}$.

For certain parameters, the equilibrium stress testing strategy of the regulator in the first period is the same as its strategy in the second period, and is illustrated in Table 1. In such an equilibrium, the posterior probabilities that the regulator is the lenient type given that the bank passes the stress test and realizes a payoff of R in the first period and given that the bank fails the stress test in the first period are given by

$$z_2^R = \frac{[\alpha + (1-\alpha)(1-d)]\, z_1}{[\alpha + (1-\alpha)(1-d)]\, z_1 + \alpha(1-z_1)}, \qquad z_2^f = 0 \tag{7}$$

In particular, since only a strategic regulator would fail a bank, the market associates a belief of $z_2^f = 0$ with the bank failing the stress test in the first period. If the bank passes the stress test and realizes a high payoff R in the first period, the market updates its belief to $z_2^R > z_1$, reflecting the fact that a lenient type regulator is more likely to pass a good bank than a strategic regulator.

The stress testing strategy described in Proposition 3 is an equilibrium if and only if $G_g \geq 0 \geq G_b$. The stress testing strategy of the regulator in the first period is affected by its reputation concerns, in addition to the expected surplus from the bank in that period (Eq. 6). While the effect through the bank's expected surplus in the first period is always positive for a bank with a good risky loan and negative for a bank with a bad risky loan, the reputation effect may be positive or negative in equilibrium. This is because, as discussed in Proposition 2, the regulator's surplus is non-monotonic in its reputation. On the one hand, a reputation of being strategic implies a tough stress test, which can induce the bank to be excessively conservative and reduce lending in the second period. On the other hand, a reputation of being the lenient type can result in excessive risk taking and excessive lending by the bank in the second period.

Proposition 3 states that an equilibrium in which the regulator adopts the same stress testing strategy in both periods exists whenever the reputation effect does not outweigh the direct effect through the expected surplus in the first period. There can be two cases, depending on whether the reputation effect of a pass is positive or negative (this depends on the second period payoff, as depicted in Figure 2 and described in Proposition 2). If the prior probability that the regulator is the lenient type (z_1) is low, the reputation effect of passing the bank in the first period is positive, and the equilibrium exists if the reputation effect (δ) is not so large that the strategic regulator wishes to pass the bank with a bad risky loan. If the prior probability that the regulator is the lenient type (z_1) is high, the reputation effect of passing the bank in the first period is negative, and the equilibrium exists if and reputation effect (δ) is not so large that the strategic regulator

wishes to fail the bank with a good risky loan.

For the 2009 U.S. SCAP stress test, one might argue that the discount factor was not high; i.e. the regulator was concerned more about the present than building their reputation. Moreover, given that the U.S. had explicitly committed to recapitalize banks that the exercise found to be distressed, the term z_1 was very low. We demonstrate that in this case, the stress test runs smoothly - the regulator conveys its information perfectly and recapitalization only take place for failing banks.

Proposition 3 shows that if the regulator's reputation concern δ is sufficiently small, the regulator's stress testing strategy is identical in both periods. However, we will show in the following sections that two other equilibria exist due to the regulator's reputation building incentives.

4.1 Reputation building to incentivize lending

Consider the incentives for the strategic regulator to pass the bank with a bad risky loan in the first period. If the strategic regulator fails the bank and recapitalizes it, the regulator reveals the fact that it is strategic to the market. The bank then faces a strong incentive to exert less loan origination effort and to invest more in the safe asset in the second period, in order to avoid failing the stress test. If the strategic regulator passes the bank in the first period, however, it pools with the lenient type regulator, increasing the incentive for the bank to exert loan origination effort and engage in risky lending in the second period. If the benefit of increased lending by the bank in the second period is sufficiently large, the regulator may want to pass the bank in the first period even when the risky loan of the bank is bad, in order to gain a reputation of leniency.

In the following proposition, we demonstrate that reputation building incentives to encourage lending by the bank in the second period lead to another equilibrium for the bank's stress test in the first period besides that of Proposition 3.

Proposition 4. *There is an equilibrium in which the strategic regulator passes the bank in the first period with positive probability even when the bank's risky loan is bad. Specifically, when the bank extends a risky loan in the first period, the regulator passes the bank with certainty if the loan is good, and passes the bank with positive probability $\pi_b^* > 0$ if the loan is bad.*

There exist $\underline{\delta}_b(z_1)$, such that the necessary and sufficient condition for this equilibrium to exist is $\beta > \underline{\beta}$ and $\delta \geq \underline{\delta}_b(z_1)$.

Table 2 depicts the equilibrium stress testing in the first period described in Proposition 4. Compared to the equilibrium in Proposition 3, the strategic regulator now follows a mixed strategy when facing a bank with a bad risky loan and passes this bank with positive probability. In equilibrium, the posterior

Table 2: Equilibrium stress testing in the first period when the strategic regulator wants to build reputation to incentivize lending by the bank in the second period

	Strategic regulator	Lenient regulator
$q_1 = g$	Pass	Pass
$q_1 = b$	Pass with probability $\pi_b^* > 0$	Pass

probabilities that the regulator is the lenient type given that the bank passes the stress test and realizes a payoff of R in the first period and given that the bank fails the stress test in the first period are given by:

$$z_2^R = \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + [\alpha + (1-\alpha)(1-d)\pi_b^*](1-z_1)}, \qquad z_2^f = 0 \tag{8}$$

In Proposition 3, the strategic regulator recapitalizes the bank with a bad risky loan in the first period, which maximizes the expected surplus from the bank. Passing the bank with a bad risky loan is costly, as it will incur a default cost with positive probability. However, by now passing the bank, the strategic regulator is able to increase the perception that it is of the lenient type, since the lenient type regulator always passes the bank. This is useful to the strategic regulator because it increases the incentives for the bank to originate risky loans in the second period. In other words, the regulator enjoys a positive reputation effect from passing the bank with a bad risky loan in the first period.

Proposition 4 identifies two necessary and sufficient conditions for an equilibrium with reputation building to incentivize lending to exist. First, the private cost of capital (β) must be sufficiently high, due to the bargaining power of capital providers. As a result, the bank may under-expend loan origination effort when it expects recapitalization by the regulator, implying that there is a positive reputation effect of passing the bank in the first period in equilibrium. Second, the reputation concern (δ) of the regulator must be sufficiently high, so that the regulator's reputational benefits outweigh the short-term efficiency loss when passing the bank with a bad risky loan.

While the initial European stress tests performed poorly (e.g. passing Irish banks and Dexia), one might argue that during crisis times, the main focus was preventing runs - and without a fiscal backstop it was hard to maintain credibility (Faria-e-Castro, Martinez, and Philippon, 2016). We argue that in normal times, a stress test may be lenient to incentivize banks to lend to the real economy. This may explain the 2016 EU stress test, which eliminated the pass/fail criteria, reduced the number of banks stress tested by about half, used less adverse scenarios than the U.S. and UK, and only singled out one bank as undercapitalized - Monti dei Paschi di Siena, which had failed the previous (2014) stress test and was well known to be in distress.

Table 3: Equilibrium stress testing in the first period when the strategic regulator wants to build reputation to reduce excessive risk-taking by the second bank in the second period

	Strategic regulator	Lenient regulator
$q_1 = g$	Pass with probability $\pi_g^* < 1$	Pass
$q_1 = b$	Fail	Pass

4.2 Reputation building to reduce excessive risk-taking

Consider the incentives for the strategic regulator to fail the bank with a good risky loan in the first period. If the regulator fails the bank and recapitalizes it, the regulator reveals the fact that it is strategic to the market. The bank then faces a strong incentive to invest in the safe asset in the second period, in order to avoid failing the stress test. If the regulator passes the bank in the first period, however, it pools with the lenient type regulator. The bank thus expects a lenient stress test in the second period and tends to invest excessively in the risky asset, because it does not internalize the social cost of a potential bank default. If the concerns about excessive risk-taking by the bank in the second period are sufficiently large, the strategic regulator may want to fail the bank with a risky loan in the first period even when it is good, in order to reveal to the market its willingness to fail a bank during the stress test.

In the following proposition, we demonstrate that the reputation building incentives to reduce excessive risk-taking by the bank leads to an equilibrium different than that of Propositions 3 and 4.

Proposition 5. *There is an equilibrium in which the strategic regulator fails the bank in the first period with positive probability even when the bank's risky loan is good. Specifically, when the bank extends a risky loan in the first period, strategic regulator fails the bank with certainty if the loan is bad, and passes the bank with probability $\pi_g^* < 1$ if the loan is good.*

There exists $\bar{\beta} > \underline{\beta}$, such that the necessary and sufficient condition for this equilibrium to exist is $\beta < \bar{\beta}$ and $\delta \geq \bar{\delta}_g(1)$.

Let $\pi_g^* \in (0, 1)$ denote the probability that the strategic regulator passes the bank with a good risky loan in equilibrium. Table 3 depicts the equilibrium stress testing in the first period described in Proposition 5. Compared to Proposition 3, the strategic regulator now follows a mixed strategy when facing a bank with a good risky loan and fails the bank with positive probability. In equilibrium, the posterior probabilities that the regulator is the lenient type given that the bank passes the stress test and realizes a payoff of R in the first period and given that the bank fails the stress test in the first period are given by:

$$z_2^R = \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + \alpha\pi_g^*(1-z_1)}, \qquad z_2^f = 0 \tag{9}$$

In Proposition 3, the strategic regulator passes the bank with a good risky loan in the first period. Failing

the bank in this case would result in a costly recapitalization of the bank with no benefit, since the good loan will not default. However, by now failing this bank, the strategic regulator is able to reveal to the market its willingness to recapitalize a bank, and thus reduce the bank's incentive to engage in excessive risk taking in the second period.

Proposition 5 identifies two necessary and sufficient conditions for an equilibrium with reputation building to reduce excessive risk-taking to exist. First, the private cost of capital (β) must be sufficiently low. The bank takes excessive risk because its cost of being recapitalized β is low. This implies a negative reputation effect of passing the bank in the first period in equilibrium, because the inefficiency stemming from excessive risk-taking dominates the potential inefficiency associated with too little lending. Second, the reputation concern (δ) of the regulator must be sufficiently high, so that the incentive for the strategic regulator to reduce excessive risk-taking by the bank in the second period is sufficiently strong.

U.S. stress tests have generally been regarded as much more strict than European ones. First, the Federal Reserve performs the stress test itself on data provided by the banks (and does not provide the model to the banks), whereas in Europe, it has been the case that the banks themselves perform the test. Second, the U.S. stress tests have regularly been accompanied by Asset Quality Reviews, whereas this has been infrequent for European stress tests. Third, one of the most feared elements of the U.S. stress tests has been the fact that there is a qualitative element that can (and has been) used to fail banks. In line with our results above, the fact that U.S. stress tests have been institutionalized as occurring on a yearly basis implies that reputation concerns are important. Furthermore, a swifter recovery from the crisis means that capital raising for banks is likely to be easier in the U.S.

5 Equilibrium summary and multiplicity

We now show formally that the three equilibria discussed in the previous sections are the only possible equilibria with an informative stress test. Figure 3 plots the regions of the parameter space (δ, z_1) for which each equilibrium exists.

Proposition 6. *There exists an equilibrium in the first period, and the only equilibria are*

A: an equilibrium without reputation building, as described by Proposition 3,

B: an equilibrium with reputation building to incentivize lending, as described by Proposition 4, and

C: an equilibrium with reputation building to reduce excessive risk-taking, as described by Proposition 5.

This result demonstrates that equilibria can only be one of three types. This arises from that fact that, in any equilibrium, the strategic regulator faces strictly greater incentives to pass a bank with a good risky

loan than a bank with a bad risky loan. This can be seen in Eq. 6. Passing a bank with a bad risky loan will result in a costly default with higher probability than passing a bank with a good risky loan. A bank default generates two costs. First, it generates a social cost of default D in the first period. Second, it leads to a loss of expected surplus $U(z_2^R)$ in the second period. Propositions 3–5 therefore represent all equilibria: every possibility where the probability with which the strategic regulator passes a bank with a good risky loan is weakly larger than the probability with which it passes a bank with a bad risky loan.

Recall that the reputation effect of the regulator's stress testing strategy for the first stress test is given by the second term of Eq. 6. Given that the lenient type regulator passes the bank, if the strategic regulator fails the bank in the first stress test, it is revealed to be strategic ($z_2^f = 0$) and have a strong willingness to recapitalize the bank if the bank's risky investment is of bad quality. Subsequently, in the second period, the bank behaves too conservatively. In contrast, if the strategic regulator passes the bank in the first stress test, it is pooled with the lenient type regulator who also passes the bank. Subsequently, the second period bank perceives the regulator's willingness to recapitalize the bank as weak, and invests excessively in extending risky loans. In equilibrium, the posterior probability that the regulator is of the lenient type, given that the bank passes the first stress test and then realizes a payoff of R is given by

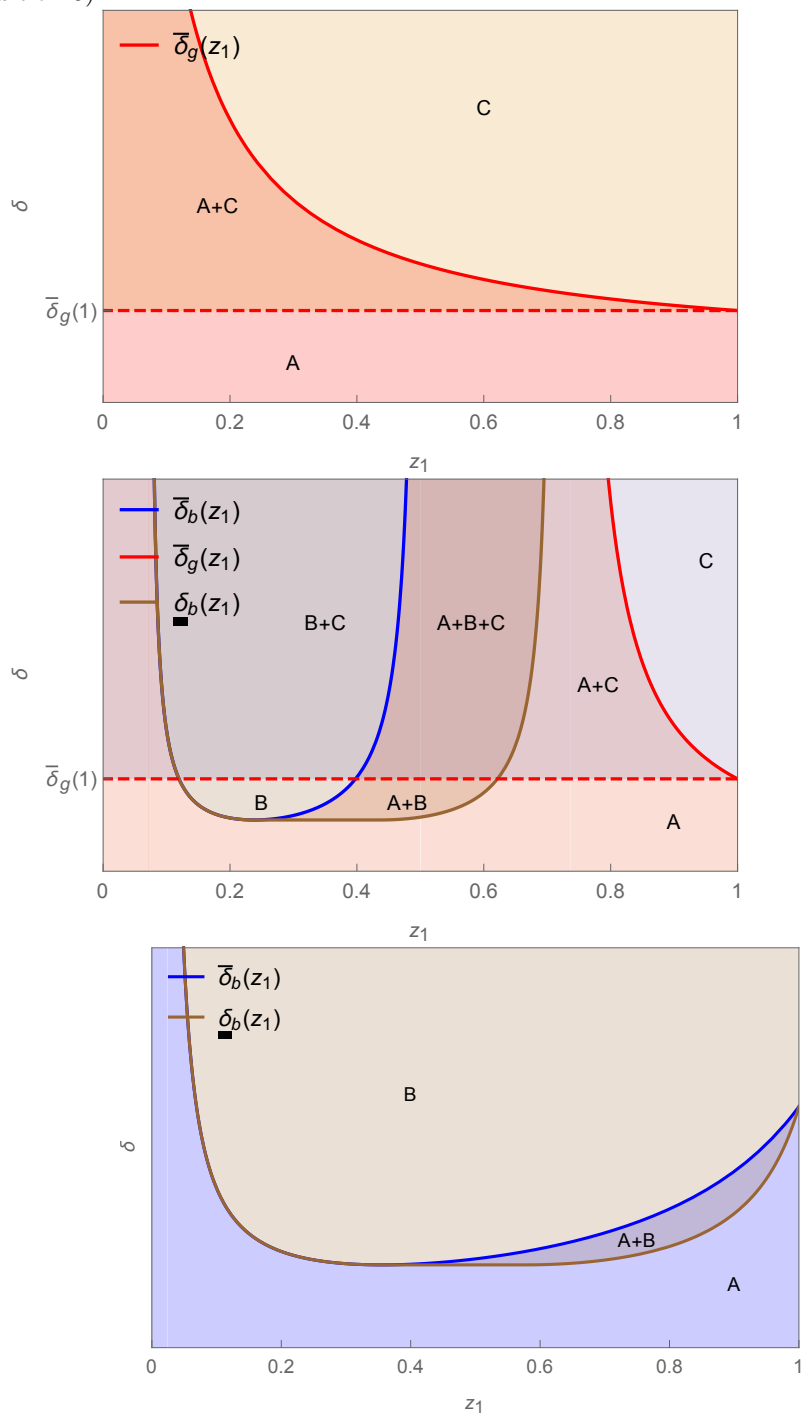
$$z_2^R = \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + [\alpha\pi_g + (1-\alpha)(1-d)\pi_b](1-z_1)} \tag{10}$$

where π_g and π_b denote the regulator's probability of passing the bank in the first stress test, given that the bank's risky loan is good and bad, respectively.

Propositions 3–5 imply that there exist parameter ranges such that more than one equilibrium exists. This is because the reputation concern of the regulator may feature a strategic complementarity between the regulator's stress testing strategy and the market's belief updating process. For certain parameters, the regulator's reputation concern is self-fulfilling.

On the one hand, the regulator's stress testing strategy and the market's belief updating process are strategic substitutes for (π_g, π_b) such that $z_2^R < \hat{z}$. This range is where this expected surplus is increasing in z_2^R (this is summarized in Proposition 2). Here, the market realizes that the strategic regulator's surplus is increasing when it is perceived to be the lenient type. If the market conjectures that the strategic regulator adopts a tougher stress testing strategy (lower π_g or π_b), the market infers that the regulator who passed the bank in the first period is more likely to be the lenient type (higher z_2^R). Consequently, the bank takes more risk in the second period after a pass result in the first period, resulting in *higher* expected surplus $U(e_2^*)$. In turn, this increases the net gain for the strategic regulator from passing the bank in the first period and the regulator should adopt a more lenient stress testing strategy.

Figure 3: Parameter space for the existence of different equilibrium. The parameter values used in this plot are the same as for Figure 2 except β. The top panel represents the case where $\beta = 0.3 < \underline{\beta}$, the middle panel represents the case where $\beta = 0.45 \in [\underline{\beta}, \bar{\beta}]$, and the bottom panel represents the case where $\beta = 0.6 > \bar{\beta} = 2\underline{\beta}$. The letters represent different equilibrium outcomes: A represents the equilibrium without reputation building (Proposition 3), B represents the equilibrium with reputation building to incentivize lending (Proposition 4) and C represents the equilibrium with reputation building to reduce excessive risk-taking (Proposition 5).



On the other hand, the strategic regulator's stress testing strategy and the market's belief updating process are strategic complements for (π_g, π_b) such that $z_2^R \geq \hat{z}$. This range is where the regulator's surplus is decreasing in z_2^R. Here, the market realizes that the strategic regulator's surplus is decreasing when it is perceived to be the lenient type. If the market conjectures that the strategic regulator adopts a tougher stress test strategy (lower π_g or π_b), the market infers that the regulator who passes the bank in the first period is more likely to be lenient (higher z_2^R). Consequently, the bank increases its risk-taking in the second period after a pass result in the first period, resulting in *lower* expected surplus $U(e_2^*)$. In turn, this further decreases the net gain for the strategic regulator from passing the bank in the first period, justifying a tougher testing strategy. It is indeed this strategy complementarity that leads to equilibrium multiplicity.

5.1 Welfare comparison of equilibria

We now analyze the welfare implication of the regulator's reputation concern during stress testing. In particular, for parameter values where multiple equilibria coexist, we compare the expected surplus of the regulator from the two banks.

Proposition 7.

- *Whenever an equilibrium with reputation building to incentivize lending (Equilibrium B) coexists with another type of equilibrium, the expected surplus of the strategic regulator for the two banks is strictly higher in Equilibrium B.*
- *Whenever an equilibrium with reputation building to reduce excessive risk-taking (Equilibrium C) coexists with another type of equilibrium, the expected surplus of the strategic regulator for the two banks is strictly lower in Equilibrium C.*

This result demonstrates that when there are multiple equilibria, we can rank the equilibria in terms of welfare. When the strategic regulator chooses the probability with which to pass a bank with credit quality q_1, it takes as constant the beliefs of the market about its choice. A first best solution wouldn't take this as constant, creating a wedge between the optimal choice and the regulator's choice. The welfare maximizing solution when there are multiple equilibria is to choose as lenient a stress test as possible. The reason for this is that, as described above, multiple equilibria occur when the strategic regulator's second period surplus is decreasing in its posterior reputation. Its posterior reputation is decreasing in the levels of leniency π_g and π_b (this can be seen in Eq. 10). Therefore the first best solution would choose as much leniency as possible. Therefore, the most lenient type of equilibrium (Equilibrium B) yields the highest welfare and the toughest equilibrium (Equilibrium C) yields the lowest welfare.

This suggests that despite heavy criticism, lenient stress tests in Europe may have been optimal given the constraints.

This equilibrium welfare ranking is sensitive to the setup of the model. For example, endogenizing the choice of the first period bank's effort could change the ordering.

Nevertheless, the benefits from leniency summarized above mean that the regulator might be even better off by not conducting stress tests for the bank in the first period. Notice that there may exist an equilibrium in which both types of regulator pass the bank in the first period with certainty. This is equivalent to an economy where the regulator does not conduct stress tests for the bank in the first period.

Corollary 1. *Among type B equilibria, there are equilibria in which the regulator passes the bank in the first period with certainty. When such an equilibrium exists, it produces the highest expected surplus for the strategic regulator among all equilibria. There exists* $\tilde{\delta}_b(z_1)$*, where* $\tilde{\delta}_b(z_1) \geq \underline{\delta}_b(z_1)$*, such that this equilibrium exists if and only if* $\beta > \underline{\beta}$ *and* $\delta \geq \tilde{\delta}_b(z_1)$*.*

This corollary shows that no stress test (or a completely uninformative stress test) for the bank in the first period yields the highest welfare among all equilibria, whenever it can be supported as an equilibrium outcome. However, because of equilibrium multiplicity, the economy may be trapped in a less efficient equilibrium. Therefore if the regulator could strategically commit to not doing a stress test in the first period, i.e. delaying the stress test, there could be substantial welfare gains. The timing of European stress tests has been quite irregular compared with the annual U.S. exercises (they were conducted in 2010, 2011, 2014, 2016, and 2018). Delay in this situation may be a way of committing to lenience.

The fact that we have three potentially coexisting equilibria raises the issue of how a particular equilibrium may be chosen. Commitment by the regulator in an ex-ante stage would facilitate this. Of course, in a crisis, committing to future actions may not be feasible. The regulator has access to several policy variables that might prove useful as commitment devices. Committing to how signals from banks are used is standard in the Bayesian Persuasion literature, but requires substantial independence from political pressure and processes that are well defined. A more practical alternative is committing to stress test scenarios. Stress test scenarios can be more or less lenient, given the effect desired. Asset quality reviews also commit more resources and reveal more information about bank positions.

6 Adding to the Model

In this section, we examine the model's properties further. First, we look at a comparative static result on the cost of bank default. Second, we show that the model's qualitative results hold when we replace the behavioral lenient-type regulator with a behavioral tough-type regulator.

6.1 The cost of bank default

Our working assumption for the paper has been that the stress tests are conducted in 'normal' times. Nevertheless, it is of interest to explore what the model says about less-than-normal times, i.e. a crisis. In order to do so, we look at how a change in the social cost of a bank default affects the equilibrium stress testing regime. An increase in the social cost of a bank default may be due to contagion, fire-sale externalities, or several other channels. We analyze this in the following proposition:

Proposition 8. *If the social cost of a bank default D increases, then*

- $\bar{\delta}_b(z_1)$ *increases, i.e. Equilibrium A exists for a larger range of parameters; and*
- $\underline{\delta}_b(z_1)$ *increases, i.e. Equilibrium B exists for a smaller range of parameters.*

Recall that Equilibrium A is the most informative equilibrium as the strategic regulator conveys its information truthfully. This proposition then implies that stress tests tends to be more informative during financial distress due to the increased downside of an unanticipated default on the market.

6.2 A tough type regulator

In the model, we assumed that one of the types of regulator was behavioral and always passed the bank in its stress test - the lenient type. In this subsection, we demonstrate the robustness of our results by considering an alternative model in which the behavioral regulator instead always fails the bank and recapitalizes it. We call this the tough type. Recapitalization can lead to a high dilution cost for the bank. In order to ensure that the bank extends a risky loan whenever the opportunity arises, we make the following assumption, which is analogous to part (i) of Assumption 1.

Assumption 4. $(1-\beta)\left[\alpha(R-1)+(1-\alpha)\left((1-d)R-1\right)\right] > R_0 - 1.$

If the bank expects that the regulator is a tough type, then the bank only captures a fraction $(1-\beta)$ of the NPV from extending the risky loan. This assumption therefore provides a sufficient condition for the bank to be willing to extending the risky loan rather than to invest in the safe asset.

The following proposition characterizes the equilibria with the tough type regulator.

Proposition 9. *Consider the model with a tough type regulator and Assumption 4 holding. In the second period, there exists a unique equilibrium in which, if the bank extends a risky loan at $t = 2$, the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

In the first period, an equilibrium exists, and the only equilibria are:

A_T: *an equilibrium without reputation building, in which the strategic regulator's stress testing strategy in the first period is the same as in the second period (i.e., fully informative),*

B_T: *an equilibrium with reputation building to incentivize lending,in which the strategic regulator passes the bank with positive probability even when the bank's risky loan is bad, and*

C_T: *an equilibrium with reputation building to reduce excessive risk-taking, in which the strategic regulator fails the bank with positive probability even when the bank's risky loan is good.*

The proposition demonstrates that the types of equilibria when the behavioral regulator is lenient are also the only types of equilibria when the behavioral regulator is tough. This, of course, relies on Assumption 4, which assumes that the tough regulator's recapitalizations are not so painful for the bank that the bank is still willing to take on risky loans.

7 Conclusion

Stress tests have been incorporated recently into the regulatory toolkit. The tests provide assessments of bank risk in adverse scenarios. Regulators respond to negative assessments by requiring banks to raise capital. However, regulators have incentives to be tough by asking even some safe banks to raise capital or to be lenient by allowing some risky banks to get by without raising capital. These incentives are driven by the weight the regulator places on lending in the economy versus stability. Banks respond to the leniency of the stress test by altering their lending policies. We demonstrate that in equilibrium, regulators may be tough and discourage lending or lenient and encourage lending. These equilibria can be self-fulfilling and the regulator may get trapped in one of them.

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8 Proofs

8.1 Proof of Proposition 1

The regulator's stress testing strategy at stage 2 is as described in the discussion in Section 3. The bank's effort choice $e_2^*(z_2)$ is given by Eq. 2. The first order condition that characterizes the bank's effort choice is

$$[\alpha + (1-\alpha)z_2(1-d)]\left(R - \tilde{R}_2^p\right) + (1-\alpha)(1-z_2)(1-\beta)[(1-d)R-1] - (R_0-1) - ke_2 = 0 \tag{11}$$

After substituting in $\tilde{R}_2^p(\alpha_2^p)$ given by Eq. 1, the bank's optimal screening choice in equilibrium satisfies Eq. 3.

An equilibrium with $e_2^*(z_2) \in (0,1)$ exists because the the LHS of Eq. 3 is strictly positive for $e_2 = 0$, and is strictly negative for $e_2 = 1$ (by Assumption 2). The equilibrium is also unique, because the LHS of Eq. 3 is strictly decreasing in e_2.

It follows that the equilibrium screening effect is increasing in the regulator's reputation z_2

$$\frac{\partial e_2^*(z_2)}{\partial z_2} = \frac{\beta(1-\alpha)[(1-d)R-1]}{k} > 0 \tag{12}$$

8.2 Proof of Proposition 2

$U(e_2)$ is given by Eq. **??**. Notice that $U(e_2)$ is increasing in e_2 if and only if $e_2 \leq \hat{e}$, where $\hat{e} \in (0,1)$ is defined by

$$\hat{e} = \frac{1}{k}\left([\alpha + (1-\alpha)(1-d)]R - R_0 - (1-\alpha)C\right), \tag{13}$$

where $\hat{e} < 1$ by Assumption 2 and $\hat{e} > 0$ by Assumption 3.

Recall that the bank's equilibrium screening effort $e_2^*(z_2)$ is increasing in z_2, where

$$e_2^*(1) = \frac{1}{k}\left([\alpha + (1-\alpha)(1-d)]\,R - R_0\right) > \hat{e}, \tag{14}$$

and

$$e_2^*(0) = \frac{1}{k}\left([\alpha + (1-\alpha)(1-d)]\,R - R_0 - (1-\alpha)\beta[(1-d)R-1]\right) \tag{15}$$

where $e_2^*(0) < \hat{e}$ if and only if $\beta > \underline{\beta}$, where $\underline{\beta}$ is defined by Eq. 5.

Therefore there exists $\hat{z} \in [0,1)$, such that $U(e^*(z_2))$ is increasing in z_2 if and only if $z_2 \leq \hat{z}$, where $\hat{z}$

is given by Eq. 4. That is, if $\beta \leq \underline{\beta}$, $\hat{z} = 0$, and if $\beta > \underline{\beta}$, $\hat{z}$ is defined by $e_2^*(\hat{z}) = \hat{e}$, which is equal to the expression given in this proposition.

8.3 Proof of Proposition 3

Before we proceed to prove this proposition, it is useful to define parameter values such that $U^*(0) > U^*(1)$.

$$\begin{aligned} U^*(1) - U^*(0) &= [e_1^*(1) - e_2^*(0)]\left([\alpha + (1-\alpha)(1-d)]R - R_0 - (1-\alpha)C\right) - \frac{1}{2}k\left[(e_2^*(1))^2 - (e_2^*(0))^2\right] \\ &= \frac{1}{k}(1-\alpha)\beta[(1-d)R-1]\left([\alpha + (1-\alpha)(1-d)]R - R_0 - (1-\alpha)C\right) \\ &\quad - \frac{1}{k}(1-\alpha)\beta[(1-d)R-1]\left(([\alpha + (1-\alpha)(1-d)]R - R_0) - \frac{1}{2}(1-\alpha)\beta[(1-d)R-1]\right) \end{aligned}$$

Therefore $U^*(0) > U^*(1)$ if and only if $\beta < \bar{\beta}$, where $\bar{\beta} > \underline{\beta}$ is defined by

$$\bar{\beta}[(1-d)R-1] = 2C \tag{16}$$

If $\beta > \bar{\beta}$, Proposition 2 then implies that $U^*(0) \leq U^*(z_2)$ for all $z_2 > 0$. If $\beta < \bar{\beta}$, Proposition 2 implies that, there exists $\tilde{z} \in (\hat{z}, 1)$, such that $U^*(0) > U^*(z_2)$ if and only if $z_2 > \tilde{z}$, where $\tilde{z}$ is defined by

$$\tilde{z} = \begin{cases} = 0, & \text{if } \beta < \underline{\beta}, \\ \text{is defined by } U^*(0) = U^*(\tilde{z}), & \text{if } \beta \in [\underline{\beta}, \bar{\beta}] \\ = 1, & \text{if } \beta > \bar{\beta} \end{cases} \tag{17}$$

Let us now derive the conditions for the equilibrium described in Proposition 3 to exist. Conjecture an equilibrium in which the strategic regulator passes the bank with a risky loan if and only if its credit quality is high. In this equilibrium, the market's posterior beliefs about the regulator's type given the bank's first period stress test results are given by Eq. 7. This is an equilibrium if and only if $G_g > 0 > G_b$.

Consider first G_g. There exists $\tilde{z}_g$ such that $U^*(\frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1+\alpha(1-z_1)}) > U^*(0)$ if and only if $z_1 < \tilde{z}_g$, where $\tilde{z}_g < \tilde{z}$ is defined by

$$\tilde{z}_g = \begin{cases} = 0, & \text{if } \beta < \underline{\beta} \\ \text{is defined by } U^*(0) = U^*(\frac{[\alpha+(1-\alpha)(1-d)]\tilde{z}_g}{[\alpha+(1-\alpha)(1-d)]\tilde{z}_g+\alpha(1-\tilde{z}_g)}), & \text{if } \beta \in [\underline{\beta}, \bar{\beta}] \\ = 1, & \text{if } \beta > \bar{\beta} \end{cases} \tag{18}$$

This implies that $G_g > 0$ for all $z_1 \leq \tilde{z}_g$. For all $z_1 > \tilde{z}_g$, there exists $\bar{\delta}_g(z_1)$, such that $G_g > 0$ if and only if

$\delta < \bar{\delta}_g(z_1)$, where $\bar{\delta}_g(z_1)$ is defined by

$$C + \bar{\delta}_g(z_1)\left[U^*(\frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1+\alpha(1-z_1)}) - U^*(0)\right] = 0 \tag{19}$$

Notice that $\bar{\delta}_g(z_1)$ as defined above exists for $z_1 > \tilde{z}_g$ and is decreasing in z_1. Moreover, $\bar{\delta}_g(z_1) \to \infty$ as z_1 approaches $\tilde{z}_g$ from above. For completeness, let us define $\bar{\delta}_g(z_1) = \infty$ for $z_1 \leq \tilde{z}_g$.

Consider next G_b. There exists $\tilde{z}_b$ such that $(1-d)U^*(\frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1+\alpha(1-z_1)}) < U^*(0)$ if and only if $z_1 > \tilde{z}_b$, where $\tilde{z}_b \leq \tilde{z}_g$ is defined by

$$\tilde{z}_b = \begin{cases} = 0, & \text{if } \beta < \underline{\beta} \\ \text{is defined by } U^*(0) = (1-d)U^*(\frac{[\alpha+(1-\alpha)(1-d)]\tilde{z}_b}{[\alpha+(1-\alpha)(1-d)]\tilde{z}_b+\alpha(1-\tilde{z}_b)}), & \text{if } \beta < \bar{\beta} \\ = 1, & \text{if } \beta \geq \bar{\beta} \end{cases} \tag{20}$$

This implies that $G_b < 0$ for all $z_1 \geq \tilde{z}_b$.

For all $z_1 < \tilde{z}_b$, then there exists $\bar{\delta}_b(z_1)$, such that $G_b < 0$ if and only if $\delta < \bar{\delta}_b(z_1)$, where $\bar{\delta}_b(z_1)$ is defined by

$$C - dD + \bar{\delta}_b(z_1)\left[(1-d)U^*(\frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1+\alpha(1-z_1)}) - U^*(0)\right] = 0 \tag{21}$$

Notice that $\bar{\delta}_b(z_1)$ as defined above exists for $z_1 < \tilde{z}_b$. Moreover, $\bar{\delta}_b(z_1) \to \infty$ as z_1 approaches $\tilde{z}_b$ from below. For completeness, let us define $\bar{\delta}_b(z_1) = \infty$ for $z_1 \geq \tilde{z}_b$.

To summarize, the equilibrium described in Proposition 3 exists if and only if $\delta \leq \min\{\bar{\delta}_g(z_1), \bar{\delta}_b(z_1)\}$, where $\bar{\delta}_g(z_1)$ and $\bar{\delta}_b(z_1)$ are defined by Eq. 19 and Eq. 21, respectively.

8.4 Proof of Proposition 4

This proof follows similar logical steps as the proof of Proposition 3. Consider first the strategic regulator facing a bank with a bad risky loan. In this case, the regulator's incentive to pass the bank is given by G_b, where z_2^R and z_2^f are given by Eq. 8. The regulator passes the bank with probability $\pi_b^* \in (0,1]$ if and only if $G_b \geq 0$. This is the case if and only if $\delta \geq \underline{\delta}_b(z_1)$, where $\underline{\delta}_b(z_1)$ is defined by

$$\begin{aligned} C - dD + \underline{\delta}_b(z_1) \max_{\pi_b \in [0,1]} \left[(1-d)U^*(z_2^R) - U^*(0)\right] = 0 \\ \text{where} \quad z_2^R = \frac{[\alpha+(1-\alpha)(1-d)]z_1}{[\alpha+(1-\alpha)(1-d)]z_1 + [\alpha+(1-\alpha)(1-d)\pi_b](1-z_1)} \end{aligned} \tag{22}$$

Notice that $\underline{\delta}_b(z_1)$ as defined above only exists for $\beta > \underline{\beta}$ and for z_1 such that

$$\max_{\pi_b \in [0,1]} \left[(1-d)U^*\left(\frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + [\alpha + (1-\alpha)(1-d)\pi_b](1-z_1)}\right) - U^*(0) \right] < 0 \tag{23}$$

For completeness, let us define $\underline{\delta}_b(z_1) = \infty$ for all other cases.

Consider next the strategic regulator facing a bank with a good risky loan. Since $G_b \geq 0$ implies that $G_g > 0$ for z_2^R define by Eq. 8, the regulator passes the bank with a good risky loan with certainty.

To summarize, an equilibrium described in Proposition 4 exists if and only if $\beta > \underline{\beta}$ and $\delta \geq \underline{\delta}_b(z_1)$, where β is defined by Eq. 5 and $\underline{\delta}_b(z_1)$ is defined by Eq. 22.

8.5 Proof of Proposition 5

This proof follows similar logical steps as the proof of Proposition 3. Consider first the strategic regulator facing a bank with a good risky loan. In this case, the regulator's incentive to pass the bank is given by G_g, where z_2^R and z_2^f are given by Eq. 9. Notice that we focus on the equilibria with $z_2^f = 0$ even in the case $\pi_g^* = 0$, for which z_2^f is not defined by the Bayes rule. This is consistent as the limit as $\pi_g^* \to 0$, since only the strategic regulator would fail a bank.[21]

The regulator fails the bank with probability $\pi_g^* \in [0, 1)$ if and only if $G_g \leq 0$. Notice that this implies that $\beta < \bar{\beta}$ and $z_1 > \tilde{z}_g$, where $\tilde{z}_g$ is defined by Eq. 18. Further, G_g is strictly increasing in π_g for all $(\pi_g, 0)$ such that $G_g \leq 0$. Therefore there exists $\pi_g^* < 1$ if and only if $G_g \leq 0$ for $\pi_g = 0$. This is the case if and only if $\beta < \bar{\beta}$ and $\delta \geq \bar{\delta}_g(1)$, where $\bar{\delta}_g(z_1)$ is defined by Eq. 19.

Consider next the regulator facing a bank with a bad risky loan. Since $G_g \leq 0$ implies that $G_b < 0$ for z_2^p defined by Eq. 9, the regulator fails the bank with a bad risky loan with certainty.

To summarize, the equilibrium described in Proposition 3 exists if and only if $\beta < \bar{\beta}$ and $\delta \geq \bar{\delta}_g(1)$, where $\bar{\delta}_g(z_1)$ is defined by Eq. 19.

8.6 Proof of Proposition 6

Suppose by way of contradiction that an equilibrium exists in which $\pi_g < 1$ and $\pi_b > 0$. This is an equilibrium if $G_g \leq 0$ and $G_b \geq 0$. However, Eq. 6 implies that $G_g > G_b$ in any equilibrium, a contradiction.

[21] In addition to the equilibria characterized in this proposition, there might exist equilibria in which $\pi_g = 0$ and $z_2^f \in (0, 1]$. However, if we endogenize the lenient type regulator as a strategic player who faces a high social cost of capital $\bar{C}$, as described in Footnote 19, then applying the D1 refinement of Cho and Kreps (1987) allows us to prune all equilibrium in which $z_2^f > 0$. This is because the conditions given in Footnote 19 ensures that the net gain from passing a bank is strictly negative for all beliefs held by the market regardless of the quality of the bank's risky loan. The D1 refinement thus requires that $z_2^f = 0$ in any equilibrium in which $\pi_g = 0$, because the set of beliefs for which the strategic type regulator is willing to deviate to failing pass the bank with a good risky loan is strictly larger than that for which the lenient type regulator is willing to do so, where the latter is empty.

8.7 Proof of Proposition 7

Let $V^A(z_1)$, $V^B(z_1)$ and $V^C(z_1)$ denote the expected surplus for the strategic regulator for the two periods given that the bank in the first period extends a risky loan, in the equilibria described in Propositions 3, 4 and 5, respectively, assuming the equilibria exist. Let $\underline{V}$ denote the expected surplus for the strategic regulator for the two banks given that the bank in the first period extends a risky loan, given that the regulator fails the first period bank with certainty (i.e. $\pi_g = \pi_b = 0$). $\underline{V}$ is given by

$$\underline{V}(z_1) = [\alpha + (1-\alpha)(1-d)]R - C + \delta U^*(0)$$

In an equilibrium in which the regulator's stress testing strategy is (π_g^*, π_b^*), the expected surplus for the strategic regulator for the bank across both periods given that the first period bank extends a risky loan is given by

$$V(z_1) = \underline{V}(z_1) + \left[\alpha\pi_g^* G_g + (1-\alpha)\pi_b^* G_b\right]$$

where G_{q_1} is given by Eq. 6. Let $G_{q_1}^A$, $G_{q_1}^B$ and $G_{q_1}^C$ denote the value of G_{q_1} in the equilibria described in Propositions 3, 4 and 5 respectively, assuming the equilibria exist.

By the proofs of Proposition 3–5, G_{q_1} satisfies the following properties in each equilibrium:

Equilibrium C	$G_g^C \leq 0$	$G_b^C < 0$
Equilibrium A	$G_g^A > 0$	$G_b^A < 0$
Equilibrium B	$G_g^B > 0$	$G_b^B \geq 0$

This and the fact that $G_g = G_b + dD$ (see Eq. 6) implies that $G_{q_1}^C < G_{q_1}^A < G_{q_1}^B$.

Let (π_g^A, π_b^A), (π_g^B, π_b^B) and (π_g^C, π_b^C) denote the strategic regulator's first period stress testing strategy for the bank in the equilibria described in Propositions 3, 4 and 5 respectively, assuming the equilibria exist. We can now rank the expected surplus for the strategic regulator in the different equilibria. First, consider Equilibrium C, in which $\pi_g^C < 1$, $\pi_b^C = 0$ and $G_g^C \leq 0$.

$$V^C(z_1) = \underline{V}(z_1)$$

This follows because in equilibrium, either $G_g^C = 0$, or $G_g^C < 0$ and $\pi_g^* = 0$.

Next, consider Equilibrium A, in which $\pi_g^A = 1$, $\pi_b^A = 0$, and $G_g^A > 0$.

$$V^A(z_1) = \underline{V}(z_1) + \alpha G_g^A$$

Clearly $V^A(z_1) > V^C(z_1)$. Therefore Equilibrium C is strictly dominated by Equilibrium A, whenever they coexist.

Finally, consider Equilibrium B, in which $\pi_g^B = 1$, $\pi_b^B > 0$ and $G_g^B > G_b^B \geq 0$.

$$V^B(z_1) = \underline{V}(z_1) + \left[\alpha G_g^B + (1-\alpha)\pi_b^B G_b^B\right] \geq \underline{V}(z_1) + \alpha G_g^B$$

It follows that $V^B(z_1) > V^0(z_1) + \alpha G_g^A = V^A(z_1)$, because $G_g^B > G_g^A$ as argued above. That is, Equilibrium B strictly dominates Equilibrium A, whenever they coexist. This implies that Equilibrium B also strictly dominates Equilibrium C, whenever they coexist.

8.8 Proof of Corollary 1

Let $V^P(z_1)$ denote the expected surplus for the strategic regulator for the two banks given that the first period bank extends a risky loan and let $G_{q_1}^P$ denote the value of G_{q_1} in an economy in which the regulator does not conduct stress tests for the first period bank.

We first derive the condition for the equilibrium described in this corollary to exist. An uninformative equilibrium exists if and only if $G_g > G_b \geq 0$ for $(\pi_g, \pi_b) = (1, 1)$. This is the case if and only if $z < \tilde{z}$ (which implies $\beta > \underline{\beta}$) and $\delta \geq \tilde{\delta}_b(z_1)$, where $\tilde{z}$ is defined by Eq. 17 and $\tilde{\delta}_b(z_1)$ is defined by

$$C - dD + \tilde{\delta}_b(z_1)\left[(1-d)U^*(z_1) - U^*(0)\right] = 0 \tag{24}$$

If this equilibrium exists, we now show that it dominates any other equilibrium (if exists) for the strategic regulator. $V^P(z_1)$ and $G_{q_1}^P$ coincides with the equilibrium quantities in the uninformative equilibrium. Proposition 7 implies that $V^P(z_1) > V^A(z_1), V^C(z_1)$. We now compare the passive equilibrium to other potential Equilibrium B. In any other Equilibrium B, if exists, $G_b^B = 0 \leq G_b^P$. Therefore $V^P(z_1) \geq V^B(z_1)$ for any other Equilibrium B.

8.9 Proof of Proposition 8

Notice that $U^*(z_2)$ is independent of D. It then follows from Eq. 21 that $\bar{\delta}_b(z_1)$ is increasing in D, because the LHS of Eq. 21 is decreasing in D and increasing in δ.

Similarly, it follows from Eq. 22 that $\underline{\delta}_b(z_1)$ is increasing in D, because the LHS of Eq. 22 is decreasing in D and increasing in δ.

8.10 Proof of Proposition 9

Let s_t denote the market's ex ante belief in period $t \in \{1, 2\}$ that the regulator is of the strategic type. That is, a higher s_t reflects a market belief that the regulator is likely to be more lenient lenient.

Following backward induction, we first solve for the equilibrium in the second period. Analogous to Proposition 1, the equilibrium in the second period is characterized in the following lemma.

Lemma 1. *In the second period, there exists a unique equilibrium in which the bank expends loan origination effort $e_2^*(s_2)$ at stage 1. If the bank extends a risky loan, at stage 2, the tough type regulator fails the bank with certainty and the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

The bank's loan origination effort in equilibrium $e^*(s_2)$ is increasing in the market's belief that the regulator is of the strategic (relatively lenient) type, s_2, where $e^*(s_2)$ is given by

$$e^*(s_2) = \frac{1}{k}\left(\underbrace{[\alpha + (1-\alpha)(1-d)]\,R - R_0}_{NPVeffect} - \underbrace{\beta\,[\alpha(1-s_2)\,(R-1) + (1-\alpha)\,((1-d)R-1)]}_{dilutioncost}\right) \tag{25}$$

Proof. Because the game ends after the bank's stress test in the second period, the regulator has no reputational incentives. The stress test strategy of the strategic regulator at stage 2 depends on the quality of the bank's risky loan $q_2 \in \{g, b\}$, as described in this lemma.

At stage 3, given the stress test result, the bank raises capital if it fails the stress test, and rolls over its maturing debt. Given the regulator's stress testing strategy described in the lemma, the market's posterior beliefs that the bank's risky loan is good, given that the bank passes and fails the stress test, respectively, are given by

$$\alpha_2^p = 1, \qquad \alpha_2^f = \frac{\alpha(1-s_2)}{\alpha(1-s_2) + (1-\alpha)} \tag{26}$$

The promised repayment when the bank rolls over its maturing debt at stage 3 depends on the stress test results and the market's posterior belief about the bank's type. First, if a bank fails the stress test, it is required by the regulator to raise 1 unit of capital at $t = 3$. The capital allows the bank to roll over its maturing debt with a promised repayment of $\tilde{R}_2^f = 1$. Because of the capital providers' bargaining power, the bank's expected payoff captures only a fraction $(1-\beta)$ of the surplus. Second, only a bank with a good risky loan passes the stress test. It then rolls over its maturing debt at stage 3 by promising a repayment $\tilde{R}_2^p(\alpha_2^p)$, given by Eq. 1.

At stage 0, the bank's chooses its screening effort, taking as given the rollover repayments $\tilde{R}_2^p$ and $\tilde{R}_2^f$ in

case it passes and fails the stress test. That is,

$$\max_{e_2} \quad \underbrace{e_2 \left(\overbrace{\alpha s_2 \left(R - \tilde{R}_2^p\right)}^{\text{pass}} + \overbrace{(1-\beta)\left[\alpha(1-s_2)\left(R - \tilde{R}_2^f\right) + (1-\alpha)\left((1-d)R - \tilde{R}_2^f\right)\right]}^{\textbf{fail}} \right)}_{\text{risky loan}}$$

$$+ \underbrace{(1-e_2)(R_0 - 1)}_{\text{safe asset}} - \underbrace{\frac{1}{2}ke_2^2}_{\text{effort cost}} \tag{27}$$

The first order condition that characterizes the bank's effort choice is

$$\alpha s_2 \left(R - \tilde{R}_2^p\right) + (1-\beta)\left[\alpha(1-s_2)\left(R - \tilde{R}_2^f\right) + (1-\alpha)\left((1-d)R - \tilde{R}_2^f\right)\right] - (R_0 - 1) - ke_2 = 0 \tag{28}$$

After substituting in $\tilde{R}_2^p(\alpha_2^p)$ given by Eq. 1, the bank's optimal screening choice in equilibrium is given by Eq. 25.

An equilibrium with $e^*(s_2) \in (0,1)$ exists because the LHS of Eq. 28 is strictly positive for $e_2 = 0$ (by Assumption 4), and is strictly negative for $e_2 = 1$ (by Assumption 2). The equilibrium is also unique because the LHS of Eq. 28 is strictly decreasing in e_2.

It follows that the equilibrium screening effort is increasing in the regulator's reputation s_2. For all $e_2^*(s_2) > 0$, we have

$$\frac{\partial e_2^*(s_2)}{\partial s_2} = \frac{\beta\alpha(R-1)}{k} > 0 \tag{29}$$

∎

Recall that the strategic regulator's expected surplus from the bank in the second period $U(e_2)$ is given by Eq. **??**. We have the following lemma, analogous to Proposition 2.

Lemma 2. *The expected surplus from the bank in the second period for the strategic regulator in equilibrium, $U^*(s_2)$, is strictly increasing in the probability that the regulator is of the strategic (relatively lenient) type s_2 for $s_2 < \hat{s}$, and decreasing in s_2 for $s_2 \geq s_2$. $\hat{s} \in [0,1)$ is defined by*

$$\hat{s} = \begin{cases} 0, & \text{if } \beta \leq \underline{\beta}_s, \\ \frac{\beta[\alpha(R-1)+(1-\alpha)((1-d)R-1)]-(1-\alpha)C}{\beta\alpha(R-1)}, & \text{if } \beta \in (\underline{\beta}_s, \underline{\beta}) \\ 1, & \text{if } \beta \geq \underline{\beta} \end{cases} \tag{30}$$

where $\underline{\beta}$ is given by Eq. 5 and $\underline{\beta}_s < \underline{\beta}$ is defined by

$$\underline{\beta}_s \left[\alpha(R-1) + (1-\alpha)\left((1-d)R - 1\right)\right] = (1-\alpha)C \tag{31}$$

Proof. $U(e_2)$ is increasing in e_2 if and only if $e_2 \leq \hat{e}$, where $\hat{e} \in (0,1)$ is defined by Eq. 13.

Recall that the bank's equilibrium screening effort $e_2^*(s_2)$ is increasing in s_2, where

$$e_2^*(0) = \frac{1}{k}\left(\left[\alpha R + (1-\alpha)(1-d)\right]R - R_0 - \beta\left[\alpha(R-1) + (1-\alpha)\left((1-d)R-1\right)\right]\right) \tag{32}$$

Notice that $e_2^*(0) \geq \hat{e}$ if and only if $\beta \leq \underline{\beta}_s$. Therefore if $\beta \leq \underline{\beta}_s$, then $U(s_2)$ is decreasing for all s_2. That is, $\hat{s} = 0$. Moreover,

$$e_2^*(1) = \frac{1}{k}\left(\left[\alpha R + (1-\alpha)(1-d)\right]R - R_0 - \beta(1-\alpha)\left[(1-d)R - 1\right]\right) \tag{33}$$

Notice that $e_2^*(1) \leq \hat{e}$ if and only if $\beta \geq \underline{\beta}$, where $\underline{\beta}$ is given by Eq. 5. Therefore if $\beta \geq \underline{\beta}_s$, then $U(s_2)$ is increasing for all s_2. That is, $\hat{s} = 1$. Finally, $\beta \in (\underline{\beta}_s, \underline{\beta})$, $\hat{s}$ is defined by $e^*(\hat{s}) = \hat{e}$, which is equal to the expression given in this lemma. ∎

Next, we analyze the equilibrium stress testing strategy of the regulator for the bank in the first period, given the equilibrium in the second period.

Let us consider the incentives of the strategic regulator to pass the first period bank. Let G_{q_1} denote the net gain of passing the bank relative to failing the bank, given the quality of the bank's risky loan $q_1 \in \{g, b\}$.

$$G_g = C + \delta\left[U^*(s_2^p) - U^*(s_2^R)\right] \tag{34}$$

$$G_b = (C - dD) + \delta\left[(1-d)U^*(s_2^p) - (1-d)U^*(s_2^R) - dU^*(s_2^0)\right] \tag{35}$$

Analogous to Eq. 6, the first term in Eq. 35 represents the net gain in terms of the expected surplus from the bank in the first period and the second term represents the reputation concern in terms of the expected surplus from the bank in the second period. In contrast to the baseline setup, passing the first period bank reveals that the regulator is strategic, i.e. $s_2^p = 1$. Subsequently, the bank does not default in the first period and thus continues to the second period with probability 1 if its risky loan is good, or with probability $(1-d)$ if its risky loan is bad. The terms s_2^R and s_2^0 are the posterior beliefs held by the market about the probability that the regulator is strategic, given that the first period bank fails the stress test and given the realized payoff R and 0, respectively. Since the bank recapitalizes after failing the stress test, it continues to

the second period regardless of the realized payoff.

We first characterize the equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period. Analogous to Proposition 3, we have

Lemma 3. *The equilibrium stress testing strategy of the strategic regulator in the second period described in Lemma 1 is an equilibrium strategy in the first period. That is, in the first period, if the bank extends a risky loan, at stage 2, the strategic regulator passes the bank if and only if the risky loan is good.*

There exists thresholds $\bar{\delta}_g(s_1)$ and $\bar{\delta}_b(s_1)$, such that the necessary and sufficient conditions for this equilibrium to exist is $\delta < \min\left\{\bar{\delta}_g(s_1), \bar{\delta}_b(s_1)\right\}$.

Proof. As in the proof of Proposition 3, we first define parameter values such that $U^*(0) > U^*(1)$.

$$\begin{aligned} U^*(1) - U^*(0) &= \left[e_2^*(1) - e_2^*(0)\right]\left(\left[\alpha + (1-\alpha)(1-d)\right]R - R_0 - (1-\alpha)C\right) - \frac{1}{2}k\left[\left(e_2^*(1)\right)^2 - \left(e_2^*(0)\right)^2\right] \\ &= \frac{1}{k}\beta\alpha(R-1)\left(\left[\alpha + (1-\alpha)(1-d)\right]R - R_0 - (1-\alpha)C\right) \\ &\quad + \frac{1}{k}\beta\alpha(R-1)\left(\left[\alpha + (1-\alpha)(1-d)\right]R - R_0 - \beta\alpha(R-1) - \frac{1}{2}\beta(1-\alpha)\left[(1-d)R - 1\right]\right) \end{aligned} \tag{36}$$

Therefore $U^*(0) > U^*(1)$ if and only if $\beta < \bar{\beta}_s$, where $\bar{\beta}_s \in (\underline{\beta}_s, \underline{\beta})$ is defined by

$$\bar{\beta}_s\left[\alpha(R-1) + \frac{1}{2}(1-\alpha)\left[(1-d)R - 1\right)\right] = (1-\alpha)C \tag{37}$$

where $\underline{\beta}_s$ and $\underline{\beta}$ are defined by Eq. 31 and Eq. 5 respectively.

We now derive the conditions for this equilibrium to exist. In such an equilibrium, the posterior probabilities that the regulator is of the strategic type given a pass result of the bank's stress test and the realized payoff of the bank in the first period are given by

$$s_2^p = 1, \qquad s_2^R = \frac{(1-\alpha)(1-d)s_1}{(1-\alpha)(1-d) + \alpha(1-s_1)}, \qquad s_2^0 = s_1 \tag{38}$$

In particular, since only a strategic (relatively lenient) regulator would fail a bank, the market associates a belief of $s_2^p = 1$ with the passing of the first bank. If the first bank fails the stress test, the market updates its beliefs to $s_2^R < s_1$ if the bank realizes a payoff of R, reflecting the fact that the strategic (relatively lenient) regulator is less likely to fail a good bank than the tough regulator, or to $s_2^0 = s_1$ if the bank realizes a payoff of 0, reflecting the fact that both types of the regulator fails a bad bank with certainty.

In such an equilibrium, $G_g > 0 > G_b$. Consider first G_g. $G_g > 0$ if and only if $\delta < \bar{\delta}_g(s_1)$, where $\bar{\delta}_g(s_1)$

is defined by

$$C + \bar{\delta}_g(s_1)\left[U^*(1) - U^*(\frac{(1-\alpha)(1-d)s_1}{(1-\alpha)(1-d)+\alpha(1-s_1)})\right] = 0 \tag{39}$$

Notice that $\bar{\beta}_g(s_1)$ as defined above only exists for $\beta < \underline{\beta}$ and for s_1 such that

$$U^*(1) - U^*(\frac{(1-\alpha)(1-d)s_1}{(1-\alpha)(1-d)+\alpha(1-s_1)}) < 0 \tag{40}$$

For completeness, let us define $\bar{\delta}_g(s_1) = \infty$ for all other cases.

Consider next G_b. $G_b < 0$ if and only if $\delta < \bar{\delta}_b(s_1)$, where $\bar{\delta}_b(s_1)$ is defined by

$$(C - dD) + \bar{\delta}_b(s_1)\left[(1-d)U^*(1) - (1-d)U^*(\frac{(1-\alpha)(1-d)s_1}{(1-\alpha)(1-d)+\alpha(1-s_1)}) - dU^*(s_1)\right] = 0 \tag{41}$$

Notice that $\bar{\delta}_b(s_1)$ as defined above only exists for s_1 such that

$$(1-d)U^*(1) - (1-d)U^*(\frac{(1-\alpha)(1-d)s_1}{(1-\alpha)(1-d)+\alpha(1-s_1)}) - dU^*(s_1) > 0 \tag{42}$$

For completeness, let us define $\bar{\delta}_b(s_1) = \infty$ for all other cases.

To summarize, the equilibrium described in Lemma 3 exists if and only if $\min\{\bar{\delta}_g(s_1), \bar{\delta}_b(s_1)\}$, where $\bar{\delta}_g(s_1)$ and $\bar{\delta}_b(s_1)$ are defined by Eq. 39 and Eq. 41, respectively. ∎

Next, we characterize the equilibrium with reputation building to incentivize lending. Analogous to Proposition 4, we have

Lemma 4. *There is an equilibrium in which the strategic regulator passes the bank in the first period with positive probability even when the bank's risky loan is bad. Specifically, when the bank extends a risky loan in the first period, the regulator passes the bank with certainty if the loan is good, and passes the bank with positive probability $\pi_b^* > 0$ if the loan is bad.*

There exists $\underline{\delta}_b(s_1)$, such that the necessary and sufficient condition for this equilibrium to exist is $\delta \geq \underline{\delta}_b(s_1)$.

Proof. In such an equilibrium, the posterior probabilities that the regulator is of the strategic type given a the result of the bank's stress test and the realized payoff of the bank in the first period are given by

$$s_2^p = 1, \qquad s_2^R = \frac{(1-\alpha)(1-d)(1-\pi_b)s_1}{(1-\alpha)(1-d)\left[(1-\pi_b)s_1 + (1-s_1)\right] + \alpha(1-s_1)}, \qquad s_2^0 = \frac{(1-\pi_b)s_1}{(1-\pi_b)s_1 + (1-s_1)} \tag{43}$$

In such an equilibrium, $G_g > 0$ and $G_b \geq 0$. Consider first G_b. $G_b \geq 0$ if and only if $\delta \geq \underline{\delta}_b(s_1)$, where $\underline{\delta}_b(s_1)$ is defined by

$$(C - dD) + \underline{\delta}(s_1) \max_{\pi_b \in [0,1]} \left[(1-d)U^*(1) - (1-d)U^*(s_2^R) - dU^*(s_2^0)\right] = 0$$
$$\text{where } z_2^R \text{ and } z_2^0 \text{ are given by Eq. 43} \tag{44}$$

Notice that $\underline{\delta}_b(s_1)$ as defined above only exists for s_1 such that

$$(1-d)U^*(1) - (1-d)U^*(s_2^R) - dU^*(s_2^0) > 0, \quad \text{where } z_2^R \text{ and } z_2^0 \text{ are given by Eq. 43} \tag{45}$$

For completeness, let us define $\underline{\delta}_b(s_1) = \infty$ for all other cases.

Consider next G_g. Since $G_b \geq 0$ implies that $G_g > 0$ for s_2^R defined by Eq. 43, the regulator passes the bank with a good risky loan with certainty.

To summarize, an equilibrium described in Lemma 4 exists if and only if $\delta \geq \underline{\delta}_b(s_1)$, where $\underline{\delta}_b(s_1)$ is defined by Eq. 44. ∎

We then characterize the equilibrium with reputation building to reduce excessive risk-taking. Analogous to Proposition 5, we have

Lemma 5. *There is an equilibrium in which the strategic regulator fails the bank in the first period with positive probability even when the bank's risky loan is good. Specifically, when the bank extends a risky loan in the first period, strategic regulator fails the bank with certainty if the loan is bad, and passes the bank with probability $\pi_g^* < 1$ if the loan is good.*

There exists $\bar{\delta}_g(s_1)$, such that the necessary and sufficient condition for this equilibrium to exist is $\beta < \underline{\beta}$ and $\delta \geq \underline{\delta}(s_1)$.

Proof. In such an equilibrium, the posterior probabilities that the regulator is of the strategic type given a the result of the bank's stress test and the realized payoff of the bank in the first period are given by

$$s_2^p = 1, \qquad s_2^R = \frac{[\alpha(1-\pi_g) + (1-\alpha)(1-d)]\, s_1}{[\alpha(1-\pi_g) + (1-\alpha)(1-d)]\, s_1 + [\alpha + (1-\alpha)(1-d)]\,(1-s_1)}, \qquad z_2^0 = s_1 \tag{46}$$

In such an equilibrium, $G_g \leq 0$ and $G_b < 0$. Consider first G_g. $G_g \leq 0$ if and only if $\delta \geq \underline{\delta}_g(s_1)$, where $\underline{\delta}_g(s_1)$ is defined by

$$C + \underline{\delta}(s_1) \max_{\pi_g \in [0,1]} \left[U^*(1) - U^*(s_2^R)\right] = 0, \quad \text{where } z_2^R \text{ is given by Eq. 46} \tag{47}$$

Notice that $\underline{\delta}_g(s_1)$ as defined above only exists for $\beta < \underline{\beta}$ and s_1 such that

$$U^*(1) - U^*(s_2^R) < 0 \quad \text{where } z_2^R \text{ is given by Eq. 46} \tag{48}$$

For completeness, let us defined $\underline{\delta}_b(s_1) = \infty$ for all other cases.

Consider next G_b. Since $G_g \leq 0$ implies that $G_b < 0$ for s_2^R and s_2^0 defined by Eq. 46, the regulator fails the bank with a bad risky loan with certainty.

To summarize, an equilibrium described in Lemma 5 exists if and only if $\beta < \underline{\beta}$ and $\delta \geq \underline{\delta}_g(s_1)$, where $\underline{\beta}$ is defined by Eq. 5 and $\underline{\delta}_g(s_1)$ is defined by Eq. 47. ∎

We have now chatacterized all three equilibria described in Proposition 9. Finally, we show that no other equilibria exist. Suppose there exists an equilibrium in which $\pi_g < 1$ and $\pi_b > 0$. $\pi_g < 1$ implies that $G_g \leq 0$, i.e. $U^*(s_2^p) < U^*(s_2^R)$. This then implies that $G_b < 0$, which implies that $p_b = 0$, a contradiction. Therefore no other equilibria exist.