Sovereign risk and bank fragility: Investment versus enforcement*

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1 Extended Abstract

How should banks' holdings of domestic sovereign debt to regulated? This question has, and continues to vex the minds of policy-makers, especially in euro area countries in the aftermath of the recent sovereign debt crisis. According to the prevailing *doom loop* theory of events (Brunnermeier et al., 2016), the increase in sovereign default risk for countries like Greece, Portugal and Spain directly impacted the solvency of banks in those countries that held large portfolios of domestic sovereign bonds. Weak banks that sought bailouts further impaired their sovereign's ability to service its debts.

Recently, new regulations have been proposed to break this doom loop, which is also seen as an impediment to economic recovery and growth in the euro area (Lane and Langfield, 2018). These include introducing (i) non-zero risk-weights (BCBS, 2017), and (ii) large exposure regimes (ESRB, 2015) for banks' holdings of domestic sovereign bonds. These proposals argue that by introducing limits on banks' domestic sovereign bond portfolios, banks' solvency will no longer be intertwined with their sovereigns' solvency, promoting prudent risk diversification and investment by banks.

While these proposals are detailed in their prescriptions, the underlying economic forces at play are less well understood. The main contribution of this paper is to offer a new general equilibrium theory of bank risk taking in the presence of strategic sovereign default risk. A benevolent domestic government seeks to maximise domestic consumption. In the baseline model, domestic households choose between investing in the real economy and buy sovereign bonds. Foreign investors can also buy sovereign bonds and, as the marginal buyers, determine the bond price.

Sovereign default is costly because it impairs households' consumption via two channels. First, households suffer losses on their sovereign bond portfolios. And second, some of the output produced from the real sector is lost. If, however, the sovereign repays, then domestic households are taxed. If the transfer of tax revenue from domestic households to foreign bondholders is less than the costs of default, then the sovereign will choose to default.

Domestic households choose how much to invest in the real economy and domestic sovereign bonds to purchase. A marginal increase in households' holdings of government bonds induces two effects. First, the cost of sovereign default is increased, and so the sovereign's incentives to repay improve. Second, investment in the real economy is crowded out. Consequently, the pre-tax resources domestic households is reduced, which increases the sovereign's incentives to default. We characterise the competitive equilibrium for households' portfolios and the sovereign's default risk.

We also derive normative implications of our model by comparing the competitive equilibrium with the allocation a social planner would choose for households' portfolios. The planner takes into account the influence of households' portfolio choice on the sovereign's default incentives, which influence the bond price. Thus, a role for regulation exists due to a pecuniary externality. It is socially optimal for households to hold more government bonds if the gains from reducing the bond price is greater than the opportunity cost from investing in the real economy. If, however, the opportunity cost is larger, then it is socially optimal for households to reduce their sovereign bond holding, which is consistent with the current regulatory proposals.

We extend our baseline model by introducing banks that are subject to limited liability. Banks attract deposits from households, invest in the real economy and purchase government bonds. Banks take into account the influence of their portfolios on their limited liability constraints, but not on the sovereign's default incentives. Despite this, the effects remain qualitatively unchanged.

Related literature. Our paper related to the growing literature on sovereign risk and bank risk-taking (see e.g., Ari, 2017 and Crosignani, 2017). They argue that riskier banks tend to buy more risky domestic sovereign debt because of their limited liability status. These papers, however assumes that sovereign risk is exogenous and non-strategic. We depart by considering how strategic sovereign default interacts with bank risk-taking.

Uhlig (2013) and Farhi and Tirole (2017) consider how banking supervision can influence banks' risk-taking in the presence of sovereign risk. Banks load up on risky domestic sovereign debt because of lax domestic financial supervision. We argue, in contrast, that banks may load up on domestic sovereign debt at the behest of the financial supervisor for whom such holdings are socially optimal.

The 'financial repression' in our model stems from a pecuniary externality: banks do internalise the effect of their portfolios on the price of sovereign bonds. In related work, Chari et al. (2016) develop a model of optimal financial repression in an open economy model. They argue that it is optimal for governments to practice financial repression when faced with sudden stops of lending by foreign investors.

Our paper also contributes to the literature on the costs of sovereign default. Gennaioli et al. (2014) present a model where sovereign defaults are costly because they adversely impact the balance sheets of banks. And, the more leveraged are banks, they greater are the costs. Broner et al. (2014) argue that even if a sovereign could perfectly discriminate between defaulting on foreign bondholders but not on domestic ones, the full costs of a sovereign default will be borne by domestic bondholders who buy bonds from foreign bondholders in a secondary market.

2 A model of sovereign risk

There are two dates, $t = 0$ and $t = 1$ and a single good that is used for both consumption and investment. The economy is populated by a large pool of risk-neutral agents who care about consuming at $t = 1$. The agents are segmented into two clientele; a large pool of 'international' investors and a continuum of mass one of 'domestic' households. There is a domestic government referred to as the sovereign.

Domestic households. Each household is endowed with a unit of the consumption good at the $t = 0$. Households can either invest their endowments using a production technology or purchase government bonds. Household i's gross return at $t = 1$ from investing $l_i \leq 1$ units in the production technology at $t = 0$ is $Y(l_i) = A\,y(l_i)$. The total factor productivity (TFP), $A \in \mathcal{A} \subset \mathbb{R}^+$, is a random variable drawn at the start of $t = 1$ according to the cumulative distribution function $F(A)$, and is common across all households.[1] The production function, $y : [0, 1] \rightarrow \mathbb{R}^+$, is increasing and concave, such that $\lim_{l \to 0} y'(l) = \infty$. If household i purchases $b_i \leq 1$ worth of government bonds, the gross return at $t = 1$ is $b_i(1 + r_g)$ if the sovereign repays. If the sovereign defaults, then households receives nothing from their bonds. The $t = 0$ portfolio allocations must satisfy the budget constraint $b_i + l_i = 1$ for all households $i \in [0, 1]$.

International investors. International investors are deep-pocketed. At $t = 0$, each investor can choose to purchase debt issued by the sovereign or invest in a safe storage technology with net return $\bar{r} > 0$.

[1] The set $\mathcal{A}$ is compact with an upper bound $\bar{A}$ and lower bound normalised to zero.

Sovereign. At $t = 0$, the sovereign has a stock, $S > 0$, of legacy debt that needs to be refinanced until $t = 1$. The sovereign issues an infinitely divisible one-period bond with face value $S(1 + r_g)$, where r_g is the net interest rate. At $t = 1$, the sovereign receives a fixed endowment $N > 0$ of the consumption good. The sovereign repays its debt by taxing domestic households. If the sovereign defaults then households are not taxed, but they suffer a loss equal to a fraction $\delta < 1$ of the output produced. The sovereign's objective is to maximise the sum of households' and its own consumption.

Timing. At $t = 0$, the sovereign issues bonds; international investors and domestic households choose how much sovereign debt to purchase; households invest using the production technology. At $t = 1$, the TFP is realised; the sovereign chooses whether to repay or default on its debts; households, investors and the sovereign consume.

2.1 Equilibrium

We solve the model by backward induction.

Definition 1. *The symmetric pure-strategy sub-game perfect equilibrium comprises of (i) for each domestic households, $i \in [0, 1]$, its portfolio allocations $\{b_i^*, l_i^*\}$ between purchasing government bonds and investing in using the production technology; (ii) the interest rate that the Sovereign must pay to roll over its debts, r_g^*, and a critical default threshold, $\widehat{A}^*$, for the sovereign, such that*

a. at $t = 1$ the Sovereign repays whenever the TFP is greater than the threshold, $A \geq \widehat{A}^*$ and defaults otherwise, given the portfolio allocation $\{b_i^*, l_i^*\}$ for each household $i \in [0, 1]$ and the interest rate on sovereign debt, r_g^*;

b. at $t = 0$, each household $i \in [0, 1]$ optimally choose its portfolio, $\{l_i^*, b_i^*\}$, given *the interest rate on sovereign debt, r_g^*, and the default threshold, $\widehat{A}^*$;*

c. *at $t = 0$, the interest rate on Sovereign debt, r_g^*, is derived from the international investors' binding participation constraint, given the portfolio allocation $\{b_i^*, l_i^*\}$ for each household $i \in [0, 1]$, and the default threshold, $\widehat{A}^*$.*

2.2 Sovereign default

At $t = 1$, following the realization of the TFP, the sovereign chooses to either tax the households and repay its debts in full or to default.[2] We treat each case in turn.

If the sovereign repays, it raises $S(1 + r_g)$ in tax revenue. Insofar domestic households purchased sovereign debt at $t = 0$, some of the tax revenue reverts to them. The remainder goes to international investors. Aggregate domestic consumption is

$$\begin{aligned} & N + A \int_0^1 y(l_i)\,\mathrm{d}i + S(1 + r_g) \int_0^1 \frac{b_i}{S}\,\mathrm{d}i - S(1 + r_g) \\ = \; & N + A \int_0^1 y(l_i)\,\mathrm{d}i - S(1 + r_g) \left[1 - \int_0^1 \frac{b_i}{S}\,\mathrm{d}i \right] . \end{aligned} \tag{1}$$

Equation (1) is decomposed into three terms. The first is the endowment that the sovereign consumes. The second term is aggregate output. The third term is domestic tax revenues transferred to international investors holding government bonds.

If the sovereign defaults then households are not taxed. But, since default is non-discriminatory, households are directly effected through their holdings of government bonds. Households are also effected indirectly: output produced using the production

[2] In the baseline model, we exclude the possibility of partial default.

technology is diminished by a fraction $\delta < 1$. Aggregate domestic consumption is

$$N + (1-\delta)\, A \int_0^1 y(\ell_i)\, \mathrm{d}i \,. \tag{2}$$

In what follows, $B \equiv \int_0^1 b_i \mathrm{d}i$ denotes the aggregate share of domestic households' $t = 0$ endowments used to purchase government bonds. Comparing Equations (1) and (2), the sovereign chooses to repay its debts at $t = 1$ as long as the TFP is greater than a critical value, $\widehat{A}$, where

$$\widehat{A} \equiv \big(1 + r_g\big) \frac{S - B}{\delta \int_0^1 y(\ell_i)\mathrm{d}i} \,. \tag{3}$$

If the losses to households following sovereign default is greater than the tax revenue that must be transferred to repay international investors, then the sovereign repays.

Next, we solve the portfolio allocation choice for each households, and determine the interest rate on government bonds. We treat each in turn.

2.3 Portfolio choice for households

Each household is small and cannot influence the sovereign's default decision. In what follows, we focus on the representative aggregate risk-neutral household's portfolio problem, which is without loss of generality. Denoting the aggregate level of investment by $L = \int_0^1 l_i \, \mathrm{d}i$, the optimisation problem is given by Equation (4).

$$\begin{aligned} \max_{L,B} \qquad & \big(1-\delta\big) \int_0^{\widehat{A}} A\, y(L)\, dF(A) + \int_{\widehat{A}}^{\bar{A}} \Big[A\, y(L) \;-\; (1 + r_g)\Big(S \;-\; B\Big)\Big]\, dF(A) \\ & \text{subject to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4) \\ & L + B = 1 \,, \end{aligned}$$

The household maximises expected consumption, subject to the budget constraint that investment in production and government bonds must sum up to unity. The expected consumption consists of two terms. The first term captures the amount consumed if the sovereign defaults. In this case, the household only obtains a fraction $1-\delta$ of the investment using the production technology. The second term reflects the amount consumed if the sovereign repays. In this case, all of the output from the production technology is consumed, and the household is repaid on its holdings of government bonds. But, at the same time, the household is taxed by the government.

The household's privately optimal level of investment in government bonds is

$$y'(1-B^*)\left[\big(1-\delta\big)\int_0^{\widehat{A}} A\,dF(A)+\int_{\widehat{A}}^{\bar{A}} A\,dF(A)\right]=\big(1+r_g\big)\big(1-F(\widehat{A})\big)\,. \tag{5}$$

2.4 Interest rate on government bonds

As the marginal buyers of government bonds, international investors determine the interest rate, g, according to their binding participation constraint, i.e.,

$$\big(1-F(\widehat{A})\big)\big(1+r_g\big)=1+\bar{r}\,. \tag{6}$$

Equations (3), (5) and (6) fully characterises the equilibrium, which is summarised in Proposition 1.

Proposition 1. *In the competitive equilibrium, households invest up to L^* where the* *marginal return from production is equal to international investors' risk-free rate,*

$$y'(L^*) = \frac{1+\bar{r}}{\left(1-\delta\right)\int_0^{\widehat{A}^*} A\,dF(A) + \int_{\widehat{A}^*}^{\bar{A}} A\,dF(A)}\,, \tag{7}$$

while the remainder, $B^* = 1 - L^*$, is invested in government bonds. The critical default threshold is implicitly defined by

$$\widehat{A}^* = \frac{S - B^*}{\delta\, y(1-B^*)}\left(\frac{1+\bar{r}}{1-F(\widehat{A}^*)}\right). \tag{8}$$

Proof. See above. □

Depending on the households' and investors' initial beliefs the model may exhibit multiple equilibria. Figure 1 plots the equilibrium sovereign default threshold.[3] If households and investors are 'pessimistic' and believe that the sovereign will default, they will require a higher rate of return to lend to the government. This in turn makes it more likely that the sovereign will default, implying the equilibrium default threshold labelled (p). While, if households and investors are 'optimistic' and believe that the sovereign will not default, the required rate of return is low. For any given level of households investment in government bonds, this means that the sovereign transfers less resources to foreign investors in order to repay its debts. This makes it less likely that the sovereign will default, leading to the equilibrium sovereign default threshold labelled (o), which is the stable fixed-point. In what follows, we focus on the stable fixed-point. Furthermore, for ease of exposition, we suppose that the probability distribution function for the TFP random variable has a constant hazard rate, $\lambda < \frac{1}{\bar{A}}$, which is bounded from above.

[3]In producing this figure, we set $S = 6$, $\bar{r} = 0$ and $\delta = 0.9$. The production technology is $y(L) = L^{\alpha}$, where $\alpha = 0.4$. The TFP random variable is exponential distributed over the interval $[0, 50]$, with rate parameter $\lambda = 0.01$. Finally, the solid black line is produced for $B = 0.1$, while the dashed red line has $B = 0.4$

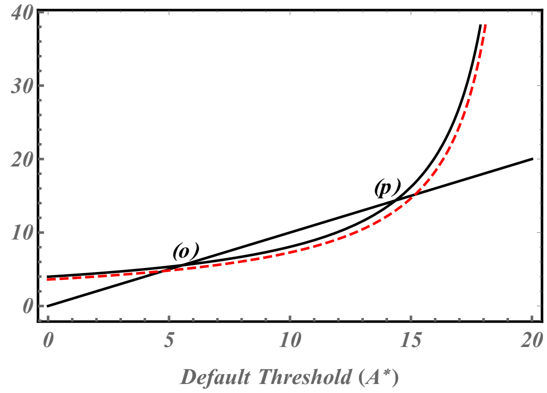

Figure 1: Multiple equilibria for the sovereign's default threshold.

Lemma 1. *Suppose that the total stock of outstanding debt is large, $S > \frac{1}{\alpha}$. If the cost of default is bounded from below, $\delta > \hat{\delta}$, then a marginal increase in S leads to a decrease in both households' holdings of government bonds and a the Sovereign's default threshold. While, if the cost of default is low, $\delta < \hat{\delta}$, then a marginal increase in S leads to an increase in both B^* and A^*.*

Proof. See Appendix A. $\square$

With a large stock of outstanding debt, a large portion will be held by foreign investors. This implies that the sovereign's incentives to default will be large, and hence the required return on the government bonds will also be high. Since households are price takers, they respond to the higher bond returns by reallocating their portfolios towards holding more government bonds and investing less using the production technology. But, to the extent that the cost of default is high, $\delta > \hat{\delta}$, the crowding out effect is relatively small. This further increases the Sovereign's incentives to repay.

Lemma 2. *Suppose that the total stock of outstanding debt is small, $S < 1$. A marginal increase in S leads to an increase in both households' holdings of government bonds and the sovereign's default threshold.*

Proof. See Appendix A. □

When the outstanding stock of debt is small to start with, an increase in S still implies that the sovereign's incentives to default are increased. This leads to a higher required return for government bonds, and so households choose to hold more of them. However, in contrast to the result in Lemma 1, the crowding out of investment into the production technology is always more pronounced. Households hold large portfolios of government bonds and generate low output in $t = 1$. The incentives for the sovereign to default are, in turn, raised.

2.5 Social optimum

Does the allocation described in Proposition 1 maximise domestic welfare? To answer this derive the portfolio allocation for the domestic social planner who maximises the sum of government consumption and expected consumption for the aggregate household. Accounting for the budget constraint, the problem can be expressed as

$$\begin{aligned} \max_{B} \quad & N + y(1-B)\left\{(1-\delta)\int_{0}^{\widehat{A}(B)} AdF(A) + \int_{\widehat{A}(B)}^{\bar{A}} A\mathrm{d}F(A)\right\} \\ - \quad & (S-B)\Big(1+r_g^*(B)\Big)\Big(1-F\big(\widehat{A}(B)\big)\Big), \end{aligned} \tag{9}$$

where

$$\widehat{A}(B) = \frac{\big(S-B\big)\big(1+r_g^*(B)\big)}{\delta y(1-B)},$$

and $r_g^*(B)$ is derived from the international investors' binding participation constraint. There are two important differences between the planner's problem and the household's private problem in Equation(4). First, the planner includes the consumption of the sovereign. And second, the planner accounts for how the household's holdings of government debt influence the sovereign's debt threshold and the interest rate.

Proposition 2 summarises the equilibrium.

Proposition 2. *The social planner chooses a level of investment given by*

$$y'(L^*) = \frac{1+\bar{r} - \frac{\partial g^*}{\partial B}(S-B^*)\big(1-F(\widehat{A}^*)\big)}{\big(1-\delta\big)\int_0^{\widehat{A}^*} A\,dF(A) + \int_{\widehat{A}^*}^{\bar{A}} A\,dF(A)}\,, \tag{10}$$

where the amount of government bonds held domestically are $B^* = 1 - L^*$, and the Sovereign's default threshold is

$$\widehat{A}^* = \frac{S-B^*}{\delta\, y(1-B^*)}\left(\frac{1+\bar{r}}{1-F(\widehat{A}^*)}\right)\,.$$

Compared with the competitive equilibrium, the planner takes into account the effects of households' holdings of government debt on the price. Insofar that an increase in households' holdings of government debt leads to a decline in the bond price, under the planner's allocation, households would hold more government bonds than in the competitive equilibrium. Lemma 3 summarises this result where we denote the levels of investment under the planner's and competitive equilibrium by L^*_{SP} and L^*_{CE}, respectively.

Lemma 3. *If the stock of outstanding debt is small, $S < 1$, then a marginal increase in domestic households' holdings of government bonds leads to a decrease in the price of debt, i.e., $\frac{dg^*}{dB} < 0$. Furthermore, if all output is lost under default, $\delta = 1$, then $L^*_{SP} < L^*_{CE}$.*

3 A model of sovereign risk and banks

In this section, we explore how banks that are subject to limited liability constraints influence a sovereign's incentives to default and analyse their portfolio choice problem. Building on the model presented in Section 2, we make three important modifications.

Penniless bankers. There is a unit mass of penniless bankers who have no endowment. At $t = 0$, banker $i \in [0, 1]$ attracts $d_i > 0$ of deposits from households, invest $\ell_i \leq d_i$ in the production technology and purchases $b_i \leq d_i$ worth of government bonds. At $t = 1$, following the realisation of the TFP random variable and the sovereign's decision to repay or default, banker i is obliged to repay depositors $1 + r_{d,i}$ where, $r_{d,i} \geq 0$ is the interest rate earned on deposits at bank i. Bankers are protected by limited liability. If the return on investment and government bonds is less than repayment due to depositors, then the bankers are unable to repay their depositors. Depositors seize the banks' assets and bankers have zero consumption. Finally, bankers' profits cannot be taxed by the sovereign to repay its debts.

Infinitely risk-averse households. There is a unit mass of infinitely risk-averse households, each of whom is endowed with a unit of the consumption good at $t = 0$. Households cannot invest directly use the production technology or purchase government bonds. Instead, household $h \in [0, 1]$ can deposit $e_{h,i} \leq 1$ with banker $i \in [0, 1]$ who, in turn, intermediates and invests using the production technology and purchase government bonds. Households can also store their endowments using a risk-free storage technology. However, for every unit stored, households obtain only $1 - \epsilon$, where $\epsilon \ll 1$ reflects the cost of using the storage technology. As before, households are subject to taxation by the sovereign to repay its debts at $t = 1$.

Deposit guarantees. The sovereign can guarantee banks deposits. In particular, for every unit deposited in a bank at $t = 0$, the sovereign credibly promises depositors that they will receive the full principle amount for sure, even if the bank fails at $t = 1$ and the value of its assets falls short. The guarantee payment is taken out of the sovereign's endowment, N, and diminishes the sovereign's consumption.

Timing. At $t = 0$, each banker $i \in [0, 1]$ offers an interest rate $r_{d,i}$ on deposits; household $h \in [0, 1]$ deposits $e_{h,i}$ with banker i (the total deposits with banker i is $d_i = \int_0^1 e_{h,i} \, \mathrm{d}h$); banker i invests ℓ_i in production and $b_i = d_i - \ell_i$ in government bonds; At $t = 1$, the TFP is realised; the sovereign chooses to repay or default; banks repay depositors; if the value of banks' assets are insufficient, then the sovereign pays depositors the shortfall; households, bankers and the sovereign consume.

3.1 Equilibrium

As before, we solve the model by backward induction.

Definition 2. *The symmetric pure-strategy sub-game perfect equilibrium comprises of (i) for each banker $i \in [0, 1]$ the interest rate, $r^*_{d,i}$, offered to households to attract their deposits, the amount of deposits, d^*_i, raised, and the portfolio allocations, $\{b^*_i, l^*_i\}$, between purchasing government bonds and investing in production; (ii) the amount $e^*_{h,i}$ each household $h \in [0, 1]$ deposits in bank i; (iii) the interest rate r^*_g the sovereign pays to roll over its debts, and (iv) the critical default threshold, $\widehat{A}^*$ for the sovereign.*

3.2 Interest rate on deposits

In choosing which bank to deposit their endowments with at $t = 0$ households compare the deposit rates, $r_{d,i}$, offered by the different banks, $i \in [0, 1]$. But, since households are infinitely risk-averse, they strictly prefer 'safe' debt, whose value is insensitive to the return on banks' investments and the sovereign's decision to repay or default. Safe debt is possible in the model because of deposit guarantees that the sovereign provides on the principle amount lent to banks. At the same time, the combination of infinite risk-aversion and deposit guarantees, implies that no banker has an incentive to offer a strictly positive net return on deposits since households' storage technology is subject to the depreciation cost, ϵ. Corollary 1 summarises our result.

Corollary 1. *The deposit rate is* $r^*_{d,i} = r^*_{d,j} = 0$*, for all banks* $i, j \in [0, 1]$*. Households never utilise the storage technology and deposit an equal share of their endowments in each bank,* $e^*_{h,i} = e^*_{h,j}$*, and* $e^*_{h,i} = e^*_{h',i}$ *for all households* $h, h' \in [0, 1]$ *and banks* $i, j \in [0, 1]$*. Consequently, the deposit base is* $d^*_i = d^*_j = 1$ *for all banks* $i, j \in [0, 1]$*.*

3.3 Sovereign default

If the sovereign repays, then aggregate domestic consumption is

$$\begin{aligned}
& \int_0^1 \max\Big\{0, A\, y(\ell_i) + b_i(1 + r_g) - d_i(1 + r_{d,i})\Big\}\, \mathrm{d}i \\
+ \quad & \int_0^1 \left[\int_0^1 e_{h,i}\Big(1 + I_i^b(A)\, r_{d,i}\Big)\, \mathrm{d}i + \left(1 - \int_0^1 e_{h,i}\, \mathrm{d}i\right)(1 - \epsilon)\right] \mathrm{d}h - S(1 + r_g) \\
+ \quad & N - \int_0^1 \max\left\{0, \int_0^1 e_{h,i}\, \mathrm{d}h - \Big[A\, y(\ell_i) + b_i(1 + r_g)\Big]\right\} \mathrm{d}i
\end{aligned}$$

Equation (11) consists of three terms. The first is the consumption of bankers who are subject to limited liability. The second is the consumption of households net of taxation. Each household can deposit a portion of their endowment with different banks and invest the remainder using the safe storage technology. If a bank repays, then the household earns the interest rate on deposits. But, if a bank fails, then households who deposited with that bank only recover their principle investment from the deposit guarantee. We capture the failure of bank $i \in [0,1]$ using the indicator function

$$I_i^b(A) = \begin{cases} 1 & \text{if} \quad A \geq \frac{d_i(1+r_{d,i}) - b_i(1+r_g)}{y(\ell_i)} \\ 0 & \text{otherwise} \end{cases} .$$

The third term is the consumption of the sovereign, which is diminished by having to pay out deposit guarantees.

Using the result of Corollary 1, we can rewrite Equation (11) as

$$\int_0^1 \Big[A\, y(\ell_i) \;+\; b_i\big(1+r_g\big)\Big]\, \mathrm{d}i + N - S(1+r_g)\,. \tag{11}$$

Aggregate domestic consumption depends only on the value of banks' assets, the sovereign's endowment, and taxation of households.

If the sovereign defaults then aggregate domestic consumption is

$$\begin{aligned} & \int_0^1 \max\Big\{0, (1-\delta)A\, y(\ell_i) - d_i(1+r_{d,i})\Big\}\, \mathrm{d}i \\ + \quad & \int_0^1 \left[\int_0^1 e_{h,i}\Big(1 + I_i^b(A)\, r_{d,i}\Big)\, \mathrm{d}i + \left(1 - \int_0^1 e_{h,i}\, \mathrm{d}i\right)\, \big(1-\epsilon\big)\right] \mathrm{d}h \\ + \quad & N - \int_0^1 \max\left\{0, \int_0^1 e_{h,i}\, \mathrm{d}h - (1-\delta)A\, y(\ell_i)\right\}\, \mathrm{d}i\,. \end{aligned} \tag{12}$$

As before, sovereign default has three implications for the domestic economy. First, there is no taxation. Second, domestic banks' suffer losses on their holdings of government bonds. Third, output from the production technology is depreciated by a fraction $\delta < 1$. Using the result of Corollary 1, Equation (12) simplifies to

$$\int_0^1 \big(1-\delta\big)\, A\, y(\ell_i)\, \mathrm{d}i \;+\; N\,, \tag{13}$$

which depends only on the level of investment undertaken by banks and the sovereign's endowment.

The sovereign repays whenever $A \geq \widehat{A}^s \equiv \frac{(S - \int_0^1 b_i \mathrm{d}i)(1+r_g)}{\delta \int_0^1 y(\ell_i)\mathrm{d}i}$, which is identical to the threshold in Equation (3). However, the nature of the trade-off with banks is subtly different. The sovereign repays by taxing households. Some of the tax revenue goes to the bankers who retain it in their profits. By choosing to default, however, the sovereign does not tax households. Instead, the bankers are adversely effected both directly through losses on their government bonds and indirectly through the loss of output generated by the production technology. Thus, the sovereign will choose to repay whenever the loss to bankers from defaulting, $\int_0^1 [\delta\, A\, y(\ell_i) + b_i(1+r_g)]\mathrm{d}i$, is greater than the loss to households from repaying, $S(1+r_g)$.

3.4 Portfolio choice for banks

At $t = 0$, each bank chooses how much to invest using the production technology and how much government bonds to purchase. While banks do not internalise the effects of their choices on the sovereign's default incentives, they nevertheless take into account the effects on their own limited liability constraints. Suppose $\xi \in [0,1]$ is the probability that banks and foreign investors ascribe to the sovereign repaying. Since banks are ex ante identical, we focus on the portfolio problem of the representative aggregate bank without loss of generality. The portfolio problem is given by

$$
\begin{aligned}
\max_{L,\,B} \qquad & \xi \int_0^{\bar{A}} \max\Big\{0, A\,y(L) \;+\; B(1+r_g) \;-\; 1\Big\}\,\mathrm{d}F(A) \\
+ \quad & \big(1 - \xi\big) \int_0^{\bar{A}} \max\Big\{0, (1-\delta)\,A\,y(L) \;-\; 1\Big\}\,dF(A) \\
& \text{subject to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (14) \\
& L + B = 1\,,
\end{aligned}
$$

Inserting the budget constraint, the first-order condition for the amount invested in government bonds is

$$
\frac{y'(1\,-\,B^*)}{1\,-\,F\big(\widehat{A}_1^b(B^*)\big)}\left[\xi\int_{\widehat{A}_1^b(B^*)}^{\bar{A}} A\,\mathrm{d}F(A) \;+\; \big(1\,-\,\xi\big)\big(1\,-\,\delta\big)\int_{\widehat{A}_0^b(B^*)}^{\bar{A}} A\,\mathrm{d}F(A)\right] = \xi\,\big(1\,+\,r_g\big)\,, \qquad (15)
$$

where

$$
\widehat{A}_1^b(B) \equiv \frac{1\,-\,B(1\,+\,r_g)}{y(1-B)}\,, \quad \text{and} \quad \widehat{A}_0^b(B) \equiv \frac{1}{\big(1\,-\,\delta\big)\,y(1-B)}
$$

are the default thresholds for the bank if the sovereign repays or defaults, respectively.

3.5 Pricing of government bonds

As before, international investors determine the price of government bonds through their participation constraint. This yields

$$
\xi\big(1\,+\,r_g\big) \;=\; 1\,+\,\bar{r}\,. \qquad (16)
$$

Finally, the belief that the sovereign repays is self consistently determined as

$$
\xi = \mathrm{Prob}\big(A > \widehat{A}^g\big) \;=\; 1\,-\,F\left(\big(1\,+\,r_g\big)\frac{S\,-\,B}{\delta\,y(L)}\right) \qquad (17)
$$

. Equations (15), (16) and (17) characterise the equilibrium, which we summarise in Proposition 3.

Proposition 3. *The representative banker's choice for government bonds to purchase is given by*

$$\frac{y'(1-B^*)}{1-F\big(\widehat{A}_1^b(B^*,\xi)\big)}\left[\xi\int_{\widehat{A}_1^b(B^*,\xi)}^{\bar{A}} A\,\mathrm{d}F(A)+\big(1-\xi\big)\big(1-\delta\big)\int_{\widehat{A}_0^b(B^*)}^{\bar{A}} A\,\mathrm{d}F(A)\right]=1+\bar{r}\,,\tag{18}$$

where

$$\widehat{A}_1^b(B^*,\xi)=\frac{1-\frac{B^*}{\xi}(1+\bar{r})}{y(1-B^*)}\,,\tag{19}$$

while the remainder, $L^*=1-B^*$ is invested using the production technology. The belief that the sovereign will repay is implicitly determined via

$$\xi^*=1-F\left(\frac{1+\bar{r}}{\xi^*}\frac{S-B}{\delta\,y(1-B)}\right).\tag{20}$$

References

Ari, A. (2017). Sovereign risk and bank risk-taking. IMF Working Papers 17/280.

BCBS (2017, December). The regulatory treatment of sovereign exposures. Discussion Paper.

Broner, F., A. Erce, A. Martin, and J. Ventura (2014). Sovereign debt markets in turbulent times: Creditor discrimination and crowding-out effects. *Journal of Monetary Economics 61*, 114–142.

Brunnermeier, M. K., L. Garicano, P. R. Lane, M. Pagano, R. Reis, T. Santos, D. Thesmar, S. V. Nieuwerburgh, and D. Vayanos (2016). The sovereign-bank diabolic loop and esbies. *American Economic Review 106*, 508–152.

Chari, V. V., A. Dovis, and P. J. Kehoe (2016). On the optimality of financial repression. Federal Reserve Bank of Minneapolis Research Department Staff Report 1000.

Crosignani, M. (2017). Why are banks not recapitalized during crises? ESRB Working Paper Series No 57.

ESRB (2015). Esrb report on the regulatory treatment of sovereign exposures.

Farhi, E. and J. Tirole (2017). Deadly embrace: Sovereign and financial balance sheets doom loops. *Review of Economic Studies Forthcoming*.

Gennaioli, N., A. Martin, and S. Rossi (2014). Sovereign default, domestic banks, and financial institutions. *Journal of Finance 69*, 819–866.

Lane, P. and S. Langfield (2018). The feasibility of sovereign bond-backed securities for the euro area. `https://voxeu.org/article/feasibility-sovereign-bond-backed-securities-euro-area`.

Uhlig, H. (2013). Sovereign default risk and banks in a monetary union. *German Economic Review 15*, 23–41.

A Proof of Lemma 1 and 2

The equilibrium $(\widehat{A}^*, B^*)$ is given by the solution to

$$
\begin{aligned}
g\big(\widehat{A}^*, B^*\big) &\equiv \widehat{A}^* - \frac{(1+\bar{r})(S-B^*)}{\delta\left(1-F\big(\widehat{A}^*\big)\right)y(1-B^*)} = 0 \\
h(\widehat{A}^*, B^*) &\equiv y'(1-B^*)\left[(1-\delta)\int_0^{\widehat{A}^*} A\,dF(A) + \int_{\widehat{A}^*}^{\bar{A}} A\,dF(A)\right] = 0\,.
\end{aligned}
$$

To derive the comparative statics for the equilibrium, we first need the determinant of the Jacobian matrix,

$$
J = \begin{pmatrix} g_{\widehat{A}} & g_B \\ h_{\widehat{A}} & h_B \end{pmatrix}.
$$

Evaluating the partial derivatives at the equilibrium, we get

$$
\begin{aligned}
g_A(\widehat{A}^*, B) &= 1 - \widehat{A}^*\xi > 0 \\
g_B(\widehat{A}^*, B) &= \widehat{A}^*\left\{\frac{1}{S-B} - \frac{y'(1-B)}{y(1-B)}\right\} \\
h_A(\widehat{A}, B) &= -\delta\,\widehat{A}\,y'(1-B)\,f(\widehat{A}) < 0 \\
h_B(\widehat{A}, B^*) &= \frac{-y''(1-B^*)}{y'(1-B^*)}(1+\bar{r}) > 0
\end{aligned}
$$

To sign g_B, note that if and only if $1 - \alpha S - (1-\alpha)B > 0$, then $g_B > 0$. A sufficient condition that yields $g_B > 0$, which is true for all values of B, is give by $S < 1$. While a sufficient condition for $g_B < 0$ is $S > \frac{1}{\alpha}$. We treat each case in turn.

Case 1 – small stock of outstanding debt ($S < 1$) The determinant of the Jacobian is positive, i.e., $|J| > 0$. By Cramer's rule, we have that

$$
\frac{dB^*}{dS} = \frac{\begin{vmatrix} g_{\widehat{A}} & -g_S \\ h_{\widehat{A}} & -h_S \end{vmatrix}}{|J|}, \quad \text{and} \quad \frac{d\widehat{A}^*}{dS} = \frac{\begin{vmatrix} -g_S & g_B \\ -h_S & h_B \end{vmatrix}}{|J|}, \tag{21}
$$

where

$$g_S(\widehat{A}, B) = \frac{-\big(1+\bar{r}\big)}{\delta\left(1-F(\widehat{A})\right)y(1-B)} < 0\,, \quad \text{and} \quad h_S(\widehat{A}, B) = 0\,.$$

It immediately follows that $\frac{dB^*}{dS} > 0$ and $\frac{d\widehat{A}^*}{dS} > 0$.

Case 2 – large stock of outstanding debt $\left(S > \frac{1}{\alpha}\right)$ The determinant of the Jacobian is

$$\begin{aligned} |J| \;&=\; \left(1-\widehat{A}^*\xi\right)(1+\bar{r})\,\frac{-y''(1-B^*)}{y'(1-B^*)} \\ &+\; \delta\,\widehat{A}^*\,y'(1-B^*)\,f(\widehat{A}^*)\,\widehat{A}^*\left\{\frac{1}{S-B^*}-\frac{y'(1-B^*)}{y(1-B^*)}\right\}\,, \end{aligned}$$

where the first term is positive, while the second term is negative. The determinant of the Jacobian is decreasing in δ and for $\delta = 0$ it is positive. If $\delta = 1$, then, in the limit $\alpha \to 1$, the determinant is negative. While, in the limit $\alpha \to 0$, the determinant is positive. This implies that there exists $\hat{\alpha}$, such that for all $\alpha > \hat{\alpha}$, the determinant is negative whenever $\delta = 1$. Consequently, there exists $\hat{\delta}$, such that if $\delta < \hat{\delta}$, then the determinant is positive, while for $\delta > \hat{\delta}$, the determinant is negative.

Finally, to derive how the equilibrium changes following a marginal increase in S using Cramer's rule, since $h_S = 0$, the signs of the terms in the numerators in Equation 21 do not depend on the sign of g_B. Therefore, if $\delta < \hat{\delta}$, then $\frac{dB^*}{dS} > 0$ and $\frac{dA^*}{dS} > 0$. While, if $\delta > \hat{\delta}$, then the signs are reversed.