Loan Insurance, Adverse Selection and Screening*

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Abstract

We examine insurance against loan default when lenders can screen in primary markets at a heterogeneous cost and learn loan quality over time. In equilibrium, low-cost lenders screen loans but some high-cost lenders insure them. Insured loans are risk-free and liquid in a secondary market, while uninsured loans are subject to adverse selection. Loan insurance reduces the amount of lemons traded in the secondary market for uninsured loans and improves liquidity and welfare. This pecuniary externality implies insufficient loan insurance in the liquid equilibrium. To achieve constrained efficiency, a regulator (i) guarantees a minimum price in the market for uninsured loans to eliminate a welfare-dominated illiquid equilibrium; and (ii) imposes Pigouvian subsidies on loan insurance in the liquid equilibrium to correct for the externality.

JEL classifications: G01, G21, G28.

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1 Introduction

Risk in credit markets is often assumed at loan origination by third parties for a fee. A typical example is loan default insurance that protects the loan owner against borrower default and is popular in mortgage markets around the world (Blood, 2001). Various guarantees and external credit enhancements provided by third parties at origination (e.g., letters of credit or bond insurance) have a similar function. Governments also offer guarantees against default for various types of loans, including mortgages, student loans, small business and export loans. A prominent example is mortgage insurance in the U.S.. In 2017, the federal government insured about 70% of newly originated residential mortgages through institutions such as the Federal Housing Administration, Fannie Mae, and Freddie Mac. Mortgages with government guarantees are traded in pools as agency mortgage-backed securities (MBS) and, in contrast to private-label MBS, maintained robust issuance and trading volumes and low spreads throughout the recent financial crisis (Vickery and Wright, 2013).

The widespread use of loan default insurance in credit markets leads to several important positive and normative questions. What is the impact of loan insurance on secondary market liquidity (allocative efficiency) and on lending standards in primary markets (productive efficiency)? And how do changes in loan characteristics, screening technology, or the liquidity risk of lenders affect the privately optimal amount of loan insurance? On the normative side, does loan insurance create a (pecuniary) externality that motivates a role for government intervention? If so, how do various regulatory interventions fare when evaluated against a welfare benchmark?

In this paper, we propose a model of lending where lenders have three options to reduce their exposure to borrower default risk. The first option is to screen borrowers at a cost. The second option is to wait in order to privately learn loan quality over time and then dump lemons in a secondary market at a depressed price because of adverse selection. Our contribution is to explore a third option – loan insurance at origination – and its interplay with the other options. Our model features a trade-off between productive efficiency – the quality of originated loans – and allocative efficiency – the final allocation of loan cash-flows. Loan insurance and screening are substitutes as they both improve allocative efficiency (reduced adverse selection) but more insurance reduces screening. Loan insurance creates a liquid secondary market for insured loans and increases liquidity in the market for uninsured loans. Due to this pecuniary externality, loan insurance is excessively low and a Pigouvian subsidy restores constrained efficiency when the uninsured loans market is liquid. To liquify an inefficiently frozen market, subsidies for purchases of uninsured loans are effective. Once the market is liquid, however, insurance subsidies dominate purchase subsidies.

In the model, there are three dates and two risk-neutral agents: lenders and deep-pocketed financiers. At an initial date, each lender has access to a pool of borrowers and chooses whether to screen at a cost that is heterogeneous across lenders. Screening identifies a high-quality borrower with low default probability and, thus, improves the probability of loan repayment. The amount of repayment is a reduced-form measure of competition in the lending market. Lenders also choose whether to insure the loan, which passes its idiosyncratic default risk to outside financiers at a competitive fee. At an interim date, all lenders privately learn the quality of their borrowers (Parlour and Plantin, 2008; Plantin, 2009). Lenders also learn whether they face a liquidity shock, such as a superior consumption or investment opportunity (or a bank run). Outside financiers do not observe the screening cost, screening choice, loan quality or liquidity shock. However, they observe whether a loan is insured. Thus, there exist separate secondary markets and the price in each is set for outside financiers break even in expectation. Because of asymmetric information between lenders and outside financiers, the secondary market for uninsured loans is subject to adverse selection. A lender sells either a performing loan because of the liquidity shock (realizing gains from trade) or a lemon (capitalizing on superior information).

We start with a benchmark without loan insurance. In equilibrium, the heterogeneous screening costs implies a threshold strategy, so only low-cost lenders choose to screen. We label an equilibrium as 'liquid' when trade in the secondary market of uninsured loans occurs. That is, lenders sell high-quality loans upon a liquidity shock. An illiquid equilibrium always exists, since a zero price and no trade of high-quality loans are mutually consistent. A liquid equilibrium exists for a sufficiently high liquidity shock. Screening in the liquid equilibrium is lower than in the illiquid equilibrium because the option to profitably sell lemons reduces screening incentives.

We next allow for loan insurance. Lenders can pass default risk to outside financiers at origination before the arrival of private information about loan quality. In equilibrium, low-cost lenders screen but never insure, while high-cost lenders do not screen but may insure. Loan insurance reveals a lender's choice of no screening and the competitive fee reflects the default cost of non-screened loans. Insured loans are always traded in a separate secondary market at a price that reflects their average quality. In sum, insured loans are free from default risk and adverse selection.

We characterize loan insurance in the liquid equilibrium and show that only some high-cost lenders insure. Insured high-costs lenders benefit from higher gains from trade upon a liquidity shock but lose the option to sell lemons absent a liquidity shock. In equilibrium, both effects equalize and high-cost lenders are indifferent about insurance. Loan insurance only reduces non-screened loans from the secondary market for uninsured loans and thus the amount of lemons in this market, which increases liquidity and reduces screening. Loan insurance occurs in equilibrium when the probability of loan default is low or competition in the lending market is high. For high competition or low default risk, the benefit of greater liquidity exceeds the cost of lower screening, so welfare increases (intensive margin). A welfare-dominant liquid equilibrium also exists for a smaller liquidity shock (extensive margin). Loan insurance eliminates part of the adverse selection and increases the price of uninsured loans up to a level consistent with high-cost lenders being indifferent about insurance.

We study the comparative statics of the liquid equilibrium. The fraction of high-cost lenders who insure increases in both the repayment probability of an average non-screened borrower and lending market competition. In both cases, the cost of insurance is lower. The fraction of high-cost lenders who insure is also non-monotone in the probability of the liquidity shock. We identify two reasons for this. First, a higher probability increases the proportion of liquidity sellers and directly reduces adverse selection. Second, a higher probability reduces screening and indirectly increases adverse selection. If the first effect dominates, insurance eliminates a lower amount of adverse selection, reducing the fraction of insured high-cost lenders.

We turn to the normative implications of loan insurance. We characterize the constrained-efficient allocation chosen by a planner who observes screening costs, chooses loan insurance for all lenders, and can select the equilibrium by guaranteeing a minimum price for uninsured loans. The planner is subject to lenders choosing screening and loan sales and outside financiers breaking even in secondary markets. In contrast to the competitive equilibrium, the planner internalizes the positive pecuniary externality of insurance, whereby insured loans trade in a separate secondary market and, therefore, reduce the amount of lemons in the market for uninsured loans. The planner chooses more loan insurance (intensive margin) and a positive amount of insurance for a larger set of parameters (extensive margin) in the liquid equilibrium. The planner also uses insurance to 'liquify' the market, creating a liquid equilibrium where the unique competitive equilibrium is illiquid. For some parameters, however, liquifying the market is feasible but the higher secondary market price reduces the screening incentives so severely that the planner prefers to keep the market frozen.

We consider a regulator with the same information as outside financiers and a balanced budget constraint. When the constrained-efficient allocation prescribes a liquid equilibrium, the regulator can promise a minimum price in the secondary market for uninsured loans to eliminate the illiquid equilibrium. This policy can be credibly implemented via a Pigouvian subsidy for outright purchases of uninsured loans.[1] Once the liquid equilibrium arises as the unique regulated equilibrium, we show that the constrained-efficient allocation can be implemented by a Pigouvian subsidy on loan insurance. It induces lenders to internalize the positive externality of their individual insurance choice on the secondary market price of uninsured loans. By contrast, the loan purchase subsidy fails to achieve constrained efficiency because it encourages the sale of lemons, while the net effect of the insurance subsidy is a reduction in the amount of lemons in the market for uninsured loans. When the constrained-efficient allocation prescribes an illiquid equilibrium, then all high-cost lenders fully insure loans, so there is no role for a Pigouvian subsidy for loan insurance.

Finally, we probe the robustness of our results by considering several generalizations and extensions of the model. In particular, we consider a general required return of outside financiers, a generalized screening technology, the option to partially insure a loan and the option to sell parts of an uninsured loan, as well as changes in the timing of the payment of the insurance fee. We show that our results extend to these cases and characterize the implications for the incentives to insure loans, screen at origination, and adverse selection in the secondary market for uninsured loans.

[1] This result was established in earlier work on the optimal intervention in frozen markets plagued by adverse selection, e.g. Tirole (2012), Philippon and Skreta (2012), and Chiu and Koeppl (2016).

Literature. Several papers highlight the trade-off between productive and allocative efficiency. Pennacchi (1988) and Gorton and Pennacchi (1995) show that a lender needs to retain sufficient risk exposure to borrowers to maintain monitoring incentives after loan sales. Parlour and Plantin (2008) study the interplay between liquidity in secondary loan markets plagued by adverse selection and the incentives of a relationship bank to monitor its borrower before loan sales. Vanasco (2017) studies the optimal risk retention by originating lenders when screening improves productive efficiency but the induced private information reduces secondary market liquidity. Daley et al. (2018) examine how credit ratings affect secondary market liquidity and screening incentives. We contribute to this literature by examining the positive and normative implications of loan insurance.

Perhaps closest in spirit is Parlour and Winton (2013), who analyse the impact of credit default swaps (CDS) as an alternative to loan sales in secondary markets. Both CDS and loan sales affect a lender's incentive to monitor its borrower but the lender retains control rights only with a CDS. There are two main differences to our approach. First, we consider the incentives to screen borrowers before loan sales as opposed to monitoring incentives after laying off credit risk. Second, whether a loan is insured is observable in our model and results in a separate secondary market, consistent with conforming mortgages sponsored by Freddie and Fannie, for example.

The remainder of the paper is organized as follows. Section 2 presents the model with loan insurance, screening, and adverse selection. We study a benchmark without loan insurance in section 3 and characterize the positive implications of loan insurance in section 4. Turning to normative implications in section 5, we derive the constrained-efficient allocation and evaluate Pigouvian subsidies for loan insurance and purchases of uninsured loans against this benchmark. Section 6 contains several extensions and generalizations. Section 7 concludes. All proofs are in the Appendix.

2 Model

There are three dates $t = 0, 1, 2$ and a single good for consumption and investment. Two groups of risk-neutral agents are protected by limited liability. Outside financiers are competitive, deep-pocketed at $t = 1, 2$, and require a return normalized to one.[2] A unit mass of lenders has one unit of funds each to make a loan at $t = 0$. Each lender has access to an individual pool of borrowers. Without screening, $s_i = 0$, lender i finds an average borrower and receives A (repayment) with probability $\mu \in (0, 1)$ or 0 (default). We interpret μ as a credit score, a public signal about the probability of loan repayment, and lower values of A as a more competitive lending market. The loan payoff is independently and identically distributed across lenders and publicly observable at $t = 2$. Screening, $s_i = 1$, improves the probability of repayment to $\psi \in (\mu, 1)$, as shown in Figure 1. We focus on $\psi \to 1$ in the main text.[3]

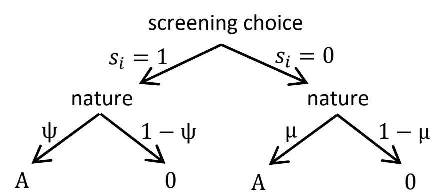

Figure 1: Screening and loan payoffs.

The non-pecuniary cost of screening, η_i, is distributed across lenders according to a density function $f(\eta) > 0$ with support $[0, \overline{\eta}]$ and cumulative distribution function $F(\eta)$. The cost and choice of screening, η_i and s_i, are private information to lender i.

At $t = 1$, lenders receive two sources of private information. First, each lender learns the loan payoff $(0, A)$. This assumption is consistent with lenders forming a relationship with their borrower and the notion of learning about an asset by holding it (Plantin, 2009). Second, lenders learn about an idiosyncratic liquidity shock, whereby

[2] We study a general required return of financiers in section 6.1.

[3] We study the general case of $\psi < 1$ in section 6.2.

their preference for interim consumption is $\lambda_i \in \{1, \lambda\}$ with $\lambda > 1$. The liquidity shock is independently and identically distributed across lenders, independent of the loan payoff, and arises with probability $Pr\{\lambda_i = \lambda\} \equiv \nu \in (0, 1)$. Thus, the preference of lender i is

$$u_i = \lambda_i c_{i1} + c_{i2} - \eta_i s_i, \tag{1}$$

where c_{it} is consumption of lender i at date t and $s_i \in \{0, 1\}$ is the screening choice.

At $t = 0$, each lender chooses whether to insure the loan against default, $\ell_i \in \{0, 1\}$. Without loss of generality, we focus on full insurance – the transferral of all default risk.[4] If a loan is insured, its idiosyncratic default risk passes to outside financiers. The insurance contract guarantees the payoff A to the loan owner for a competitive fee k. Both the insurance payoff and fee are charged at $t = 2$, resulting in a safe payoff $\pi = A - k$.[5] Whether a loan is insured is publicly observable at $t = 1$.

At $t = 1$, each lender can sell the loan in secondary markets to outside financiers. These potential buyers are uninformed about the screening cost and choice, liquidity shock, and loan quality but observe whether a loan is insured. Segmented markets for insured and uninsured loans may exist, with respective prices p_I and p_U and sales $q_i^I \in \{0, \ell_i\}$ (insured) and $q_i^U \in \{0, 1 - \ell_i\}$ (uninsured).[6] Figure 2 shows the timeline.

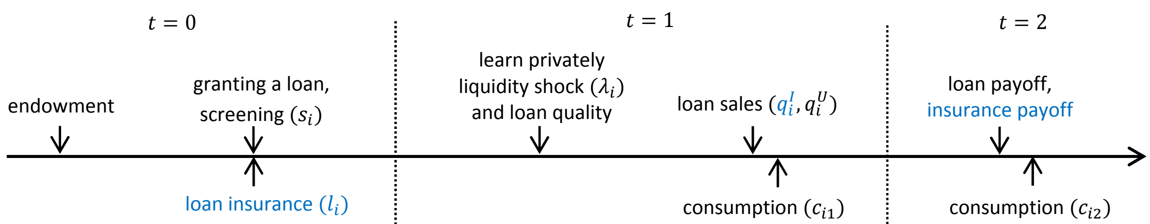

Figure 2: Timeline of events.

[4] We allow for partial insurance in section 6.3 and show that full insurance is optimal and efficient.

[5] This approach parallels the non-pecuniary screening cost and does not affect the lending volume. It is feasible because contracts can be written on the observable realization of the loan payoff at $t = 2$. For an extension with an insurance fee that must be paid upfront (at $t = 0$), see section 6.4.

[6] We allow for partial sales in section 6.5 and show that our results are qualitatively unchanged.

3 Equilibrium without loan insurance

We start with the benchmark without loan insurance. This setup corresponds to the timeline without the actions highlighted in blue in Figure 2.

Definition 1. *A symmetric pure-strategy equilibrium comprises screening choices* $\{s_i\}$*, sales* $\{q_i^U\}$*, and a secondary market price for uninsured loans* p_U *such that:*

1. *At* $t = 1$*, each lender* i *optimally chooses sales for each realized liquidity shock* $\lambda_i \in \{1, \lambda\}$ *denoted by* $q_i^U(s_i, \lambda_i)$*, given the price* p_U *and screening choice* s_i*.*

2. *At* $t = 1$*, the price* p_U *is set for outside financiers to break even in expectation, given the screening choices* $\{s_i\}$ *and sales schedules* $\{q_i^U(\cdot)\}$ *of all lenders.*

3. *At* $t = 0$*, each lender* i *chooses screening* s_i *to maximize expected utility, given the price* p_U *and the sales schedule* $q_i^U(\cdot)$*:*

$$\begin{aligned}
\max_{s_i, c_{i1}, c_{i2}} \quad & \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - s_i \eta_i\right] \qquad \textit{subject to} \\
c_{i1} \quad &= \quad q_i^U(s_i, \lambda_i)\, p_U \\
c_{i2} \quad &= \quad \left[1 - q_i^U(s_i, \lambda_i)\right] \times \begin{cases} A & \textit{with probability} \quad s_i + \mu(1 - s_i) \\ 0 & \qquad\qquad\qquad\quad (1 - \mu)(1 - s_i). \end{cases}
\end{aligned}$$

We exclude the unstable asymmetric equilibrium in which a fraction of a high-quality loans are traded in the secondary market. In the symmetric equilibrium, lenders use a threshold strategy (without loss of generality). Each lender with a screening cost below the threshold η chooses to screen, where $\mathbf{1}\{\cdot\}$ is the indicator function:

$$s_i = \mathbf{1}\Big\{\eta_i \leq \eta\Big\}. \tag{2}$$

If $\eta \leq 0$, no lender screens, $F(\eta) = 0$, while if $\eta \geq \bar{\eta}$, all lenders screen, $F(\eta) = 1$.

Sales in the secondary market for uninsured loans. Since there is asymmetric information between lenders and outside financiers at $t = 1$, all lenders choose to sell low-quality loans (worth 0). As a result, the participation constraint of financiers implies a price $p_U \in [0, A)$. Hence, lenders choose not to sell high-quality loans (worth A) when not hit by a liquidity shock. A defining feature of the equilibrium is whether lenders sell high-quality loans upon a liquidity shock:

$$p_U \lambda \geq A. \tag{3}$$

If condition (3) holds, the equilibrium is 'liquid', i.e. the equilibrium features a liquid secondary market for uninsured loans. In a liquid equilibrium, sales at $t = 1$ are:[7]

$$q_i^U(s_i, \lambda_i) = \mathbf{1}\Big\{\lambda_i = \lambda\Big\} + \mathbf{1}\Big\{\lambda_i = 1\Big\} \begin{cases} 0 & \text{with probability} \quad s_i + \mu(1 - s_i) \\ 1 & \qquad\qquad\qquad (1-\mu)(1-s_i). \end{cases} \tag{4}$$

Secondary-market price. In a liquid equilibrium, all lenders sell their loans upon a liquidity shock (liquidity sellers). Due to private learning about loan payoffs, a fraction $1-\mu$ of high-cost lenders (informed sellers) also sells low-quality loans ('lemons'). Thus, the break-even condition of outside financiers ensures that the price equals the value of high-quality loans sold by liquidity sellers, $\nu F(\eta)$ loans sold by high-cost lenders and $\nu\mu(1 - F(\eta))$ loans by low-cost lenders, divided by the total quantity of loans sold:

$$p_U = \nu A \frac{F(\eta) + \mu(1 - F(\eta))}{\nu + (1-\nu)(1-\mu)(1 - F(\eta))} \equiv p_U(\eta). \tag{5}$$

More screening leads to fewer aggregate investment in low-quality loans at $t = 0$, which reduces the degree of adverse selection at $t = 1$ and supports the price, $\frac{dp_U}{d\eta} > 0$.

[7] Similar to Parlour and Plantin (2008), the binary choice of loan sales and limited liability preclude signaling in this market. See section 6.5 for an analysis of partial loan sales.

Screening. The marginal lender is indifferent between screening and not screening. Screening allows a lender to identify a high-quality loan sold only after a liquidity shock, yielding $\nu\lambda p_U + (1-\nu)A - \eta$ in expectation. Not screening results in loan sales except for a high-quality loan without a liquidity shock, yielding $\nu\lambda p_U + (1-\nu)[\mu A + (1-\mu)p_U]$ in expectation. Equating both payoffs determines the screening cost threshold η:

$$\eta = (1-\nu)(1-\mu)(A - p_U) \equiv \eta(p_U). \tag{6}$$

Intuitively, a low-cost lender benefits from the higher payoff, $A - p_U$, only when there is no liquidity shock and when the loan is of low quality. A higher price (e.g., due to lower degree of adverse selection) reduces the benefit of screening, $\frac{d\eta}{dp_U} < 0$.

Figure 3 shows the construction of the liquid equilibrium.

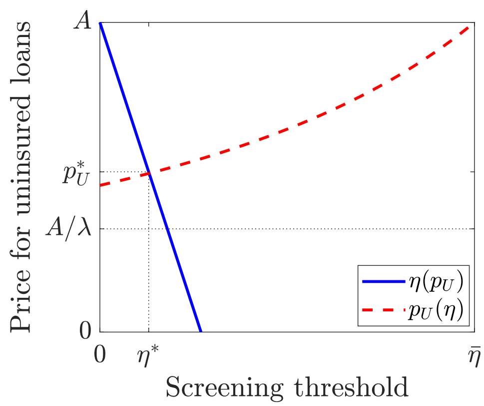

Figure 3: Existence of a unique equilibrium in the class of liquid equilibria. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\mu = 0.9$, $\lambda = 3$, $A = 3$.

Lemma 1. ***Liquid equilibrium without loan insurance.*** *If $\lambda \geq \underline{\lambda}_U$, then there exists a unique interior equilibrium in the class of equilibria in which lenders sell a high-quality loan after a liquidity shock. It is characterized by a secondary market price, $p_U^* \in \left[\frac{A}{\lambda}, A\right)$, and a cost threshold, $\eta^* \in (0, \bar{\eta})$, below which each lender screens.*

Proof. See Appendix A.1 (which also defines the bound $\underline{\lambda}_U$). ■

Lemma 2 summarizes the equilibrium without sales of high-quality loans.

Lemma 2. ***Illiquid equilibrium without loan insurance.*** *There always exists an illiquid equilibrium, $p_U^* = 0$. The screening threshold is $\eta^* = (1-\mu)A$. If $\lambda < \underline{\lambda}_U$, the illiquid equilibrium is the unique equilibrium without loan insurance.*

An equilibrium with illiquid market always exists. If the price is zero, lenders do not sell high-quality loans. Only low-quality loans are traded, which is consistent with the zero price.[8] The screening threshold is again obtained from the indifference condition of the marginal lender. With an illiquid market, loans are kept until maturity and the payoff from screening is $A - \eta$, and the payoff from not screening is μA. Equalizing these payoffs yields the stated threshold. All lenders screen if $\bar{\eta} \leq (1-\mu)A$.

Figure 4 shows the areas for which a liquid equilibrium exists.

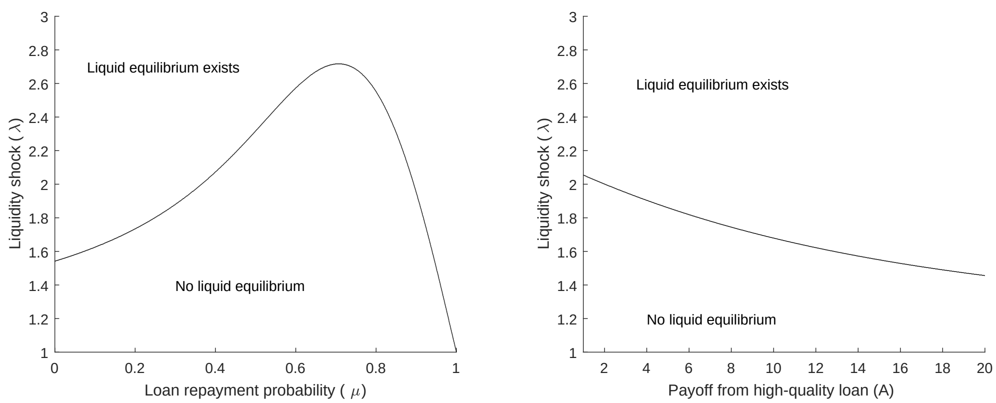

Figure 4: Existence of liquid equilibrium without loan insurance.
Parameters: $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, left panel: $A = 3$, right panel: $\mu = 0.9$.

[8]Imperfect screening, $\psi < 1$, ensures the existence of an illiquid equilibrium, $p_U = 0$. If screening were perfect and sufficiently cheap, $\bar{\eta} < (1-\mu)A$, then each lender would choose to screen, $F(\eta^*) = 1$, because $\eta^* = (1-\mu)A > \bar{\eta}$ and no lemons would be sold in the secondary market. Therefore, the competitive price would be $p_U = A$, contradicting the supposed illiquid equilibrium. As a result, no illiquid equilibrium would exist with perfect screening for parameters $\bar{\eta} < (1-\mu)A$. In the limit $\psi \to 1$, a positive but vanishing amount of lemons is sold in the secondary market at price $p_U^* = 0$.

4 Equilibrium with loan insurance

Having reviewed the benchmark case, we turn to the equilibrium with loan insurance. Insured loans are risk-free and, therefore, not subject to adverse selection in the secondary market. Whether a loan is insured is publicly observable at $t = 1$, so the secondary market for insured loans is separate from the market for uninsured loans.

Definition 2. *An equilibrium with loan insurance comprises choices of screening* $\{s_i\}$*, insurance* $\{\ell_i\}$*, sales of insured and uninsured loans* $\left\{q_i^I, q_i^U\right\}$*, secondary market prices* p_I *and* p_U*, and an insurance fee* k *such that:*

1. *At* $t = 1$*, each lender* i *optimally chooses sales of insured and uninsured loans for each realized liquidity shock* $\lambda_i \in \{1, \lambda\}$*, denoted by* $q_i^I(s_i, \lambda_i, \ell_i)$ *and* $q_i^U(s_i, \lambda_i, \ell_i)$*, given the prices* p_I *and* p_U *and choices of screening* s_i *and insurance* ℓ_i*.*

2. *At* $t = 1$*, prices* p_I *and* p_U *are set for outside financiers to expect to break even, given screening* $\{s_i\}$ *and insurance* $\{\ell_i\}$ *choices, the fee* k*, and sales* $\{q_i^I, q_i^U\}$*.*

3. *At* $t = 0$*, each lender* i *chooses its screening* s_i *and loan insurance* ℓ_i *to maximize expected utility, given prices* p_I *and* p_U*, the fee* k*, and sales* q_i^I *and* q_i^U*:*

$$\max_{s_i, \ell_i, c_{i1}, c_{i2}} \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - s_i \eta_i\right] \qquad \textit{subject to}$$
$$c_{i1} = q_i^U(s_i, \lambda_i, \ell_i)\, p_U + q_i^I(s_i, \lambda_i, \ell_i)\, p_I,$$
$$c_{i2} = \left[\ell_i - q_i^I\right](A - k) + \left[1 - \ell_i - q_i^U\right] \times \begin{cases} A & w.\ p. \quad s_i + \mu(1 - s_i) \\ 0 & \quad\quad\ (1 - \mu)(1 - s_i). \end{cases}$$

4. *At* $t = 0$*, the fee* k *is set for outside financiers to break even in expectation, given screening* $\{s_i\}$ *and insurance* $\{\ell_i\}$ *choices.*

Let m denote the fraction of insured loans among high-cost lenders, $\eta_i > \eta^*$.

Proposition 1. ***Loan insurance.*** *Low-cost lenders screen but never insure:* $s_i^* = 1$ *and* $\ell_i^* = 0$ *if* $\eta_i \leq \eta^*$*. Competitive loan insurance charges* $k^* = (1-\mu)A$*, so we have* $\pi^* = \mu A = p_I^*$*. In a liquid equilibrium, at most some high-cost lenders insure,* $m^* \in [0, 1)$*. In an illiquid equilibrium, all high-cost lenders insure,* $m^* = 1$*.*

Proof. See Appendix A.2. ■

Insurance converts the risky loan payoff to a risk-free payoff π independent of the unobserved screening choice. Since screening is costly, lenders who insure loans do not screen them. As a result, only non-screened loans may be insured and the competitive fee for them is $(1-\mu)A$, the expected cost of guaranteeing the loan. Whether a loan is insured is observable at $t = 1$, so the competitive price in the secondary market of insured loans equals its risk-free payoff at $t = 2$: $p_I^* = \pi^* = \mu A$.

There does not exist a liquid equilibrium in which all high-cost lenders insure, $m^* < 1$. If they did, $m = 1$, only high-quality loans are sold (the quantity of lemons sold in secondary markets vanishes for $\psi \to 1$), so the price for uninsured loans would be $p_U = A$. This price, however, would contradict the equilibrium condition for high-cost lenders preferring to insure. The payoff from insurance, $\kappa\mu A$, would be lower than the payoff from not insuring, which would be κA at the implied price, where $\kappa \equiv \nu\lambda + 1 - \nu > 1$ is the expected marginal utility of consumption at $t = 1$.

Therefore, in the liquid equilibrium with insurance, $m^* > 0$, high-cost lenders are indifferent about loan insurance. When they insure, they benefit from higher gains from trade upon a liquidity shock, $\nu\lambda(p_I - p_U)$, but loose the option to sell lemons without the liquidity shock, $(1-\nu)(1-\mu)p_U$. Both effects are equalized in equilibrium, where an indifference condition determines the price of uninsured loans:

$$\nu\lambda\left(\mu A - p_U^*\right) = (1-\nu)\left(1-\mu\right)p_U^*. \tag{7}$$

In any illiquid equilibrium, uninsured loans must be kept until maturity and gains from trade at $t=1$ cannot be realized. As a result, the payoff of a high-cost lender is μA. Since market for insured (and thus risk-free) loans remains liquid, insurance allows such lenders to exploit the gains from trade after the liquidity shock. Thus, the payoff with insurance, $\kappa\mu A$ are strictly higher. It follows that $m^*=1$.

Proposition 2. ***Liquid equilibrium when loan insurance is available.***

1. *If $A \geq \bar{A}$ (equivalently, $\mu \leq \underline{\mu}$) and $\lambda \geq \underline{\lambda}_U$, then all loans are uninsured, $m^*=0$, and the liquid equilibrium described in Lemma 1 exists.*

2. *If $A < \bar{A}$ and $\lambda \geq \underline{\lambda}_I$, some loans are insured, $m^* = 1 - \frac{\kappa F(\eta^*)}{\mu(\lambda-1)(1-\nu)(1-F(\eta^*))} \in (0,1)$, where the screening threshold is $\eta^* \equiv \frac{(1-\nu)(1-\mu)^2\kappa A}{\nu\lambda+(1-\nu)(1-\mu)}$, and the price of uninsured loans is $p_U^* \equiv \frac{\nu\lambda\mu A}{\nu\lambda+(1-\nu)(1-\mu)} \in [\frac{A}{\lambda}, p_I^*)$. Loan insurance increases the price for uninsured loans, lowers the screening threshold, and increases welfare.*

   a. *If $\lambda \geq \underline{\lambda}_U$, the liquid equilibrium exists irrespective of loan insurance.*

   b. *If $\underline{\lambda}_U > \lambda \geq \underline{\lambda}_I$, the liquid equilibrium exists only with loan insurance.*

Proof. See Appendix A.3 (which also defines the bounds $\bar{A}$, $\underline{\mu}$, and $\underline{\lambda}_I$). ∎

The liquid equilibrium again requires a high liquidity shock (high λ). Loan insurance occurs for a high degree of competition in the lending market (low A) or a high credit score (high μ), as shown in Figure 5. When insurance is used, welfare exceeds the level without loan insurance. The social benefit of insurance is that there is no asymmetric information between insurers and lenders at $t=0$, as lenders

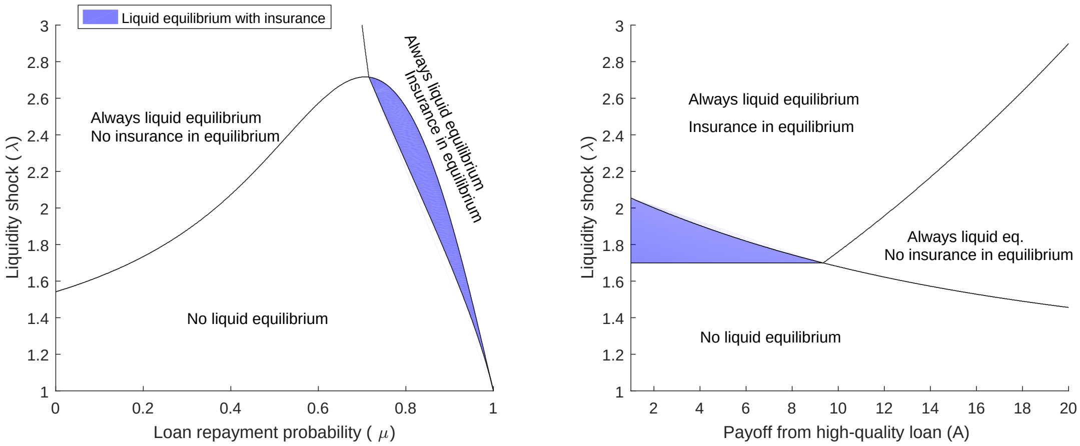

Figure 5: Existence of liquid equilibrium with loan insurance.
Parameters: $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, left panel: $A = 3$, right panel: $\mu = 0.9$.

have not yet learned loan quality. By insuring, a lender effectively commits to not exploiting future private information, reducing the severity of adverse selection in the market for uninsured loans at $t = 1$. Insurance increases the average quality of uninsured loans because only non-screened loans are insured in equilibrium. As a result, the price of uninsured loans is higher (a beneficial direct effect of higher gains from trade) that indirectly reduces screening incentives (a detrimental effect). When loan insurance is used in equilibrium, the positive effect unambiguously dominates and increases welfare. Figure 6 and 7 show the effect of the availability of loan insurance on the secondary market price for uninsured loans and on screening.

We turn to the comparative statics of the liquid equilibrium.

Proposition 3. ***Comparative statics of the liquid equilibrium.***

1. *When loan insurance is used, $m^* > 0$, then:*

   (a) the screening threshold η^* increases in A, decreases in μ, and ν;

   (b) the price for uninsured loans p_U^* increases in A, μ, and ν; and

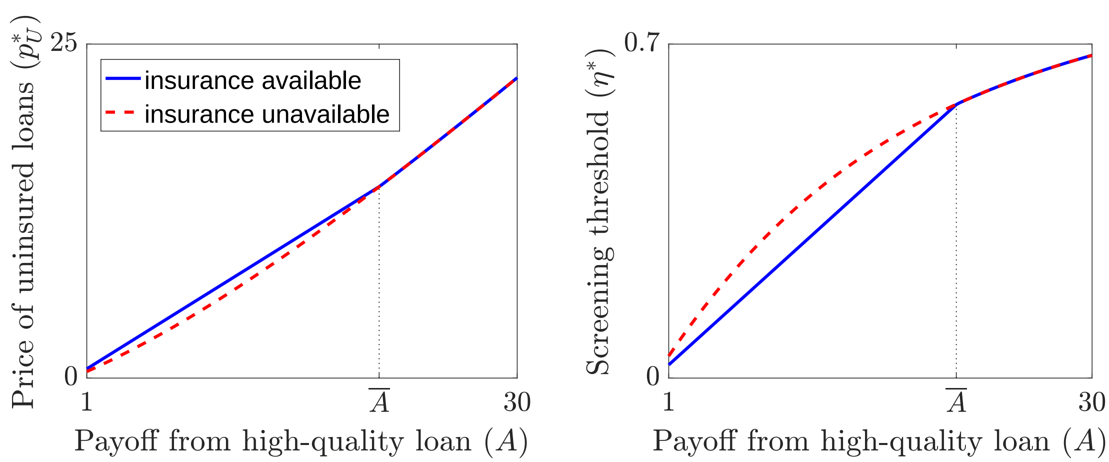

Figure 6: Loan insurance, the screening threshold, and the price for uninsured loans. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $\mu = 0.9$.

(c) *the proportion of high-cost lenders who insure, m^*, increases in μ but it decreases in A. If $\frac{F(\eta_\nu^*)(1-F(\eta_\nu^*))}{f(\eta_\nu^*)} < (1+(\lambda-1)\mu)A\lambda^{-1}$, where $\eta_\nu^* \equiv \lim_{\nu\to 0}\eta^*$, then m^* is non-monotonic in ν: it first increases and then decreases.*

2. *If loan insurance is not used, $m^* = 0$, then*

   (a) *the screening threshold η^* increases in A and decreases in μ and ν; and*

   (b) *the price for uninsured loans p_U^* increases in A. If $\eta_\mu^* \frac{f(\eta_\mu^*)}{1-F(\eta_\mu^*)} > 1$, where $\eta_\mu^* \equiv \lim_{\mu\to 0}\eta^*$, then the price is non-monotonic in μ, decreasing first. The price can also be non-monotonic in ν.*

Proof. See Appendix A.4, which also includes comparative statics of the size of the liquidity shock λ and a stochastic dominance shift of the cost distribution F. ■

The screening threshold increases in lower competition (higher final payoff A), a lower credit score μ, and a lower probability of a liquidity shock ν. A higher payoff increases screening benefits and, thus, raises the price directly and indirectly through a higher screening threshold. A lower credit score and a lower liquidity shock probability reduce the opportunity cost of screening, raising the screening threshold.

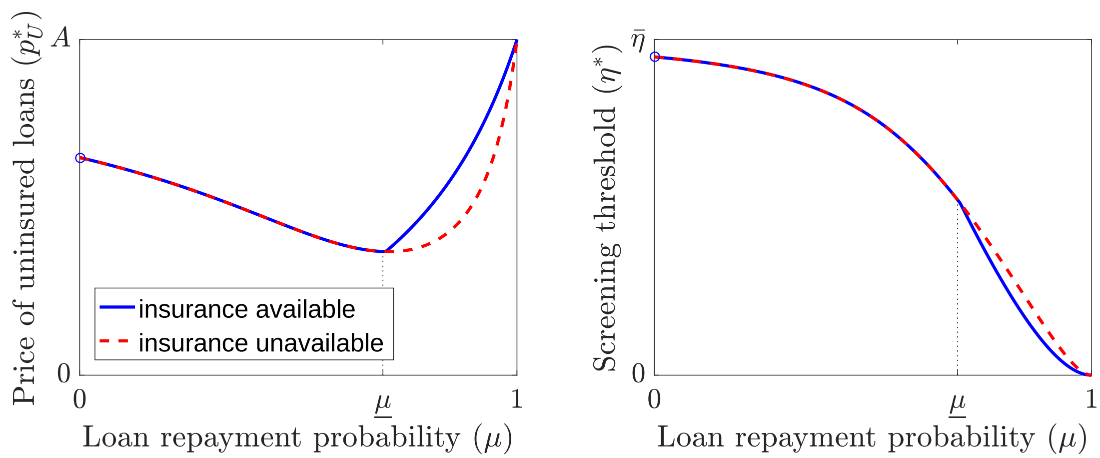

Figure 7: Loan insurance, the screening threshold, and the price for uninsured loans. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $A = 3$.

Without loan insurance, the price can be non-monotonic in the credit score μ. A higher score reduces the benefit of screening, which indirectly lowers the price, but a higher score also directly raises the price because of fewer lemons traded. Figure 7 shows how the price and screening threshold vary with the credit score. For $\mu \to 1$, only high-quality loans are financed, so the secondary market price reaches maximum $p_U \to A$. The price is lowest for an intermediate score that satisfies $(1-\mu)\left(-\frac{d\eta^*}{d\mu}\right) = \frac{1-F(\eta^*)}{f(\eta^*)}$. In the limit $\mu \to 0$, the low average quality of loans provides lenders with maximum incentives to screen and, therefore, the indirect effect on price through higher screening threshold dominates under the stated sufficient condition.

With loan insurance, the price for uninsured loans is determined by the indifference condition of high-cost lenders about insurance (7). Lower competition (higher A), a higher credit score (μ), and a higher probability of a liquidity shock (ν) increase the attractiveness of insurance, and thus the price p_U^* increases to maintain the insurance indifference. A higher credit score and probability of liquidity shock also reduce the screening incentives both indirectly through higher price of uninsured loans and directly through lower probability of benefiting from screening.

Both a higher proportion of insured high-cost lenders, m^*, and a higher screening threshold reduce the adverse selection and thus increase the price for uninsured loans. Screening achieves that by lower issuance of lemons and insurance by the sale of lemons in a separate market for insured loans, which itself is free of adverse selection. But loan insurance eliminates adverse selection only up to the price level consistent with the insurance indifference condition (7). As a result, insurance and screening are substitutes. For example, for lower competition or lower credit scores, the screening threshold is higher, so less insurance is needed to maintain the price high enough to satisfy the indifference condition (7).

The proportion of insured high-cost lenders m^* can be non-monotonic in ν. First, higher ν increases the proportion of liquidity sellers in the secondary market and directly reduces the adverse selection which increases the uninsured loan price. Thus less insurance is required to keep price high enough to maintain the indifference condition (7). Second, higher ν reduces screening by lenders, which indirectly increases adverse selection, and then needs to be substituted by higher insurance. In the limit $\nu \to 1$, the price effect is dominant. In the limit $\nu \to 0$, the second effect through screening threshold dominates under the stated sufficient condition.

We turn to the illiquid equilibrium. It is shown in Figure 15 in Appendix A.5.

Proposition 4. ***Illiquid equilibrium when loan insurance is available.*** *There always exists an illiquid equilibrium, $p_U^* = 0$. If $\lambda < \min\{\underline{\lambda}_U, \underline{\lambda}_I\}$, the illiquid equilibrium is unique. If $(1 - \kappa\mu)A < \bar{\eta}$, screening is partial and lower than in the illiquid equilibrium without loan insurance. Since all high-cost lenders insure and realize gains from trade, welfare exceeds the level in the illiquid equilibrium without insurance.*

Proof. See Appendix A.5. ■

5 Constrained Efficiency and Regulation

We turn to the normative implications of loan insurance. We define and characterize the constraint-efficient allocation as a planner's choice of loan insurance and equilibrium. We proceed by showing that a regulator subject to a balanced budget and with no information advantage over outside financiers can achieve this benchmark with a combination of subsidies to loan insurance and to purchases of uninsured loans.

5.1 Constrained efficiency

We consider a constrained planner, P, who observes the screening costs of lenders, chooses loan insurance $\{\ell_i\}$, and can pick the preferred equilibrium in secondary market for uninsured loans (liquid or illiquid) by guaranteeing a minimum price in this market. The planner maximizes utilitarian welfare subject to the individually optimal screening and loan sale choices of lenders and subject to outside financiers breaking even. In the remainder of this section, we first study the planner's problem subject to a liquid and illiquid equilibrium, respectively, and then compare these.

Suppose the planner picks the liquid equilibrium (L) when it exists in the decentralized equilibrium, $\lambda \geq \min\{\underline{\lambda}_U, \underline{\lambda}_I\}$. The planner, however, internalizes the impact of loan insurance on the price of uninsured loans. This positive pecuniary externality arises because lenders insure their loans before they learn about loan quality, effectively committing to not acting on future private information. Insurance reduces the quantity of lemons sold in the market for uninsured loans by $(1-\mu)(1-F(\eta))m$, and changes the break-even condition of outside financiers to equation (9), whereby the price of uninsured loans equals the value of uninsured loans sold by liquidity sellers divided by the quantity of uninsured loans from liquidity sellers, $\nu\left(1-\left(1-F\left(\eta\right)\right)m\right)$, and informed sellers, $(1-\nu)(1-\mu)(1-F(\eta))(1-m)$. The planner solves:[9]

$$
\begin{aligned}
W^L \quad &\equiv \quad \max_m \overbrace{\nu(\lambda-1)\left[p_U\left(1-\left(1-F\left(\eta\right)\right)m\right)+\mu A\left(1-F\left(\eta\right)\right)m\right]}^{\text{Gains from trade}} \\
&\quad + \underbrace{\left[F(\eta)+\mu(1-F(\eta))\right]A}_{\text{Fundamental value}} - \underbrace{\int_0^\eta \tilde{\eta}dF(\tilde{\eta})}_{\text{Screening costs}} \equiv W \qquad (8) \\
\text{s.t.} \quad & (6), \quad p_U\lambda \geq A, \quad \text{and} \\
p_U \quad &= \quad \nu A \frac{F\left(\eta\right)+\mu\left(1-F\left(\eta\right)\right)\left(1-m\right)}{\nu\left(1-\left(1-F\left(\eta\right)\right)m\right)+\left(1-\nu\right)\left(1-\mu\right)\left(1-F\left(\eta\right)\right)\left(1-m\right)}. \qquad (9)
\end{aligned}
$$

Proposition 5. ***Efficient insurance in the liquid equilibrium.*** *For $\lambda \geq \min\{\underline{\lambda}_U, \underline{\lambda}_I\}$, there exists an allocation with efficient insurance level, $m^P \in [m^*, 1)$, $p_U^P \in [p_U^*, A)$, $\eta^P \in (0, \eta^*]$. Loan insurance exceeds the unregulated level at the intensive and extensive margins: $m^P > m^* > 0$ for $A < \bar{A}$ and $m^P > 0 = m^*$ for $\bar{A} \leq A < \bar{A}^P$. More insured loans by high-cost lenders, $m^P > m^*$, implies a higher price in the market for uninsured loans, $p_U^P > p_U^*$, and less screening, $\eta^P < \eta^*$.*

Proof. See Appendix A.6. ■

Figure 8 shows how the efficient level of loan insurance in the liquid equilibrium exceeds the unregulated level at the intensive margin (case $A < \bar{A}$). Figure 9 and 10 show the effect of the efficient level of loan insurance on the secondary market price for uninsured loans and screening. Higher insurance level on the interval $A < \bar{A}^P$ ($m^P > m^*$) increases the average quality of uninsured loans, resulting in higher price, which in turn reduces the incentives to screen. Figure 11 shows the parameter subspace where insurance is higher at both the intensive and the extensive margin.

[9] It is constrained efficient that low-cost lenders, who choose to screen, do not insure. Therefore, the planner's choice of insurance for each lender, $\{\ell_i\}$, is equivalent to choosing the proportion m of high-cost lenders who insure.

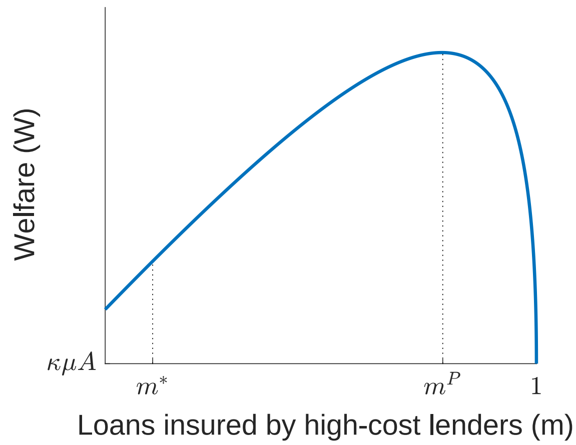

Figure 8: Welfare in the liquid equilibrium as a function of the fraction of insured loans by high-cost lenders. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $\mu = 0.9$, $A = 3$.

Next, suppose that the planner picks the liquid equilibrium when it does not exist in the decentralized equilibrium, $\lambda < \min\{\underline{\lambda}_U, \underline{\lambda}_I\}$. The planner can *liquify the market* by exploiting the pecuniary externality of loan insurance. That is, the planner can create a liquid equilibrium by choosing a price $p_U \geq A/\lambda$ and a high enough loan insurance m to support such a price. The existence of a constrained-efficient allocation then extends from Proposition 5 to this case.

We turn to the illiquid or frozen (F) equilibrium. The price of uninsured loans is zero, so insurance has no positive pecuniary externality. The planner's problem reduces to maximizing welfare subject to the individually optimal screening choice:

$$W^F = \max_m W \text{ s.t. } \eta = (1-\kappa\mu)A \text{ and } p_U = 0.$$

The fraction of insured high-cost lenders, m, only appears within the gains from trade, $\nu(\lambda - 1)\mu A\,(1 - F(\eta))\,m$, because the only the market for *insured* loans is liquid at $t = 1$. Hence, full insurance is constrained-efficient in the illiquid equilibrium, which is the same corner solution as in the unregulated equilibrium, $m^P = m^* = 1$. The implied

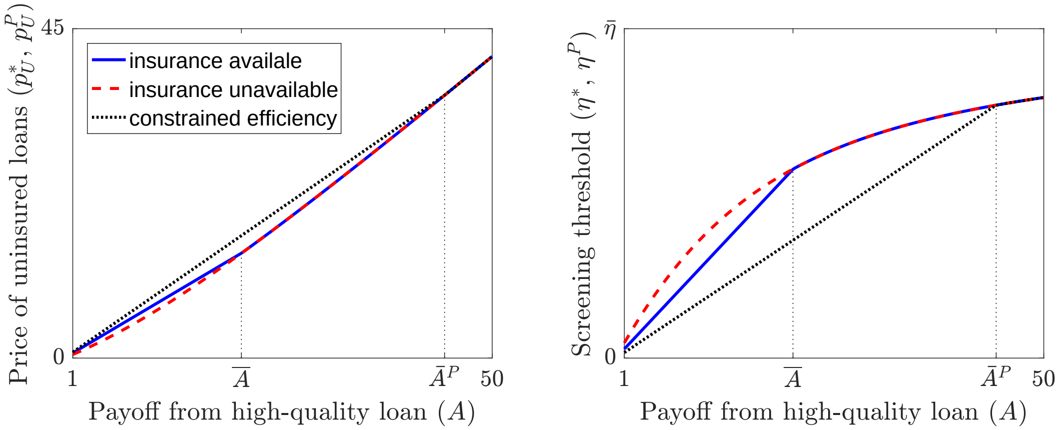

Figure 9: Constrained efficient levels of screening and prices of uninsured loans. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $\mu = 0.9$.

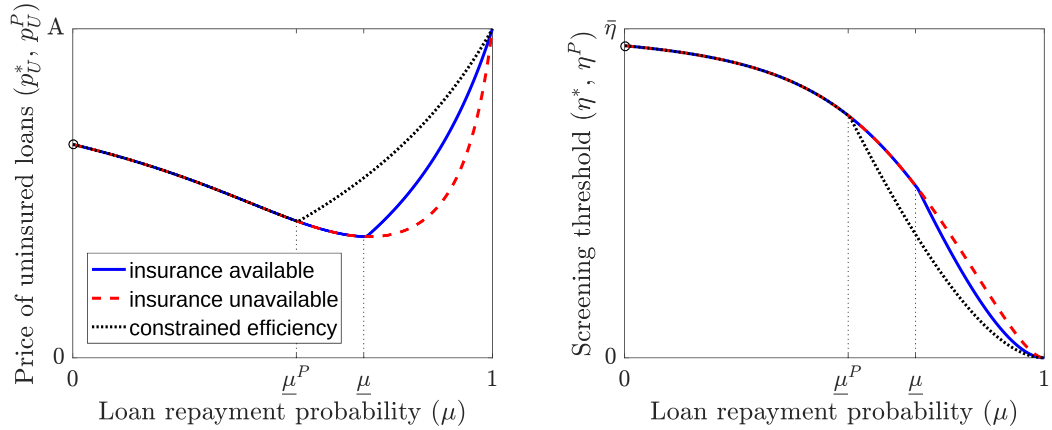

Figure 10: Constrained efficient levels of screening and prices of uninsured loans. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $A = 3$.

welfare is $W^F = \nu(\lambda - 1)\mu A \left(1 - F\left(\eta^*\right)\right) + \left[F(\eta^*) + \mu(1 - F(\eta^*))\right] A - \int_0^{\eta^*} \tilde{\eta} dF(\tilde{\eta})$.

Finally, we consider whether the planner prefers the liquid or the illiquid equilibrium. The liquid equilibrium is constrained efficient if it is superior to the illiquid equilibrium, $W^L \geq W^F$. This occurs whenever the gains from trade in the secondary market for uninsured loans exceed the welfare loss due to lower ex-ante screening incentives. Figure 11 shows the parameter spaces where the planner liquifies the market and where the illiquid equilibrium is welfare-superior and Proposition 6 summarizes.

Proposition 6. ***(Global) constrained efficiency.***

1. *For $\tilde{\lambda}_I < \lambda$, the planner chooses the welfare-dominant liquid equilibrium.*

   a. *For $\tilde{\lambda}_I < \lambda < \min\{\underline{\lambda}_U, \underline{\lambda}_I\}$, the planner liquifies the market by choosing a price $p_U \geq A/\lambda$ and a high enough loan insurance consistent with this price in order to create a welfare-dominant liquid equilibrium.*

   b. *For $\min\{\underline{\lambda}_U, \underline{\lambda}_I\} \leq \lambda$ and $A < \bar{A}^P$, Proposition 5 applies.*

2. *For $\tilde{\lambda}_I \geq \lambda$, the planner chooses the welfare-dominant illiquid equilibrium.*

Proof. See Appendix A.7 (which also defines the bound $\tilde{\lambda}_I$). ■

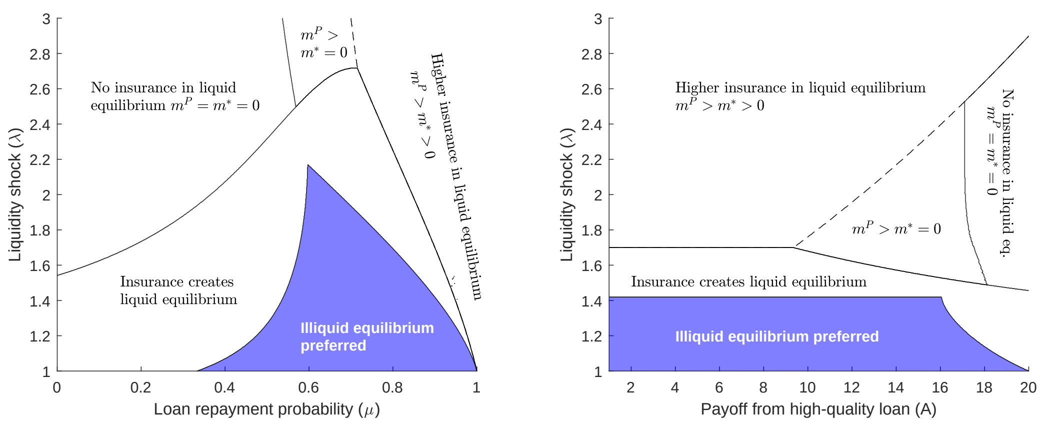

Figure 11: Constrained-efficient allocation: liquid and illiquid markets. In the shaded area, the planner prefers to keep the market illiquid/frozen. Parameters: $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, left panel: $A = 3$, right panel: $\mu = 0.9$.

5.2 Regulation via subsidizing loan insurance

We consider a regulator with a balanced budget and no information advantage over uninformed outside financiers. Hence, direct implementation by choosing insurance ℓ_i for each lender is infeasible because only high-cost lenders should insure but the screening costs of lenders are private information. We consider two regulatory tools. First, a Pigouvian subsidy, $b_I \geq 0$, to the owner of insured loans allows the regulator to select an optimal level of insurance in a given equilibrium. Second, we consider a minimum price guarantee in the secondary market for uninsured loans implemented with purchase subsidy for uninsured loans. This tool allows the regulator to rule out the illiquid equilibrium whenever it is welfare-dominated by the liquid one.

We turn to the loan insurance subsidy. This subsidy is funded by a lump-sum tax T on lenders after trade occurred at $t = 1$ and before they consume. To ensure that lenders can always pay the tax, we introduce an additional non-pledgeable and perishable endowment n at $t = 1$ that can be used to pay taxes or for consumption.[10] The timeline is shown in Figure 12, where the subsidy is received at $t = 1$.[11]

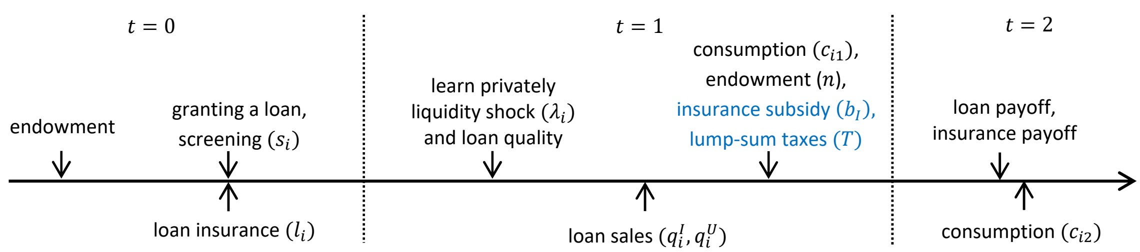

Figure 12: Timeline with loan insurance subsidy.

Definition 3. *A competitive equilibrium in the regulated economy with a loan insurance subsidy comprises lender choices of screening $\{s_i\}$, insurance $\{\ell_i\}$, sales of insured and uninsured loans in secondary markets $\{q_i^I, q_i^U\}$, insurance subsidy b_I and lump-sum taxes T, market prices p_I and p_U, and insurance fee k such that:*

1. *At $t = 1$, each lender i chooses sales of insured and uninsured loans for each*

[10] An endowment $n \geq (1-\mu)A$ covers any meaningful subsidy policy. When $b_I = (1-\mu)A$, all lenders insure $m^* = 1$, $p_U^* = A$, $\eta^* = 0$ and $\int T di = (1-\mu)A$. But Proposition 5 shows that full insurance is not optimal, therefore $b_I^R < (1-\mu)A$ and $T < (1-\mu)A$.

[11] The subsidy could also be received at $t = 0$. One advantage of the chosen timing is the comparability with a loan purchase subsidy studied in section 5.3.

realized liquidity shock $\lambda_i \in \{1, \lambda\}$, *denoted by* $q_i^I(s_i, \lambda_i, \ell_i)$ *and* $q_i^U(s_i, \lambda_i, \ell_i)$, *given prices* p_U *and* p_I, *subsidy* b_I, *and choices of screening* s_i *and insurance* ℓ_i.

2. *At* $t = 1$, *prices* p_I *and* p_U *are set for outside financiers to break even in expectation, given the choices of screening* $\{s_i\}$ *and insurance* $\{\ell_i\}$, *sales schedules* $\{q_i^I(\cdot), q_i^U(\cdot)\}$ *of all lenders, and the insurance subsidy* b_I.

3. *At* $t = 0$, *each lender* i *chooses its screening* s_i *and loan insurance* ℓ_i *to maximize expected utility, given* p_I, p_U, $q_i^I(\cdot)$, $q_i^U(\cdot)$, b_I, *and* T:

$$\max_{s_i, \ell_i, c_{i1}, c_{i2}} \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - s_i \eta_i\right] \qquad \textit{subject to}$$

$$\begin{aligned} c_{i1} &= q_i^U(s_i, \lambda_i, \ell_i)\, p_U + q_i^I(s_i, \lambda_i, \ell_i)\, p_I + b_I \ell_i + n - T, \\ c_{i2} &= \left[\ell_i - q_i^I\right](A - k) + \left[1 - \ell_i - q_i^U\right] \times \begin{cases} A & \textit{w. p.} \quad s_i + \mu(1 - s_i) \\ 0 & \qquad\quad (1 - \mu)(1 - s_i). \end{cases} \end{aligned}$$

4. *At* $t = 0$, *the fee* k *is set for outside financiers to break even in expectation, given screening* s_i *and insurance* ℓ_i *choices.*

5. *At* $t = 0$, *regulator chooses insurance subsidy* b_I *to maximize welfare of all lenders subject to the balanced budget constraint,* $T = b_I m^*(1 - F(\eta^*))$.

This policy makes insurance more attractive and increases the fraction of insured loans, m, which in turn indirectly increases the secondary market price for uninsured loans, p_U. In the equilibrium with regulation, the price for uninsured loans p_U is determined by an indifference condition of high-cost lenders between insurance and no insurance, $(\mu A + b_I)\kappa = \nu\lambda p_U + (1 - \nu)\left(\mu A + (1 - \mu)\, p_U\right)$, which is rewritten as:

$$p_U(b_I) = \frac{\nu\lambda\mu A + \kappa b_I}{\nu\lambda + (1 - \nu)(1 - \mu)}. \tag{10}$$

A higher price p_U, in turn, decreases the screening threshold η, as given in equation (6). Conditional on implementing the liquid equilibrium, the regulator chooses an optimal insurance subsidy, b_I, to maximize welfare:[12]

$$\begin{aligned}\max_{b_I} W \quad = \quad & \max_{b_I} \overbrace{\nu(\lambda-1)\left[p_U\left(1-(1-F(\eta))\,m\right)+\mu A\left(1-F(\eta)\right)m\right]}^{\text{gains from trade}} \qquad (11)\\ & + \underbrace{\left(F(\eta)+\mu(1-F(\eta))\right)A}_{\text{fundamental value}} - \underbrace{\int_0^{\eta}\tilde{\eta}dF(\tilde{\eta})}_{\text{Screening costs}} + \kappa(n+\underbrace{b_I m(1-F(\eta))-T}_{\text{policy redistribution }(=0)})\\ & \text{s.t.}(6),(9),(10) \text{ and } p_U\lambda \geq A.\end{aligned}$$

The last term in equation (11) expresses the welfare effect of redistributing resources from all tax-paying lenders to lenders who choose to insure their loan. Since regulator has balanced budget and the average marginal utilities of all lenders and insured lenders is the same (κ), this term collapses to zero.

Proposition 7. ***Loan insurance subsidy.*** *Consider parameters for which the constrained planner wishes to implement a liquid market with positive loan insurance, that is* $\tilde{\lambda}_I \leq \lambda < \min\{\underline{\lambda}_U, \underline{\lambda}_I\}$, *or* $\min\{\underline{\lambda}_U, \underline{\lambda}_I\} \leq \lambda$ *and* $A < \bar{A}^P$. *The constrained-efficient allocation can be implemented with a Pigouvian subsidy for loan insurance:*

$$b_I^R = \frac{[\kappa-(1-\nu)\mu]p_U^P - \nu\lambda\mu A}{\kappa} \qquad (12)$$

Proof. See Appendix A.8. ■

[12] We focus on the interval $b_I \leq (1-\mu)A$ without loss of generality. Higher subsidies have no effect on welfare, as the payoff of insured loans $\mu A + b_I$ would exceed the payoff from high-quality loans, so all lenders insure and do not screen, resulting in lower welfare $W = \kappa(\mu A + n)$. See Figure 8.

5.3 Subsidizing uninsured loan purchases

Despite the insurance subsidy, there are still multiple equilibria that can be welfare-ranked. In order to eliminate the illiquid equilibria when it is welfare-dominated, the regulator can directly intervene in the market for uninsured loans by guaranteeing a minimum price for these loans. This policy is implemented with loan purchase subsidies. The regulator buys uninsured loans at $t = 1$ at a price above the fair price by a Pigouvian subsidy b_U. Its fundamental value is recovered by selling these loans to outside financiers. The cost of the subsidy, $b_U q_i^U$, is again financed by lump-sum taxes at $t = 1$. This policy is equivalent to guaranteeing a minimum price for uninsured loans, $p_{min} \equiv p_U + b_U$. Figure 13 shows the timeline with purchase subsidies.

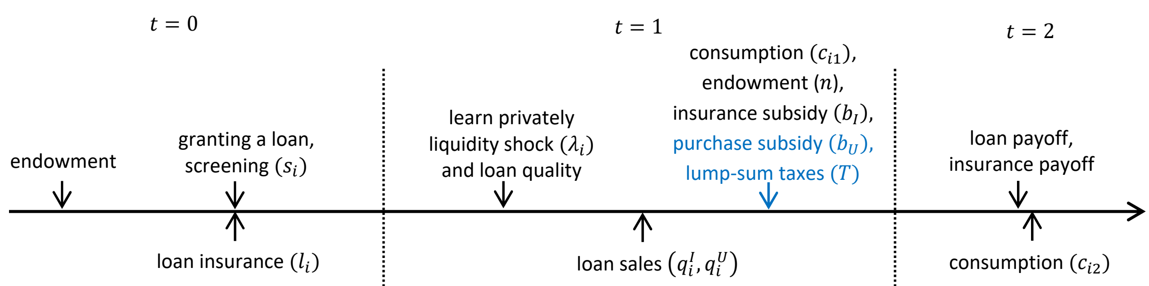

Figure 13: Timeline with subsidized purchases of uninsured loans.

Proposition 8. ***Purchase subsidies for uninsured loans.***

1. *To eliminate the illiquid equilibrium when it is welfare-dominated, $\lambda > \tilde{\lambda}_I$, the regulator uses loan purchase subsidies to guarantee a minimum price, $p_U = A/\lambda$. Then, the liquid equilibrium is the unique equilibrium.*

2. *In the liquid equilibrium, subsidies on loan purchases are not used because they are dominated by loan insurance subsidies.*

Proof. See Appendix A.9. ■

Only subsidizing loan purchases can eliminate the illiquid equilibrium. Conditional on the liquid equilibrium, loan purchase subsidies can keep the markets liquid, $b_U = A/\lambda - p_U^*$, for a low price in the unregulated equilibrium, $p_U^* < A/\lambda$. However, insurance subsidies are superior to uninsured loan purchase subsidies because of the positive pecuniary externality of insurance. The policy of subsidized purchases of uninsured loans does not take advantage of an externality and, therefore, is more expensive. Hence, loan purchase subsidies are not used in the liquid equilibrium, $b_U^R = 0$, when insurance subsidies are available. Both subsidies reduce screening incentives in the liquid equilibrium but have the opposite effects on the fraction of insured loans.

Figure 14 compares welfare in the liquid equilibrium under both policies when they target the same price p_U^T. This price arises either directly with subsidized purchases of uninsured loans, $p_U^T = p_U + b_U$, or indirectly with subsidized loan insurance, $p_U^T = p_U(b_I)$. We find that subsidizing insurance is better, $p_U^P = p_U(b_I) > p_U^*$. Purchase subsidies in isolation can have a positive effect on welfare due to the redistribution of wealth from taxed lenders to sellers of uninsured loans, where the latter have a higher marginal utility of consumption. However, purchase subsidies lower welfare due to the elimination of insurance and reduced screening. In the example shown in Figure 14, the regulator chooses not to subsidize uninsured loan sales, $p_U(b_U^R) = p_U^*$, in the absence of insurance subsidies. When both policies are available, only insurance subsidies are used and they achieve the constrained efficiency.[13]

[13] The discontinuity between no intervention and an effective purchase subsidies for uninsured loans, at $p_U^T = p_U^*$, arises because this policy eliminates insurance and its positive pecuniary externality that is compensated by costly purchases. At $p_U^T = A$, all lenders receive a subsidy under both policy options and, therefore, the overall welfare levels are equalized: $W \mid_{p_U^T = A} = \kappa(\mu A + n)$.

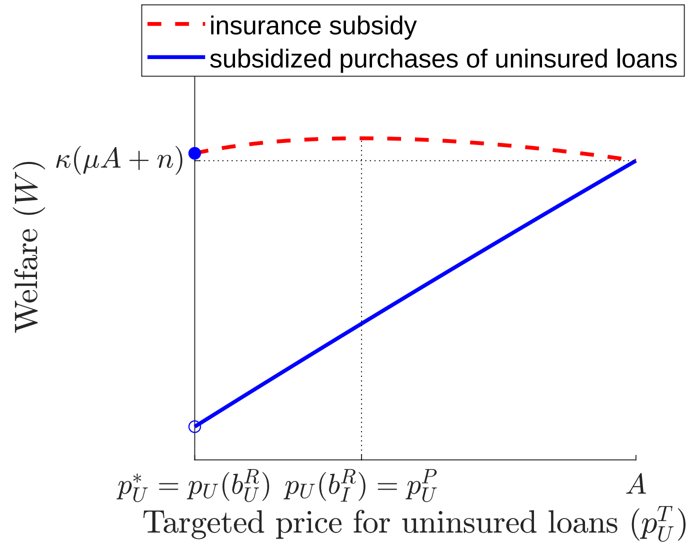

Figure 14: Comparison of welfare under the two policies. Parameters: $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $\mu = 0.9$, $A = 3$ (so $\lambda > \min\{\underline{\lambda}_U, \underline{\lambda}_I\}$ and $A < \bar{A}$).

6 Generalizations and Extensions

6.1 Required return of outside financiers

Consider a general required return of outside financiers, $R > 0$. It may reflect, in reduced form, the competition in secondary markets or the impact of monetary policy.

Proposition 9. *If $\underline{R} < R < \overline{R}$, then a lower required return of outside financiers (i) increases prices for insured and uninsured loans; (ii) decreases the screening threshold; and (iii) increases loan insurance at both the intensive and the extensive margin.*

Proof. See Appendix A.10 (which also defines the bounds $\underline{R}$ and $\overline{R}$). ■

When outside financiers require a lower return, they pay more for loans in the secondary market, increasing the proceeds of lenders from selling loans. The payoff of low-quality loans is particularly affected, since they are always sold in the market—

unlike high-quality loans. As a result, the incentives to screen at loan origination are reduced, which implies that choosing insurance becomes relatively more attractive.

This comparative static result is broadly consistent with US financial markets prior to the financial crisis. The lower required return corresponds to higher competition among outside financiers, perhaps due to large capital inflows ('savings glut'), which implies that lending standards are low and loan insurance is popular.

6.2 Imperfect screening

Consider imperfect screening, whereby the success probability is $\psi \in (\mu, 1)$. As a result, some low-cost lenders also sell low-quality loans in the secondary market.

Proposition 10. ***Imperfect screening.*** *If $\psi > \underline{\psi}$, better screening ($\psi \uparrow$) lowers loan insurance on both the intensive and extensive margin: fewer high-cost lenders insure, $\frac{dm^*}{d\psi} < 0$, and the parameter range for insurance shrinks, $\frac{d\bar{A}}{d\psi} < 0$.*

Proof. See Appendix A.4 (which also contains the definition of $\underline{\psi}$). ■

A better screening technology implies higher incentives to screen. The threshold cost is $\eta = (1-\nu)(\psi - \mu)(A - p_U)$. When insurance is used, the price of uninsured loans is given by the indifference condition for high-cost lenders, stated in equation (17), which is not affected by ψ. Thus, higher screening incentives result in a higher screening threshold, $\frac{d\eta^*}{d\psi} > 0$. Hence, less insurance is needed to achieve the same price for uninsured loans that makes high-cost lenders indifferent about insurance.

6.3 Partial insurance

Suppose insurance contracts allow lenders to choose the fraction ω of default costs covered by the insurance. Such insurance contracts are equivalent to guaranteeing the non-default payment A with a deductible $(1-\omega)A$, where the owner of the loan pays the insurance fee at the time of maturity ($t=2$). As proven in Appendix A.11, only high-cost lenders insure, so the competitive insurance fee is actuarially fair and reflects the average cost of insurance, $k=\omega(1-\mu)A$. We have the following result.

Proposition 11. *Full insurance, $\omega^*=1$, is privately and socially optimal.*

Proof. See Appendix A.11. ■

With partial insurance, $\omega<1$, the value of an insured loan of low quality is $\omega A-k=\omega\mu A$, which is below the value of an insured loan of high quality, $A-k=A\left(1-(1-\mu)\omega\right)$. (This result is in contrast to the full-insurance case.) Hence, there is adverse selection in the market for partially insured loans, since lenders without liquidity shock decide to sell only low-quality loans. Adverse selection redistributes wealth from lenders with liquidity shock, who sell all loans, to lenders without liquidity shock, who sell only lemons. Since lenders have higher utility of consumption in states with liquidity shock, they choose full coverage, $\omega^*=1$, to avoid the negative effect of adverse selection. As for social optimality, a higher insurance coverage has a positive externality on the price of uninsured loans, so a planner also chooses full coverage.[14]

[14] An alternative interpretation of partial insurance is insurer default. We have assumed so far that the insurer has deep-pockets, perhaps because of (implicit) government backing. In contrast, suppose that the insurer defaults on its liabilities after the fee is paid at $t=2$ with exogenous probability $1-\omega$. The expected value of an insured loan is $\omega A-k$ upon loan default ($-k$ when insurer defaults and $A-k$ otherwise) and $A-k$ upon loan repayment (irrespective of insurer default). The insurance fee is $k=\omega(1-\mu)A$. Since the expected payoffs are equal to those for partial insurance, the problem with insurer default is identical. Proposition 11 implies that welfare decreases in insurer default risk.

6.4 Upfront insurance fee

In this extension, we suppose that the insurance fee k has to be paid at $t = 0$. Hence, a lender who insures can only fund a fraction $1 - k$ of the loan. Despite this negative effect on lending volume, we show that our qualitative results remain unchanged.

Proposition 12. ***Upfront fee.*** *Suppose the insurance fee is paid at $t = 0$.*

1. *For $A < \bar{A}'$ and $\lambda \geq \underline{\lambda}_I'$, some loans are insured, $m^{*\prime} = 1 - \frac{\kappa F(\eta^{*\prime})(1-\delta)}{(1-F(\eta^{*\prime}))\left[\mu(\lambda-1)(1-\nu)-\kappa\delta\frac{\nu+(1-\nu)(1-\mu)}{\nu}\right]} \in (0,1)$, the screening threshold is $\eta^{*\prime} \equiv \frac{(1-\nu)(1-\mu)^2\kappa A}{\nu\lambda+(1-\nu)(1-\mu)}(1+\delta)$, and the price of uninsured loans is $p_U^{*\prime} \equiv \frac{\nu\lambda\mu A-\kappa(1-\mu)A\delta}{\nu\lambda+(1-\nu)(1-\mu)}$, where $\delta \equiv \frac{\mu A-1}{1+A(1-\mu)}$. Loan insurance increases the price $p_U^{*\prime}$, reduces screening, and increases welfare.*

2. *The constrained efficient level of loan insurance, $m^{P\prime} \in [m^{*\prime}, 1)$, exceeds the unregulated level at both the intensive and the extensive margin.*

3. *If $\mu A > 1$, then insurance is less beneficial under upfront fee payment, $m^{*\prime} \leq m^*$ and $\bar{A}' < \bar{A}$ and $m^{P\prime} \leq m^P$, which implies $p_U^{*\prime} \leq p_U^*$, $\eta^{*\prime} \geq \eta^*$, $\underline{\lambda}_I' > \underline{\lambda}_I$.*

Proof. See Appendix A.12 (which also defines the bounds $\bar{A}'$ and $\underline{\lambda}_I'$). ■

If the return from non-screened loans exceeds the intertemporal rate of substitution of insurers, $\mu A > 1$, then the net individual benefit of insurance is negatively affected by the reduced lending volume. Compared to payment at the final date, less insurance occurs at both the intensive margin, $m^{*\prime} \leq m^*$, and the extensive margin, $\bar{A}' < \bar{A}$. There is a weaker positive effect on the price of uninsured loans, which is lower than under final-date payment. The lower price, in turn, implies a higher screening threshold and a higher required threshold for the existence of liquid equilibria, $\underline{\lambda}_I' > \underline{\lambda}_I$. But insurance continues to have a positive pecuniary externality in the market for uninsured loans, so our normative results go through qualitatively.

6.5 Partial loan sales

We allow for partial sales of uninsured loans, $q_i^U \in [0, 1 - \ell_i]$, and study whether retaining default risk on the uninsured loan, $1 - \ell_i - q_i^U$, can signal loan quality. The quantity of uninsured loans not sold is a continuous choice that can be used by financiers to update their beliefs about loan quality and may result in a continuum of perfect Bayesian equilibria.

Proposition 13. ***Partial loan sales.***

1. *For $\bar{\eta} < (1-\mu)A$, risk retention (partial loan sales) induces the existence of perfect Bayesian equilibria with full screening characterized by $q^{U*} \in \left(0, 1 - \frac{\bar{\eta}}{(1-\mu)A}\right]$, $\eta^*(q^{U*}) \geq \bar{\eta}$ and sustained by out-of-equilibrium beliefs interpreting $q^U > 1 - \frac{\bar{\eta}}{(1-\mu)A}$ as a signal of low quality. All originated loans are of high quality and adverse selection in the secondary market for uninsured loans disappears.*

2. *For $\bar{\eta} \geq (1-\mu)A$, all perfect Bayesian equilibria are pooling characterized by $q^{U*} \in (\bar{q}^U, 1]$, partial screening, $\eta^*(q^{U*}) < \bar{\eta}$, and sustained by out-of-equilibrium beliefs interpreting $q^U \neq q^{U*}$ as a signal of low quality. That means that the quality of uninsured loans remains private information, adverse selection in the secondary market remains and our results are qualitatively unchanged:*

   a. *When $A < \bar{A}(q^U)$, some loans are insured in the liquid equilibrium, $m^* > 0$.*

   b. *The constrained-efficient level of loan insurance in the liquid equilibrium is higher at both the intensive margin, $m^P > m^* > 0$ when $A < \bar{A}(q^U)$, and the extensive margin, $m^P > m^* = 0$ when $\bar{A}^P(q^U) > A \geq \bar{A}(q^U)$.*

Proof. See Appendix A.13 (which formally defines the equilibrium and $\bar{q}^U$). ■

Since lenders have limited liability, any loan sale q^U would be mimicked by sellers of low-quality loans (similar to Parlour and Plantin 2008). Thus, the quality of uninsured loans is public information only in the corner solution in which everyone screens, $\eta^* \geq \bar{\eta}$, which arises for $\bar{\eta} < (1-\mu)A$. In this case, the upper bound on screening costs is low enough so that sufficient default risk retention incentivizes all lenders to screen and, therefore, all loans are of high-quality. When $\bar{\eta} \geq (1-\mu)A$, however, some lenders do not screen and the screening choice of sellers of uninsured loans and the quality of uninsured loans remain private information in this pooling equilibrium. This results in adverse selection in the market for uninsured loans and our results from the main text extend to partial loan sales. Loan insurance reduces such adverse selection and the competitive level of loan insurance is inefficiently low.

7 Conclusion

We have studied insurance against loan default in a parsimonious model of lending with costly screening of borrowers in primary markets and adverse selection in secondary markets. A key result is that loan insurance reduces the adverse selection in secondary markets for uninsured loans. This raises the gains from trade, reduces screening incentives, and increases welfare. Loan insurance is excessively low in the competitive equilibrium because its positive effect on the price of uninsured loans is not internalized by lenders. A Pigouvian subsidy on loan insurance restores constrained efficiency. A subsidy for purchases of uninsured loans eliminates the illiquid equilibrium when it is welfare-dominated by the liquid equilibrium. In the liquid equilibrium, however, the insurance subsidy dominates the loan purchase subsidy.

Our results contribute to a debate on government intervention in credit markets.

After the recent financial crisis, this debate has been especially intense regarding mortgage guarantees by Government Sponsored Enterprises and analyzed some of its negative implications. In contrast, our analysis suggests that subsidies for loan default insurance are efficient for loans with low default risk (e.g., borrowers with high credit scores) and in competitive lending markets. The beneficial welfare effect of loan insurance uncovered in our analysis contributes to the debate about the reform of Government Sponsored Enterprises such as Fannie Mae and Freddie Mac.

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A Appendix

A.1 Proof of Lemma 1

For the liquid equilibrium, we obtain expressions for the price (5) and screening threshold (6) for a general $\psi \in (\mu, 1)$. The price reflects that low-cost lenders sell some (but fewer) lemons:

$$p_U = \nu A \frac{\psi F(\eta) + \mu(1 - F(\eta))}{\nu + (1-\nu)\left[(1-\psi) F(\eta) + (1-\mu)(1 - F(\eta))\right]} \equiv p_U(\eta) \tag{13}$$

The screening threshold is again obtained by equalizing payoff when screening, $\nu\lambda p_U + (1-\nu)(\psi A + (1-\psi)p_U) - \eta$, and when not-screening, $\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)$:

$$\eta = (1-\nu)(\psi - \mu)(A - p_U) \equiv \eta(p_U). \tag{14}$$

The equilibrium value of the screening threshold, η^*, is obtained by substituting equation (13) in equation (14). It is implicitly given by:

$$\eta^* = \frac{(1-\nu)(\psi-\mu)\left[1 - \mu - F(\eta^*)(\psi - \mu)\right]}{\nu + (1-\nu)\left[1 - \mu - F(\eta^*)(\psi-\mu)\right]} A. \tag{15}$$

Within the class of liquid equilibria, does a unique equilibrium exist? Regarding uniqueness, the left-hand side (LHS) of equation (15) increases in η and the right-hand side (RHS) decreases in it, so at most one intersection exists. Regarding existence, we evaluate both sides of (15) at the bounds of the screening cost, using $F(0) = 0 < 1 = F(\bar{\eta})$. Since $LHS(0) < RHS(0)$ and $LHS(\bar{\eta}) > RHS(\bar{\eta})$ for $\bar{\eta} > \frac{(1-\nu)(\psi-\mu)(1-\psi)}{\nu+(1-\nu)(1-\psi)}$, there exists a unique interior screening threshold $\eta^* \in (0, \bar{\eta})$. (For $\psi \to 1$, the above condition always holds.) Thus:

$$p_U^* \equiv p_U(\eta^*) = \nu A \frac{\psi F(\eta^*) + \mu(1 - F(\eta^*))}{\nu + (1-\nu)\left[(1-\psi) F(\eta^*) + (1-\mu)(1 - F(\eta^*))\right]}. \tag{16}$$

To verify the supposition of a liquid equilibrium (in which high-quality loans are sold in the secondary market for uninsured loans), we combine conditions (13) and (3). Thus, a necessary condition for the liquid equilibrium is $\lambda \geq \underline{\lambda}_U \equiv \frac{\nu+(1-\nu)[(1-\psi)F(\eta^*)+(1-\mu)(1-F(\eta^*))]}{\nu(\psi F(\eta^*)+\mu(1-F(\eta^*)))}$, whose right-hand side is independent of λ and η^* is given in equation (15).

A.2 Proof of Proposition 1

Since insurance transforms the loan payoff at $t = 2$ from risky to risk-free, $\pi = A - k$, outside financiers break even for a price equal to this payoff, $p_I = \pi$. Next, the payoff from an insured loan is independent of the screening choice because outside financiers cannot observe the screening choice. A lender i who insures has a payoff $\nu\lambda p_I + (1-\nu)\pi = \kappa p_I$ when not screening and a payoff $\kappa p_I - \eta_i$ when screening. Thus, lenders who insure prefer not to screen for any $\eta_i \geq 0$. As a result, low-cost (screening) lenders never insure loans.

Recall that only high-cost lenders screen and the market for insured loans is not subject to adverse selection (when insuring at $t = 0$, lenders do not yet know loan quality). Thus, outside financiers break even when the insurance costs reflect the costs of guaranteeing the payoff A and the probability of loan repayment is μ, so $k = A - \mu A$. This implies that the payoff and market price of insured loans is $\pi = A - k = \mu A = p_I^*$.

For $m^* \in (0, 1)$, high-cost lenders must be indifferent between insuring and not insuring:

$$p_I^* \kappa = \nu\lambda p_U^* + (1-\nu)\left(\mu A + (1-\mu)\, p_U^*\right), \tag{17}$$

which simplifies to $\nu\lambda\left(\mu A - p_U^*\right) = (1-\nu)(1-\mu)\, p_U^*$. The benefits of insurance on the left are a higher price upon a liquidity shock. The cost of insurance on the right is giving up the option to sell only low-quality loans at $t = 1$ while keeping high-quality loans.

Next, we show that some non-screened loans are uninsured, $m^* < 1$. Proof by contradiction. If $m = 1$, then no high-cost lenders would sell lemons in the market for uninsured loans (all loans of high-cost lenders are indeed insured and sold in a separate market) and, for $\psi \to 1$, the quantity of lemons sold by low-cost lenders vanishes. Hence, there would be no adverse selection, only high-quality loans are sold, $p_U = A$. However, $m = 1$ requires high-cost lenders to prefer insurance over no insurance, or $p_I^* \kappa \geq \nu\lambda p_U^* + (1-\nu)\left(\mu A + (1-\mu)\, p_U^*\right)$ instead of equation (17), which simplifies to $\mu \geq 1$, a contradiction.

In an illiquid equilibrium, high-cost lenders have higher payoff when insuring, $\kappa\mu A$, than when not insuring, μA, since they must keep uninsured loans until maturity and cannot exploit the gains from trade. Thus, $m^* = 1$ in any illiquid equilibrium.

A.3 Proof of Proposition 2

We start by constructing the secondary market price of insured loans. The results in Proposition 1 equip us to derive the price, sales volume and screening threshold. First, condition (17) pins down the price for uninsured loans at $t = 1$:

$$p_U^* = \frac{\nu\lambda\mu A}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{18}$$

Substituting equation (18) into equation (14), we obtain the threshold screening effort:

$$\eta^* = \frac{(1-\nu)(1-\mu)(\psi-\mu)\kappa A}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{19}$$

The price in equation (18) must satisfy condition (3) to ensure a liquid equilibrium, that is high-quality loans are sold after a liquidity shock. Thus, a necessary condition for a liquid equilibrium when insurance is used is $\mu\nu\lambda^2 - \nu\lambda - (1-\mu)(1-\nu) \geq 0$. Since only the larger root of this quadratic condition is positive, the condition collapses to $\lambda \geq \underline{\lambda}_I \equiv \frac{1}{2\mu} + \sqrt{\frac{1}{4\mu^2} + \frac{(1-\mu)(1-\nu)}{\mu\nu}}$. When insurance option is available, the liquid equilibrium exists if $\lambda > \max\{\underline{\lambda}_U, \underline{\lambda}_I\}$. To pin down the fraction of loans insured by high-cost lenders m^*, the price of uninsured loans is also given by the break-even condition of the outside financiers:

$$p_U = \nu A \frac{\psi F(\eta) + \mu(1-F(\eta))(1-m)}{\nu(1-(1-F(\eta))m) + (1-\nu)\big[(1-\psi)F(\eta) + (1-\mu)(1-F(\eta))(1-m)\big]}. \tag{20}$$

Combined with (18), we obtain the fraction of loans insured by high-cost lenders:

$$m^* = 1 - \frac{(\kappa(1-\mu)\psi - (1-\psi)\lambda\mu) F(\eta^*)}{\mu(\lambda-1)(1-\nu)(1-\mu)(1-F(\eta^*))}. \tag{21}$$

Next, the condition to ensure $m^* > 0$ can be expressed as $A < \bar{A}$, where

$$\bar{A} \equiv \frac{\nu\lambda + (1-\nu)(1-\mu)}{(1-\nu)(1-\mu)(\psi-\mu)\kappa} F^{-1}\left(\frac{\mu(\lambda-1)(1-\nu)(1-\mu)}{\kappa(1-\mu)\psi - (1-\psi)\lambda\mu + \mu(\lambda-1)(1-\nu)(1-\mu)}\right) \tag{22}$$

and we substituted η^* from (19) in (21). This constraints is also expressed as $\mu > \underline{\mu}$. For $\psi \to 1$, the bound $\underline{\mu}$ is implicitly but uniquely defined by

$$\underline{\mu} \equiv \frac{\kappa}{(\lambda-1)(1-\nu)} \frac{F\left(\eta^*(\underline{\mu})\right)}{1 - F\left(\eta^*(\underline{\mu})\right)}. \tag{23}$$

We turn to the impact of loan insurance on the equilibrium allocation and welfare. There are two cases: whether or not the liquid equilibrium exists without loan insurance.

A.3.1 Liquid equilibrium exits irrespective of loan insurance option.

For the effect on the price, the total derivative of (20) is:

$$\frac{dp_U^*}{dm^*} = \frac{\partial p_U^*}{\partial m^*} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dp_U^*}\frac{dp_U^*}{dm^*} = \frac{\frac{\partial p_U^*}{\partial m^*}}{1 - \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dp_U^*}} > \frac{\partial p_U^*}{\partial m^*} > 0, \tag{24}$$

since $\frac{\partial p_U^*}{\partial m^*} = \frac{\nu A F(\eta^*)(1-(\eta^*))(1-\mu)}{\left[\nu(1-(1-F(\eta^*))m)+(1-\nu)\left[(1-\psi)F(\eta^*)+(1-\mu)(1-F(\eta^*))(1-m^*)\right]\right]^2}$, $\frac{dp_U^*}{d\eta^*} > 0$, and $\frac{d\eta^*}{dp_U^*} = -(1-\nu)(\psi-\mu) < 0$. Since the price increases in loan insurance, the screening threshold falls, $\frac{d\eta^*}{dm^*} = \frac{d\eta^*}{dp_U^*}\frac{dp_U^*}{dm^*} < 0$. Using that outside financiers and insurers break even in expectation—so welfare adds up the expected value to lenders—welfare can be expressed as:

$$W(m=0) = \underbrace{\nu\lambda p_U^*}_{\text{Shock}} + \underbrace{(1-\nu)\big\{F(\eta^*)[\psi A + (1-\psi)p_U^*] + \big[1-F(\eta^*)\big](\mu A + (1-\mu)p_U^*)\big\}}_{\text{No liquidity shock}} - \underbrace{\int_0^{\eta^*} \eta dF(\eta)}_{\text{Screening costs}} \tag{25}$$

without loan insurance ($A \geq \bar{A}$). With loan insurance, in contrast, welfare is:

$$W(m>0) = \overbrace{\nu\lambda\Big\{p_U^*(F(\eta^*)+(1-F(\eta^*))(1-m^*))+m^*(1-F(\eta^*))p_I^*\Big\}}^{\text{Liquidity Shock}} + \overbrace{(1-\nu)m^*(1-F(\eta^*))p_I^*}^{\text{No shock insured}}$$

$$+ \overbrace{(1-\nu)[F(\eta^*)\big[\psi A+(1-\psi)p_U^*\big]+(1-m^*)(1-F(\eta^*))(\mu A+(1-\mu)p_U^*)]}^{\text{No shock uninsured}} - \overbrace{\int_0^{\eta^*}\eta dF(\eta)}^{\text{Screening costs}}. \tag{26}$$

For $A \geq \bar{A}$, there is no loan insurance and equation (26) collapses to equation (25). This implies that welfare is not affected by the availability of the insurance option. For $A < A$, however, some loans are insured at $t=0$ and sold at $t=1$, and we can substitute p_I^* from condition (17) to get the same functional form as (25)—but with different arguments (p_U^* is higher and $\eta^*(p_U^*)$ is lower). To prove that the introducing the option of loan insurance increases welfare, we show that the function $W(p_U^*)$ increases in p_U^*, using equation (6):

$$\frac{dW}{dp_U^*} = \overbrace{\frac{\partial W}{\partial p_U^*}}^{>0} + \overbrace{\frac{\partial W}{\partial \eta^*}}^{=0}\overbrace{\frac{d\eta^*}{dp_U}}^{<0} > 0, \tag{27}$$

where $\frac{\partial W}{\partial p_U^*} = \nu\lambda + (1-\nu)\big[(1-\mu)(1-F(\eta^*))+(1-\psi)F(\eta^*)\big] > 0$ and $\frac{\partial W}{\partial \eta^*} = [(1-\nu)(\psi-\mu)(A-p_U)-\eta^*]f(\eta^*) = 0$ by an envelope-theorem-type argument (the lender with the threshold screening cost is indifferent for any secondary market price of uninsured loans).

A.3.2 Liquid equilibrium only with loan insurance.

We compare welfare in a liquid equilibrium with insurance in (26) with welfare in a frozen market without insurance:

$$W^F = F(\eta^F)\psi A + (1-F(\eta^F))\mu A - \int_0^{\eta^F}\eta dF(\eta), \tag{28}$$

where $\eta^F = (\psi - \mu\kappa)A$ exceeds the screening threshold when insurance is available, $\eta^L = (1-\nu)(\psi-\mu)(A-p_U^*)$, where $p_U^* > 0$. To compare welfare, we decompose it in three parts:

1. Lenders of mass $1-F(\eta^F)$ are high-cost irrespective of the availability of loan insurance. Those lenders strictly prefer the payoff in equilibrium with insurance, $\kappa\mu A$, to the payoff in equilibrium without insurance, μA, because $\kappa > 1$.

2. Lenders of mass $F(\eta^L)$ are low-cost irrespective of the availability of loan insurance. Those lenders weakly prefer the payoff in equilibrium with insurance, $\nu\lambda p_U^* + (1-\nu)(\psi A + (1-\psi)p_U^*) - \eta_i$, to payoff in equilibrium without insurance $\psi A - \eta_i$ since $p_U^*\lambda \geq A$ in the liquid equilibrium.

3. Lenders of mass $F(\eta^F) - F(\eta^L)$ are low-cost when insurance is available and high-cost otherwise. Those lenders choose not screen with insurance option, so $\kappa p_I > \nu\lambda p_U^* + (1-\nu)(\psi A + (1-\psi)p_U^*) - \eta_i$. Since $p_U^*\lambda \geq A$, this payoff strictly exceeds the payoff from screening, when insurance is unavailable, and an illiquid market, $\psi A - \eta_i$.

In sum, lenders who are always screening weekly prefer and all remaining lenders strictly prefer the equilibrium with insurance option, so aggregate welfare is higher.

Finally, we compare $\underline{\lambda}_U$ and $\underline{\lambda}_I$. When insurance is used, $A < \bar{A}$, the price for uninsured loans is higher, so the liquid market condition (3) is easier to satisfy, resulting in $\underline{\lambda}_I < \underline{\lambda}_U$. Conversely, when insurance is not used, the payoff from insurance is lower than the payoff from not insuring, $p_I^*\kappa = \nu\lambda p_U^* + (1-\nu)\left(\mu A + (1-\mu)\, p_U^*\right)$, so $p_U^* > \frac{\nu\lambda\mu A}{\nu\lambda+(1-\nu)(1-\mu)}$. Thus, when insurance is not used, the price for uninsured loans is higher than what it would be with insurance, which implies $\underline{\lambda}_U < \underline{\lambda}_I$.

A.4 Proof of Propositions 3 and 10

First, we state the remaining formal results. When loan insurance is used, η^* and p_U^* increase in λ; η^* and p_U^* are independent of F; and m^* increases in λ and after a first-order stochastic dominance shift in $F(\cdot)$. When no loan insurance is used, η^* increases after a first-order stochastic dominance shift in $F(\cdot)$ and is independent of λ; and p_U^* decreases after a first-order stochastic dominance shift in $F(\cdot)$ and is independent of λ.

Next, we consider the liquid equilibrium without insurance, $A \geq \bar{A}$ and $\lambda \geq \underline{\lambda}_U$. For the indirect effect on the screening threshold, we use equation (15) to define

$$H \equiv \eta - \frac{(1-\nu)(\psi-\mu)\left[1-\mu-F(\eta)(\psi-\mu)\right]A}{\nu+(1-\nu)\left[1-\mu-F(\eta)(\psi-\mu)\right]} \equiv \eta - \frac{N}{D}, \tag{29}$$

with $H(\eta^*) = 0$ and N and D are the numerator and denominator, respectively. To use the implicit function theorem, we obtain the following partial derivatives of H:

$$\begin{aligned}
H_\eta &= 1 + \frac{(1-\nu)(\psi-\mu)^2\nu A f}{D^2} > 0, \qquad H_\nu = \frac{(\psi-\mu)(1-\mu-F[\psi-\mu])A}{D^2} > 0 \qquad (30)\\
H_\mu &= \frac{(1-\nu)A}{D^2}\Big\{\left[(1-\psi)F + (1+\psi-2\mu)(1-F)\right]\nu + \left[(1-\psi)F + (1-\mu)(1-F)\right]^2(1-\nu)\Big\} > 0,\\
\frac{\partial H}{\partial \lambda} &= 0, \qquad \frac{\partial H}{\partial A} = -\frac{(1-\nu)(\psi-\mu)\left[1-\mu-F(\eta)(\psi-\mu)\right]}{D} < 0.
\end{aligned}$$

These partial derivatives implies the following comparative statics, where we use the notation g_x to denote the partial derivative of some generic function $g(x;\cdot)$:

$$\frac{d\eta^*}{d\nu} = -\frac{H_\nu}{H_\eta} < 0, \;\; \frac{d\eta^*}{d\mu} = -\frac{H_\mu}{H_\eta} < 0, \;\; \frac{d\eta^*}{d\lambda} = -\frac{H_\lambda}{H_\eta} = 0, \;\; \frac{d\eta^*}{dA} = -\frac{H_A}{H_\eta} > 0. \tag{31}$$

For the direct effect on the price, we use equation (5) and obtain

$$\frac{\partial p_U^*}{\partial_U \lambda} = 0, \quad \frac{\partial p_U^*}{\partial A} = \frac{p_U^*}{A} > 0, \quad \frac{dp_U^*}{d\eta^*} = \frac{(\psi-\mu)\, A\nu f}{D^2} > 0, \quad \frac{\partial p_U^*}{\partial \mu} = \frac{\nu(1-F)A}{D^2} > 0, \tag{32}$$

$$\frac{\partial p_U^*}{\partial \nu} = \frac{[\mu + (\psi-\mu)\, F(\eta^*)]\, [(1-\psi)F(\eta^*) + (1-\mu)(1-F(\eta^*))]\, A}{D^2} > 0. \tag{33}$$

The total derivatives for A and λ yield unambiguous results, while the total derivatives for ν and μ may yield ambiguous results:

$$\frac{dp_U^*}{dA} = \frac{\partial p_U^*}{\partial A} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{dA} > 0, \qquad \frac{dp_U^*}{d\lambda} = \frac{\partial p_U^*}{\partial_U \lambda} + \frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\lambda} = 0, \tag{34}$$

$$\frac{dp_U^*}{d\nu} = \underbrace{\frac{\partial p_U^*}{\partial \nu}}_{>0} + \underbrace{\frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\nu}}_{<0} \lesseqqgtr 0, \qquad \frac{dp_U^*}{d\mu} = \underbrace{\frac{\partial p_U^*}{\partial \mu}}_{>0} + \underbrace{\frac{dp_U^*}{d\eta^*}\frac{d\eta^*}{d\mu}}_{<0} \lesseqqgtr 0.. \tag{35}$$

Increases in these parameters increase the price directly but decrease it indirectly via a negative effect on the screening threshold. A set of sufficient condition for the non-monotonicity of p_U^* in μ is $\frac{dp_U^*}{d\mu} \mid_{\mu\to 1} > 0$ and $\frac{dp_U^*}{d\mu} \mid_{\mu\to 1} < 0$. Substituting into (35) from conditions (32) and (30), we evaluate derivatives for the two limits:

$$\frac{dp_U^*}{d\mu} \mid_{\mu\to 1} = \frac{A}{\nu} > 0, \frac{dp_U^*}{d\mu} \mid_{\mu\to 0} = \frac{\nu A(1-F(\eta^*))}{D^2}\left\{1 - \frac{A(1-\nu)f(\eta^*)\,[2\nu + (1-\nu)(1-f(\eta^*))]}{D^2 + (1-\nu)\nu A f(\eta^*)}\right\}.$$

The second derivative is negative for $\psi \to 1$ if $\eta_\mu^* \frac{f(\eta_\mu^*)}{1-F(\eta\mu^*)} > 1$, where $\eta_\mu^* = \eta^* \mid_{\mu\to 0}$.

The result for the first-order stochastic dominance improvement, $\tilde{F} \leq F$, arises as the pricing schedule shifts down, $\frac{dp_U}{dF(\eta)} = \frac{(\psi-\mu)\nu A}{D^2} > 0$. Thus, the screening threshold increases.

Next, we derive the lower bound on ψ in the liquid equilibrium with loan insurance, $A < \bar{A}$ and $\lambda \geq \underline{\lambda}_I$. The comparative statics below hold for $\psi > \underline{\psi}$. For $\psi \leq \underline{\psi}$, however, insurance is strictly preferred by high-cost lenders, $m^* = 1$:

$$\kappa p_I > \nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U). \tag{36}$$

Substituting $p_I = \mu A$ and the price for uninsured loans, $p_U(m=1) = \frac{\nu\psi A}{\nu + (1-\nu)(1-\psi)}$, into (36) yields:

$$\psi > \frac{\lambda\mu}{(1-\nu)(1-\mu) + \lambda(\mu + \nu(1-\mu))} \equiv \underline{\psi} \in (\mu, 1). \tag{37}$$

For the screening threshold, we use equation (19) and $D \equiv \nu\lambda + (1-\nu)\,(1-\mu)$:

$$\frac{d\eta^*}{dA} = \frac{(1-\nu)\,(1-\mu)\,(\psi-\mu)\kappa}{D} > 0, \quad \frac{d\eta^*}{d\lambda} = -\frac{\nu(1-\nu)^2\mu(1-\mu)(\psi-\mu)A}{D^2} < 0,$$

$$\frac{d\eta^*}{d\mu} = -\frac{(1-\nu)\,\kappa A\left((1+\psi-2\mu)\nu\lambda + (1-\nu)(1-\mu)^2\right)}{D^2} < 0,$$

$$\frac{d\eta^*}{d\nu} = -\frac{(1-\mu)(\psi-\mu)A\left[\kappa^2+\mu(1-\nu)^2(\lambda-1)\right]}{D^2} < 0, \quad \frac{d\eta^*}{d\nu} = \frac{(1-\nu)(1-\mu)\kappa A}{D}.$$

For the direct effect on the price, we use equation (18) to obtain:

$$\begin{aligned}
\frac{dp_U^*}{dA} &= \frac{\nu\lambda\mu}{D} > 0, \quad \frac{dp_U^*}{d\mu} = \frac{\nu\lambda A\kappa}{D^2} > 0, \quad \frac{dp_U^*}{d\psi} = 0, \\
\frac{dp_U^*}{d\lambda} &= \frac{\nu(1-\nu)\mu(1-\mu)A}{D^2} > 0, \quad \frac{dp_U^*}{d\nu} = \frac{\lambda\mu(1-\mu)A}{D^2} > 0.
\end{aligned}$$

Both equilibrium price and screening threshold are independent of the cdf F.

A.4.1 Effect on fraction of high-cost lenders who insure

Equation (21) defines m^* as a function of η^*. Therefore, the total effect of parameters $\alpha \in \{\nu, \lambda, \mu\}$ on m^* consists of a direct and indirect effect through screening threshold, $\frac{dm^*}{d\alpha} = \frac{\partial m^*}{\partial\alpha} + \frac{dm^*}{d\eta^*}\frac{d\eta^*}{d\alpha}$. We have:

$$\begin{aligned}
\frac{dm^*}{d\eta^*} &= -\frac{[\kappa(1-\mu)\psi-(1-\psi)\lambda\mu]f}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)^2} < 0, \quad \frac{\partial m^*}{\partial\lambda} = \frac{(1-\mu)\psi-(1-\psi)\mu}{\mu(1-\mu)(1-\nu)(\lambda-1)^2(1-F)}F > 0, \\
\frac{\partial m^*}{\mu} &= \frac{\kappa\psi(1-\mu)^2+(1-\psi)\lambda\mu^2}{\mu^2(1-\mu)^2(\lambda-1)(1-\nu)(1-F(\eta^*))}F(\eta^*) > 0, \qquad \frac{\partial m^*}{\partial A} = 0, \\
\frac{\partial m^*}{\partial\nu} &= -\frac{(1-\mu)\psi-(1-\psi)\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)^2(1-F(\eta^*))}\lambda F(\eta^*) < 0 \\
\frac{\partial m^*}{\partial F} &= -\frac{\kappa(1-\mu)\psi-(1-\psi)\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)^2} < 0, \quad \frac{\partial m^*}{\partial\psi} = -\frac{\kappa(1-\mu)+\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-\nu)(1-F)}F < 0.
\end{aligned}$$

The following total derivatives are unambiguous, $\frac{dm^*}{d\mu} > 0$, $\frac{dm^*}{dA} < 0$, $\frac{dm^*}{d\psi} < 0$, and the first-order stochastic dominance improvement, $\frac{dm^*}{dF} < 0$. The total effect of ν on m^* can be ambiguous since the direct effect is negative and the indirect one is positive. A sufficient condition for non-monotonicity is the opposite sign of derivatives at both limits, $\nu \to \{0, 1\}$:

$$\begin{aligned}
\lim_{\nu\to 0}\frac{dm^*}{d\nu} &= -\frac{(1-\mu)\psi-(1-\psi)\mu}{\mu(1-\mu)(\lambda-1)(1-F)}\lambda F + A(1+(\lambda-1)\mu)\frac{\psi-\mu}{1-\mu}\frac{(1-\mu)\psi-(1-\psi)\lambda\mu}{\mu(1-\mu)(\lambda-1)(1-F)^2}f \\
\lim_{\nu\to 1}\frac{dm^*}{d\nu} &= -\infty.
\end{aligned}$$

The sufficient condition for non-monotonicity is $\frac{F(1-F)}{f}\mid_{\nu\to 0} < A(1+(\lambda-1)\mu)\frac{\psi-\mu}{1-\mu}\frac{(1-\mu)\psi-(1-\psi)\lambda\mu}{(1-\mu)\lambda\psi-(1-\psi)\lambda\mu}$.

Finally, to complete the proof of Proposition 10, we need to establish that $\frac{d\bar{A}}{d\psi} < 0$, which is straightforward using equation (22) since F^{-1} is increasing.

A.5 Proof of Proposition 4

The illiquid equilibrium always exists. If the price for uninsured loans is zero, $p_U^* = 0$, only lemons are sold in this market, which justifies the zero price. When the liquid equilibrium does not exist, $\lambda < \min\{\underline{\lambda}_U, \underline{\lambda}_I\}$, the illiquid equilibrium is unique. We have already shown that high-cost lenders always insure, $m^* = 1$, because of the gains from trade on the market for insured loans. The screening threshold is given by the indifference condition of the marginal lender who compares payoffs from screening, $A - \eta$, and not screening but insuring, $\kappa\mu A$. Equating those yields the threshold $\eta^{FI} = (1 - \kappa\mu)A$ that is below the threshold in the illiquid equilibrium where insurance is unavailable, $\eta^F = (1 - \mu)A$.

When $(1 - \kappa\mu)A \geq \bar{\eta}$, screening is full, $\eta^F > \eta^{FI} \geq \bar{\eta}$, irrespective of loan insurance. When $(1 - \kappa\mu)A < \bar{\eta}$, screening is partial, $\eta^{FI} < \bar{\eta}$, and there are three types of lenders. First, lenders of mass $1 - F(\eta^F)$ are high-cost irrespective of the availability of loan insurance. Those lenders strictly prefer the payoff in equilibrium with insurance, $\kappa\mu A$, to the payoff in equilibrium without insurance option, μA. Second, lenders of mass $F(\eta^{FI})$ are low-cost irrespective of the availability of loan insurance. Those lenders are indifferent about insurance since their payoff is always $A - \eta_i$. Third, lenders of mass $F(\eta^F) - F(\eta^{FI})$ are high-cost when insurance is available and low-cost otherwise. Those lenders do not screen because $\kappa\mu A > A - \eta_i$ and since the payoff when screening is the same in both equilibria. Thus, they strictly prefer the equilibrium with insurance.

In sum, all high-cost lenders in equilibrium with insurance are better off than in the equilibrium without the insurance option and no lender is worse off. Therefore, the aggregate welfare is superior in the equilibrium with insurance option.

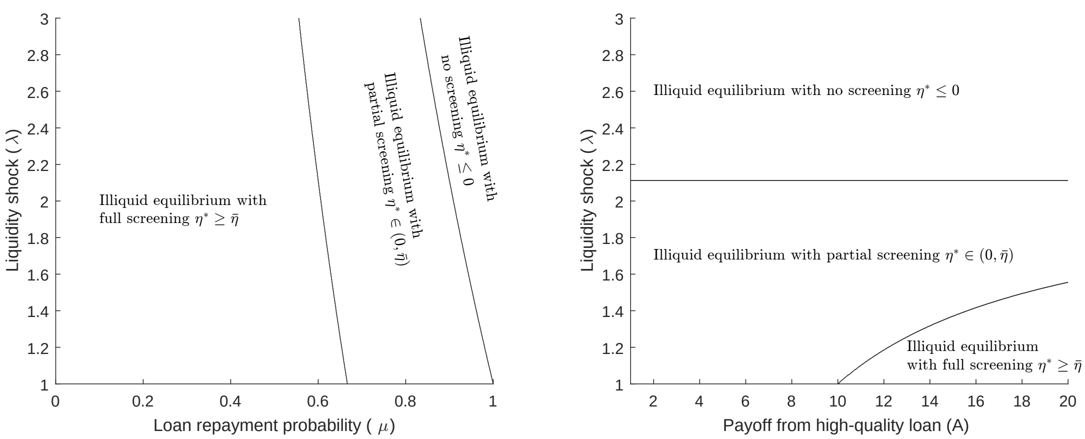

Figure 15: Type of illiquid equilibrium: all high-cost lenders insure, $m^* = 1$. Parameters: $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, left panel: $A = 3$, right panel: $\mu = 0.9$.

A.6 Proof of Proposition 5

We prove existence and the result on the intensive margin by showing that (i) welfare increases in m on the interval $m \in [0, m^*]$; and (ii) welfare decreases in m for $m \to 1$. Since the welfare function in equation (8) is continuous and defined everywhere in the interval $m \in (0,1)$, the planner's choice satisfies $m^P \in (m^*, 1)$, thus exceeding the competitive m^*. The total derivative of welfare, $\frac{dW}{dm} = \frac{\partial W}{\partial m} + \frac{\partial W}{\partial p_U^*}\frac{dp_U^*}{dm} + \frac{\partial W}{\partial \eta^*}\frac{d\eta^*}{dm}$, is evaluated using (26):

$$\begin{aligned}
\frac{\partial W}{\partial m} &= (1-F)\Big[\kappa\mu A - \nu\lambda p_U^* - (1-\nu)(\mu A + (1-\mu)p_U^*)\Big] = 0 \\
\frac{\partial W}{\partial p_U^*} &= \nu\lambda(F + (1-F)(1-m)) + (1-\nu)\left[F(1-\psi) + (1-F)(1-m)(1-\mu)\right] > 0 \\
\frac{\partial W}{\partial \eta^*} &= f\Big[(1-\nu)(\psi-\mu)(A-p_U^*) - \eta^* + m^*\left[\nu\lambda p_U^* + (1-\nu)(\mu A + (1-\mu)p_U^*) - \kappa p_I^*\right]\Big] = 0.
\end{aligned}$$

Since $\frac{dp_U^*}{dm} > 0$ and $\frac{d\eta^*}{dm} < 0$, the total derivative is positive at the decentralized equilibrium due to the positive pecuniary externality, $\frac{dW}{dm}\mid_{m=m^*} = \frac{\partial W}{\partial p_U^*}\frac{dp_U^*}{dm} > 0$. The total derivative is also positive for any $\tilde{m} < m^*$ since $\frac{\partial W}{\partial m}\mid_{\tilde{m}} > 0$, $\frac{\partial W}{\partial p}\mid_{\tilde{m}} > 0$, $\frac{\partial W}{\partial \eta^*}\mid_{\tilde{m}} < 0$.

In the remainder of the proof, we focus on $\psi \to 1$. We consider the limit $m \to 1$, so the price of uninsured loans equals payoff of high-quality loans and there is no screening, $\lim_{m\to 1} p_U = A$, $\lim_{m\to 1}\eta = 0$. Hence, we can evaluate partial derivatives, $\lim_{m\to 1}\frac{\partial W}{\partial m} = -\kappa(1-\mu)A$, $\lim_{m\to 1}\frac{\partial W}{\partial p} = 0$ and $\lim_{m\to 1}\frac{\partial W}{\partial \eta^*} = f\kappa(1-\mu)A$. This implies that total derivative is negative, $\lim_{m\to 1}\frac{dW}{dm} < 0$. From the proof of Proposition 1 (see equation 24), a higher m increases price in secondary markets for uninsured loans and decreases screening, $p_U^P > p_U^*$ and $\eta^P < \eta^*$.

To prove the result on the extensive margin, we compare the threshold of A at which insurance is zero in the decentralized equilibrium, $\bar{A}$, and in the constrained efficient case, $\bar{A}^P$. $\bar{A}$ satisfies $m^* = 0$ and $\frac{\partial W}{\partial m} = (1-F)(\kappa\mu A - \nu\lambda p_U^* - (1-\nu)(\mu A + (1-\mu)p_U^*)) = 0$. Substituting p_U^* from the break-even condition in $\frac{\partial W}{\partial m} = 0$, we get:

$$\frac{\nu(F+(1-F)\mu)}{\nu(F+(1-F)\mu) + (1-\mu)(1-F)} = \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{38}$$

The payoff $\bar{A}^P$ has to satisfy $m^P = 0$ and $\frac{dW}{dm} = 0$. After substituting p_U^*, we get

$$\frac{\nu(F+(1-F)\mu)}{\nu(F+(1-F)\mu) + (1-\mu)(1-F)} = \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)} + \frac{\frac{\partial W}{\partial p}\frac{dp_U}{dm}}{(1-F)(\nu\lambda + (1-\nu)(1-\mu))\bar{A}^P}. \tag{39}$$

Combining this condition with (38), we obtain

$$\frac{\nu(F+(1-F)\mu)}{\nu(F+(1-F)\mu) + (1-\mu)(1-F)}\mid_{A=\bar{A}} < \frac{\nu(F+(1-F)\mu)}{\nu(F+(1-F)\mu) + (1-\mu)(1-F)}\mid_{A=\bar{A}^P}.$$

Since $\frac{d\eta^*}{dA} > 0$, this condition implies $\bar{A}^P > \bar{A}$.

A.7 Proof of Proposition 6

Welfare for an illiquid market is $W^F = \nu(\lambda-1)\mu A(1-F(\eta^F)) + \left(F(\eta^L) + (1-F(\eta^L))\mu\right) A - \int_0^{\eta^F} dF(\tilde{\eta})$, where $\eta^F = (1-\kappa\mu)A$. Welfare is $W^L = \nu(\lambda-1)\left(p_U + (\mu A - p_U)(1-F(\eta^L))m\right) + \left(F(\eta^L) + (1-F(\eta^L))\mu\right) A - \int_0^{\eta^L} dF(\tilde{\eta})$ for a liquid market, where $\eta^L = \eta$ is given by (6), (9), and $p_U\lambda \geq A$. At some $\tilde{\lambda}_I$, the planner is indifferent between the illiquid equilibrium and equilibrium liquified with intervention, $W^F = W^L$:

$$\overbrace{\nu(\tilde{\lambda}_I - 1)\left(p_U + (\mu A - p_U)(1-F(\eta^L))m - \mu A(1-F(\eta^F))\right)}^{\text{higher gains from trade in liquid equilibrium}(>0)}$$
$$= \underbrace{(1-\mu)A\left(F(\eta^F) - F(\eta^L)\right) - \left(\int_0^{\eta^F} dF(\tilde{\eta}) - \int_0^{\eta^L} dF(\tilde{\eta})\right)}_{\text{higher net benefits of screening in illiquid equilibrium}(>0)}. \tag{40}$$

Next, we show that the above equation implicitly and uniquely defines a $\tilde{\lambda}_I \in (1,\infty)$. For existence, the gains from trade term dominates for $\lambda \to \infty$, so $\tilde{\lambda}_I < \infty$, while the gains from trade vanish for $\lambda \to 1$. The existence of $\tilde{\lambda}_I$ follows. For uniqueness, we start with an intermediate result. At $\lambda = \tilde{\lambda}_I$, the liquid equilibrium can be sustained only with a level of insurance that exceeds the level in the decentralized equilibrium, because the decentralized market was illiquid. Thus, the first-order condition for the optimal insurance level m^P is

$$\frac{dW^L}{dm} + \gamma\frac{dp_U}{dm} = 0, \tag{41}$$

where γ is the Lagrange multiplier for $p_U\lambda \geq A$. At $\lambda = \tilde{\lambda}_I$, the planner is indifferent between frozen and liquid markets (via a subsidy $p_U = A/\lambda$), so $\gamma \geq 0$. Since $\frac{dp_U}{dm} > 0$ and $\frac{dW}{dm} < 0$, the planner would chose fewer insured lenders without the liquidity constraint. The total derivative of the welfare difference, $W^L\,|_{p_U=A/\lambda} - W^F$, with respect to λ is:

$$\underbrace{\frac{dW^L}{dm}}_{<0}\underbrace{\frac{dm\,|_{p_U=A/\lambda}}{d\lambda}}_{<0} + \underbrace{\nu\left(\frac{A}{\lambda} + \left(\mu A - \frac{A}{\lambda}\right)(1-F(\eta^L))m - \mu A(1-F(\eta^F))\right)}_{\text{higher gains from trade in liquid eq.}(>0)} > 0. \tag{42}$$

Equation (41) implies $\frac{dW^L}{dm} < 0$ at $\lambda = \tilde{\lambda}_I$ and (40) implies that the gains from trade in the liquid equilibrium are higher. The sign, $\frac{dm|_{p_U=A/\lambda}}{d\lambda} < 0$, is due to the positive effect of insurance on price, $\frac{dp_U}{dm} > 0$, (proven already) and that a higher λ reduces the price needed for liquifying the market. Hence, the welfare difference between a liquid and frozen market increases monotonically in λ and, thus, (40) defines $\tilde{\lambda}_I$ uniquely.

A.8 Proof of Proposition 7

The objective functions of the planner in (8) and the regulator in (11) are identical—except for the interim-date endowment term—and so are the indifference condition for screening and the break-even condition of outside financiers. Therefore, a Pigouvian subsidy is set to achieve the constrained efficient price in the secondary market for uninsured loans, thus achieving the constrained efficiency. Solving equation (10) and evaluating at $p(b^{I*}) = p_U^P$ yields the efficient value of b_I^*, as stated.

A.9 Proof of Proposition 8

Definition 4. *A competitive equilibrium with both subsidies comprises screening* $\{s_i\}$*, insurance* $\{\ell_i\}$*, the sales of insured and uninsured loans* $\left\{q_i^I, q_i^U\right\}$*, an insurance subsidy* b_I*, an uninsured loans purchase subsidy* b_U*, lump-sum taxes* $\{T_i\}$*, and prices* p_I *and* p_U *such that:*

1. *At* $t = 1$*, each lender* i *optimally chooses its sales of insured and uninsured loans in secondary markets for each realized shock* $\lambda_i \in \{1, \lambda\}$*, denoted by* $q_i^I(s_i, \lambda_i, \ell_i)$ *and* $q_i^U(s_i, \lambda_i, \ell_i, b_U)$*, given* p_U, p_I*,* s_i*,* ℓ_i*,* b_I*, and* b_U*.*

2. *At* $t = 1$*, prices* p_I *and* p_U *are set for outside financiers to break even in expectation, given* $\{s_i\}$*,* $\{\ell_i\}$*,* $\{q_i^I(\cdot), q_i^U(\cdot)\}$*, and* b_I*.*

3. *At* $t = 0$*, the fee* k *is set for outside financiers to break even in expectation, given screening* s_i *and insurance* ℓ_i *choices.*

4. *At* $t = 0$*, each lender* i *chooses its screening* s_i *and loan insurance* ℓ_i *to maximize expected utility, given* p_I *and* p_U*,* $q_i^I(\cdot)$ *and* $q_i^U(\cdot)$*,* b_I*,* b_U*, and* T_i*:*

$$\max_{s_i, \ell_i, c_{i1}, c_{i2}} \quad \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - s_i \eta_i\right] \qquad \textit{subject to}$$
$$\begin{aligned} c_{i1} &\leq q_i^U(s_i, \lambda_i, \ell_i, b_U)\,(p_U + b_U) + q_i^I(s_i, \lambda_i, \ell_i)\, p_I + b_I + n - T, \\ c_{i2} &\leq A\big[\mu + (1-\mu)\, s_i\big]\big[1 - \ell_i - q_i^U(s_i, \lambda_i, \ell_i, b_U)\big] + \mu A\big[\ell_i - q_i^I(s_i, \lambda_i, \ell_i)\big]. \end{aligned}$$

5. *At* $t = 0$*, regulator chooses insurance subsidy* b_I *and uninsured loans purchase subsidy* b_U *to maximize welfare of all lenders subject to the balance budget constraint, specifically* $T = b_U q^U + b_I m^*(1 - F(\eta^*))$ *for each lender.*

It is immediate that an illiquid equilibrium, $p_U^* = 0$, can be eliminated with a subsidy $b_U^R = A/\lambda$. It breaks the existence of an illiquid equilibrium, $p_U^* + b < A/\lambda$. Appendix A.7 defines $\tilde{\lambda}_I$ for the dominance of the liquid equilibrium. Next, we compare the welfare of achieving the same target price, $p_U^T < A$, after an insurance subsidy, $p_U^T = p_U(b_I)$, and after an uninsured loan purchase subsidy, $p_U^T = p_U + b_U$. Using the insurance indifference condition (10), the welfare with an insurance subsidy (11) can be rewritten as

$$
\begin{aligned}
W(b_I) &= \overbrace{\nu\lambda p_U^T + (1-\nu)\left(F(\eta)+\mu\left(1-F(\eta)\right)\right)A + (1-\nu)\left(1-F(\eta)\right)(1-\mu)\,p_U^T + n}^{\text{value to lenders}} \\
&\quad - \underbrace{\int_0^{\eta} \tilde{\eta}\, dF(\tilde{\eta})}_{\text{screening costs}} - \underbrace{\kappa b_I m\left(1-F(\eta)\right)}_{\text{policy costs}},
\end{aligned}
$$

where $b_I(p_U^T), \eta(p_U^T), m(\eta(p_U^T))$ are given by (10), (6), and (9), respectively. In contrast, welfare with effective subsidized purchases of uninsured loans, $p_U^T > p_U^*$, is

$$
\begin{aligned}
W(b_U) &= \overbrace{\nu\lambda p_U^T + (1-\nu)\left(F(\eta)+\mu\left(1-F(\eta)\right)\right)A + (1-\nu)\left(1-F(\eta)\right)(1-\mu)\,p_U^T + n}^{\text{value to lenders}} \\
&\quad + n - \underbrace{\int_0^{\eta} \tilde{\eta}\, dF(\tilde{\eta})}_{\text{screening costs}} - \underbrace{\kappa b_U q_U}_{\text{policy costs}},
\end{aligned}
$$

where p_U is given by (5), $b_U = p_U^T - p_U$, and $\eta = (1-\nu)(1-\mu)(A - p_U^T)$.

Since the screening threshold is the same in both cases, these welfare expressions differ only in the policy cost term. Welfare under insurance subsidy exceeds welfare under subsidized purchases if $b_I m(1-F) < b_U q_U$, which holds for $p_U^T < A$:

$$
b_I m\,(1-F) < p_U^T\left(\nu + (1-\nu)(1-\mu)(1-F)\right) - \nu\left(F + \mu(1-F)\right)A. \tag{43}
$$

Substituting for b_I from (10) and for $(1-F)m = \frac{p_U^T(\nu+(1-\nu)(1-\mu)(1-F)) - \nu(F+\mu(1-F))A}{p_U^T(1-\mu+\mu\nu)-\mu\nu A}$ from (20), we can rewrite (43) as $1 > \frac{1}{\kappa}\frac{[\nu\lambda+(1-\nu)(1-\mu)]p_U^T - \nu\mu A\lambda}{p_U^T(1-\mu+\mu\nu)-\nu\mu A}$, which collapses to $p_U^T < A$.

A.10 Proof of Proposition 9

We focus on the interval in which (i) lenders sell high-quality loans upon a liquidity shock (where $R < \overline{R} \equiv \frac{\nu\mu\lambda}{\nu+(1-\nu)(1-\mu)}$ suffices for $p_U^*\lambda > A$), and (ii) adverse selection remains, so lenders do not sell high-quality projects without a liquidity shock ($p_U^* < A$ implies $R > \underline{R} \equiv \frac{\nu\mu}{\nu+(1-\nu)(1-\mu)} > \overline{R}$). The break-even conditions are $p_I^* = \frac{\mu A}{R}$ for insured loans and

$$
p_U^* = \frac{1}{R}\frac{\nu\left(F(\eta) + (1-F(\eta))(1-m)\mu\right)A}{\nu\left(F(\eta) + (1-F(\eta))(1-m)\mu\right) + (1-\mu)(1-F(\eta))(1-m)} \tag{44}
$$

for uninsured loans. Without insurance, combining (44) with (6) and substituting $m = 0$ gives $\eta^* = (1-\nu)(1-\mu)\big(A - \frac{1}{R}\frac{\nu A(\mu+(1-\mu)F(\eta^*))}{\nu+(1-\nu)(1-\mu)(1-F(\eta^*))}\big)$, so $\frac{d\eta^*}{dR} > 0$ and $\frac{dp_U^*}{dR} < 0$. With insurance, high-cost lenders are indifferent between payoff when insuring $\nu\lambda\frac{\mu A}{R} + (1-$

$\nu)\mu A \max\left\{\frac{1}{R},1\right\}$ and when not, $\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U)$. Thus:

$$p_U = \frac{1}{R}\frac{(\nu\lambda + \max\{1-R,0\}(1-\nu))\mu A}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{45}$$

Thus, $\frac{dp_U^*}{dR} < 0$ and $\frac{d\eta^*}{dR} > 0$ because of (6). Combining equations (45) and (44) yields

$$m^* = 1 - \frac{(\kappa - \frac{\max\{1-R,0\}(1-\nu)\mu}{(1-\mu)})F}{(1-F)\big(\mu(\lambda-1)(1-\nu) + \frac{\max\{1-R,0\}(1-\nu)\mu(1-\mu+\nu\mu)}{(1-\mu)\nu}\big)}, \tag{46}$$

resulting in $\frac{dm^*}{dR} < 0$ since $\frac{d\eta^*}{dR} > 0$, $\frac{\partial m^*}{\partial R} < 0$ and $\frac{\partial m^*}{\partial \eta^*} < 0$. The payoff threshold below which insurance takes place, $\bar{A}$, is obtained by solving for $m^* = 0$:

$$\bar{A} = \frac{\nu\lambda + (1-\nu)(1-\mu)F^{-1}\left(\frac{\mu(1-\mu)\nu(1-\nu)(\lambda-1)+\mu\nu(1-\mu+\mu\nu)\max(1-R,0)}{\kappa(1-\mu)\nu+(1-\mu)\mu\nu(1-\nu)(\lambda-1)-\mu^2(1-\nu)^2\max(1-R,0)}\right)}{(1-\nu)(1-\mu)\left[\left(1-\frac{\mu}{R}\right)\nu\lambda + (1-\nu)\left(1-\mu\left[1+\max\left(\frac{1}{R}-1,0\right)\right]\right)\right]}. \tag{47}$$

Since $F^{-1}(\cdot)$ is a non-decreasing function, we find that $\frac{d\bar{A}}{dR} < 0$.

A.11 Proof of Proposition 11

We derive the privately optimal insurance coverage ω^*. The price in secondary markets for insured loans is $p_I^* = \frac{\nu+(1-\nu)(1-\mu)\omega}{\nu+(1-\nu)(1-\mu)}\mu A$, which implies that p_I^* monotonically increases in insurance coverage, $\frac{dp_I^*}{d\omega} > 0$. Lenders who insure do not screen, so their problem is

$$\max_{\omega} \nu\lambda p_I + (1-\nu)\left(\mu(A-k) + (1-\mu)p_I\right) = \frac{\nu\kappa + (1-\nu)(1-\mu)\left(\kappa + \nu(\omega-1)(\lambda-1)\right)}{\nu + (1-\nu)(1-\mu)}\mu A,$$

which increases in ω. Thus, the corner solution $\omega^* = 1$ is optimal.

Next, we consider the socially optimal choice of insurance coverage. The payoff of uninsured low-cost lenders $(\nu\lambda p_U + (1-\nu)A - \eta_i)$ and high-cost lenders $(\nu\lambda p_U + (1-\nu)(\mu A + (1-\mu)p_U) - \eta_i)$ also increases in ω due to the positive externality of insurance coverage on price of uninsured loans $\frac{dp_U}{d\omega} = \frac{dp_U}{dp_I}\frac{dp_I}{d\omega} > 0$. Therefore, a planner who maximizes aggregate welfare also chooses full insurance coverage, $\omega^{SP} = 1$:

$$\begin{aligned}
\omega^{SP} &= \arg\max_{\omega} \overbrace{\nu\lambda p_U\left(F + (1-F)(1-m)\right) + (1-\nu)\left(FA + (1-F)(1-m)(\mu A + (1-\mu)p_U)\right)}^{\text{Value to uninsured lenders}} \\
&\quad + \underbrace{\left(\nu\lambda p_I + (1-\nu)\left(\mu(A-k) + (1-\mu)p_I\right)\right)m(1-F)}_{\text{Value to insured lenders}} - \underbrace{\int_0^{\eta} \tilde{\eta}\, dF(\tilde{\eta})}_{\text{Screening costs}} \\
&= \arg\max_{\omega} \nu\lambda p_U + (1-\nu)\left(FA + (1-F)(\mu A + (1-\mu)p)\right) - \int_0^{\eta} \tilde{\eta}\, dF(\tilde{\eta}) \qquad (48)
\end{aligned}$$

subject to (6) and (20), where equation (48) obtains after substituting the indifference condition (17). The solution follows from $\frac{dW}{d\omega} = \Big(\underbrace{\frac{\partial W}{\partial p_U^*}}_{>0} + \underbrace{\frac{\partial W}{\partial \eta^*}}_{=0} \underbrace{\frac{d\eta^*}{dp_U^*}}_{<0} \Big) \underbrace{\frac{dp_U^*}{dp_I^*}}_{>0} \underbrace{\frac{dp^{I*}}{d\omega}}_{>0} > 0$.

A.12 Proof of Proposition 12

First, we study the equilibrium with insurance. Insurers' break even condition, $k = (1-k)A(1-\mu)$, determines the insurance fee: $k = \frac{A(1-\mu)}{1+A(1-\mu)}$. The high-cost lenders are indifferent between the insurance payoff $(1-k)A(\nu\lambda+1-\nu) = \kappa A(\mu-\delta(1-\mu))$ and the non-insurance payoff $\nu\lambda p'_U + (1-\nu)(\mu A + (1-\mu)p'_U$. Equating those payoffs gives a condition for the price of uninsured loans:

$$p_U^{*\prime} = \frac{\nu\lambda\mu A - \kappa A\delta(1-\mu)}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{49}$$

Combining this equation with the break-even condition of outside financiers (20) gives

$$m^{*\prime} = 1 - \frac{\kappa F(1-\delta)}{(1-F)\left[\mu(1-\nu)(\lambda-1) - \frac{\kappa}{\nu}\delta(\nu+(1-\nu)(1-\mu)\right]}. \tag{50}$$

Finally, substituting p'_U from (49) into (6) gives:

$$\eta^{*\prime} = \frac{(1-\nu)(1-\mu)^2 A\kappa(1+\delta)}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{51}$$

The price in equation (49) must satisfy condition (3) to ensure a liquid equilibrium. Thus, a necessary condition for a liquid equilibrium when insurance is used is $\nu\left[\mu-\delta(1-\mu)\right]\lambda^2 - \left[\nu+\delta(1-\mu)(1-\nu)\right]\lambda - (1-\nu)(1-\mu) \geq 0$. Since only the larger root of this quadratic condition is positive, the condition collapses to $\lambda \geq \underline{\lambda}'_I \equiv \frac{\nu+\delta(1-\mu)(1-\nu)}{2\nu[\mu-\delta(1-\mu)]} + \sqrt{\frac{[\nu+\delta(1-\mu)(1-\nu)]^2}{4\nu^2[\mu-\delta(1-\mu)]^2} + \frac{(1-\mu)(1-\nu)}{\nu[\mu-\delta(1-\mu)]}}$. For $\mu A > 1$, $\delta > 0$ and the threshold for the existence of liquid equilibrium is higher, $\underline{\lambda}'_I > \underline{\lambda}_I$. Insurance takes place on the subset $A < \bar{A}'$, where the threshold $\bar{A}'$ is implicitly defined by a combination of (49) and (20), where $m^{*\prime} = 0$:

$$\kappa(1-\delta)F(\eta) = (1-F(\eta))\left[\mu(1-\nu)(\lambda-1) - \frac{\kappa}{\nu}\delta(\nu+(1-\nu)(1-\mu))\right], \tag{52}$$

where $\eta(\bar{A}')$ and $\delta(\bar{A}')$. For $\mu A > 1$, $\delta > 0$ which imply $\bar{A}' < \bar{A}$. $\bar{A}'$ is unique since $\frac{dm}{dA} < 0$. To prove this we define Ξ as the difference of the two expressions for price, (49) and (20):

$$\Xi \equiv \frac{F + (1-F)(1-m)\mu}{\nu F + (1-F)(1-m)(\nu+(1-\nu)(1-\mu))} - \frac{\lambda\mu - \frac{\kappa}{\nu}(1-\mu)\delta}{\nu\lambda + (1-\nu)(1-\mu)} = 0$$

and then show that $\frac{dm}{dA} = -\frac{\partial\Xi/\partial A}{\partial\Xi/\partial m} < 0$. If $\mu A > 1$ and $A < \bar{A}'$, then $\delta > 0$ and $\eta^{*\prime}$ given by (51) exceeds η^* given by (19), $p_U^{*\prime}$ given by (49) is smaller than price p_U^* given by (18), which together with dp_U/dm implies that $m^{*\prime} < m^*$.

Second, we consider normative implications. We proceed similarly as in proof of Proposition 5. The welfare when insurance is used can be expressed as

$$W(m>0)=\overbrace{\nu\lambda\Big\{p_U^{*\prime}(F(\eta^{*\prime})+(1-F(\eta^{*\prime}))(1-m))+m(1-F(\eta^{*\prime}))(1-k)A\Big\}}^{\text{Liquidity Shock}}-\overbrace{\int_0^{\eta^{*\prime}}\eta dF(\eta)}^{\text{Screening costs}}$$

$$+\overbrace{(1-\nu)m(1-F(\eta^{*\prime}))(1-k)A}^{\text{No shock insured}}+\overbrace{(1-\nu)[F(\eta^{*\prime})A+(1-m)(1-F(\eta^{*\prime}))(\mu A+(1-\mu)p_U^{*\prime})]}^{\text{No shock uninsured}}. \tag{53}$$

The result on the intensive margin is proved by showing that (i) welfare increases in m on the interval $m\in[0,m^{*\prime}]$; and (ii) welfare decreases in m for $m\to 1$. Since the welfare function in equation (53) is continuous and defined everywhere in the interval $m\in(0,1)$, the planner's choice satisfies $m^P\in(m^{*\prime},1)$, thus exceeding the competitive $m^{*\prime}$. The total derivative of welfare, $\frac{dW}{dm}=\frac{\partial W}{\partial m}+\frac{\partial W}{\partial p_U^{*\prime}}\frac{dp_U^{*\prime}}{dm}+\frac{\partial W}{\partial \eta^{*\prime}}\frac{d\eta^{*\prime}}{dm}$, is evaluated:

$$\begin{aligned}
\frac{\partial W}{\partial m} &= (1-F)\Big[\kappa(1-k)A-\nu\lambda p_U^{*\prime}-(1-\nu)(\mu A+(1-\mu)p_U^{*\prime})\Big]=0\\
\frac{\partial W}{\partial p_U^{*\prime}} &= \nu\lambda(F+(1-F)(1-m))+(1-\nu)(1-F)(1-m)(1-\mu)>0\\
\frac{\partial W}{\partial \eta^{*\prime}} &= f\Big[(1-\nu)(1-\mu)(A-p_U^{*})-\eta^{*\prime}+m\left[\nu\lambda p_U^{*\prime}+(1-\nu)(\mu A+(1-\mu)p_U^{*\prime})-\kappa(1-k)A\right]\Big]=0.
\end{aligned}$$

Since $\frac{dp_U^*}{dm}>0$ and $\frac{d\eta^*}{dm}<0$, the total derivative is positive at the decentralized equilibrium due to the positive pecuniary externality, $\frac{dW}{dm}\mid_{m=m^*}=\frac{\partial W}{\partial p_U^*}\frac{dp_U^*}{dm}>0$. The total derivative is also positive for any $\tilde{m}<m^*$ since $\frac{\partial W}{\partial m}\mid_{\tilde{m}}>0$, $\frac{\partial W}{\partial p}\mid_{\tilde{m}}>0$, $\frac{\partial W}{\partial \eta}\mid_{\tilde{m}}<0$. Next, we consider the limit $m\to 1$, so the price of uninsured loans equals payoff of high-quality loans and there is no screening, $\lim_{m\to 1}p_U=A$, $\lim_{m\to 1}\eta=0$. Hence, we can evaluate partial derivatives, $\lim_{m\to 1}\frac{\partial W}{\partial m}=-\kappa kA$, $\lim_{m\to 1}\frac{\partial W}{\partial p}=0$ and $\lim_{m\to 1}\frac{\partial W}{\partial \eta}=f\kappa kA$. This implies $\lim_{m\to 1}\frac{dW}{dm}<0$.

To prove the result on the extensive margin, we compare the threshold of A at which insurance is zero in the decentralized equilibrium, $\bar{A}'$, and in the constrained efficient case, $\bar{A}^{P\prime}$. $\bar{A}'$ satisfies $m^*=0$ and $\frac{\partial W}{\partial m}=(1-F)(\kappa\mu A-\nu\lambda p_U^{*\prime}-(1-\nu)(\mu A+(1-\mu)p_U^{*\prime}))=0$. Substituting $p_U^{*\prime}$ from the break-even condition in $\frac{\partial W}{\partial m}=0$, we get:

$$\frac{\nu(F+(1-F)\mu)}{\nu(F+(1-F)\mu)+(1-\mu)(1-F)}=\frac{\nu\lambda\mu-\kappa\delta(1-\mu)}{\nu\lambda+(1-\nu)(1-\mu)}. \tag{54}$$

The payoff $\bar{A}^{P\prime}$ has to satisfy $m^{P\prime}=0$ and $\frac{dW}{dm}=0$. After substituting $p_U^{*\prime}$, we get

$$\frac{\nu(F+(1-F)\mu)}{\nu(F+(1-F)\mu)+(1-\mu)(1-F)}=\frac{\nu\lambda\mu-\kappa\delta(1-\mu)}{\nu\lambda+(1-\nu)(1-\mu)}+\frac{\frac{\partial W}{\partial p}\frac{dp_U}{dm}}{(1-F)(\nu\lambda+(1-\nu)(1-\mu))\bar{A}^{P\prime}}. \tag{55}$$

Combining this condition with (54) yields $\frac{\nu(F+(1-F)\mu)}{\nu(F+(1-F)\mu)+(1-\mu)(1-F)}\mid_{A=\bar{A}'}<\frac{\nu(F+(1-F)\mu)}{\nu(F+(1-F)\mu)+(1-\mu)(1-F)}\mid_{A=\bar{A}^{P\prime}}$.

Since $\frac{d\eta^*}{dA} > 0$, this condition implies $\bar{A}^{P\prime} > \bar{A}'$.

A.13 Proof of Proposition 13

Definition 5. *A perfect Bayesian equilibrium with loan insurance comprises choices of screening $\{s_i\}$, insurance $\{\ell_i\}$, loan sales $\{q_i^I, q_i^U\}$, financier beliefs about loan quality ϕ_t, secondary market prices p_I and p_U, and an insurance fee k such that:*

1. *At $t = 1$, each lender i optimally chooses sales of insured and uninsured loans for each realized liquidity shock $\lambda_i \in \{1, \lambda\}$, denoted by $q_i^I(s_i, \lambda_i, \ell_i)$ and $q_i^U(s_i, \lambda_i, \ell_i)$, given the prices p_I and p_U and choices of screening s_i and insurance ℓ_i.*

2. *At $t = 1$, outside financiers use the Bayes' rule to update their beliefs $\phi_1(q_U, q^I, \ell)$ on equilibrium path and prices p_I and p_U are set for outside financiers to expect to break even, given screening $\{s_i\}$ and insurance $\{\ell_i\}$ choices, the fee k, and sales $\{q_i^I, q_i^U\}$.*

3. *At $t = 0$, each lender i chooses its screening s_i and loan insurance ℓ_i to maximize expected utility, given prices p_I and p_U, the fee k, and sales q_i^I and q_i^U:*

$$\begin{aligned}
&\max_{s_i, \ell_i, c_{i1}, c_{i2}} \mathbb{E}\left[\lambda_i c_{i1} + c_{i2} - s_i \eta_i\right] \qquad \textit{subject to} \\
&c_{i1} = q_i^U(s_i, \lambda_i, \ell_i)\, p_U + q_i^I(s_i, \lambda_i, \ell_i)\, p_I, \\
&c_{i2} = \left[\ell_i - q_i^I\right](A - k) + \left[1 - \ell_i - q_i^U\right] \times \begin{cases} A & w.\ p. \quad s_i + \mu(1 - s_i) \\ 0 & \qquad\quad (1 - \mu)(1 - s_i). \end{cases}
\end{aligned}$$

4. *At $t = 0$, outside financiers use the Bayes' rule to update their beliefs $\phi_0(\ell)$ on equilibrium path and the fee k is set for financiers to break even in expectation, given screening $\{s_i\}$ and insurance $\{\ell_i\}$ choices.*

Risk retention as signal of loan type. In a separating equilibrium with both high-cost and low-cost lenders, sellers of high-quality loans choose $q^U \in (0, 1]$ (since $\ell_i \in \{0, 1\}$) and sellers of low-quality loans choose $q^{U\prime} \neq q^U$, such that $p_U(q^U) = A$ and $p_U(q^{U\prime}) = 0$. Since lenders cannot commit to negative consumption, high-cost lenders with lemons will always want to mimic sellers with high-quality loans since $q^U p_U(q^U) = q^U A > q^{U\prime} p_U(q^{U\prime}) = 0$. Hence, there exists no separating equilibrium with partial screening, $\eta^* < \bar{\eta}$.

However, there could exist an equilibrium with $q^U < 1$, where all lenders screen and, therefore, loan quality becomes public information. We derive the threshold screening costs by equating the payoff from screening, $\nu[\lambda p_U q^U + (1 - q^U)A] + (1 - \nu)A - \eta$, and payoff when not screening, $\nu[\lambda p_U q^U + (1 - q^U)\mu A] + (1 - \nu)(\mu A + (1 - \mu) p_U q^U)$:

$$\begin{aligned}
\eta &= (1 - \mu)[\nu(1 - q^U)A + (1 - \nu)(A - p_U q^U)] && (56) \\
&= (1 - \mu)(1 - q^U)A, && (57)
\end{aligned}$$

where the second equality comes from $p_U = A$. Equation (57) implies that there are no high-cost lenders, $\eta \geq \bar{\eta}$, if retention is large enough, $(1-q^U) \geq \frac{\bar{\eta}}{(1-\mu)A}$. Thus, a sufficient condition for ruling out this equilibrium is $\bar{\eta} \geq (1-\mu)A$.

Pooling equilibria with partial sales. The rest of the proof focuses on the pooling equilibria with partial sales and shows that our main results are qualitatively unchanged. Define $\bar{q}^U \equiv \min\left\{0, 1-\frac{\bar{\eta}}{(1-\mu)A}\right\}$ as the maximum loan sales consistent with full screening, $\eta^* \geq \bar{\eta}$. Then there exist a continuum of pooling perfect Bayesian equilibria with $q^U \in (\bar{q}^U, 1]$ in the appropriately generalized liquid equilibrium, $\lambda > \hat{\lambda}(q^U)$, where the out-of-equilibrium beliefs of financiers that a sold loan is of high quality is $\phi_1 = 0$ if $q_i^U \neq q^U$.

If insurance is used in this equilibrium, high-cost lenders have to be indifferent between payoff when not-insuring, $\nu\lambda p_U q^U + \nu(1-q^U)\mu A + (1-\nu)(\mu A + (1-\mu)p_U q^U)$, and insurance when insuring, $\kappa\mu A$. Equating those payoffs determines the price of uninsured loans:

$$p_U^* = \frac{\nu\mu A\left[\lambda + \frac{(\lambda-1)(1-q^U)}{q^U}\right]}{\nu\lambda + (1-\nu)(1-\mu)}, \tag{58}$$

which is a generalization of (18). This price decreases in q^U, $dp_U/dq^U < 0$, because higher uninsured loan sales make insurance relatively less attractive and a lower price of uninsured loans satisfies the insurance indifference equation. Using (56), the screening threshold is

$$\eta^* = \frac{(1-\mu)\kappa A}{\nu\lambda + (1-\nu)(1-\mu)}\left[(1-\nu)(1-\mu) + \nu(1-q^U)\right], \tag{59}$$

which is a generalization of (19). The screening threshold decreases with q^U, $d\eta/dq^U < 0$, since higher q^U lowers the net benefits of screening from loans hold to maturity in case of liquidity shock, term $(1-\mu)\nu(1-q^U)A$, and increase the payoff from sale of lemons when not screening, term $(1-\nu)p_U q^U$, where $dp_U q^U/dq^U > 0$. Combining (58) with the break-even condition of outside financiers (20), the fraction of insured high-cost lenders is

$$m^* = 1 - \frac{F(\eta^*)\left[\kappa q^U(1-\mu) - \mu(\lambda-1)(1-q^U)\right]}{\mu(1-F(\eta^*))(\lambda-1)[(1-\nu)(1-\mu) + (1-q^U)\nu]}, \tag{60}$$

which is a generalization of (21). Hence, $m^* > 0$ whenever

$$A < \bar{A}(q^U) \equiv \frac{\nu\lambda + (1-\nu)(1-\mu)}{(1-\mu)\kappa[(1-\nu)(1-\mu) + (1-q^U)\nu]}F^{-1}\left(\frac{\mu(\lambda-1)[(1-\nu)(1-\mu) + (1-q^U)\nu]}{\kappa(1-\mu)q^U + \mu(\lambda-1)(1-\nu)(q^U-\mu)}\right). \tag{61}$$

It is straightforward to show that the constrained efficient level of insurance exceeds the unregulated level at both the intensive margin and the extensive margin by following the same steps as in the proof for case $q^U = 1$ in Appendix A.6.