Persuasion in Relationship Finance*

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Abstract

Relationship finance as seen in bank lending and venture capital features incumbent financiers' observing interim information after initial investment but before continuation decision. We model the entrepreneurs' endogenous information production and subsequent issuance of securities to both the incumbent insider and competitive outsider investors as persuasion games with heterogeneous receivers and contingent transfers. Entrepreneurs' endogenous experimentation reduces insider investors' information monopoly, but holds up initial relationship investments. Insider financiers' own information production and interim competition from outsiders can mitigate the hold-up, and jointly explain the empirical non-monotone patterns linking competition and relational lending. Optimal relationship contracts restore first-best outcomes using convertible securities for insiders and residual claims for outsiders. Our findings do not rely on full commitment to information disclosure, and remain robust under restricted information design and continuum action space.

JEL Classification: D47, D82, D83, G14, G23, G28

Keywords: Bank Competition, Bayesian Persuasion, Contracting, Experimentation, Hold-up, Information Design, Relationship Lending, Security Design, Venture Capital.

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1 Introduction

What is the benefit of raising capital from intermediaries such as banks or venture funds instead of issuing securities in public markets? A large literature on relationship finance reveals that intermediaries can mitigate informational asymmetry and moral hazard when they form relationships with entrepreneurs to become "insider" financiers (e.g., Diamond, 1984, 1991; Fama, 1985; Ramakrishnan and Thakor, 1984; Kerr, Nanda, and Rhodes-Kropf, 2014). Existing literature recognizes how insiders may hold up entrepreneurs due to information monopoly, but ignores the endogenous nature of information production and design.[1] Yet in reality entrepreneurs' actions not only shape project cash flows, but also alter the informational environment.[2]

Meanwhile, as a confluence of work on Bayes correlated equilibria and Bayesian persuasion, information design is arguably the most active area of research in information economics in recent years (Kamenica, 2017). It has been quickly adopted to address issues in banking regulation, online advertising, entertainment, etc. Yet its application in corporate finance has been limited because contingent transfers prevalent in security design and contracting are typically absent in extant Bayesian-persuasion models, despite the fact that relationship finance provides a most natural setting for a sender to commit to an information structure.

Several questions naturally arise. Do the endogenous production of information matter for sequential fund-raising? Do they affect the link between bank orientation and competition? How would the persuasion game play out in the presence of contingent transfers and receiver heterogeneity? What are their implications for designing securities for venture investors?

Motivated by these questions, we model interim experimentation in relationship financing, contracting, and security design as a Bayesian persuasion game with contingent transfers and heterogeneously-informed receivers. We show that the entrepreneur's endogenous information production reduces a relationship financier's rent from her interim bargaining power, but inefficiently holds up her initial investment in the relationship. We then derive

[1] Sharpe (1990) and Rajan (1992) discuss the hold-up issues in the context of relationship banking; Admati and Pfleiderer (1994) argue that staged financing in venture capital can also produce conflicts of interest and Fluck, Garrison, and Myers (2006) discuss hold-ups in staged venture financing.

[2] Pharmaceutical firms can affect FDA's and investors' decisions by providing additional information and tests (e.g., "Guidance for Industry and FDA Staff. Post market Surveillance Under Section 522 of the Federal Food, Drug and Cosmetic Act" May 16, 2016); Software startups decide on different markets for beta launches because the media attention generated is different, and thus information communicated to potential users ("Why Is Canada Such A Good Testing Ground For Game Releases?" Forbes Tech. Nov 27, 2012); entrepreneurs choose the specific prototype or trial market to work on which produces disparate forms of information; career concerns and managerial choices of projects exhibit similar features.

a contractual solution to fully restore efficient information production: the entrepreneur optimally promises early insider investors—relationship financiers who provide the initial funding— convertible securities in future at a pre-specified price and quantity, and upon the insider's continued financing, issues residual securities to outsider investors. We also show that our results hold even after we introduce relationship investors' proprietary information technology.[3]

Specifically, our baseline model considers a capital-constrained entrepreneur with a project that requires two rounds of financing. The first round requires a fixed investment that enables the entrepreneur to "experiment"—broadly interpreted as conducting early-stage activities such as hiring key personnel, acquiring initial users, and developing product prototypes—to produce interim information to persuade investors for continued financing. The entrepreneur lacks ex-ante commitment to any specific experiment and thus the production technology of interim information. But by monitoring and having a relationship access, the insider observes and can verify (better than arms-length outsiders) interim signals from the experiment, which is informative of the eventual profitability.

After forming the financing relationship and conducting interim experimentation, the entrepreneur raises capital in another round by issuing securities to this insider and potentially outsider investors. The insider's informational advantage relative to outsiders gives her bargaining power, and outsiders may learn from her decisions to continue or terminate the project. Following standard persuasion games that feature divergent objectives of the sender and the receiver, we assume that the entrepreneur enjoys a private benefit of continuing the project which is difficult to verify or contract upon. The entrepreneur's limited liability and the security choice also contribute to the divergence of sender-receiver objectives.

We first show that the entrepreneur follows a threshold strategy for experimentation. Through designing the informational environment, the entrepreneur makes the insider investor indifferent between termination and continuation. On the one hand, this reduces the insider's bargaining power from her information monopoly. On the other hand, the entrepreneur's interim information production is inefficient, rendering the insider incapable of recovering the initial investment in forming the relationship. Projects may not get initial financing to start with—a phenomenon we call Information Production Hold-up (IPH). We are the first to make these observations and our findings are in sharp contrast to existing

[3]We should remark that our findings do not rely on the entrepreneur's ability to commit to a disclosure policy or designing information to be arbitrarily informative. Instead, we require that the entrepreneur can design simple experiments such as one producing binary, threshold signals, and that within a financing relationship an insider verifies interim experiment outcomes better than outsiders.

theories on bank monitoring and hold-ups on entrepreneurial effort that ignore endogenous information production.

In industries requiring less entrepreneur-specific knowledge, "sophisticated" relationship investors can use their own information technology to evaluate projects' prospect, thus extracting positive interim rent, partially restoring the feasibility of the initial relationship investment. Relationship financing also becomes viable with moderate interim competition, because selling to competitive outsiders encourages more efficient information production by the entrepreneur. Investor competition (reflected through the insider's interim bargaining power) and sophistication (captured by the informativeness of her independent signal) jointly impact the dependence of relationship financing on competition, which is non-monotone in general. In particular, for intermediate levels of investor sophistication, the ease of relationship formation proxied by the initial funding capacity can therefore depend on interim competition in a U-shaped pattern, consistent with empirical findings that other models cannot explain (Elsas, 2005; Degryse and Ongena, 2007).

To explore contractual solutions to this general problem of information production hold-up, we recognize that the entrepreneur is biased towards continuation and gets all the ex-ante surplus. At the time of forming a financing relationship, he wants to but cannot commit to efficient interim information production—an inter-temporal wedge between the ex-ante entrepreneur (who shares the objective of a social planner at $t = 0$) and the interim entrepreneur. Inefficient continuation could also come from a continuation-payoff wedge between the insider and a social planner. Moreover, the mix of inside and outside finance at the interim date not only affects how surplus is shared (as in Rajan, 1992), but also how much information is produced. The ex-post expropriation (hold-up) generates suboptimal investment decisions by distorting incentives to produce information. The first-best security design for the entrepreneur therefore should align both the entrepreneur's interim information production incentive and the insider's continuation incentive with a social planner's. The problem involves infinitely-dimensional nested optimization, prompting us to take a novel constructive-proof approach.

Specifically, we propose a set of contracts that restore social efficiency, and show that they are the only optimal contracts. Because the entrepreneur's private benefit cannot be contracted upon, such a security has to fully expose the entrepreneur to the cost of inefficient continuation. Giving the insider debt-like securities in all bad states of the world (when the entrepreneur tends to overcontinue) and reserving a certain level of the securities to be sold to outsiders ensure that the entrepreneur's payoff is the most sensitive to the project's

payoff. This follows from that outsider investors are competitive and pays the entrepreneur fair prices, effectively rendering the entrepreneur the residual claimant of the project's social surplus regardless of the information revealed during the interim. Meanwhile, relationship financing is feasible as long as the contract yields the insider enough interim rent to recover her initial investment, leaving the security design partially determinate in good states of the world. Consequently, this optimal design uses warrants that specify both the quantity and terms for the insider to purchase some form of convertible securities, and issues the residual securities such as equity to arms-length outsiders, broadly consistent with real-life observations.

Finally, we discuss how our findings remain robust when we allow restricted experimentation space, partial commitment to information design, investor sophistication, and scalable investment, among others. We also discuss who should enjoy the right for experimentation design. The information production hold-up problem manifests itself under alternative security forms as well, and the economic mechanism apply even beyond relationship lending and staged venture financing. Our study helps underscore and formalize this practical issue, and develops potential contractual solutions. From a theory perspective, our study also sheds light on Bayesian Persuasion games with contingent transfers and sequential heterogeneous receivers, and deepens our understanding of contracting under endogenous information production.

Literature — Our theory foremost contributes to the large literature on relationship financing. Boot (2000), Gorton and Winton (2003), and Srinivasan et al. (2014) survey relationship lending. Theoretical studies on relationship banking focus on information production and control (e.g., Diamond, 1984, 1991; Fama, 1985): while relationship financing can improve financing efficiency (e.g., Petersen and Rajan, 1994), it naturally induces information monopoly (Berger and Udell, 1995; Petersen and Rajan, 2002; Rajan, 1992), potentially holding up the entrepreneur's effort in relationship lending (e.g., Santos and Winton, 2008; Schenone, 2010)) and venture capital (Burkart, Gromb, and Panunzi, 1997; Ewens, Rhodes-Kropf, and Strebulaev, 2016). We inform the debate by endogenizing the informational environment and analyzing the information production hold-up problem.[4]

[4]In this regard, our paper broadly relates to incomplete contracting and hold-up problems (Hart and Moore, 1988; Aghion, Dewatripont, and Rey, 1994; Nöldeke and Schmidt, 1995) and whether long-term contracts can mitigate investment inefficiencies (Von Thadden, 1995; Nöldeke and Schmidt, 1998). We differ primarily in endogenizing interim information production and deriving the optimal design without requiring contractibility of the entrepreneurs' bias of continuation — the non-contractibility of entrepreneurs' private

Empirically, the effect of competition on bank orientation has received significant attention, and Elsas (2005) and Degryse and Ongena (2007) document a puzzling U-shaped effect of market concentration on relationship lending. Extant theories predict either opposing monotone patterns (e.g., Petersen and Rajan (1995) and Dell'Ariccia and Marquez (2004) versus Boot and Thakor (2000) and Dinc (2000)) or suggest a hump-shaped pattern (e.g., Yafeh and Yosha, 2001; Anand and Galetovic, 2006). Our theory offers an information-based explanation for the empirical findings.

Our paper also sheds light on the role of intermediaries and contracting in entrepreneurial finance (Da Rin, Hellmann, and Puri, 2011; Kortum and Lerner, 2001). Importantly, we add to earlier studies on optimal securities in relationship finance or venture capital (e.g., Gompers, 1997; Kaplan and Strömberg, 2004; Hellmann, 2006; Inderst and Vladimirov, 2017) by endogenizing information production, and allowing general feasible securities and information production technology in the joint security and information design. Our discussion on security design also relates to seminal studies such as Holmstrom (1979) and Innes (1990), which concern agents' actions that alter the distribution of cash flows only.[5] The agent in our setting shapes the informational environment, and securities are issued to heterogeneous agents. While first-best outcomes are typically unattainable in conventional settings, our contractual solution restores efficient information production and investment.

Rather than deriving the optimality of convertible securities from ex-ante asymmetry between the issuer and investors (e.g., Stein, 1992; Brennan and Schwartz, 1988), our model features symmetrically informed entrepreneurs and investors, and studies informational issues in staged financing. Instead of examining hidden manipulations of non-verifiable signals under an exogenously given information structure (Cornelli and Yosha, 2003), we emphasize endogenous information production (verifiable to the relationship financier) and sequential securities for heterogeneous investors.[6] To our best knowledge, we are the first to show

benefit is realistic in typical relationship finance. See footnote 10 for further discussions.

[5] Although extant studies on the agency issues in costly experimentation such as Hörner and Samuelson (2013) and Bergemann and Hege (1998) consider information-acquisition effort, the principal cannot isolate information produced by the agent (hidden effort and hidden information).

[6] Cornelli and Yosha (2003) focuses on conversion as a tool for disciplining the entrepreneur's communication. The rationale for using convertible securities differ in our setting in that it not only helps the entrepreneur internalize inefficient continuation but also ensures the insider to recover initial investment cost. Arms-length investors serve to discipline the entrepreneur's communication as well. Moreover, Cornelli and Yosha (2003) show that *one* design—convertible securities—can address the issue of window dressing, we characterize *all* optimal designs under general security and contract space, and demonstrate they all exhibit debt-likeness in bad states and convertibility feature in good states. Chang and Szydlowski (2018) provides another setting of information design with heterogeneous agents.

that issuing convertible securities to early insider investors and equities to later outsider investors is optimal and robust to investor sophistication. Different from the prior literature, we emphasize the importance of both convertible securities for the insider financier and the presence of competitive outsider investors in the optimal contract design.

From a theory perspective, our paper contributes to the field of information design (Bergemann and Morris, 2017; Hörner and Skrzypacz, 2016), especially Bayesian Persuasion (Kamenica and Gentzkow, 2011; Ely, 2017; Dworczack and Martini, 2017). We take a linear programming approach similar to Bergemann and Morris (2017), but allow infinite payoff-relevant states and different types of informed receivers.[7] More importantly, we incorporate security design that endogenizes the dependence of the sender and receiver' payoffs on the state, allowing interactions of multiple asymmetrically informed receivers.

By so doing, our paper advances the emerging applications of information design in finance. Most closely related is Szydlowski (2016) that similarly allows contingent transfers and derives an irrelevant result of security choice when the entrepreneur jointly designs disclosure and security. Also related is Jiang and Yang (2018) which studies accounting requirements and firm investment. Other recent studies concern topics on capital structure (Trigilia, 2017), government intervention (Cong, Grenadier, and Hu, 2017), market for advice (Chang and Szydlowski, 2018), and stress tests (Bouvard, Chaigneau, and Motta, 2015; Goldstein and Leitner, 2015; Orlov, Zryumov, and Skrzypacz, 2017; Inostroza and Pavan, 2017). Our paper instead constitutes a first examination of relationship finance and contracting under information production hold-up. One main challenge of the field has been the strong assumption of commitment in disclosure policy or communication strategy. We overcome the challenge both by using dynamic relationship to naturally induce the insider investor's observing the interim experiment and signals, and by demonstrating the robustness of our findings to partial commitment. In fact, it is the lack of commitment to information design during entrepreneurs and investors' initial interactions that calls for security design to mitigate inefficient information production.

[7] We relax the assumption that the sender's utility from a message completely depends on the expected state (Kolotilin, 2017), or the sender's payoff over the receiver's actions is independent of the state (Gentzkow and Kamenica, 2016). Our discussion on investor sophistication also expands our knowledge about persuasion games with various receiver types. Kolotilin (2017) finds a non-monotone effect of the receiver's signal informativeness on sender and receiver's utilities. We differ in that we do not restrict the insider's and the entrepreneur's information production to be independent. When the receiver's type can be flexibly correlated with the signal of a sender's experiment, we find that the sender's and receiver's utilities are monotone in the receiver's signal precision.

2 A Model of Relationship Finance and Information

2.1 Financing of Projects

Consider a three-period economy with time index $t = 0, 1, 2$. There is no time discounting. A risk-neutral entrepreneur has a project that requires a fixed investment $I \in (0, 1)$ at $t = 1$, and produces an uncertain cash-flow $X \in [0, 1]$ at $t = 2$ with a prior distribution denoted by a continuous and atomless pdf $f(X)$. The entrepreneur can raise I by issuing securities at $t = 1$ to competitive, risk-neutral investors to finance the project.

In addition, an investor may invest K in the project at $t = 0$ to become a "relationship financier," or "insider." The initial investment enables the entrepreneur to experiment and generate interim information about the distribution of the cash-flow. One may also think of $K + I$ as the total investment needed, but raised in stages whereby early experimentation generates interim information (Kerr, Nanda, and Rhodes-Kropf, 2014). For simplicity, we assume that the seed investment K generates negligible initial cash flows compared to the final payoff expected by the investor and the entrepreneur, which is similar to normalizing the liquidation value to zero (Diamond, 1993) and can be equivalently interpreted as K representing the investment net of initial cash flows.

2.2 Experimentation and Information Production

The interim experimentation essentially allows the entrepreneur (the sender) to choose a finite set of messages $\mathcal{Z}$ and a mapping between the messages and the outcomes, which can be specified as conditional probabilities $\pi(z|X)$ for $z \in \mathcal{Z}$ and $X \in [0, 1]$. Consistent with the Bayesian persuasion literature and for expositional ease, we only require the mapping to be Bayes Plausible (Kamenica and Gentzkow, 2011).[8] That said, our baseline results all go through had we assumed a more restrictive space of information design (potentially due to different information costs or frictions, e.g., Hellwig, Kohls, and Veldkamp, 2012), as long as we allow binary experiments with a threshold scheme that generate a high signal for X sufficiently large and low signal otherwise. That is to say, full flexibility of information

[8] A distribution of posteriors is Bayes plausible if the expected posterior probability equals the prior. The literature typically only requires Bayes plausibility which is equivalent to allowing $(\mathcal{Z}, \pi)$ to be arbitrarily informative. This is justified by interpreting X as the most informative signal the entrepreneur can generate while conducting its usual entrepreneurial activities (Kamenica and Gentzkow (2011)). Under this interpretation, we should view the security payoff $s(X)$ as the expected payoff upon observing an abstract intermediate signal of value X. More explicitly, if Y is the set of all verifiable physical outcomes and the contract used is $\tilde{s}(Y)$, then $s(X) = \mathbb{E}[\tilde{s}(Y)|X]$.

design is not crucial and can be easily relaxed, which we do in Section 6.1.

Following the literature of relationship finance, we assume that even though $(\mathcal{Z}, \pi)$ is common knowledge, with probability $\mu \in [0, 1]$ only the insider observes the outcome of the experiment, and with probability $1 - \mu$ all investors publicly observe the outcome. μ essentially indicates the extent of the insider's informational advantage through close monitoring and repeated interactions (Megginson and Weiss, 1991), and we can interpret $1 - \mu$ as a reduced-form measure of the "interim competition" between the insider and the outsider commonly modeled in the relationship lending literature (Petersen and Rajan, 1995). $\mu = 0$ corresponds to perfect interim competition and $\mu = 1$ corresponds to information monopoly by the insider.

We should emphasize that unlike many papers on Bayesian persuasion, we do not require the entrepreneur to be able to commit to a disclosure policy or communication strategy after the experiment. Instead, we only need the experiment (a prototype, a trial run, a beta launch, etc.) to be more observable and its outcome to be more verifiable to the insider than to the outsiders, which is natural in relationship finance. For example, the experiment could be the building and testing of a prototype, over which the entrepreneur has full control. All we require is that a relationship financier understands the prototype design and testing, and effectively observes the mapping π because the relationship built at $t = 0$ allows close monitoring and interaction.

2.3 Misalignment of Incentives

As the project develops after the initial funding, the entrepreneur develops a private benefit of continuing the project. Specifically, the entrepreneur receives a private benefit ε if the project is financed at $t = 1$, where $\varepsilon \in (0, \bar{\varepsilon})$, and $\bar{\varepsilon}$ satisfies $\mathbb{E}\left[(X - I)\mathbb{I}_{\{X \geq I - \bar{\varepsilon}\}}\right] = K$, so that the project is always financed through relationship financing (positive NPV ex ante to the financier) under efficient interim information production. ε has an atomless distribution described by the pdf $g(\cdot)$ ex ante ($t = 0$) and is realized at the start of $t = 1$.[9] As would become clear shortly, financiers can infer from the interim experimentation the realization of ε, effectively rendering its interim value common knowledge.

[9]This resolution of ε is a simple way to capture that some uncertainty at $t = 0$ (about the misalignment or final project cash flow) resolves through the course of relationship (starting from $t = 1$). Our main results obtain even with a deterministic ε at $t = 0$, which we show in Section 6.1. But it is realistic that only after having started on the project that the entrepreneur realizes the exact misalignment of incentives in continuing the project.

Here ε could correspond to the entrepreneur's utility from the "non-assignable control rent"(Diamond, 1993), or payoff from assets- or business-in-place (Myers and Majluf, 1984), or perquisites that managers appropriate (Jensen and Meckling, 1976; Winton and Yerramilli, 2008; Jiang and Yang, 2018). We assume ε to be non-contractible because in practice it is hard to quantify or verify (Aghion and Bolton, 1992; Dyck and Zingales, 2004).[10]

As we show later, the entrepreneur's limited liability drives most of our results, and therefore we could have alternatively allowed negative values of X that the entrepreneur does not need to bear (e.g., Rajan, 1992). That said, using a private bias ε to capture the misalignment of incentives is standard in the literature and it constitutes a realistic source of agency conflict and eases our subsequent exposition of optimal security design.

To best illustrate our economic mechanism and match reality for early business start-ups, we assume that $\mathbb{E}[X - I + \bar{\varepsilon}] < 0$ which implies the absence of direct financing by arms-length investors ex ante. This innocuous assumption allows us to focus on the case in which relationship financing and informational considerations are indispensable.

2.4 Contracting Environments

In addition to the informational advantage, a relationship financier can potentially contract with the entrepreneur at the formation of the financing relationship at $t = 0$. In the paper, we consider three contracting environments.

First, to highlight the information production hold-up, we assume in the baseline case (Section 3) that the entrepreneur only uses an exogenously given security $s(X)$. We assume limited liabilities of the entrepreneur and insider investor, that is, $s(X) \in [0, X]$, and double-monotonicity for the security, that is, both $s(X)$ and $X - s(X)$ are weakly increasing in X.[11]

After the realization of z, the insider makes a take-it-or-leave-it offer to purchase a λ fraction of the security $s(\cdot)$, i.e. $s_I(X) = \lambda s(X)$, at a total price p^I.[12] The entrepreneur decides whether to accept and then if he still needs financing, he sells the remaining security, $s_O(X) = (1 - \lambda)s(X)$, to outsiders who offer a competitive total price p^O for the remaining securities. The investment takes place if and only if I is successfully raised, otherwise the

[10]The investor cannot promise to pay ε upon terminating the project because otherwise it would lead to entry of fly-by-night firms.

[11]See, for example, Nachman and Noe (1994), DeMarzo and Duffie (1999), DeMarzo, Kremer, and Skrzypacz (2005) and more recently, Cong (2017). If such monotonicity is violated, either the entrepreneur or the investor can be better off destroying some surplus for some state X, as Hart and Moore (1995) point out.

[12]The TIOLI assumption is not crucial, but is natural and consistent with the literature's assumption that relationship financiers have information monopoly and thus the interim bargaining power.

pledged capital is returned to investors.[13]

Second, to understand the importance of security design, we consider in Section 4 first a scenario wherein the entrepreneur can contract with the insider investor at time $t = 0$ on the fraction λ of security issuance at $t = 1$ that the insider can purchase before it is offered to arms-length investors. Then in the remainder of Section 4, without loss of generality, we allow the entrepreneur to contract at $t = 0$ on both λ and the design of securities to be offered at $t = 1$ to the insider financier and arms-length investors.[14]

2.5 Interim Payoffs and Relationship Formation

In general, the players' interim payoffs after the formation of a financing relationship are as follows:

$$p^O = \mathbb{E}[(1-\lambda)s_O(X)|\mathcal{F}^O] \tag{1}$$

$$u^E(X; p^I, p^O, \varepsilon) = \left(\varepsilon + X - s_I(X) - s_O(X) + p^I + p^O - I\right) \mathbb{I}_{\{p^I + p^O \geq I\}} \tag{2}$$

$$u^I(X; p^I, p^O) = \left(\lambda s_I(X) - p^I\right) \mathbb{I}_{\{p^I + p^O \geq I\}}, \tag{3}$$

where $\mathcal{F}^O$ denotes the outsiders' information set after having observed the insider's continuation or termination action.[15]

If the project is not financed, all players receive outside options normalized to zero. Intuitively, relationship financing is feasible only if the insider can recover in expectation at least the initial investment K, i.e.,

$$\mathbb{E}[u^I(X; p^I, p^O)] \geq K. \tag{4}$$

Finally, we assume the project would always be financed through relationship financing (positive NPV ex ante to the financier) if the interim information production is socially efficient. This holds under endogenous security design because $\bar{\varepsilon}$ satisfies $\mathbb{E}\left[(X - I)\mathbb{I}_{\{X \geq I - \bar{\varepsilon}\}}\right] = K$, but requires $\mathbb{E}[s(X) - I | X \geq I - \bar{\varepsilon}] \geq K$ in the baseline when $s(\cdot)$ is exogenous. This

[13] This scheme is often referred to as "all-or-nothing." Regarding its wide applications and impact on project implementation and information aggregation, see Cong and Xiao (2018).

[14] This is equivalent to allowing any form of contracts over X, including one that promises the insider $s_0(X)$ at $t = 0$. The insider's getting a total of $s_0(X) + s_I(X)$ upon continuation and nothing otherwise is equivalent to getting some other $s_I(X)$ alone.

[15] One can interpret the signal being either public or private as the receiver's private type. Note that the entrepreneur's experimentation affects both the insider and the outsiders' actions, different from Kolotilin, Mylovanov, Zapechelnyuk, and Li (2017) which studies the case of a single receiver with private types.

assumption allows us to focus on failures of financing relationship formation purely driven by the entrepreneur's endogenous information production, which generally differs from the social optimal.

2.6 Investor Sophistication and Information Production

In reality, a relationship financier may dictate entrepreneur's information-production activities, or receive additional information beside what the entrepreneur produces. For example, the insider may experiment herself, dictate to some extent what the entrepreneur must do, or use her proprietary business experience and expertise to predict the market demand or project valuation in future financing rounds. Importantly, the insider can set milestones in the initial contract, in which the insider conditions the next round of funding on pre-specified achievements and accomplishments. In fact, we can interpret the milestones as some "interim signals" that give information about the final cash-flow. We collectively refer to the ability of a relationship financier to utilize such information production technology exogenous to the entrepreneur's design as "*investor sophistication*."

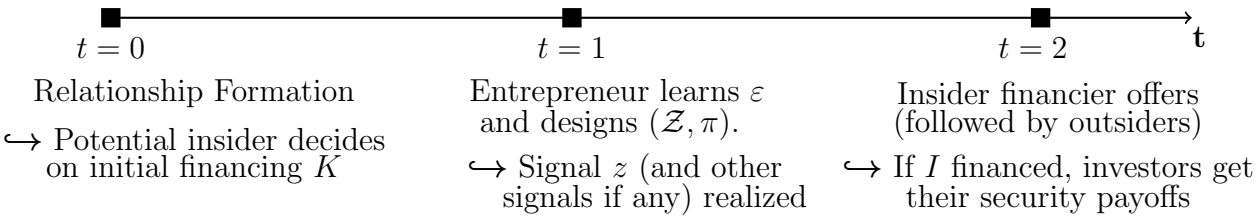
Investor sophistication can be incorporated in our baseline framework by allowing the insider investor to use an exogenous technology to produce an interim signal about the final project cash-flow X only through the financing relationship (not observable to the outsiders). Nevertheless, to best understand IPH, Sections 3 and 4 assume that only the entrepreneur has the relevant skill and expertise to design $(\mathcal{Z}, \pi)$ after raising K. This happens when the lender either has no previous experience on the project or it is too costly for him to extract information, e.g. the firm is located in a hardly accessible location, or the investor has no relevant expertise to generate independent signals. Section 5 then relaxes the assumption and models investor sophistication in detail.

3 Equilibrium and Information Production Hold-up

To clearly highlight the stark effect of the **information production hold-up (IPH)** problem and the way it drastically alters our understanding of relationship finance, we abstract away from contracting in this section and take the security design as exogenous, before allowing contracting at $t = 0$ and highlighting the role of security design in Section 4. Figure 1 summarizes the timeline of the baseline game.

To analyze the equilibrium, we work backward by first analyzing the interim persua-

Figure 1: Timeline of the game.



sion game after the formation of the financing relationship. In order to underscore the IPH problem under endogenous information production, it is useful to discuss exogenous experimentation as a benchmark, which we do next.

3.1 Benchmark with Exogenous Information

Consider an exogenously given $(\mathcal{Z}, \pi)$ that earlier studies specialize to. If signal z is privately observed by the insider, then when $\mathbb{E}[s(X)|z] \geq I$, the insider offers $p^I = I$ to finance the project entirely ($\lambda = 1$ endogenously), assuming any indifference (when $\mathbb{E}[s(X)|z] = I$) is resolved by financing $\lambda = 1$; else when $\mathbb{E}[s(X)|z] < I$, the insider terminates the project, leading to the outsiders negatively updating their priors and not investing either. The insider's information monopoly essentially gives her full bargaining power over the contractible interim surplus generated, $\max\{\mathbb{E}[s(X) - I|z], 0\}$, which corresponds to the well-known information hold-up in earlier models such as Rajan (1992).

In the case where z is publicly observable (which happens with probability $1 - \mu$), then both the insider and the outsiders would offer the competitive $p^I = p^O = \mathbb{E}[s(X)|z]$ when $\mathbb{E}[s(X)|z] - I \geq 0$. The entrepreneur extracts the whole interim surplus.

The entrepreneur's expected payoff for a given realization of ε is thus

$$\begin{aligned} U^E(\mathcal{Z}, \pi; \varepsilon) &= \mathbb{E}[u^E] = \mu \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - s(X))\pi(z|X) f(X) dX \\ &\quad + (1-\mu) \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - s(X) + \mathbb{E}[s(X)|z] - I)\pi(z|X) f(X) dX \\ &= \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - I)\pi(z|X) f(X) dX - \mu \int_0^1 \sum_{z \in \mathcal{Z}^+} (s(X) - I)\pi(z|X) f(X) dX, \end{aligned} \tag{5}$$

where $\mathcal{Z}^+ = \{z | \mathbb{E}[s(X)|z] \geq I\}$ corresponds to the set of signals that justify the in-

terim investment. Equation (5) follows from applying the law of iterative expectation to $\mathbb{E}[\mathbb{E}[s(X)|z]|z \in \mathcal{Z}^+]$. Correspondingly, the insider's payoff is:

$$U^I(\mathcal{Z}, \pi) = \mathbb{E}[u^I] = \mu \int_0^1 \sum_{z \in \mathcal{Z}^+} (s(X) - I)\pi(z|X)f(X)dX \tag{6}$$

Equations (5) and (6) reveal that for a given experiment, the entrepreneur's expected payoff is decreasing and the insider's expected payoff is increasing in μ — a measure of the insider's information monopoly (opposite to competition). By juxtaposing equations (4) and (6), we see that financing relationship is feasible only when the interim rent is sufficiently high, i.e., $\mu\mathbb{E}[(s(X) - I)\mathbb{I}_{\{\mathbb{E}[s(X)|z] \geq I\}}] \geq K$. These observations confirm the results in Petersen and Rajan (1995) that less interim competition leads to higher possibility of financing relationship.

We next show that these celebrated results have to be modified once we model the endogenous interim information production and the persuasion game played by the entrepreneur.

3.2 Endogenous Experimentation and Information Production

Here we show that the insider's payoff is no longer monotone in μ with endogenous information production. In particular, when the insider enjoys information monopoly (μ approaches 1), her *initial investment* to form the relationship is held-up. This *reverse* hold-up is in sharp contrast with the traditional hold-up under exogenous information in the previous literature on relationship finance—in the traditional hold-up, the possibility of the relationship financing is the highest when $\mu = 1$. The entrepreneur's *effort* is instead held up.

According to (5), the entrepreneur solves the following maximization problem

$$\max_{(\mathcal{Z},\pi)} \mathbb{E}[(\varepsilon + X - \mu s(X) - (1-\mu)I)\mathbb{I}_{\{\mathbb{E}[s(X)|z] \geq I\}}]. \tag{7}$$

Proposition 1 (**Endogenous Information Production**). *Entrepreneur's optimal experiments use two signals, i.e., $|\mathcal{Z}| = 2$. A signal h induces investment if $X \geq \max\{\bar{X}, \hat{X}(\mu)\}$,*

and otherwise a signal l induces termination, where $\bar{X}$ and $\hat{X}(\mu)$ solve:

$$\begin{aligned} \mathbb{E}[s(X)|X \geq \bar{X}] - I &= 0 && (8) \\ \varepsilon + \hat{X}(\mu) - \mu s(\hat{X}(\mu)) - (1-\mu)I &= 0 \quad \textit{if} \;\; \varepsilon - (1-\mu)I < 0 && (9) \\ \hat{X}(\mu) &= 0 \quad \textit{if} \;\; \varepsilon - (1-\mu)I \geq 0 && (10) \end{aligned}$$

Moreover, all optimal experiments lead to the same investment and payoffs, rendering the equilibrium essentially unique.

Proposition 1 characterizes the optimal experimentation after relationship formation. (8) indicates that the continuation based on $X \geq \max\{\bar{X}, \hat{X}(\mu)\}$ makes the insider financier at least break even; (9) indicates that if the private benefit is small relative to the competition, the entrepreneur rationally induces the continuation at X if and only if he can break even; (10) just says that if the private benefit is large relative to interim competition, the entrepreneur always benefit from the continuation, and the threshold is again pinned down by $\bar{X}$ in (8).

In equilibrium, while all profitable projects receive continued financing, some negative NPV ones do as well due to the entrepreneur's persuasion. When the insider privately observes the experiment outcome, she bears the cost of inefficient continuation and the entrepreneur would like to lower the threshold for investment for his private benefit; but as μ decreases, the entrepreneur's chance of getting a competitive price is higher, helping him better internalize the cost of inefficient continuation. These trade-offs determine the optimal threshold. In the extreme case $\mu = 0$, the entrepreneur sends a high signal for $X \geq I - \varepsilon$, which implements the socially efficient outcome; at the other extreme $\mu = 1$, the entrepreneur decreases the threshold to make the insider indifferent between investment and termination ($\mathbb{E}[s(X)|z = h] = I$).[16]

The proposition reflects the general phenomenon in persuasion games that a sender can fully "squeeze" a receiver's rent. The intuition does not rely on the TIOLI bargaining protocol: if there are outcomes in which the insider gets a positive expected payoff, then the entrepreneur could pool lower types of X with the higher ones in a way that increases the probability of investment and yields him a higher expected payoff. Therefore, such outcomes cannot possibly constitute an equilibrium.

Two questions are immediately and particularly relevant for relationship finance: First,

[16] In Appendix B4, we show that IPH can also reduce the effort distortion introduced by Rajan (1992). Here we focus on information production.

what does this information design power imply for the relationship formation? The next corollary reveals that information production hold-up may severely preclude (relationship) financing, both when the relationship financier enjoys strong information monopoly over the interim signal and when she faces intense interim competition.

Corollary 1 (**Information Production Hold-up** (IPH)). *For any K, there exist $0 < \mu^l < \mu^h < 1$ such that when $\mu \in [0, \mu^l) \cup (\mu^h, 1]$, relationship financing is infeasible.*

Recall that in Section 2.5 we have that with efficient information production, the financing relationship is feasible and socially efficient. But that is not the case under endogenous information production. It is straightforward that too much interim competition prevents initial investment from potential insiders, because the insider does not accrue enough interim rent to cover the initial investment K. Surprisingly, with full information monopoly the insider financier could also be held-up. This is because the entrepreneur produces imperfect information to inefficiently continue projects, making the insider again just break even during the interim and fail to recover K.[17]

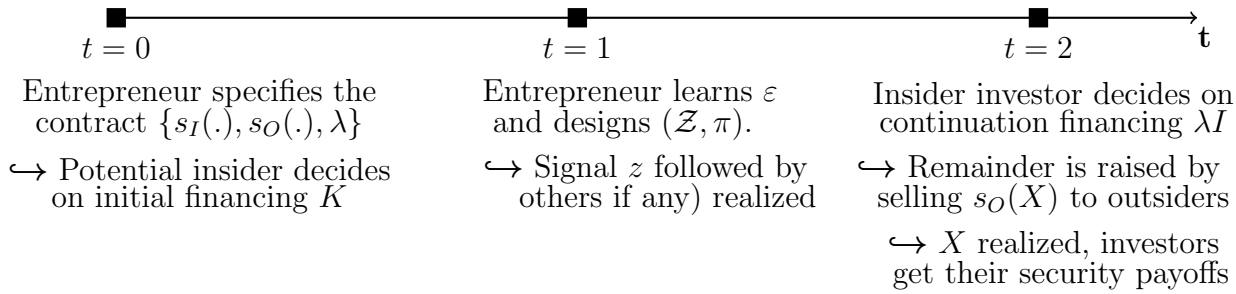
Given this IPH, the second question pertinent to finance is whether contingent transfers in the form of security payments, which is typically not considered in persuasion games, would solve the problem. As we show in the next section in Lemma 1, IPH is not driven by the fact that we are not contracting on the verifiable cash flow X, but is driven by the endogenous nature of information production. Contrasting the Corollary with (5) and (6), it should also be apparent that taking information as exogenous in relational financing is not an innocuous assumption—whether information production is endogenous determines whether the entrepreneur or the investor has interim bargaining power.

4 Contractual Solution for IPH

So far we have assumed that the entrepreneur only issues securities at $t = 1$. In this section, we allow long-term contracts at $t = 0$ that specify the amount of the securities the insider can purchase at $t = 1$ and their payoff.

[17] Our results are robust to allowing agents to renegotiate the security after $t = 0$, as long as the renegotiation protocols do not depend on agents' posterior beliefs on X (otherwise it is equivalent to contracting on experiment outcome). For any informational environment, the entrepreneur may extract the full interim surplus no matter what the security is. Therefore the entrepreneur always designs information to make the relationship financier break even.

Figure 2: Timeline of the game with optimal security design.



We assume that the security payoff can depend on X, but not on $(\mathcal{Z}, \pi)$ or the interim signals, which is standard in the Bayesian persuasion literature. It is also realistic to assume that contracts can be contingent on verifiable cash flows but cannot be contingent on the actions that generate information in a particular structure, because the information design space is rich and complex such that the entrepreneur cannot commit to it ex ante due to contract incompleteness. For example, an angel investor does not know what kinds of team members a founder is going to assemble, or what kind of field-specific experiments to conduct.

Moreover, interim signals are often not well-defined and too costly to verify to be useful for security contracts. For example, in Kaplan and Strömberg (2003), the sample of venture capital financing contracts involve contingencies only on an IPO taking place and satisfying certain conditions (e.g. regarding the issue price, proceeds, or market capitalization), but hardly ever on other observable and potentially verifiable interim information such as sales revenue, market share, or profitability.

4.1 Long-term Contracts

By long-term contracts, we mean that at time 0, the entrepreneur can contract on λ, the fraction of investment I to be financed from the insider financier in the second round, and the corresponding payment to the insider $s_I(\cdot)$. Outsiders observe the insider's decision, and finance the remaining $(1 - \lambda)I$ by purchasing security $s_O(\cdot)$ at a competitive price p^O. As such, every contract can be summarized by the triplet $\{s_I(\cdot), s_O(\cdot), \lambda\}$. Because the contracts specify both how the cost I is shared and how the contingent payments depend on future cash flows, they constitute a general contracting space. Figure 2 displays the timing of the interactions.

We first restrict our attention to long-term contracting with exogenous security types in

the next lemma. For example, if the security type is exogenously restricted to debt contracts, then $s(\cdot)$ is simply pinned down by the face value; if it is restricted to equity contracts, then $s(\cdot)$ is specified by the number of shares. These restricted contracts partially resolve the traditional information hold-up problem (e.g., Von Thadden, 1995), but not IPH.

Lemma 1 (Contracting with Exogenous Security Design)**.**

(a) For any given security $s(\cdot)$, without outsider investors ($\lambda = 1$, $s_I(X) = s(X)$), the equilibrium is unique and no project is financed at $t = 0$ unless $K = 0$.

(b) For any given security $s(\cdot)$, $\exists \lambda \in (0, 1)$ such that the insider receives positive interim rent under the contract $\{\lambda s(\cdot), (1 - \lambda)s(\cdot), \lambda\}$.

(c) No contract with a single form of security (including debt, equity, or call options) for both the insider and outsiders implements the first-best social outcome.

Recall that contract $\{\lambda s(\cdot), (1 - \lambda)s(\cdot), \lambda\}$ allows the insider financier to purchase λ fraction of the securities issued at $t = 1$, and sells the remainder to competitive arms-length investors. Part (a) extends Proposition 1 and Corollary 1 to the case where the entrepreneur commits to a long-term contract and consequently the insider does not have full bargaining power from making TIOLI offers during the interim. IPH is thus robust to whether the insider has information monopoly. More generally, one can prove that when the contracts can be renegotiated to leave the insider an intermediate level of bargaining power, IPH persists, just like our baseline case without long-term contracts.

Part (b) reveals that the insider's interim rent becomes positive for some $\lambda < 1$. This reflects the importance of "second-sourcing" the investment I (e.g., Farrell and Gallini (1988)), i.e., committing to $\lambda < 1$, because the interim rent is decreasing in λ when λ is big. Even the insider investor has the incentive to decrease his shares of the surplus somewhat to enable the entrepreneur to internalize the cost of inefficient continuation, and thus creating a larger social surplus. Note that although λ here has a similar effect as μ, it is a contract design parameter rather than an exogenous friction.

Finally, Part (c) reveals that long-term contracting with a single form of security cannot restore social efficiency. Given that the entrepreneur gets all the expected social surplus at $t = 0$, her ex-ante payoff is not maximized either. As we show next, only a set of carefully designed securities different for the insider and outsiders can resolve the problem associated with IPH.

4.2 Optimal Security Design and Contracting

Now we endogenize the design $s_I(X)$ for the insider and $s_O(X)$ for the outsiders, and show that the first-best outcomes are restored.

First, consider the designs that result in the outsiders investing only if the insider continues. We later show that designs outside this set cannot be optimal. Note that the entrepreneur's expected payoff from the contract (s_I, s_O, λ) is then given by:

$$U^E = \int_0^{\bar{\varepsilon}} \int_0^1 (\varepsilon + X - s_I(X) - s_O(X) + p^O(\varepsilon) - (1-\lambda)I)\mathcal{I}(X;\varepsilon)f(X)g(\varepsilon)dXd\varepsilon, \quad (11)$$

where p^O is the amount raised from the outsiders by selling the security $s_O(X)$, and $\mathcal{I}$ is the probability of investment at state $X \in [0,1]$ and the realized ε. The outsiders' information set contains the public signal z (if any) they receive with probability $1-\mu$, and the inference from the insider's action of continuation or termination.

Outsiders, being competitive, pay a "fair price" p^O given by

$$p^O(\varepsilon) = \frac{1}{\mathbb{E}[\mathcal{I}(X;\varepsilon)]} \int_0^1 s_O(X)\mathcal{I}(X;\varepsilon)f(X)dX \quad (12)$$

Combining (11) and (12) we rewrite

$$U^E = \int_0^{\bar{\varepsilon}} \int_0^1 M(X; s_I, s_O, \lambda, \varepsilon)\mathcal{I}(X;\varepsilon)f(X)g(\varepsilon)dXd\varepsilon,$$

where

$$M(X; s_I, s_O, \lambda, \varepsilon) = \varepsilon + X - s_I(X) - (1-\lambda)I. \quad (13)$$

The entrepreneur's optimal design then corresponds to the following maximization problem:

$$\begin{aligned} &\max_{s_I(.), s_O(.), \lambda} \mathbb{E}[M(X; s_I, s_O, \lambda, \varepsilon)\mathcal{I}^*(X;\varepsilon)] \\ s.t. \quad &\mathbb{E}[(s_I(X) - \lambda I)\mathcal{I}^*(X;\varepsilon)] \geq K \quad \textit{and} \quad s_I(X) + s_O(X) \leq X \quad \forall X \in [0,1], \end{aligned} \quad (14)$$

where the optimization is over the set of designs and the option to walk away from the financing relationship, and the constraints are the IC of the insider to form relationship and the entrepreneur's limited liability. $\mathcal{I}^*(\cdot;\varepsilon)$ is the equilibrium investment function under the

optimal experiment $(\mathcal{Z}^*(\varepsilon), \pi^*(\varepsilon))$, and is given by

$$\mathcal{I}^*(X;\varepsilon) = \sum_{z\in\mathcal{Z}} \pi^*(z|X,\varepsilon)\mathbb{I}_{\{\mathbb{E}[s_I(X)-\lambda I|z]\geq 0\}\cap\{\mathbb{E}[s_O(X)-(1-\lambda)I|\mathcal{F}^O]\geq 0\}}$$

and the optimal experiment given the contract $\{s_I(\cdot), s_O(\cdot), \lambda\}$ and ε solves the following:

$$\begin{aligned} &\max_{(\mathcal{Z},\pi)} \int_0^1 M(X; s_I, s_O, \lambda, \varepsilon)\mathcal{I}(X;\varepsilon)f(X)dX \\ \textit{where} \quad &\mathcal{I}(X;\varepsilon) = \sum_{z\in\mathcal{Z}} \pi(z|X,\varepsilon)\mathbb{I}_{\{\mathbb{E}[s_I(X)-\lambda I|z]\geq 0\}\cap\{\mathbb{E}[s_O(X)-(1-\lambda)I|\mathcal{F}^O]\geq 0\}\}} \end{aligned} \tag{15}$$

The problem involves infinitely-dimensional nested optimization, and the solution methodologies in conventional contracting and security design problems do not easily apply. We instead take a constructive-proof approach by conjecturing the optimal designs and show this set of designs uniquely achieve the first-best outcome and are indeed optimal in the sense that they maximize the entrepreneur's ex-ante payoff for all realizations of ε.

Proposition 2. *(**Optimal Design**) An optimal design exists and implements the first-best social outcome. All optimal designs entail experiments that generate continuation if and only if $X \geq I - \varepsilon$. Moreover, they essentially all involve the use of convertible securities, and are described by $\{s_I(\cdot), s_O(\cdot), \lambda\}$ satisfying the following conditions*

$$\lambda \in \left[0, \frac{I-\bar{\varepsilon}}{I}\right] \tag{16}$$

$$s_I(X) = \min\{\lambda I, X\}, \quad \forall\, X < I \tag{17}$$

$$\mathbb{E}[(s_I(X) - \lambda I)\mathbb{I}_{\{X\geq I\}}] = K \tag{18}$$

$$s_O(X) = X - s_I(X), \quad \forall\, X \in [0,1] \tag{19}$$

Equation (16) reflects the partial indeterminacy of the optimal design because λ can take on a range of values. Equation (17) requires that in the bad states of the world, the security is debt-like. In fact, the shape of the security in the region $X < I - \bar{\varepsilon}$ is indeterminate, but in terms of payoffs, they are equivalent, thus the word "essentially" in how we characterize the securities. (18) ensures that the insider breaks even ex-ante, but leaves the security shape indeterminate. Notice when we achieve the first-best social outcome, the insider gets paid only when the project is continued $X \geq I - \varepsilon$. But payoff is zero in $[I - \bar{\varepsilon}, I)$ anyway as ε is a $t = 0$ random variable that cannot be contracted upon, therefore the security payment

to the insider is necessarily independent of ε, consistent with the fact that the relationship is formed at $t = 0$ before ε is realized. Finally, (19) is driven by the entrepreneur's limited liability: As we argue shortly, she can use outsiders to commit to internalizing the cost inefficient continuation, but cannot give outsiders more than $X - s_I(X)$.

Figure 3 provides a concrete illustration using convertible notes for the insider and equities for the outsiders. The dotted line indicates the threshold for continuation vs termination signals.[18] We next provide the intuition for Proposition 2

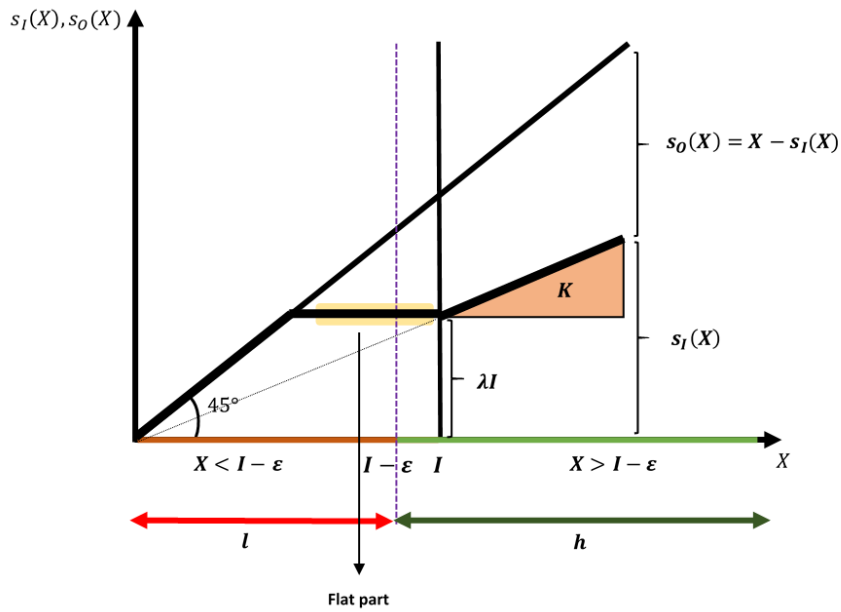


Figure 3: The optimal security under flexible security design.

Because the inefficiency lies in the continuation of bad projects ($X + \varepsilon < I$), what matters is how the security makes the information producer (the entrepreneur) internalize the cost of such inefficiency (partially through outsider investors that are present). From the insider investor's perspective, efficient continuation then entails her paying exactly the fair value for the security λI when the investment is socially marginal (zero-NPV), lest there is either a form of debt overhang resulting in underinvestment (when $s_I(I - \varepsilon) > \lambda I$ and the insider gets more than her fair share of the surplus) or a subsidy from the insider that leads to overinvestment (when $s_I(I - \varepsilon) < \lambda I$).[19]

[18] In Appendix B1, we show equity or debt could be optimal when the entrepreneur is constrained to issue the same security for the insider and the outsiders ($s_I(X) = \lambda s(X)$, $s_O(X) = (1 - \lambda)s(X)$, for some $s(X)$) due to some regularity reasons.

[19] We thank an anonymous referee for pointing out this intuitive argument.

On a first look, it seems that an optimal security only requires $s_I(I-\varepsilon) = \lambda I$, which makes the entrepreneur (i) indifferent between continuing and not continuing at $X = I - \varepsilon$ (social NPV is zero), (ii) strictly preferring continuation when $X > I - \varepsilon$, and (iii) strictly preferring termination when $X < I - \varepsilon$. Note that (ii) and (iii) follow from the monotonicity of $M(X; s_I, s_O, \lambda, \varepsilon)$ in X. However, ε represents an uncertainty resolved at $t = 1$ which then affects the entrepreneur's information production, and the entrepreneur cannot design the security at $t = 0$ to be dependent on one particular value of ε. As we discuss in Section 6.1, this is a general phenomenon when the information design space (from experimentation) differs from the contracting space (which is on the cash flow only), not an artifact of baseline assumption on ε.

As a result, committing to a large enough $1 - \lambda$ to expose the entrepreneur's payoff to inefficient continuation for the entire region of $X = I - \varepsilon$ yields the debt-like flat region in Figure 3.[20] Meanwhile, the security design also needs to ensure the insider earns enough from the second round to cover the initial investment K.[21] This restricts the security somewhat, but as long as the area under s_I but above the horizontal line λI reaches K, its shape to the right of the debt-like region is indeterminate. That said, the endogenous experimentation leads to a determinate informational environment that is also socially optimal.

we demonstrate that optimal designs have to entail some form of debt-likeness and convertibility, consistent with real practice. Information production is crucial in determining the optimal design because the entrepreneur does not control project continuation directly. Note that the debt-likeness is not driven by risk-bearing capacity or informational asymmetry, but by the fact that entrepreneurial bias and information production are difficult to contract on. As a result, it is crucial that an optimal design aligns the entrepreneur's preferences in information production with that of the social planner in a robust manner.

Optimal Security and Contract in Practice

Our goal here is not to introduce an alternative mechanism or competing theory for the predominant use of convertible securities in VC financing (Kaplan and Strömberg, 2003) or to proclaim the information design channel to be the most dominant. Besides proving the

[20] We prove in Section 6.1 that even with a deterministic ε at $t = 0$, optimal security design entails this debt-like flat region as long as there is some uncertainty at $t = 0$ (about the misalignment or final project cash flow) that resolves through the course of relationship (starting from $t = 1$), because then pinning down one single point of the security based on ε does not trivially correct the divergence of incentives.

[21] In Appendix B3, we discuss how our contractual setting relates to the classical literature on contracting and moral hazard.

optimality of convertible securities, we emphasize the role of arms-length outsiders and the need for the joint optimal design for both initial insider investors and outsiders, which extant theories do not discuss (Inderst and Vladimirov, 2017, is a notable exception). Moreover, Proposition 2 indicates that (i) earlier studies' conclusions are robust to introducing endogenous and flexible information production, and (ii) the optimal security for the insider not only has a debt-like region, but can also include a large class of securities, consistent with empirical observations.

We can interpret the initial-round optimal contract as warrants to the insider to purchase convertible securities in the later round. Conditional on the insider's follow-on investment, the entrepreneur raises the remaining $(1-\lambda)I$ from outsiders by issuing residual securities. Alternatively, we can view the initial round as issuing $\frac{K}{K+\lambda I}$ fraction of convertible securities to the insider, together with warrants allowing the insider to purchase the remaining fraction at a price λI in a later round. One more interpretation is that $K+\lambda I$ is the total amount of financing at the initial round, but paid out in stages, and $(1-\lambda)I$ is a separate issuance to the outsiders. In all these scenarios, although round financing and stage financing in general differ (Cuny and Talmor (2005)), the effect of the optimal design remains the same.

In addition, setting λ can be viewed as designing a later-stage syndication. While the lead investor finances λI, the remainder is financed by syndicate members that observe the leader's action and are competitive.[22]

No matter which interpretation we take, as the cost of initial experimentation goes up, the entrepreneur needs to commit to financing a higher fraction of the follow-on investment from the insider. Take for example a commonly observed security — convertible notes with conversion rate 1:I, i.e., a bond with face value λI can be converted to λ share of equity. One can show that λ is strictly increasing in K.

Finally, we note that the optimal design is partially indeterminate and include most convertible securities used in real-life practice, a fact other models do not account for.

[22]Indeed, with a few exceptions all later-stage venture capital investments are syndicated (Lerner (1994)), and partnership agreements often pre-commit venture capitalists to syndicate later-stages of investments (Sahlman (1990)). In this sense, syndication not only can protect the entrepreneur from ex post hold-up by investors and thereby encourage effort (Fluck, Garrison, and Myers (2006)), it also encourages more efficient information production.

5 Relationship Finance under Investor Sophistication

In this section, we show investor sophistication—the ability of a relationship financier to utilize a given information production technology other than the entrepreneur's experimentation to generate interim information—mitigates IPH. Interestingly, its interaction with interim competition helps us rationalize puzzling empirical patterns in the formation of lending relationships. We also prove that the contractual solution in Proposition 2 is robust to the level of the insider's sophistication. By doing so, we essentially develop the solution for a Bayesian persuasion game involving multiple heterogeneous receivers with private types.

To incorporate the insider's information production, we allow her to use an exogenous technology to produce an interim signal about the final project cash-flow X through the financing relationship (not observable to the outsiders). In particular, the insider uses experiment $(\mathcal{Y}, \omega_q)$, where $\mathcal{Y} = \{y_1, y_2, \ldots, y_m\}$ is a finite set with m signals. $\omega_q(y_i|X)$ represents conditional distributions. $q \in [0,1]$ is an index that ranks the informativeness of different experiments, with $q = 0$ for an uninformative experiment. Specifically, for $q > q' \in [0,1]$, the experiment $(\mathcal{Y}, \omega_q)$ is more informative than $(\mathcal{Y}, \omega_{q'})$ in the sense of Blackwell (1951).[23]

Moreover, for every q, we assume the signals in $\mathcal{Y}$ can be ranked: for every $m \geq i > i' \geq 1$, distribution $f(X|y_i)$ dominates distribution $f(X|y_{i'})$ in the sense of the first order stochastic dominance, i.e. for every $X \in (0,1)$, we have $F(X|y_i) < F(X|y_{i'})$. This assumption directly implies that $\mathbb{E}_q[s(X)|y_i, z] > \mathbb{E}_q[s(X)|y_{i'}, z]$ for every security $s(\cdot)$ and signal z with a non-degenerate distribution. The following instance illustrates the information structure.

Example 1. *The insider's experiment generates a binary signal, i.e.* $\mathcal{Y} = \{\tilde{h}, \tilde{l}\}$*, with the following information structure for* $(\{\tilde{h}, \tilde{l}\}, \omega_q)$*:*

$$\omega_q(\tilde{h}|X) = \begin{cases} \frac{1+q}{2} & I \leq X \leq 1 \\ \frac{1-q}{2} & 0 \leq X < I. \end{cases}$$

The investor receives a signal $y = \tilde{h}$ with probability $\frac{1+q}{2} \geq \frac{1}{2}$ if the project is profitable, and $y = \tilde{l}$ with probability $\frac{1-q}{2} \leq \frac{1}{2}$ otherwise. Clearly $\mathbb{E}_q[s(X)|\tilde{h}] \geq \mathbb{E}_q[s(X)|\tilde{l}]$ for every $q > 0$

[23] Recall the Blackwell Informativeness Criterion: An experiment provides the decision maker with an updated set of probabilities over the states of nature. One signal is at least as informative as another if, for all utility functions and action sets, the decision maker's expected utility from implementing the optimal action (which may vary with his preferences and with the signal he observes) is at least as high with that signal.

and security $s(\cdot)$. Moreover, if $1 > q > q' > \frac{1}{2}$, then the following inequalities also hold:

$$\mathbb{E}_q[s(X)|\tilde{h}] > \mathbb{E}_{q'}[s(X)|\tilde{h}] > \mathbb{E}_{q'}[s(X)|\tilde{l}] > \mathbb{E}_q[s(X)|\tilde{l}]$$

It means that experiments with higher values of q generate relatively more extreme signals.

Considering the insider's information production, the entrepreneur now designs an experiment $(\mathcal{Z} \times \mathcal{Y}, \pi)$, where $\pi(.,.|X) : \mathcal{Z} \times \mathcal{Y} \to [0,1]$ is the joint conditional probability of observing the signals. Note that the marginal distributions for $y \in \mathcal{Y}$ must be consistent with the insider's experiment, i.e. $\sum_{z \in \mathcal{Z}} \pi(z, y|X) = \omega_q(y|X)$ for every $X \in [0,1]$ and $y \in \mathcal{Y}$. Moreover, we allow the signals in $\mathcal{Z}$ and $\mathcal{Y}$ to be correlated conditional on the true state of the world.[24] Similar to the baseline case, the outsiders observe signal z with probability $1 - \mu$, while signal y is always privately observed by the insider financier.

5.1 Information Production with Investor Sophistication

Similar to our discussions in Sections 3 and 4, we first derive the optimal experiment and the equilibrium payoffs for the entrepreneur and the insider for a given $q \in [0,1]$ when long-term contracting is not feasible, and then discuss contracting on interim events and how investor sophistication interacts with competition to rationalize intriguing empirical facts in relationship lending. Section 5.4 analyzes the effect of long-term contracting and security design. We first prove a useful lemma:

Lemma 2. *The optimal (Z, π) is at least as informative as (q, ω_q), in the Blackwell sense. In other words, the outsiders perfectly infer $y \in \mathcal{Y}$ by observing $z \in \mathcal{Z}$ from the endogenous experimentation.*

Lemma 2 essentially says that the entrepreneur optimally conveys the insider's private signal to the outsiders. This does not matter when z is only observed by the insider (with probability μ). But when z is public (with probability $1-\mu$), the entrepreneur prefers to level the playing field by informing the outsiders of y and eliminating the insider's informational advantage. A priori, one might think this could hurt the entrepreneur's payoff because a

[24] To think about this correlation, note that the entrepreneur can simply include the results from the insider's experiment or a noisy version of them in his experiment report to the investors. Even when the entrepreneur does not observe y, he can still enable the correlation as long as he knows $(\mathcal{Y}, \omega_q)$. Guo and Shmaya (2017) and Kolotilin (2017) derive general results for the case that the sender's and the receiver's experiments must generate *independent* signals conditional on X. In Appendix B2, we illustrate how results are robust to such a requirement.

negative y signal may decrease the probability of continued financing from outsiders. But obfuscating the signal is not helpful here because the insider's termination action upon seeing a negative y already conveys the information to the outsiders.

Denote the signal that $X \geq \max\{\hat{X}(\mu), \bar{X}(y_i)\}$ by x_H^i, where $\bar{X}(y_i)$ is the solution to $\mathbb{E}_q[s(X) - I|y_i, X \geq \bar{X}(y_i)] = 0$ if a solution exists and is zero otherwise. Denote the signal for the opposite $X < \max\{\hat{X}(\mu), \bar{X}(y_i)\}$ by x_L^i. Now we can characterize the optimal experiment in Proposition 3.

Proposition 3 (**Endogenous Information under Investor Sophistication**). *An optimal experiment exists and requires at most $2|\mathcal{Y}| = 2m$ signals. For every signal $y_i \in \mathcal{Y}$, the entrepreneur sends either $z_i^h = \{y_i, x_H^i\}$ or $z_i^l = \{y_i, x_L^i\}$.*

Proposition 3 extends Proposition 1 to allow insider sophistication. Lemma 2 implies that the entrepreneur can split the information design problem into m separate information design problem, each following Proposition 1.[25] Since the optimal experiment in Proposition 1 has at most two signals, the optimal experiment in presence of a sophisticated investor has at most $2m$ signals.

Before we investigate the impact of investor sophistication on relationship formation, the following lemma reveals that the first-best outcomes cannot be achieved under investor sophistication alone.

Corollary 2. *For any given $(\mathcal{Y}, \omega_q)$, the equilibrium investment decision is not ex-post socially optimal with a positive probability.*

Therefore, initial long-term contracts with the right security design is integral to achieving the socially optimal investment. To that end, we next discuss whether setting milestones resolves IPH, before discussing optimal contracting and security design under investor sophistication in Section 5.4.

5.2 Setting Milestones

Contracting on interim events is related to "milestones" commonly used in venture financing. For example, the entrepreneur can commit to reaching a pre-specified scale of

[25] A priori, it is not obvious that the entrepreneur should design the experiment in correspondence to y type by type. For instance, if the insider and outsiders bid for the security simultaneously, the entrepreneur might find it optimal to only partially disclose the insider's signal to the outsiders. The simple design follows only after taking into consideration that outsiders learn from the insider's interim action.

customer base before seeking continued finance. Would that solve the IPH problem? Perhaps surprisingly, we show the insider cannot increase her expected payoff by contracting on the interim events.

First note that contracting on milestones is different from contracting on the experiment $(\mathcal{Z}, \pi)$. Instead, it is about how continuation is contingent on other information the insider observes. Take the example of user base, the information structure about user base is already given, i.e., the insider can observe the number of users which is informative about the future profitability of the project.

To proceed, we use $\mathcal{Y}^b \subset \mathcal{Y}$ to denote binding signals following which the insider commits to either continue or terminate financing ex ante. We denote the set of non-binding signals by $\mathcal{Y}^{nb} = \mathcal{Y} \setminus \mathcal{Y}^b$.

Corollary 3 (**Milestone Futility**). *The insider cannot gain from setting milestones. In particular, the insider's expected payoff is maximized when* $\mathcal{Y}^b = \emptyset$.

Corollary 3 shows that the insider cannot resolve IPH by setting milestones. The reason is that the entrepreneur can flexibly control how the insider updates her prior for every $y \in \mathcal{Y}^{nb}$, regardless of the choice of $\mathcal{Y}^b$. Therefore, the insider's information set following signals in $\mathcal{Y}^{nb}$ does not change, while she potentially makes suboptimal decisions following signals in $\mathcal{Y}^b$ due to the binding commitment.

This also explains why milestones are seldom binding in practice. Besides relying on projections in early stages that are hard to make or enforce, binding milestones render endogenous information production irrelevant, and thus do not help the insider in extracting more interim rent.

5.3 Relationship Formation, Sophistication, and Competition

How does investor sophistication impact relationship formation? For simplicity, let us focus on the case without security design or long-term contract at $t = 0$. At least in the relationship banking literature, the borrower takes the debt contract as given. Adding long-term contract without security design results in similar patterns, and we therefore leave that out.

Intuitively, the insider always benefits from having a more informative experiment, as the next corollary shows.

Corollary 4 (Relationpship and Sophistication). *For every $\mu \in [0,1]$, the insider's expected payoff $U^I(\mu; q)$ is weakly increasing in q.*

This result is in contrast with Kolotilin (2017) which shows the receiver's expected payoff to be non-monotone in the informativeness of her private signal. Because the insider's expected interim payoff $U^I(\mu; q)$ is higher with higher q, insider sophistication directly facilitates relationship formation.

Furthermore, investor sophistication interacts with interim competition, which helps us rationalize an empirical observation in relationship banking. Prior theoretical predictions on the effect of competition on bank orientation so far has been ambiguous. The investment theory (e.g., Petersen and Rajan, 1995; Dell'Ariccia and Marquez, 2004)) argues that as the credit market concentration decreases, the firms' borrowing options expand, rendering banks less capable to recoup in the course of the lending relationship the initial investments in building relationship, which hinders relationship banking; the strategic theory (e.g., Boot and Thakor, 2000; Dinc, 2000) says fiercer interbank competition drives local lenders to take advantage of their competitive edge and reorient lending activities towards relational-based lending to small, local firms, which strengthens relationship banking. Others (e.g., Yafeh and Yosha, 2001; Anand and Galetovic, 2006) suggest that competition can have ambiguous effects on lending relationships, but typically predict an inverted U-shape pattern.

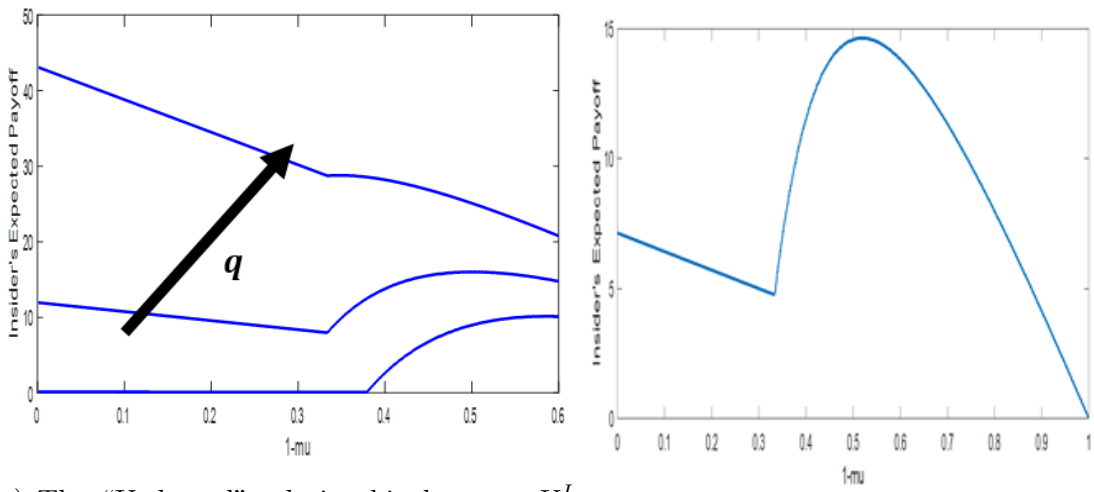
Yet empirically Elsas (2005) and Degryse and Ongena (2007) document a U-shape relationship between likelihood of the lending relationship and the level of competition in the credit market. These two studies stand out because they measure relationship banking directly in terms of duration and scope of interactions, thus improve upon and complement indirect measures such as loan rate (Petersen and Rajan, 1995) or credit availability over firms' life time (Black and Strahan, 2002), for which the impact of competition could be ambiguous in equilibrium (Boot and Thakor, 2000). Our theory offers an explanation.

Proposition 4 (Relationship and Competition). $\exists\ \underline{\mu}(q) \in (0,1)$ *such that for $\mu \in [\underline{\mu}(q), 1]$, the insider's payoff from the relationship financing, $U^I(\mu; q)$, is increasing in the level of interim competition $(1-\mu)$ for unsophisticated investors ($q = 0$), decreasing for sophisticated investors (sufficiently large q), and U-shaped for investors with intermediate sophistication.*

On the one hand, for a fixed level of private benefit of continuation, lower levels of competition increases the insider's share of the surplus, and is preferred by more sophisticated

investors who can produce her own information. On the other hand, higher levels of competition can encourage more efficient information production from the entrepreneur which increases total surplus, thus is preferred by the less sophisticated investors who have no other means to obtain information rent. For intermediate values of sophistication, competition hurts the insider's profit until it replaces the investor's independent information as her main source of interim rent, leading to the local U-shape.

Figure 4(a) illustrates the relationship between $U^I(\mu; q)$ and μ, the *inverse* measure of competition in our model. In particular, when q takes intermediate values and μ is exogenous, our model thus helps rationalize the findings of Elsas (2005) and Degryse and Ongena (2007).



(a) The "U-shaped" relationship between U^I and $1 - \mu$

(b) Non-monotone relationship between $U^I(\mu; q)$ and $1 - \mu$

Figure 4: Illustration of the equilibrium capacity of the financing in the initial round as a function of the level of ex-post competition

To the extent that small banks are able to collect and act on soft information whereas large banks rely on hard information produced by credit bureaus and alternative sources (e.g., Berger, Miller, Petersen, Rajan, and Stein, 2005)—higher q in our model, our theory predicts that competition would reduce relationship formation in regions where large and out-of-market banks are pre-dominant, and the opposite holds for regions with mostly small banks and mutual banks, consistent with empirical evidence in Presbitero and Zazzaro (2011).

Finally, our model reveals that while low levels of interim competition means an insider can recoup the initial cost of forming the relationship, the ease of forming the relationship involves a more complex non-monotonicity: In addition to the U-shape, when the market

becomes extremely competitive (μ gets much closer to 0), relationship formation eventually decreases in competition, as seen in Figure 4(b).

5.4 Optimal Contracting and Security Design

Now we examine whether security design and contracting at $t = 0$ mitigates IPH under investor sophistication. The next proposition shows that Proposition 2 is robust to insider sophistication and still achieves the first-best outcome.

Proposition 5 (Optimal Contracts with Investor Sophistication). *Regardless of $(\mathcal{Y}, \omega_q)$, all optimal securities are characterized by (17)-(19).*

The intuition behind Proposition 5 is the following: Because of the flat part of the security, the insider's security entails no risk when the entrepreneur reveals that $X > I - \varepsilon$. Therefore, the insider always continues the project following $z = h$, regardless of her own signal. Hence the insider's action only reveals whether $X > I - \varepsilon$ or not. As a result, by the design of the security, the outsiders invest if and only if the insider invests.

In contrast to the case without long-term contracts, the entrepreneur's experiment under the optimal long-term contract does not depend on the insider's information production. In other words, the entrepreneur might find it optimal to hide some of the insider's information from the outsiders. As a result, information asymmetry exists between the insiders and the outsiders but it does not prevent implementing the socially optimal investment, because the contractual solution resolves all inefficiencies.

6 Discussion and Extension

6.1 Restricted Information Design Space

Thus far we have assumed that the contracting space and information design space coincide, that is, the entrepreneur is able to generate arbitrarily informative signals about the eventual cash flow. We next show that our main results, IPH and the optimality of convertible securities, continue holding when we relax this assumption. Doing so also demonstrate that all our results are robust to having a deterministic ε at $t = 0$.

Specifically, we depart from the baseline setup to assume that the final cash flow is $Y(X, \delta_1, \delta_2)$, where $Y(\cdot, \cdot, \cdot)$ is strictly increasing in X and δ_t, $t = 1, 2$, are random variables

being realized and publicly observed at the beginning of period t. We denote their support by $[\underline{\delta}_t, \bar{\delta}_t]$, $t = 1, 2$. We further denote the probability density of Y by $f_Y(\cdot)$.

The security used to finance the project is $s(Y)$ when it is exogenous and $\{\lambda, s_I(Y), s_O(Y)\}$ when it is endogenously designed at $t = 0$. In other words, the contracting is over the cash flow Y while the information production and design is over a restricted space X. To be consistent with our baseline model, we further assume that $\mathbb{E}[\varepsilon + Y - I|\delta_1] < 0$ for all realizations of δ_1, that is, experimentation is necessary to justify the interim investment.

Note that this specification also allows the entrepreneur's private benefit $\varepsilon \in (0, \bar{\varepsilon})$ to be deterministic at $t = 0$, that is, $g(\varepsilon)$ can be an atom. To see this correspondence, we can simply set $\delta_1 = \varepsilon$ and $\delta_2 = 0$. This matters little for Proposition 6, but allows us to demonstrate in Proposition 7 that convertible securities are optimal even when ε is common knowledge at the time of relationship formation.

In Proposition 6, we first verify the presence of IPH without long-term contracting.

Proposition 6. *Consider the setting illustrated in Figure 1 with the modifications mentioned above. For any K and $s(\cdot)$, relationship financing is infeasible for sufficiently high and sufficiently low levels of interim competition (captured by $1 - \mu$).*

Proposition 6 shows that IPH does not require the entrepreneur's having access to arbitrarily informative experiments. In fact, as long as the entrepreneur's information input is marginal for the continuation decision, the insider investor would be held up by the entrepreneur's inefficient information production, especially when the level of interim competition is low.

Proposition 7 shows that under general conditions, the optimal contract involves a security with a debt-like flat region to help the entrepreneur internalize the cost of inefficient continuation. The optimality of convertible securities therefore does not hinge on our specific baseline assumptions.

Proposition 7. *Suppose $X^*(\delta_1)$ is the solution to $\mathbb{E}[Y|X = X^*(\delta_1); \delta_1] = I - \varepsilon$. Furthermore, define the probability density $\tilde{f}_{Y,\delta_1}(Y) \equiv f_Y(Y|X = X^*(\delta_1); \delta_1)$. If $\tilde{f}_{Y,\delta_1}(Y) \succ_{FOSD} \tilde{f}_{Y,\delta_1'}(Y)$ for $\delta_1 > \delta_1'$, then the optimal security for the insider financier, $s_I(Y)$, is flat in the region $[\underline{Y}, \bar{Y}]$, where $\bar{Y} \equiv \sup\{Y|\tilde{f}_{Y,\delta_1}(Y) > 0\}$ and $\underline{Y} \equiv \inf\{Y|\tilde{f}_{Y,\delta_1}(Y) > 0\}$.*

To maximize his expected payoff when forming the financing relationship at $t = 0$, the entrepreneur would like to generate continuation signal if and only if $X \geq X^*(\delta_1)$ because doing so generates the maximum social surplus which ex ante all accrue to him. He therefore

needs a security that helps him internalize the cost of inefficient continuation during the interim regardless of the realization of δ_1. In other words, the optimal security has to be flat over the entire region of Y that is realizable conditional on any realization of δ_1 and $X = X^*(\delta_1)$.

6.2 Scalable Investment and Continuum Range of Actions

Instead of the binary investment decision, we now allow scalable investment at $t = 1$ after observing the experimentation outcome.

Specifically, we assume that investing θI generates a stochastic cash flow $r(\theta)X$, where $r(\cdot) : [0, 1] \to [0, 1]$ is weakly increasing. For consistency with the baseline model, we assume $r(0) = 0$ and $r(1) = 1$; our baseline model corresponds to $r(\theta) = 0$ for $\theta < 1$ and $r(1) = 1$. Without any interim competition ($\mu = 1$), the final payoffs following investment level θ and realization of cashflow $r(\theta)X$ are $u^E = r(\theta)X - s(\theta, r(\theta)X) + r(\theta)\varepsilon$ and $u^I = s(\theta, r(\theta)X) - \theta I$ respectively, where $s(\cdot, \cdot)$ is a security payment that is generally contingent both on the project scale (or equivalently, level of investment) and the final cash flow, and private benefit of continuation depends on project scale.

For simplicity, we only require the entrepreneur's experimentation to satisfy Bayes Rule and allow him to choose experiments with infinitely many signals. In particular, we show there are situations that the entrepreneur perfectly discloses X to the investors, since their action set is not finite anymore.

First note that if $\frac{r(\theta)}{\theta}$ is weakly increasing in $\theta \in (0, 1]$, that is, the project has the full benefit of scale, the investment decision reduces to a binary decision and apparently all previous results apply. As such, we focus on diminishing return to scale (DRS). Proposition 8 extends our main results to this case.

Proposition 8 (**Scalable Investment**).

(a) The social welfare of investment, $\mathbb{E}[r(\theta)(\varepsilon+X)-\theta I]$, is increasing in interim competition $1-\mu$. In particular, when the entrepreneur issues equities, i.e., $s(\theta, r(\theta)X) = \beta r(\theta)X$ for some $\beta \in (0, 1)$ and $r(\theta) = \theta^\gamma$ for some $\gamma \in (0, 1)$, the informativeness of the entrepreneur's experiment weakly decreases in the Blackwell sense, as μ increases.

(b) Suppose $\theta^*(X; \varepsilon)$ is the optimal level of investment for X and ε, i.e. it is the solution of $\max_\theta r(\theta)(X + \varepsilon) - \theta I$. If $r(\theta^*(X; \varepsilon))X - \theta^*(X; \varepsilon)I \geq 0$ with probability one for all values of $\varepsilon \in (0, \bar{\varepsilon})$, then a convertible security for insiders and the residual claim for outsiders

implement the first-best outcome, with the entrepreneur choosing the scale after the realization of signal z to achieve the desirable level of investment, and perfectly disclosing information.

Proposition 8(a) extends the main result in Section 3 that the insider's interim rent is not increasing in her information monopoly, to continuous (non-binary) investment decisions. More information monopoly (higher μ) means less alignment of the entrepreneur's payoff with the social planner's, which leads to inefficient information production. However, the scalable investment helps the insider to recover some interim rent, even if she fully holds up the entrepreneur ($\mu = 1$). When the insider chooses the scale of investment from a continuous set, she should get zero expected rent from the marginal level of investment. The DRS assumption implies she gets positive interim rent overall, if the investment takes place.

Proposition 8(b) extends Proposition 2. To implement the first-best outcome under DRS, the security should not only encourage the entrepreneur to experiment efficiently but also incentivize the insider to efficiently scale the project. We thus choose the insider's security to perfectly align the insider's and the social planner's utilities over different scales of investment. We describe the security in greater detail in the appendix.

The key takeaway is that with DRS and a continuum of investment levels, investment decisions are no longer binary (investing I or terminating), and the insider derives partial rent from her informational monopoly. Endogenous information production still leads to reduced rent which potentially renders relationship financing infeasible, consistent with Lemma 1. Moreover, similar to the case of binary investments levels, a contractual solution entailing giving convertible securities to initial investors still achieves the first-best outcome, and is robust to investor sophistication.

6.3 Commitment to Information Design and Partial Observation

Ex-ante Commitment to Experimentation

Commitment issues are common in applications of information design and Bayesian persuasion. Our setup relaxes this strong assumption that the entrepreneur can commit in the initial round to a disclosure policy later in the relationship.[26] That said, the insider has

[26]In other words, the entrepreneur cannot commit to an experimentation when raising K. Otherwise, the insider can extract rent (as seen in Section 5), and the problem in Section 4 becomes a joint optimization problem on security design and information disclosure for a total issuance of $K + I$, which Szydlowski (2016) addresses. He shows that both equity and debt can implement the optimal design, and the optimal security is indeterminate.

limited ability in dictating the entrepreneur's experimentation at the time of initial financing, but can observe it after becoming an insider. In essence, the lack of commitment and contractibility of the experimentation at $t = 0$ in our setting (partially) breaks the indeterminacy in Szydlowski (2016) which has only one round of financing by fully competitive investors.

For illustration, suppose we use debt security to finance the project without investor sophistication, the insider investor receives less reward from high realizations. Therefore, the entrepreneur promises a larger λ to the insider in the second round to compensate for her initial investment K. The entrepreneur, less exposed to the downside (smaller $1 - \lambda$), then chooses less informative signals ex post, which decreases the financing capacity in the first round. Clearly, the choice of security ex ante affects the choice of information design ex post.

Partial Observation of Experimentation

Without requiring the entrepreneur's commitment to a disclosure policy, the key assumption we have then is that the insider observes the entrepreneur's experiment and its signal realization better than outsiders, before making the continuation decision. In the baseline, we have assumed that not only the investor's monitoring technology rules out misreporting, but the investor also commits to perfectly monitoring the entrepreneur's experiment, while she might be better off randomizing between monitoring and not monitoring. We now relax these assumptions and show that while the first-best outcome becomes no longer achievable, our results on IPH and optimal contracting and security design still hold.

To incorporate the partial observation, we here allow the possibility of the entrepreneur's misreporting and the investor's partial commitment to monitoring:

1. With probability $\alpha \in [0, 1]$, the entrepreneur can misreport the signal z without getting caught even when monitored by the investor.

2. At time $t = 0$, the investor commits to verifying the experiment and its signal realization with probability $\beta \in [0, 1]$.[27]

Note that our baseline model corresponds to $\alpha = 0$ and $\beta = 1$. In general, the following variant of Corollary 1 and Proposition 2 holds.

[27]Note that if the investor monitors the experiment, she makes the continuation decision based on both the reported and the monitored signal. In this case, we allow the investor to commit to punishing the entrepreneur by not investing in the case of misreporting.

Proposition 9 (**Partial Observation and Commitment**).

(a) *Absent long-term contracts, the insider receives no interim rent for extreme values of μ regardless of the values of $\alpha, \beta \in [0, 1]$.*

(b) *For $\alpha > 0$ and $\beta < 1$, there exists no contractual solution that implements the first-best outcome. For small enough values of α and large enough values of β, the convertible securities are still optimal for all ε, provided an optimal contract exists. Relationship financing is infeasible for large enough values of α and low enough values of β.*

Proposition 9 validates our prior knowledge that the investor's monitoring is essential to relationship financing. But Part (a) also shows that information production hold-up is robust to monitoring when competition is too low or too high. Furthermore, note that in contrast to costly state verification models that the investor optimally randomizes between monitoring and not monitoring, Proposition 9(b) shows that the full observation of the investor is required to implement the socially optimal outcome. The main difference is that the investor has no way to punish the entrepreneur for misreporting, as he is protected by his limited liability. Therefore, the entrepreneur always benefit from ex-post misreporting.

6.4 Security Design Right

Section 4 mainly focuses on the entrepreneur's optimal security design problem. It turns out that the insider investor would design the contract and security rather differently.

Proposition 10 (**Security Design Right**). *Under both investor sophistication and unsophistication, it is more socially efficient that the entrepreneur designs the security.*

The intuition for Proposition 10 is the following: In order to raise the initial K, the entrepreneur understands he has to provide at least a minimum amount of expected cash flow to the insider investor. Therefore, among all designs that generate this amount, he chooses the one that makes him commit to the most informative disclosure policy, which maximizes the total surplus. In other words, ex ante he is the residual claimant and his incentives are more aligned with a social planner. Moreover, if the insider has the security design right, she would choose a less socially optimal security, because she does not consider the entrepreneur's private benefit from the investment.

7 Conclusion

We model the dynamic financing of innovative projects by relationship and arms-length investors as a mechanism design problem with an embedded Bayesian persuasion game whereby the entrepreneur endogenously produces interim information to seek continued financing. We show that the entrepreneur's (sender's) endogenous experimentation typically reduces the relationship financier's (receiver) informational-monopoly rent, holding up that financier's incentives to form the relationship in the first place. Investor sophistication (receiver's type) and interim competition can mitigate the problem, and they interact to produce non-monotone patterns of relationship formation and interim competition. We then derive optimal sequential securities to resolve the information production hold-up: the entrepreneur contracts with investors in the initial round to allow them to purchase convertible securities in a later round, and issue residual claims to competitive outsiders later.

Our theory applies to similar hold-up problems in persuasion games with contingent transfers and heterogeneous receivers, and is immediately relevant for at least two major areas of finance. First, it underscores the impact of endogenous information production and clarifies its interactions with investor sophistication and competition, rationalizing the U-shaped link between bank orientation and interim competition documented in *relationship lending* that extant theories could not account for. Second, it underscores a new form of information hold-up and rationalizes the use of a large variety of convertible securities in *venture capital*. Given that the solutions to many of the world's biggest problems such as Alzheimer's disease, global warming, and fossil-fuel depletion require initial funding for experimentation and reliable financing relationship (Nanda and Rhodes-Kropf, 2013; Hull, Lo, and Stein, 2017), the cost of inefficient information production could be tremendous. Our study constitutes a first attempt to underscore and formalize this practical issue, and develops potential contractual solutions.

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Appendix A: Proofs of Lemmas and Propositions

A1. A Technical Lemma

The entrepreneur endogenously designs the experiment to maximize his payoff, subject to the insider investors' second-round participation constraint. With finite state space, the signal space as the range of a deterministic mapping from the state space is necessarily finite. Consequently, we can apply the method of Lagrange multipliers directly. But alas, we are dealing with infinite dimensional state space, and in unrestricted signal generation space, we cannot always apply the method of Lagrange multipliers.

That said, the optimal experimentation function is given by the characteristic function of a countable sup-level set of payoff densities, for some cutoff value "multiplier". In other words, the experimentation we look at are conditional probabilities and therefore their space is a Banach space. With this insight, we first prove the following mathematical lemma that allows us to use the method of Lagrange multipliers in the proofs of our lemmas and propositions (see also Ito (2016) for an abstract generalization).

Lemma (A1). *Suppose $w_i(x), m_i(x) : [0,1] \to \mathfrak{R}$ $(1 \leq i \leq N)$ are continuous and bounded functions. Suppose the following maximization problem has a solution:*

$$\begin{aligned} &\max_{\alpha_i(.)\in\mathcal{A}} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx \\ s.t. \quad &\int_0^1 m_i(x)\alpha_i(x)dx \geq 0 \quad \forall\ 1 \leq i \leq N, \quad \textit{and} \quad \sum_{i=1}^N \alpha_i(x) \leq 1 \quad \forall x \in [0,1], \end{aligned} \tag{20}$$

where $\mathcal{A}$ is the set of all measurable functions over $[0,1]$ that take value from $[0,1]$. Then, there exist non-negative real numbers $\{\mu_i\}_{i=1}^N$, such that the solution to (20) is a solution to the following maximization problem:

$$\max_{\alpha_i(.)\in\mathcal{A}} \int_0^1 \sum_{i=1}^N (w_i(x) + \mu_i m_i(x))\alpha_i(x)dx \quad s.t. \quad \sum_{i=1}^N \alpha_i(x) \leq 1 \quad \forall x \in [0,1] \tag{21}$$

Proof. Let $\tilde{\mathcal{A}}^N$ be the set of all $N-$tuples of functions $(\alpha_1(.), \ldots, \alpha_N(.))$ in $\mathcal{A}$ that satisfy $\sum_{i=1}^N \alpha_i(x) \leq 1$.

Since all functions are bounded and measurable, then it is easy to check that $\tilde{\mathcal{A}}^N$ constitutes a closed set in ${\mathcal{L}^1}^N$. Therefore, the following maximization problem is well-defined.

$$\max_{(\alpha_1(.),\ldots,\alpha_N(.))\in\tilde{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx \quad s.t. \quad \int_0^1 m_i(x)\alpha_i(x)dx \geq 0 \quad \forall\ 1 \leq i \leq N \tag{22}$$

Suppose $a^* \in \tilde{A}^N$ is the solution to the problems (20) and (22). It is easy to see that Slater condition, and correspondingly, strong duality holds. Therefore, there exists a vector of non-negative real numbers

$\{\mu_i\}_{i=1}^N$ such that a^* solves the following maximization problem as well:

$$\max_{(\alpha_1(.),\ldots,\alpha_N(.))\in\tilde{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx + \sum_{i=1}^N \mu_i \int_0^1 m_i(x)\alpha_i(x)dx \tag{23}$$

Note that (21) is equivalent to (23). □

In fact, for most Bayesian persuasion settings studied in the literature, the signal generation function corresponds to mapping to probability space that is bounded, and therefore lies in a Banach space. This allows us to apply the approach in Bergemann and Morris (2017) beyond finite-state-space settings.

A2. Proof of Proposition 1

First, we show that two signals are enough to implement the optimal experiment. In particular, there is an optimal experiment that induces investment when the outcome exceeds some threshold. Then, we show all optimal experiments induce the same investment decisions. Hence, the equilibrium is unique in terms of equilibrium payoffs and investment decisions.

Optimality of a binary experiment. According to (5), the entrepreneur is indifferent between experiment $(\mathcal{Z}, \tilde{\pi})$ and a binary experiment $(\{h, l\}, \pi)$ that pools all signals in $\mathcal{Z}^+$ in h and all signals in $\mathcal{Z}^- = \mathcal{Z} \setminus \mathcal{Z}^+$ in l. In other words, for every optimal experiment $(\mathcal{Z}, \tilde{\pi})$, it is straighforward that binary experiment $(\{h, l\}, \pi)$, where $\pi(h|X) = \sum_{z\in\mathcal{Z}^+} \pi(z|X)$ and $\pi(l|X) = \sum_{z\in\mathcal{Z}^-} \pi(z|X)$, is optimal as well, since it would lead to the same conditional probability of investment for all states $X \in [0, 1]$. Furthermore, note that the expected payoffs would not be affected by this change in the choice of experiment.

Optimality of threshold schemes. Given that the experiment generates a binary signal, the entrepreneur solves the following optimization problem.

$$\max_{\pi(h|X)} \int_0^1 [\varepsilon + X - \mu s(X) - (1-\mu)I]\pi(h|X)f(X)dX \tag{24}$$

$$s.t. \int_0^1 (s(X) - I)\pi(h|X)f(X)dX \geq 0, \quad and \quad \pi(h|X) \in [0, 1]$$

In (24), the entrepreneur maximizes his expected payoff given the participation constraint for the investors. Note that $\mathbb{E}[s(X)|X \geq I - \varepsilon] \geq I$. Therefore, $s(X)$ exceeds I with a positive probability. Furthermore, the set of measurable functions satisfying the constraint in (24) is a closed and bounded subset of $\mathcal{L}^1$. As a result, (24) has a solution.

The participation constraint in (24) could be either binding or non-binding. When it is non-binding, the optimal experiment sets $\pi^*(h|X) = 1$ for all values of X for which the value in the bracket is non-negative. It corresponds to the set $\{X \geq \hat{X}(\mu)\}$. When the constraint is binding, we apply Lemma A1 with $N = 1$.

Let $\hat{\lambda}$ be the corresponding multiplier. Then the optimal experiment $\pi^*(h|X)$ solves:

$$\max_{\pi(h|X)} \int_0^1 [\varepsilon + X - I + (\hat{\lambda} - \mu)(s(X) - I)]\pi(h|X)f(X)dX \tag{25}$$

The term in the bracket is strictly increasing in X, because

$$\frac{d_+}{d_+X}[\varepsilon + X - I + (\hat{\lambda} - \mu)(s(X) - I)] = 1 + (\hat{\lambda} - \mu)\frac{d_+}{d_+X}s(X) > 1 - \mu\frac{d_+}{d_+X}s(X) \geq 1 - \mu \geq 0,$$

where $\frac{d_+}{d_+X}$ denotes the right derivative. Therefore, the optimal experiment has a threshold scheme, where the threshold $\bar{X}$ satisfies $\int_{\bar{X}}^1 (s(X) - I)f(X)dX = 0$. Since the constraint in this case is binding, $\hat{X}(\mu) \leq \bar{X}$ whereas the opposite holds in the first case. The entrepreneur thus always follows a threshold strategy where the threshold is given by $\max\{\bar{X}, \hat{X}(\mu)\}$.

Uniqueness of investments and payoffs. Next, we show that the payoffs and investment decisions in equilibrium are essentially unique. The optimal experiment, itself, is not unique, because the entrepreneur can split each one of the signals h and l in the optimal experiment $(\{h, l\}, \pi^*)$ into more signals and implement the same outcome. That said, suppose there is another optimal experiment $(\mathcal{Z}', \pi')$ that induces investment with probability $\mathcal{I}'(X)$ for state $X \in [0, 1]$. As discussed earlier, the entrepreneur can replace the experiment with a binary experiment. It implies that the optimization problem in (24) has two different solutions. It is a contradiction. Therefore, $\mathcal{I}(X) = \mathbb{I}_{\{X \geq \max\{\bar{X}, \hat{X}(\mu)\}\}}$. Consequently, the entrepreneur's and the insider's payoff have to be the same across all optimal experiments.

Uniqueness under mixed strategies. So far, we have assumed that the insider follows a pure strategy. As follows, we prove that even if the insider randomizes between continuation and termination following some interim signals, the equilibrium payoffs and investments are still unique.

To see this, we show that there exists no mixed-strategy Nash Equilibrium in which the insider financier terminates the project with positive probability when she is indifferent between continuation and termination. Suppose the contrary that the entrepreneur uses experiment $(\mathcal{Z}', \pi')$ and the insider uses the investment function $i'(.) : \Delta([0, 1]) \to [0, 1]$. If the insider randomizes following some signal $z' \in \mathcal{Z}'$, i.e. $i'(z') \in (0, 1)$, it implies that the insider should be indifferent between continuation and termination after observing z', i.e. $\mathbb{E}[s(X) - I|z'] = 0$. If z' realizes with a positive probability, then there exists $X_{z'} \in (0, 1)$ such that $P(X \geq X_{z'}|z') > i'(z')$. Then, consider an alternative experiment $(\mathcal{Z}'', \pi'')$ that splits signal z' to signals z'_h and z'_l, where $\mathcal{Z}'' = (\mathcal{Z}' \setminus \{z'\}) \cup \{z'_h, z'_l\}$ and $\pi''(z'_h|X) = \pi'(z'|X)\mathbb{I}_{\{X \geq X_{z'}\}}$, $\pi''(z'_l|X) = \pi'(z'|X)\mathbb{I}_{\{X < X_{z'}\}}$ and $\pi''(z''|X) = \pi'(z''|X)$ for all $z'' \in \mathcal{Z}' \setminus \{z'\}$. We have:

$$\mathbb{E}[s(X) - I|z'_h] > \mathbb{E}[s(X) - I|z'] = 0 > \mathbb{E}[s(X) - I|z'_l], \quad \textit{and}$$
$$U^E(\mathcal{Z}'', \pi'') - U^E(\mathcal{Z}', \pi') = P(z')[P(X \geq X_{z'}|z')\mathbb{E}[\varepsilon + X - s(X)|z', X \geq X_{z'}] - i'(z')\mathbb{E}[\varepsilon + X - s(X)|z']] > 0,$$

since $P(X \geq X_{z'}|z') > i'(z')$ by construction and $\mathbb{E}[\varepsilon + X - s(X)|z', X \geq X_{z'}] \geq \mathbb{E}[\varepsilon + X - s(X)|z']$ due to

the monotonicity of $X - s(X)$. It contradicts the optimality of $(\mathcal{Z}', \pi')$, and as a result, any outcome that involves randomization by the insider cannot emerge as an equilibrium outcome.

A3. Proof of Corollary 1

We remind the readers that the corollary does not assert that relationship financing is feasible in $[\mu^l, \mu^h]$. The key message is that there are regions in which relationship financing breaks down.

The insider's equilibrium interim payoff is as follows:

$$U^I(\{h, l\}, \pi^*(\mu); \mu) = \mu \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\bar{X}, \hat{X}(\mu)\}\}}], \tag{26}$$

where $\pi^*(\mu)$ denotes the optimal experiment for μ. It is easy to see that the insider's payoff is zero for $\mu = 0$ and $\mu = 1$. Given that U^I is continuous in μ, the insider's expected payoff is less than K for a neighborhood around $\mu = 0$ and $\mu = 1$. The corollary follows.

A4. Proof of Lemma 1

Proof for Part (a)

When $\lambda = 1$, either the insider invests, or she does not and nor do the outsiders after observing the insider's action. Therefore, the insider can fully squeeze the entrepreneur, which is equivalent to the case of $\mu = 1$ in Proposition 1. As is shown in Corollary 1, the relationship financing is infeasible in this case.

Proof for Part (b)

We first show that λ enters into the entrepreneur's payoff in a way similar to that of μ in (5). We then show the insider gets a positive expected interim payoff (after investing K) for values of $\lambda > 0$ such that $\hat{X}(\lambda) > \bar{X}$.

Note that following signal $z \in \mathcal{Z}$, the insider chooses to invest by paying $p^I = \lambda I$ if and only if $\lambda \mathbb{E}[s(X) - I|z] \geq 0$. Following the insider's action, the outsiders learn if the project has a positive conditional expected payoff, that is, $z \in \mathcal{Z}^+$. Therefore, following the insider's investment, the outsiders pay the entrepreneur $p^O = (1 - \lambda)\mathbb{E}[s(X)|z \in \mathcal{Z}^+]$. Therefore, the entrepreneur's payoff from experiment $(\mathcal{Z}, \pi)$ is given by:

$$\begin{aligned}
U^E(\mathcal{Z}, \pi; \lambda, \varepsilon) &= \sum_{z \in \mathcal{Z}^+} \int_0^1 (\varepsilon + X - I - s(X) + p^I + p^O)\pi(z|X)f(X)dX \\
&= \sum_{z \in \mathcal{Z}^+} \int_0^1 (\varepsilon + X - I - s(X) + \lambda I + \mathbb{E}[s(X)|z \in \mathcal{Z}^+])\pi(z|X)f(X)dX \\
&= \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - I)\pi(z|X)f(X)dX - \lambda \int_0^1 \sum_{z \in \mathcal{Z}^+} (s(X) - I)\pi(z|X)f(X)dX
\end{aligned}$$

Note that the entrepreneur's preference over different experiments exactly coincides with (5), had we replaced μ with λ. Similarly, the insider's expected interim payoff is:

$$U^I(\mathcal{Z}, \pi; \lambda) = \lambda \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\bar{X}, \hat{X}(\lambda)\}\}}] \tag{27}$$

By definition, $\mathbb{E}[(s(X)-I)\mathbb{I}_{\{X\geq\bar{X}\}}]=0$ and $\hat{X}(0)=I-\varepsilon>\bar{X}$. Hence, for $\lambda>0$ sufficiently close to zero, $\hat{X}(\lambda)>0$ and consequently (27) is positive, which completes the proof.

Proof for Part (c)

The investment is socially efficient if and only if $X\geq I-\varepsilon$. However, for contracts of the form $\{\lambda s(\cdot),(1-\lambda)s(\cdot),\lambda\}$, investment takes place if $X\geq\max\{\bar{X},\hat{X}(\lambda)\}$, as shown in the previous part. Since both $\bar{X}$ and $\hat{X}(\lambda)$ are less than $I-\varepsilon$ for $\lambda>0$, the investment is inefficient with a positive probability (overinvestment for $X\in[\max\{\bar{X},\hat{X}(\lambda)\},I-\varepsilon)$) for $\lambda>0$. Moreover, relationship financing is impossible for $\lambda=0$, when $K>0$. It completes the proof.

A5. Proof of Proposition 2

A design implements the socially optimal outcome when the investment takes place iff $X\geq I-\varepsilon$. We introduce a security that maximizes the entrepreneur's expected payoff and implements the socially optimal outcome. Then, we characterize the set of optimal designs that achieves the first-best regardless of the realization of ε.

Social optimality of optimal designs. Note that the social surplus from the relationship financing for a given ε is bounded by

$$U_{FB}(\varepsilon)=\mathbb{E}[(\varepsilon+X-I)\mathbb{I}_{\{\varepsilon+X-I\geq 0\}}]-K. \tag{28}$$

We show that this bound is achievable for a contract that satisfies the constraints in (16)-(19). Note that (17) implies that $M(X;s_I,s_O,\lambda)=\varepsilon+X-I$ for all $X\in[\lambda I,I]$, where $M(\cdot)$ is defined in (13). Since $M(\cdot)$ is increasing in X and $M(I-\varepsilon;s_I,s_O,\lambda,\varepsilon)=0$, the entrepreneur sends a high signal for $X\geq I-\varepsilon$, provided the security can cover the investment cost for the insiders and outsiders. Condition (18) ensures that is the case for the insider and for the outsiders. We thus have:

$$\begin{aligned}
\int_{I-\varepsilon}^{1}(s_O(X)-(1-\lambda)I)f(X)dX &= \int_{I-\varepsilon}^{1}(X-s_I(X)-(1-\lambda)I)f(X)dX \\
&= \int_{I-\varepsilon}^{1}(X-I)f(X)dX-\int_{I-\varepsilon}^{1}(s_I(X)-\lambda I)f(X)dX \qquad (29)\\
&= \int_{I-\varepsilon}^{1}(X-I)f(X)dX-K=\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq I-\varepsilon\}}]-K\geq 0\\
\Rightarrow \mathbb{E}\left[s_O(X)-(1-\lambda)I|X\geq I-\varepsilon\right] &\geq 0 \quad \forall\varepsilon\in(0,\bar{\varepsilon}),
\end{aligned}$$

where we used the conditions (17) and (18) in the third equation. As a result, the entrepreneur receives expected payoff $U_{FB}(\varepsilon)$ for all $\varepsilon\in(0,\bar{\varepsilon})$ and the socially optimal outcome is implemented. We have proven that all optimal designs implement the socially optimal outcome.

The set of optimal designs. We now argue that the set of contracts specified in (16)-(19) are the only optimal designs. In order for a contract to implement the socially efficient outcome for all values of ε, we need

to have $M(I-\varepsilon; s_I, s_O, \lambda) = 0$ for all $\varepsilon \in (0, \bar{\varepsilon})$. Therefore, we should have $s_I(I-\varepsilon) = \lambda I$ for all $\varepsilon \in (0, \bar{\varepsilon})$. It proves the necessity of condition (17) for $X > I - \bar{\varepsilon}$. Furthermore, there would be no investment for $X < I - \bar{\varepsilon}$, as it would be inefficient for any value of ε. It implies the contingent transfers for these states are irrelevant.

According to (17), we need to have $\lambda I = s_I(I-\varepsilon) \leq I - \varepsilon$ for all $\varepsilon \in (0, \bar{\varepsilon})$, which implies that (16) needs to hold as well. (18) ensures that the insider breaks even over the course of the relationship. Finally, the inequality in (29) becomes equality as ε goes to $\bar{\varepsilon}$. Therefore, the limited liability condition has to bind in order to implement the socially optimal investment decision when the entrepreneur's private benefit is large.

A6. Proof of Lemma 2

We show the entrepreneur designs the experiment in a way that the realized signal z fully reveals the insider's signal y. In other words, for a given signal $z \in \mathcal{Z}$ in the optimal experiment, there exists signal $y \in \mathcal{Y}$ such that $P(y|z) = \frac{\int_0^1 \pi(z,y|X) f(X) dX}{\sum_{z \in \mathcal{Z}} \int_0^1 \pi(z,y|X) f(X) dX} = 1$.

Consider experiment $(\mathcal{Z}, \pi)$ and signal $z \in \mathcal{Z}$. Suppose there are $l \geq 2$ distinct signals $\tilde{\mathcal{Y}}(z) = \{y_1, y_2, \ldots, y_l\} \subset \mathcal{Y}$ such that $P(y_i|z) > 0$ for all $1 \leq i \leq l$. We show that the entrepreneur can increase his expected payoff by splitting signal z into signals $z_1, z_2, \ldots, z_l$, where $\pi(z_i, y_j|X) = \pi(z, y_j|X)\mathbb{I}_{i=j}$.

Let us examine the entrepreneur's expected payoff in these two scenarios. It should be apparent that the insider either chooses $\lambda = 1$ or $\lambda = 0$ because her expected payoff is linear in her amount of investment. In the first scenario that the signals are pooled, suppose the insider makes the investment for $y \in \tilde{\mathcal{Y}}^+(z) \subsetneq \tilde{\mathcal{Y}}(z)$, following signal z. When signal z is public, the outsiders offer $p^O = \mathbb{E}[s(X)|z, y \in \tilde{\mathcal{Y}}(z) \setminus \tilde{\mathcal{Y}}^+(z)]$ if it exceeds the cost of investment I, otherwise they do not make any offer. We argue that $p^O < I$, that is the outsiders never make an offer when the insider has a strictly more informative signal.

Consider the contrary and suppose $p^O = \mathbb{E}[s(X)|z, y \in \tilde{\mathcal{Y}}(z) \setminus \tilde{\mathcal{Y}}^+(z)] \geq I$ for some z. Then there should exist $\tilde{y} \in \tilde{\mathcal{Y}}(z) \setminus \tilde{\mathcal{Y}}^+(z)$ such that $\mathbb{E}[s(X)|z, \tilde{y}] \geq I$. It implies that the insider should also continue after (z, y), or equivalently, y needs to be in $\tilde{\mathcal{Y}}^+(z)$ as well, which is a contradiction. Therefore, the insider never pays more than I following such signal z, even if z is publicly observed.

In contrast, if the entrepreneur splits the signals into $z_1, z_2, \ldots, z_l$ as mentioned above, he gets strictly more than I for the realization of z_i and y_i that $\mathbb{E}[s(X)|z_i, y_i] > I$ when z_i is publicly observed. Therefore, the entrepreneur would indeed be better off by splitting the signal into $z_1, z_2, \ldots, z_l$ in the way described.

A7. Proof of Proposition 3

Lemma 2 reveals that the entrepreneur essentially conveys the insider's information to outsiders. As such, the insider's signal alters the problem as if all investors have a different prior belief. Consequently, the entrepreneur essentially faces m different experiment design problems each specified by (7) with the priors $f(X|y_i)$. Proposition 1 then leads us to the optimal experiments under investor sophistication.

The only exceptions are the cases in which either $P(s(X) \geq I|y_i) = 0$ or $\mathbb{E}[X|y_i] > 0$, where $\bar{X}(y_i)$ does not exist. In the first case, there would be no investment by the insider, and consequently the outsiders (according to Lemma 2), regardless of the entrepreneur's choice of signals. In the second case, the entrepreneur optimally induces invest only when $X \geq \hat{X}(\mu)$.

A8. Proof of Corollary 2

Suppose the contrary that there exists an insider's experiment $(\mathcal{Y}, \omega_q)$ that leads to the socially optimal investment decisions. To implement the socially optimal outcome, the threshold for all m signals should be $I - \varepsilon$. We thus need to have $\max\{\hat{X}(\mu), \bar{X}(y)\} = I - \varepsilon$ for all $y \in \mathcal{Y}$. Since $\hat{X}(\mu) < I - \varepsilon$ for all $\mu > 0$, then we need to have $\bar{X}(y) = I - \varepsilon$ for all signals in $\mathcal{Y}$. It implies $\mathbb{E}[s(X) - I | z_i^h] = 0$ for $1 \leq i \leq m$. Therefore, the insider receives zero interim expected payoff, failing to recover the initial cost K. Then the insider would not start the relationship financing in the first place, contradicting the outcome being socially optimal.

A9. Proof of Corollary 3

It is easy to show that Lemma 2 still holds: the entrepreneur chooses an experiment strictly more informative than $(\mathcal{Y}, \omega_q)$. It means the entrepreneur still solves m independent information design problem for every signal in $\mathcal{Y}$ to determine the additional information to reveal.

This independence implies that the entrepreneur does not choose a more informative experiment for signals in $\mathcal{Y}^{nb}$, compared to the benchmark case without setting milestone. Moreover, the insider's action following signals in $\mathcal{Y}^b$ is weakly dominated by that without the commitment, because in the latter she can respond to the additional information that the entrepreneur provides for these states. Therefore, the insider does not gain from setting milestones.

A10. Proof of Proposition 5

For every insider's experiment $(\mathcal{Y}, \omega_q)$, (17)-(19) characterize the set of optimal long-term contracts. In particular, we show that under these conditions the entrepreneur optimally designs a binary experiment that sends a high signal if $X \geq I - \varepsilon$, which induces investment. First, suppose the entrepreneur chooses this experiment. (17) implies that the insider always invests if she learns that $X \geq I - \varepsilon$. (18) and (19) together imply that the outsiders also invest if and only if the entrepreneur's experiment sends a high signal. The reason is that the insider's action is binary and only reveals the signal of the entrepreneur's experiment. Therefore, the project is invested if and only if $X \geq I - \varepsilon$. Moreover, by an argument similar to the proof of Proposition 2, it is the optimal experiment for the entrepreneur and these contracts give the entrepreneur the whole social surplus. Now to show that the optimal design has to satisfy (17)-(19), we can use the argument almost verbatim in the proof of Proposition 2.

A11. Proof of Corollary 4

As follows, we show that the insider financier earns higher expected payoff from a more informative experiment. Consider two experiments $(\mathcal{Y}, \omega_q)$ and $(\mathcal{Y}, \omega_{q'})$ with $q' > q$. Therefore, there exists an $m \times m$ Markovian matrix T such that $f_q(X|y_i) = \sum_{j=1}^m T_{ij} f_{q'}(X|y_j)$. Moreover, we can write the insider's expected payoff from experiment $(\mathcal{Y}, \omega_q)$ as:

$$U^I(\mu; q) = \mu \sum_{y \in \mathcal{Y}} P_q(y_i) \mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\hat{X}(\mu), \bar{X}(y)\}\}}\right]. \tag{30}$$

According to the definition of $\bar{X}(y)$ introduced in Proposition 3, $\bar{X}(y) > 0$ implies $\mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \bar{X}\}}] = 0$. We can thus rewrite (26) as

$$U^I(\mu; q) = \mu \sum_{i=1}^{m} P_q(y_i) \max\{\int_{\hat{X}(\mu)}^{1} (s(X) - I) f_q(X|y_i) dX, 0\} \tag{31}$$

Substituting $f_q(X|y_i)$ by $\sum_{j=1}^{m} T_{ij} f_{q'}(X|y_j)$, we have

$$\begin{aligned} U^I(\mu; q) &= \mu \sum_{i=1}^{m} P_q(y_i) \max\{\int_{\hat{X}(\mu)}^{1} (s(X) - I) \sum_{j=1}^{m} T_{ij} f_{q'}(X|y_j) dX, 0\} \\ &\leq \mu \sum_{i=1}^{m} P_q(y_i) \sum_{j=1}^{m} T_{ij} \max\{\int_{\hat{X}(\mu)}^{1} (s(X) - I) f_{q'}(X|y_j) dX, 0\} \\ &= \mu \sum_{i=1}^{m} P_{q'}(y_i) \max\{\int_{\hat{X}(\mu)}^{1} (s(X) - I) f_{q'}(X|y_j) dX, 0\} = U^I(\mu; q'), \end{aligned}$$

where the last inequality follows from the identity $P_{q'}(y_j) = \sum_{i=1}^{m} T_{ij} P_q(y_i)$.

A12. Proof of Proposition 4

The derivative of (30) with respect to μ (when it exists) is

$$\begin{aligned} \frac{d}{d\mu} U^I(\mu; q) = &\sum_{y \in \mathcal{Y}} P_q(y) \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\hat{X}(\mu), \bar{X}_q(y)\}\}}] + \\ &\mu \sum_{y \in \mathcal{Y}} P_q(y)(s(\hat{X}(\mu)) - I) f(\hat{X}(\mu)|y) \mathbb{I}_{\{\hat{X}(\mu) > \bar{X}_q(y)\}} \end{aligned} \tag{32}$$

To derive the relation between the insider's expected payoff and μ, fix q and consider the following two cases:

1. Suppose $\mu \geq 1 - \frac{\varepsilon}{I}$, which implies $\hat{X}(\mu) = 0$. Then (26) implies that $U^I(\cdot; q)$ is weakly increasing in μ for $\mu \in [1 - \frac{\varepsilon}{I}, 1]$.
2. Suppose $\mu < 1 - \frac{\varepsilon}{I}$, then $\hat{X}(\mu) > 0$. In this range of values of μ, if $\bar{X}_q(y) < \hat{X}(\mu)$ for some $y \in \mathcal{Y}$, then the first term in the right hand side of (32) is positive and the second term is negative. For small enough values of μ the derivative is strictly positive, since the first term dominates the second term. Moreover, the derivative is weakly decreasing, since both of the terms are decreasing in μ. It implies the insider's expected payoff is concave in μ for $\mu \in [0, 1 - \frac{\varepsilon}{I}]$.

Denote $\bar{\mu} \in [0, 1 - \frac{\varepsilon}{I}]$ the maximizer of $U^I(\cdot; q)$. If $\bar{\mu} < 1 - \frac{\varepsilon}{I}$, then the insider's expected payoff is U-shape in μ for $\mu \in [\bar{\mu}, 1]$, which completes the proof.

A13. Proof of Proposition 6

The steps of the proof are similar to that of Proposition 1 and Corollary 1. In fact, we modify the expressions for the expected payoffs and, similar to the baseline case, we show that the entrepreneur chooses

a threshold scheme for disclosing X, for every realization of δ_1. Then, we verify the presence of IPH by showing that for low enough levels of interim competition (large enough μ), the threshold is chosen such that the the insider just breaks even at the interim period, and hence, she cannot recoup the initial cost K.

Similar to (7), the entrepreneur designs an experiment that solves the following optimization problem, given δ_1:

$$\max_{(\mathcal{Z},\pi)} \mathbb{E}\left[\mathbb{E}\left[\varepsilon + Y - \mu s(Y) - (1-\mu)I | z, \delta_1\right] \mathbb{I}_{\{\mathbb{E}[s(Y)|z,\delta_1] \geq I\}}\right] \tag{33}$$

The solution to (33) is a binary experiment with a threshold scheme, since the entrepreneur's indirect payoff from the continuation at X, i.e. $\mathbb{E}[\varepsilon + Y - \mu s(Y) - (1-\mu)I|X, \delta_1]$, is weakly increasing in X. To see this, note that:

$$\mathbb{E}[\varepsilon + Y - \mu s(Y) - (1-\mu)I|X, \delta_1] = \mathbb{E}[\varepsilon + (1-\mu)(Y - I)|X, \delta_1] + \mathbb{E}[\mu(Y - s(Y))|X, \delta_1]$$

Both terms on the RHS are weakly increasing in Y and Y is strictly increasing in X. Therefore, the LHS is weakly increasing in X. When $\mu \in (0, 1)$, the LHS is strictly increasing in X.

Furthermore, note that for sufficiently large values of μ, more specifically if $\varepsilon > (1-\mu)I$, the entrepreneur's payoff from continuation would be strictly positive for all values of Y, since both $Y - \mu s(Y)$ and $\varepsilon - (1-\mu)I$ are positive. Therefore, for such values of μ, the entrepreneur chooses the threshold $\bar{X}(\delta_1)$ such that $\mathbb{E}[s(Y)|X \geq \bar{X}(\delta_1), \delta_1] = I$, that is, the project is financed when $X \geq \bar{X}(\delta_1)$ and the insider merely breaks even from the continuation, which means she cannot cover the initial cost K for sufficiently low levels of interim competition. Here we note that the condition $\mathbb{E}[\varepsilon + Y - I|\delta_1] < 0$ implies that $\bar{X}(\delta_1) > 0$ for all realizations of δ_1. Hence the threshold is interior.

Next, the insider cannot recover K for sufficiently intense interim competition (sufficiently low μ) either because her information monopoly rent vanishes as μ goes to 0. The proposition ensues.

A14. Proof of Proposition 7

To prove the proposition, we first find the entrepreneur's expected continuation payoff for every X, for a given contract $\{s_I(\cdot), s_O(\cdot), \lambda\}$. The M function introduced in (13) would be modified. Then, we show that to implement the optimal information design, which requires sending a high signal iff $X \geq X^*(\delta_1)$, $s_I(\cdot)$ needs to be flat in $[\underline{Y}, \bar{Y}]$. Note that the expected social surplus for a given pair of (X, δ_1) is $\mathbb{E}[Y - I + \varepsilon|X, \delta_1]$. Since Y is strictly increasing in X and $\mathbb{E}[Y - I + \varepsilon|X = X^*(\delta_1), \delta_1] = 0$ by definition, the investment is socially optimal if and only if $X \geq X^*(\delta_1)$.

The entrepreneur's expected payoff from continuation at X, for a given δ_1, is:

$$\tilde{M}(X; s_I, s_O, \lambda, \delta_1) = \mathbb{E}[\varepsilon + Y - s_I(Y) - (1-\lambda)I|X, \delta_1]. \tag{34}$$

A necessary condition for the threshold experiment $X \geq X^*(\delta_1)$ to be optimal for the entrepreneur is that $\tilde{M}(X) \geq 0$ if and only if $X \geq X^*(\delta_1)$, for all realizations of δ_1. Since $\tilde{M}(\cdot)$ is strictly increasing in X, this necessary condition can be translated to $\tilde{M}(X^*(\delta_1)) = 0$, for all δ_1, which implies:

$$\mathbb{E}[\varepsilon + Y - s_I(Y) - (1-\lambda)I|X = X^*(\delta_1), \delta_1] = 0 \Rightarrow \mathbb{E}[s_I(Y)|X = X^*(\delta_1), \delta_1] = \lambda I. \tag{35}$$

Noting that $\tilde{f}_{Y,\delta_1}(Y) \equiv f_Y(Y|X = X^*(\delta_1), \delta_1)$, we can rewrite (35) as

$$\int_{\underline{Y}}^{\bar{Y}} s_I(Y)\tilde{f}_{Y,\delta_1}(Y)dY = \lambda I, \tag{36}$$

where $\tilde{f}_{Y,\delta_1}(\cdot)$ is strictly monotone in the sense of first order stochastic dominance. Furthermore, note that $s_I(Y)$ is weakly increasing in Y. Therefore, if $s_I(Y_1) > s_I(Y_2)$ for some $Y_1 > Y_2$, where $Y_1, Y_2 \in [\underline{Y}, \bar{Y}]$, then the integral in (36) would be increasing in δ_1, which is a contradiction. Therefore, $s_I(Y)$ should be constant over $[\underline{Y}, \bar{Y}]$ in order to implement the socially optimal information design, and hence, the socially optimal investment decision.

A15. Proof of Proposition 8

Proof for Part (a)

We first characterize the entrepreneur's information design problem and show how the optimal experiment changes with μ. For a given security $s(\theta, r(\theta)X)$ and the entrepreneur's experiment $(\mathcal{Z}, \pi)$, let $F_\pi(dz)$ denote the implied measure of unconditional probabilities over signals $z \in \mathcal{Z}$. We further denote the insider's optimal and the socially optimal action given the signal z by $\theta^I(z)$ and $\theta^S(z)$ respectively. In other words,

$$\begin{aligned} \theta^I(z) &\in argmax_{\theta\in[0,1]} \quad \mathbb{E}[s(\theta, r(\theta)X) - \theta I|z], \\ \theta^S(z) &\in argmax_{\theta\in[0,1]} \quad \mathbb{E}[r(\theta)(X+\varepsilon) - \theta I|z] \quad s.t. \quad \mathbb{E}[s(\theta, r(\theta)X) - \theta I|z] \geq 0. \end{aligned} \tag{37}$$

When the signal is privately observed by the insider, she chooses $\theta^I(z)$. When the signal is publicly observed, all investors invest at the welfare-maximizing level, provided the security covers the investment cost. Thus, the entrepreneur solves the following optimization problem

$$\begin{aligned} \max_{(\mathcal{Z},\pi)} \mu \int_{z\in\mathcal{Z}} &\mathbb{E}[r(\theta^I(z))(X+\varepsilon) - s(\theta^I(z), r(\theta^I(z))X)|z]F_\pi(dz) \\ &+(1-\mu)\int_{z\in\mathcal{Z}} \mathbb{E}[r(\theta^S(z))(X+\varepsilon) - \theta I|z]F_\pi(dz) \end{aligned} \tag{38}$$

Equation (38) shows that the entrepreneur faces a trade-off between gaining more rent when the signal is private (the first term) and increasing the efficiency of investment (the second term). As μ increases, the entrepreneur puts smaller weight on the social efficiency of investment, which clearly leads to experiments that implement less socially efficient outcomes.

Now, we characterize the optimal experiment when the entrepreneur uses equities ($s(\theta, r(\theta)X) = \beta r(\theta)X$) and $r(\theta) = \theta^\gamma$. First we find the investors' optimal level of investment given a posterior $f(X|z)$. Following the definition of $\theta^I(\cdot)$ and $\theta^S(\cdot)$ provided in (37), we can easily see:

$$\theta^I(z) = \left(\frac{\gamma\beta\mathbb{E}[X|z]}{I}\right)^{\frac{1}{1-\gamma}}$$

$$\theta^S(z) = \min\left\{\left(\frac{\gamma\mathbb{E}[X+\varepsilon|z]}{I}\right)^{\frac{1}{1-\gamma}}, \left(\frac{\beta\mathbb{E}[X|z]}{I}\right)^{\frac{1}{1-\gamma}}\right\}$$

Moreover, since the scale of the investment only depends on the expected state $\mathbb{E}[X|z]$, we can WLOG assume that the signal only has information about the expected state. We can then denote the distribution of signals by $G(X)$, where $F(X)$ is a mean-preserving spread of $G(X)$. As a result, we can rewrite the entrepreneur's problem as follows:

$$\max_{G(X)\prec_{SOSD}F(X)} \int_0^1 u_\mu(X)dG(X), \tag{39}$$

where

$$u_\mu(X) = \mu a_I(X) + (1-\mu)a_S(X)$$
$$a_I(X) = (\frac{\gamma\beta}{I})^{\frac{\gamma}{1-\gamma}} X^{\frac{\gamma}{1-\gamma}}((1-\beta)X+\varepsilon)$$
$$a_S(X) = \begin{cases} \left(\frac{\beta}{I}\right)^{\frac{\gamma}{1-\gamma}} X^{\frac{\gamma}{1-\gamma}}((1-\beta)X+\varepsilon) & \text{if } \beta X \le \gamma(X+\varepsilon) \\ I^{-\frac{\gamma}{1-\gamma}}(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}})(X+\varepsilon)^{\frac{1}{1-\gamma}} & \text{if } \beta X > \gamma(X+\varepsilon) \end{cases}$$

The following lemma is useful in showing $G(\cdot)$ becomes less informative in the Blackwell sense as μ increases.

Lemma (A2). *For every $\mu \in [0,1]$, there exists $X^*(\mu)$ such that the optimal experiment entails pooling all the types below $X^*(\mu)$ and separating all the types above $X^*(\mu)$.*

Proof. First we show that there exists a threshold value $T(\mu)$ such that $u''_\mu(X) > 0$ for $X > T(\mu)$ and $u''_\mu(X) < 0$ for $X < T(\mu)$. Then, the proof follows from Theorem 1 in Dworczack and Martini (2017).

By taking the second order derivative of $a_I(\cdot)$ and $a_S(\cdot)$, we get:

$$a''_I(X) = \left(\frac{\gamma\beta}{I}\right)^{\frac{\gamma}{1-\gamma}} \left(((1-\beta)\left(2X^2 + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon\right) X^{\frac{3\gamma-2}{1-\gamma}}$$
$$a_S(X) = \begin{cases} (\frac{\beta}{I})^{\frac{\gamma}{1-\gamma}} \left(((1-\beta)\left(2X^2 + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon\right) X^{\frac{3\gamma-2}{1-\gamma}} & \text{if } \beta X \le \gamma(X+\varepsilon) \\ I^{-\frac{\gamma}{1-\gamma}} \left((\frac{\gamma}{(1-\gamma)^2}\right)\left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right)(X+\varepsilon)^{\frac{2\gamma-1}{1-\gamma}} & \text{if } \beta X > \gamma(X+\varepsilon) \end{cases} \tag{40}$$

For $\gamma \ge \frac{1}{2}$, $u_\mu(\cdot)$ is convex and full-disclosure is optimal. Moreover, if $\gamma(1+\varepsilon) \ge \beta$, $u_\mu(X)$ just scales with the change of μ, hence the optimal information disclosure is independent of μ. For $\gamma < \min\{\frac{1}{2}, \frac{\beta}{1+\varepsilon}$, there is $T(\mu)$ such that $\left(2X^2 + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon < 0$ if and only if $X < T'$ over the range of $X \in [0,1]$. Therefore, there is a threshold $T(\mu) \in [\frac{\gamma\varepsilon}{\beta-\gamma}, T']$, for which $u''_\mu(X) < 0$ if and only if $X < T(\mu)$. Q.E.D.

□

This characterization of optimal disclosure implies that we only need to show $X^*(\mu)$ is weakly increasing in μ. For $\gamma \ge \frac{1}{2}$, $X^*(\mu) = 0$, hence the statement is straightforward. For $\gamma < \frac{1}{2}$, $X^*(\mu) \in [T(\mu), 1]$, since $u''_\mu(X) < 0$ for $X < T(\mu)$ and they should be pooled. Moreover, note that $X^*(\mu)$ is the solution to $\mathbb{E}[u_\mu(X)|X \le X^*(\mu)] = u_\mu(\mathbb{E}[X|X \le X^*(\mu)])$. If such a solution does not exist, then $X^*(\mu) = 1$, i.e. no-disclosure is optimal. Finally, it is easy to check that $\frac{\partial^2}{\partial\mu\partial X}u_\mu(X) < 0$ for $X \in [\frac{\gamma\varepsilon}{\beta-\gamma}, T(\mu)]$. Therefore, $\mathbb{E}[u_{\mu_2}(X)|X \le X^*(\mu_1)] \le u_{\mu_2}(\mathbb{E}[X|X \le X^*(\mu_1)])$ for any $\mu_2 > \mu_1$. Consequently, $X^*(\mu_2) \ge X^*(\mu_1)$. Q.E.D.

Proof for Part (b)

Consider $s^I(\theta, r(\theta)X) = \lambda \max\{r(\theta)X, \theta I\}$, where λ is defined such that $\mathbb{E}[r(\theta)X - \theta I] = \frac{K}{\lambda}$. Moreover, suppose the outsiders receive the residue $s^O(\theta, r(\theta)X) = r(\theta)X - s^I(\theta, r(\theta)X)$. We want to show that if the entrepreneur fully discloses X and chooses the optimal level of investment $\theta^*(X;\varepsilon)$, then both the insider and the outsiders are willing to invest their share of investment, which are $\lambda\theta^* I$ and $(1-\lambda)\theta^* I$, respectively.

The insider is always willing to invest θ^* as $s^I(\theta, r(\theta)X) - \lambda\theta I \geq 0$ for all values of θ. Moreover, the assumption in the proposition implies that $r(\theta^*)X \geq \theta^* I$ with probability one. Therefore, the outsiders are also willing to participate because

$$s^O(\theta^*, r(\theta^*)X) - (1-\lambda)\theta^* I = r(\theta^*)X - \lambda \max\{r(\theta^*)X, \theta^* I\} - (1-\lambda)\theta^* I = (1-\lambda)(r(\theta^*)X - \theta^* I) \geq 0$$

It is easy to see that the solution works for all ε and the insider's private experimentation.

A16. Proof of Proposition 9

Proof for Part (a)

Since the insider's payoff is continuous in μ, we only need to prove that the insider receives zero expected payoff for $\mu = 0$ and $\mu = 1$. For $\mu = 0$, the insider has no information rent and clearly gets zero expected payoff. We now discuss the case of $\mu = 1$.

Secret manipulation. First suppose the entrepreneur can secretly change the signal realization with probability $\alpha > 0$. Similar to Proposition 1, the entrepreneur follows a threshold strategy, i.e., there exists $\bar{X}_\alpha \in [0,1]$ such that the experiment generates a high signal for $X \geq \bar{X}_\alpha$. The high signal induces investment if the investor receives non-negative expected payoff from the investment following the high signal, which is equivalent to

$$\alpha \int_0^{\bar{X}_\alpha} (s(X) - I) f(X) dX + \int_{\bar{X}_\alpha}^1 (s(X) - I) f(X) dX \geq 0. \tag{41}$$

The first term in (41) shows the probability that the experiment generates a low signal, but the entrepreneur finds the chance to send a high signal. Note that for $\alpha = 1$, the inequality does not hold for any threshold value, because:

$$\mathbb{E}[s(X) - I] < \mathbb{E}[\varepsilon + X - I] < 0$$

Therefore, there exists $\bar{\alpha} \in [0,1]$ above which the investment is not feasible, as the entrepreneur's commitment problem to truthful reporting of the signal is sufficiently serious. However, for $\alpha \leq \bar{\alpha}$, the entrepreneur chooses $\bar{X}_\alpha$ such that the inequality (41) binds, which implies the investor becomes indifferent between investment and not investment after receiving the high signal. As a result, the investor receives zero interim rent for all values of $\alpha \in [0,1]$.

Random monitoring. Now consider the case that the insider investor verifies the signal realization with probability $\beta < 1$. This case involves two subcases:

First, the investor cannot commit to punish the entrepreneur for misreporting. In this subcase, there is no signal such as h that always induces investment, because otherwise the entrepreneur would optimally always report h, which leads to negative expected payoff for the investor when she does not monitor. As such, the investor only invests when she monitors, which makes the entrepreneur's reporting strategy irrelevant. Hence the entrepreneur would follow the threshold strategy at $\bar{X}$, in which the investor does not get interim rent.

Now consider the subcase that the investor commits to punish misreporting. In this case, the entrepreneur might use 4 different kinds of signals in his experiment: 1. low signals, such as l_0, that never induce investment. 2. Low signals, such as l_1, that only induce investment when the investor does not monitor. 3. high signals, such as h_0, that only induce investment if they are verified. 4. High signals, such as h_1, that always induce investment. The probability of investment is $(1-\beta)P(l_1) + \beta P(h_0) + P(h_1)$. We next check the incentive constraints for truthful reporting for the entrepreneur.

In particular, we show that there is no equilibrium that the insider invests without monitoring. Consider the contrary. If $\beta < \frac{1}{2}$, then types l_0 and h_0 prefer to report h_1 instead of truthfully reporting because the probability of investment strictly for l_0 and h_0 increases from 0 and β respectively to $1-\beta$. Note that in all cases, the insider pays exactly I upon continuation. Therefore, only the probability of investment affects the entrepreneur's payoff at every state X. Because of these, we should have $P(l_0) = P(h_0) = 0$, which implies that the investor always invests when she is not monitoring. Thus, she would be better off by not investing at all when she does not monitor, as her investment has a negative NPV conditional on not monitoring. It implies $P(l_1) = P(h_1) = 0$ as well. It contracts that the equilibrium involves investment with positive probability absent monitoring.

For $\beta \geq \frac{1}{2}$, $P(l_0) = 0$, because he would be strictly better off by reporting h_1. If the investor receives positive payoff from investment following $h_1 \cup l_1$, she should make a loss by investing following h_0, since $P(h_0) + P(l_1) + P(h_1) = 1$, and the project has an unconditional negative NPV. Therefore, the investor would be better off by not investing following h_0, even after verifying it, which is a contradiction with its definition. Therefore, the investor never invests without monitoring.

Proof for Part (b)

Secret manipulation. We first show that the socially optimal outcome cannot be implemented for $\alpha > 0$, then we show that the optimal contracts feature securities with a flat region for the insider, i.e., condition (17) holds.

Note that once the entrepreneur raises K, he always wants to continue the project, since he receives strictly positive payoff from continuation. Consequently, the entrepreneur misreports the bad signals whenever it is possible, which leads to inefficient continuation. Therefore, the socially optimal outcome is not implementable for $\alpha > 0$.

Now we solve for the optimal contract. If the project is invested with positive probability, then the expected payoff of the entrepreneur from the contract $\{s_I(.), s_O(.), \lambda\}$ is

$$U_\alpha^E(s_I(\cdot), s_O(\cdot), \lambda) = \mathbb{E}[M(X; s_I, s_O, \lambda)(\alpha + (1-\alpha)\mathcal{I}(X))],$$

where $\mathcal{I}(\cdot)$ is the investment function for the case that the entrepreneur cannot secretly manipulate the signal. If $\mathbb{E}[\max\{X - I + \varepsilon, 0\}] > K$, then with an argument similar to the proof of Proposition 2, the following set of convertible securities are optimal and independent of ε, for small enough values of α:

$$\lambda I \leq I - \bar{\varepsilon}, \quad s_I(X) = \min\{\lambda I, X\} \ \forall\, X < I, \quad \mathbb{E}[(s_I(X) - \lambda I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \geq I - \varepsilon\}})] = K, \tag{42}$$

$$\mathbb{E}[(s_O(X) - (1-\lambda)I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \geq I - \varepsilon\}})] = 0, \quad 0 \leq s_O(X) \leq X - s_I(X) \ \forall\, X \in [0,1] \tag{43}$$

Note that the design might not be robust to the insider's experiment $(\mathcal{Y}, \omega_q)$ because the insider's payoff becomes sensitive to the downside realization of the final cash-flow. Moreover, for big enough values of α, no security can satisfy condition (42). Therefore, relationship financing is infeasible for such big values of α.

Random monitoring. Consider the case that the investor cannot credibly threaten the entrepreneur to terminate the project when he misreports. The argument for the other case is similar. As it is discussed earlier, in equilibrium, the entrepreneur always reports a high signal and the investor invests if and only if she verifies the signal is truthfully reported. Therefore, the socially optimal outcome cannot be implemented when $\beta < 1$. Moreover, the following convertible securities are optimal.

$$\lambda I \leq I - \bar{\varepsilon}, \quad s_I(X) = \min\{\lambda I, X\} \ \forall\, X < I, \quad \beta\mathbb{E}[(s_I(X) - \lambda I)\mathbb{I}_{\{X \geq I - \varepsilon\}}] = K,$$
$$\mathbb{E}[(s_O(X) - (1-\lambda)I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \geq I - \varepsilon\}})] = 0, \quad 0 \leq s_O(X) \leq X - s_I(X) \ \forall\, X \in [0,1]$$

Clearly, these securities are implementable for large enough values of β. For the case of credible punishments, an optimal design might not exist, as the optimal experimentation involves three signals for large values of ε, while it involves two signals for the smaller values. However, for smaller values of ε, the convertible securities specified above are optimal and the equilibrium outcomes are similar.

A17. Proof of Proposition 10

According to Propositions 2 and 5, the entrepreneur's design implements the first-best outcome. It is easy to show that the insider receives $\mathbb{E}[(X - I)\mathbb{I}_{\{X \geq I\}}]$ in her optimal design, which implements investment only for $X \geq I$. Therefore, the insider's optimal design does not incorporate the entrepreneur's private benefit of investment, resulting in a lower social welfare.

Appendix B: Extended Discussions

B1. Optimal Design of a Single Security

Due to regulatory concerns, issuance costs, or high cost of communicating with outsiders in the later rounds, the entrepreneur often can only issue one single form of security to both insiders and outsiders. For example, in venture financing the right of first refusal gives its holder the contractual rights but not the obligation to purchase new security issuance before others can purchase that *same security with the same terms*; banks by regulation can only use debt contracts.

As follows, we show that if the entrepreneur can commit to financing λI from the outsider, the unique optimal security is equity. In the absence of such commitment, the optimal security is indeterminate. However, the set of optimal securities include debt contracts, but not necessarily equity contracts. For notational simplicity, we write ε as if it is deterministic. What is key is that some interim information (ϵ included) affecting the entrepreneur's information production is non-contractible at $t = 0$.

Specifically, the entrepreneur issues security contract $s(X)$ and determines the fraction to the insiders λ, i.e., $s_I(X) = \lambda s(X)$ and $s_O(X) = (1 - \lambda)s(X)$. Proposition 11 characterizes the equilibrium security, optimal level of commitment to outsider competition, and the optimal information production.

Proposition 11 (Optimality of Equity)**.** *Under the single-security constraint and for a given insider's experiment* $(\mathcal{Y}, \omega_q)$*, the entrepreneur optimally issues equities to both the insider and the outsiders. In particular,* $s(X) = X$*, hence* $s_I(X) = \lambda^* X$ *and* $s_O(X) = (1 - \lambda^*)X$ *for some* $\lambda^* \in (0, 1)$*. The optimal security is unique and it does not implement the first-best outcome if* $K > 0$*.*

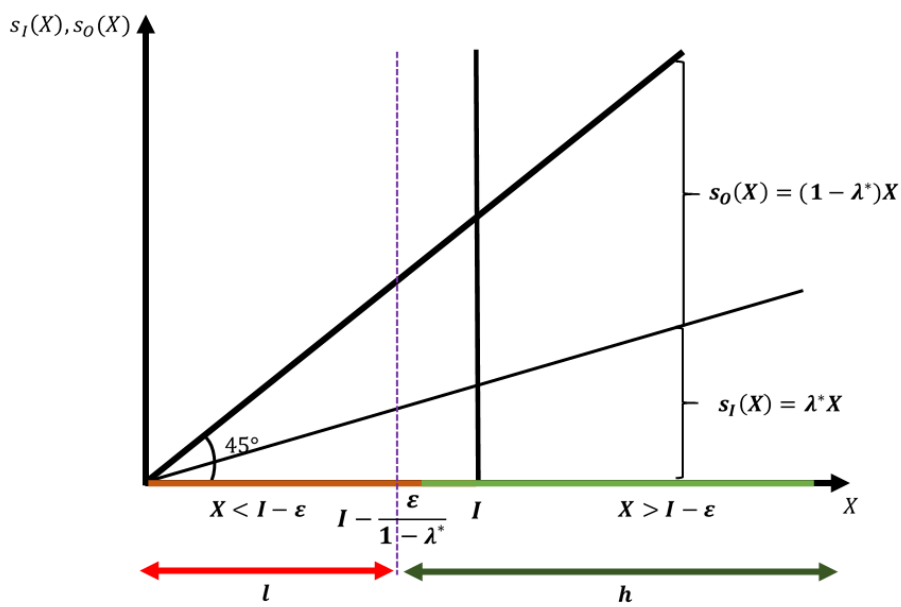

Figure 5: The optimal securities for the entrepreneur under one-security condition; specific example of without investor sophistication.

Proof. We prove the proposition for the case of unsophisticated insider, where she has no proprietary information about the outcome, other than what is provided by the entrepreneur's experiment. The proof for

the case of sophisticated investors is similar. The single-security constraint implies that the entrepreneur chooses from contracts in the form of $\{\lambda s(X), (1-\lambda)s(X), \lambda\}$. Therefore, every contract can be represented by the pair of $\{s(X), \lambda\}$.

Denote the entrepreneur's indirect utility from investment at X by $M^s(s, \lambda)$, where:

$$M^s(X; s, \lambda) = \varepsilon + X - \lambda s(X) - (1-\lambda)I = M(X; \lambda s(.), (1-\lambda)s(.), \lambda) \tag{44}$$

The entrepreneur then solves the following maximization problem:

$$\begin{aligned} &\max_{s(.),\lambda,X_c} \mathbb{E}[M^s(X; s, \lambda)\mathbb{I}_{\{M^s(X;s,\lambda)\geq 0\}}] \\ &s.t. \quad \lambda\mathbb{E}[(s(X)-I)\mathbb{I}_{\{M^s(X;s,\lambda)\geq 0\}}] \geq K \end{aligned} \tag{45}$$

Note that the objective function in (45) is bounded by $\mathbb{E}[\max\{\varepsilon+X-I, 0\}]$ and the constraint constitutes a closed and bounded subset in a $\mathcal{L}^1 \times [0,1]$ space that contains all combination of regular securities and λ. Therefore, the optimal contract exists. We denote the security $s(X) = X$, by $s_i(\cdot)$.

Definition 1. *Consider the contract $\{s(.), \lambda\}$. If $M^s(0; s, \lambda) < 0$, then $\hat{X}(s(.), \lambda)$ is defined to be the solution to $M^s(\hat{X}(s(.), \lambda); s, \lambda) = 0$; otherwise, we define $\hat{X}(s(.), \lambda) = 0$.*

The next lemma follows immediately from the expression (44) and Definition 1.

Lemma (B1). *Suppose $s_1(.)$ and $s_2(.)$ are two regular securities such that $s_1(X) \geq s_2(X)\ \forall\ X \in [0,1]$.*
a) *$\hat{X}(s_1, \lambda)$ is weakly decreasing in λ.*
b) *$\hat{X}(s_1, \lambda) \geq \hat{X}(s_2, \lambda)$, for every $\lambda \in [0,1]$.*

Now, we are ready to prove the optimality of equity for implementing the payoff channel. Consider a contract $\{s_1, \lambda_1\}$ that satisfies the constraint in (45). Part (b) in Lemma B1 implies:

$$\lambda_1\mathbb{E}[(s_1(X)-I)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\lambda_1)\}}] \leq \lambda_1\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq\hat{X}(s_i(.),\lambda_1)\}}] \tag{46}$$

Then there exists $\lambda_i \leq \lambda_1$ such that $\lambda_i\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq\hat{X}(s_i(.),\lambda_i)\}}] = K$. Next, we show that the entrepreneur prefers the contract $\{s_i(.), \lambda_i\}$ to $\{s_1, \lambda_1\}$:

$$\begin{aligned} &\mathbb{E}[M(X; s_i, \lambda_i)\mathbb{I}_{\{X\geq\hat{X}(s_i(.),\lambda_i)\}}] = \mathbb{E}[(\varepsilon + (1-\lambda_i)(X-I))\mathbb{I}_{\{X\geq\hat{X}(s_i,\lambda_i)\}}] = \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X\geq\hat{X}(s_i,\lambda_i)\}}] - K \\ &\geq \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\mu_1)\}}] - \lambda_1\mathbb{E}[(s_1(X)-I)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\lambda_1)\}}] = \mathbb{E}[M(X; s_1, \lambda_1)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\lambda_1)\}}] \end{aligned}$$

Therefore, the optimal contract under the single security constraint is equity. For the case with sophisticated investors, λ^* depends on the insider's experiment, although an equity is still uniquely optimal.

□

Proposition 11 shows that the optimal single security is equity when commitment in λ is feasible. Recall that the entrepreneur uses the security to best commit himself to efficient interim information production. With a single security, the entrepreneur cannot fully allocate the downside exposure to himself because both insiders and outsiders get the same security, and the outsiders are just a pass-through of interim surplus to the entrepreneur. However, by reducing λ, we naturally reduce the insider's exposure relative to the entrepreneur. Equity is thus optimal when λ is endogenous because it gives the insider the largest upside, allowing her to get enough rent to cover K with the smallest λ. Figure 5 displays the optimal security for the entrepreneur.

Proposition 12 (Optimality of Debt Contracts)**.** *Under the single security-constraint and when committing to raising λI from the insider is not feasible, the optimal security is indeterminate but the set of optimal securities includes debt.*

Proof. When the entrepreneur cannot commit to the way he finances I, the insider gets positive rent only if the signal is private. Therefore, she demands all issues when the signal is good. To give the insider lower rent, the entrepreneur use securities that make the insider break even, i.e.

$$U^I(s(\cdot)) = \mu\mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq \hat{X}(\mu; s(\cdot))\}}\right] = K \tag{47}$$

The entrepreneur's expected payoff for a given choice of $s(\cdot)$ from this set of securities is

$$U^E(s(\cdot)) = \mathbb{E}\left[(\varepsilon + X - \mu s(X) - (1-\mu)I)\mathbb{I}_{X \geq \hat{X}(\mu; s(\cdot))}\right] = \mathbb{E}\left[(\varepsilon + X - I)\mathbb{I}_{\{X \geq \hat{X}(\mu; s(\cdot))\}}\right] - K. \tag{48}$$

To maximize (48), the entrepreneur chooses a security that maximizes $\hat{X}(\mu; s(\cdot))$. Note that $\hat{X}(\mu; s(\cdot))$ is the solution to the following equality:

$$\hat{X}(\mu; s(\cdot)): \quad \varepsilon + \hat{X}(\mu) - \mu s(\hat{X}(\mu)) - (1-\mu)I = 0 \Rightarrow \hat{X}(\mu) \leq I - \frac{\varepsilon}{1-\mu}$$

and equality holds if and only if $s(X) = X$ for $X \leq [I - \frac{\varepsilon}{1-\mu}]$. Therefore, all double-monotone securities with $\mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq I - \frac{\varepsilon}{1-\mu}\}}\right] = K$ and $s(X) = X$ for $X \leq I - \frac{\varepsilon}{1-\mu}$ implement the optimal design for the entrepreneur. Clearly, a debt or a convertible security can implement these two conditions, but equity or call options cannot.

□

In Proposition 12, we study the case that the entrepreneur can only commit to the single security he issues in future. For example, the entrepreneur might commit to the form of security by choosing his institutional investor (Banks, Private Equities, Venture Capitalist, etc). In this case, if the commitment to the future round of investments is not feasible, then debt contracts become optimal. The intuition is the following: The security should be the most sensitive to the losses to discipline the entrepreneur the most. Since a debt contract is the most sensitive security in the down-side, it always can implement the optimal security and it is regardless of μ and the insider's experiment.

B2. Insider's Independent Experimentation

We revisit the entrepreneur's optimal information design, the entrepreneur and the insiders' equilibrium expected payoff and our contractual solution to IPH for the case that the signals of the entrepreneur's experiment are restricted to be independent of the insider's experiment, conditional on the outcome X. In Proposition 13, we show that the U-shape pattern in the insider's payoff as a function of μ still persists. In Corollary 5, consistent with Proposition 5 in Kolotilin (2017), we show the insider's payoff is not increasing in the informativeness of her signal. In Proposition 14, we examine the effect of interim competition on the efficiency of investment decisions and the insider's interim payoff for $q \in \left[\frac{1}{2}, 1\right]$. For notational simplicity, we write ε as if it is deterministic and remind the reader that our results only rely on the non-contractibility at $t = 0$ of some interim information (ϵ included) affecting the entrepreneur's information production.

The results are provided for the specific case that the information structure of the insider's signal follows the one introduced in Example 1, with the only difference that q represents the probability of receiving signal $\tilde{h}_q$ ($\tilde{l}_q$) when $X - I \geq 0$ ($X - I < 0$). As a reminder, we repeat the structure of the insider's private information:

Assumption 1. *The insider's experiment generates a binary signal, i.e.* $\mathcal{Y} = \{\tilde{h}, \tilde{l}\}$*, of the following information structure:*

$$\omega_q(\tilde{h}|X) = \begin{cases} q & I \leq X \leq 1 \\ 1-q & 0 \leq X < I. \end{cases}$$

Proposition 13 (General IPH)**.** *Suppose* $\bar{X}(q)$ *is the solution to the following equation:*

$$q \int_0^I (s(X) - I) f(X) dX + (1-q) \int_I^1 (s(X) - I) f(X) dX = 0 \tag{49}$$

Moreover, assume $q \geq \bar{q}$*, where* $\bar{q}$ *is the solution to equation* $\bar{q}^2 + \bar{q} = 1$*. (a) The entrepreneur chooses an experiment that sends a high signal if* $X \geq \max\{\bar{X}(q), \hat{X}(\mu)\}$ *and sends a low signal, otherwise.*

(b) Denote $\bar{K}_C^{\varepsilon}(\mu, q) \equiv \mu \mathbb{E}[(s(X) - I)\mathbb{I}_{X \geq \max\{\bar{X}(q), \hat{X}(\mu)\}}]$ *as the capacity of relationship formation for given* q *and* μ*. Then, there exists level of sophistication* q_1 *such that for every* $q \in (\frac{1}{2}, q_1)$*,* $\bar{K}_C^{\varepsilon}(\mu, q)$ *is U-shaped in* $[\hat{\mu}(q), 1]$ *for some* $\hat{\mu}(q) \in (0, 1)$*,.* $\bar{K}_C^{\varepsilon}(\mu, q)$ *is decreasing in* μ *for* $q \in [q_1, \bar{q}]$*.*

(c) As $\varepsilon \to 0$*, function* $\bar{K}_C^{\varepsilon}(\mu; q)$ *converges to* $\bar{K}_C^{0}(\mu; q)$*, which is increasing in* μ*.*

Proof. Similar to the proof of Lemma 1, we prove the lemma for regular securities $s(X)$. First we note that if $\mathbb{E}[s(X) - I | X > I] \leq 0$, then $\bar{X} \geq I$, where $\bar{X}_s$ is the threshold introduced in Proposition 1. In this case, the investor's signal y is not used and the equilibrium experiments, investment functions and payoffs are the same as the ones provided in Proposition 1. As such, the remaining proof is centered on the case $\mathbb{E}[s(X) - I | X > I] > 0$ for simplicity in exposition.

Proof for Part (a)

Similar to the proof of Proposition 1, first we show that for every experiment, there is another experiment with bounded number of signals that gives the entrepreneur the same expected payoff. It helps us to prove the existence of optimal experiments and then characterize them. We refer to the private information (broadly defined) the insider has as *investor type*.

Lemma (B2). *Denote* T *as the set of investor type and* A *the set of actions. As long as* A *and* T *are finite, for every experiment* $(\mathcal{Z}, \pi)$*, there exists experiment* $(\mathcal{Z}', \pi')$ *such that* $|\mathcal{Z}'| \leq |A|^{|T|}$ *and* $U^E(\mathcal{Z}', \pi') = U^E(\mathcal{Z}, \pi)$*.*

Proof. For every pure strategy of the investor, such as $\mathfrak{a}(\cdot) : T \to A$, define $\mathcal{Z}(\mathfrak{a})$ as the set of signals in $\mathcal{Z}$, such as z, that the investor chooses $\mathfrak{a}(t)$ when her type is t and she receives z. Note that if $\mathcal{Z}(\mathfrak{a})$ is non-empty, then:

$$\begin{aligned} &\mathbb{E}[u^I(\mathfrak{a}(t), X) | t, z] \geq \mathbb{E}[u^I(a', X) | t, z] \quad \forall t \in T, z \in \mathcal{Z}(\mathfrak{a}) \\ \Rightarrow &\mathbb{E}[u^I(\mathfrak{a}(t), X) | t, \mathcal{Z}(\mathfrak{a})] \geq \mathbb{E}[u^I(a', X) | t, \mathcal{Z}(\mathfrak{a})] \quad \forall t \in T \end{aligned}$$

where $u^I(a, X)$ is the final payoff of the investor from the action a in state X. Now define the experiment $(\mathcal{Z}', \pi')$ as follows: $\mathcal{Z}' = \{z_{\mathfrak{a}}\}_{\mathfrak{a} \in A^T}$, for all $\mathfrak{a}$ that $\mathcal{Z}(\mathfrak{a})$ is non-empty. Moreover, define

$$\pi'(z_{\mathfrak{a}} | X) = \sum_{z \in \mathcal{Z}(\mathfrak{a})} \pi(z | X) \quad \text{if } \mathcal{Z}(\mathfrak{a}, t) \text{ is non-empty}$$

Note that by definition, $z_{\mathfrak{a}}$ is the signal in $\mathcal{Z}'$ that induces the strategy profile $\mathfrak{a}(t)$. We only need to show that $U^E(\mathcal{Z}', \pi') = U^E(\mathcal{Z}, \pi)$. To see this,

$$U^E(\mathcal{Z}', \pi') = \int_0^1 \sum_{t \in T} \sum_{z_{\mathfrak{a}} \in \mathcal{Z}'} u^E(\mathfrak{a}(t), X) \pi'(z_{\mathfrak{a}}|X) g(t|X) f(X) dX$$
$$= \int_0^1 \sum_{t \in T} \left[\int_{z \in \mathcal{Z}} u^E(a(z,t), X) \pi(z|X) dz \right] g(t|X) f(X) dX = U^E(\mathcal{Z}, \pi)$$

where $a(z,t)$ is the investor's action for type t upon receiving signal z. □

Building from the above result, the next lemma proves the existence of an optimal experiment and characterizes it.

Lemma (B3). ***(a)*** *Suppose the investor's action is binary ($A = \{0,1\}$) and investor types are ordered by $T = \{t_1, t_2, \ldots, t_{|T|}\}$ such that posteriors $u^I(1,X) - u^I(0,X)|t_i$ are ranked by first-order stochastic dominance. Then for every experiment, there is an experiment that implements the same expected payoffs and uses at most $|T|+1$ signals.* ***(b)*** *Under these conditions, an optimal experiment exists.*

Proof.

Proof of Part (a)

Note that under single-property condition, $a(z,t)$ is weakly increasing in t. Therefore, there are at most $|T|+1$ functions of the form $\mathfrak{a} : T \to A$ that $\mathcal{Z}(\mathfrak{a})$ is non-empty. With a similar argument as the one in the proof of the lemma, one can construct an experiment with at most $|T|+1$ signals that implements the same expected payoffs.

Proof of Part (b)

To show the existence of the optimal experiment, we only need to look at experiments with at most $|T|+1$ signals. In particular, we only need to show that the conditional distributions $\pi(z|X)$, $z \in \mathcal{Z}$, constitute a closed bounded set in $\mathcal{L}^{1^{|T|+1}}$.

For an experiment $(\mathcal{Z}, \pi)$, we can assume it has at most one signal $z_i \in \mathcal{Z}$ such that the entrepreneur chooses $a = 1$ only if her types is t_i or above. $z_{|T|+1}$ is the signal following which no types would invest. Therefore, the part (a) shows that every experiment implements the same expected payoffs with an experiment with following conditions:

$$\begin{aligned} &\int_0^1 (u^I(1,X) - u^I(0,X)) \pi(z_i|X) g(t_j|X) f(X) \geq 0 \quad \text{iff } j \geq i, \forall 1 \leq i \leq |T|+1 \\ &\sum_{i=1}^{|T|+1} \pi(z_i|X) = 1 \quad \forall X \in [0,1], \quad \textit{and} \quad \pi(z_i|X) \geq 0 \quad \forall X \in [0,1], z_i \in \mathcal{Z} \end{aligned} \tag{50}$$

It is easy to check that the set of experiments satisfying (50) is closed and bounded. Therefore, an optimal experiment exists.

□

As a result of the lemma and the corollary, the optimal experiment exists and has at most three signals. An exhaustive list of candidate signal ranges for such an optimal experiment is $\{m, l\}$, $\{h, l\}$ or $\{m, h, l\}$, where the investor only invests if she receives $(m, \tilde{h})$, $(h, \tilde{l})$ or $(h, \tilde{h})$. We show that for $q \leq \bar{q}$, the optimal experiment is essentially unique and it is of the second form.

Lemma (B4). *No two-signal experiment in the form of $(\{l, m\}, \pi)$, where the investor invests iff she receives $(m, \tilde{h})$, is optimal.*

Proof. Suppose the contrary and suppose that there exists an optimal experiment $(\{m,l\},\pi_M^*)$. Therefore, π_M^* solves the following maximization problem:

$$\max_{\pi(m|X)} (1-q)\int_0^I (\varepsilon+X-s(X))\pi(m|X)f(X)dX + q\int_I^1 (\varepsilon+X-s(X))\pi(m|X)f(X)dX$$

$$s.t. \quad (1-q)\int_0^I (s(X)-I)\pi(m|X)f(X)dX + q\int_I^1 (s(X)-I)\pi(m|X)f(X)dX \geq 0$$

$$\pi(m|X)\in[0,1] \qquad \forall X\in[0,1]$$

Let κ be the multiplier corresponding to the constraint. π_M^* then maximizes the following objective function, given the constraint $\pi_M^*(m|.)\in[0,1]$.

$$\max_{\pi(m|X)} (1-q)\int_0^I [\varepsilon+X-s(X)+\kappa(s(X)-I)]\pi(m|X)f(X)dX$$
$$+q\int_I^1 [\varepsilon+X-s(X)+\kappa(s(X)-I)]\pi(m|X)f(X)dX$$

Since both $X-s(X)$ and $s(X)-I$ are weakly increasing functions, there is a threshold value $X_m\in[0,1]$ such that for all $X\geq X_m$, the expression in the bracket is non-negative. According to lemma A1, the optimal experiment among those that only implement signals m and l has a threshold scheme, where the threshold X_m satisfies the following:

$$(1-q)\int_{X_m}^I (s(X)-I)f(X)dX + q\int_I^1 (s(X)-I)f(X)dX = 0$$

In this case, the expected utility of the entrepreneur from $(\{m,l\},\pi_M^*)$ is:

$$U^E(\{m,l\},\pi_M^*) = (1-q)\int_{X_m}^I (\varepsilon+X-s(X))f(X)dX + q\int_I^1 (\varepsilon+X-s(X))f(X)dX \tag{51}$$

By comparing the recent equality with (49), it is easy to see that $X_m<\bar{X}(q)$. Now, we show how the entrepreneur can improve upon π_M^* by introducing signal h (a signal that induces investment regardless of the signal the investor receives). To show this, we consider two cases:

- $s(I)<I$: In this case, we can find a subset $A\subset[I,1]$ such that $\int_A (s(X)-I)f(X)dX=0$. Then an experiment that sends h for the members of A ($\pi(h|X)=1$ iff $X\in A$) and sends m for $[X_m,1]\setminus A$ implements higher payoff for the entrepreneur by $(1-q)\int_A(\varepsilon+X-s(X))f(X)dX$.
- $s(I)=I$: Since $X-s(X)$ is a weakly increasing function, it implies that $s(X)=X$ for all $X\leq I$. Consider small positive values $\eta_1,\eta_2,\eta_3\geq 0$ that satisfy the following:

$$(1-q)\int_{X_m+\eta_1}^{I-\eta_2} (s(X)-I)f(X)dX + q\int_{I+\eta_3}^1 (s(X)-I)f(X)dX \geq 0$$

$$q\int_{I-\eta_2}^I (s(X)-I)f(X)dX + (1-q)\int_{I+\eta_3}^1 (s(X)-I)f(X)dX \geq 0$$

and introduce the following alternative experiment $(\{l,m,h\},\tilde{\pi}_M)$:

$$\tilde{\pi}_M(h|X)=\begin{cases}1 & X\in[I-\eta_2,I+\eta_3)\\ 0 & X\in[0,I-\eta_2)\cup[I+\eta_3,1]\end{cases} \qquad \tilde{\pi}_M(m|X)=\begin{cases}1 & X\in[X_m+\eta_1,I-\eta_2)\cup[I+\eta_3,1]\\ 0 & X\in[0,X_m+\eta_1)\cup[I-\eta_2,I+\eta_3)\end{cases}$$

It is easy to verify that the experiment $\tilde{\pi}_M$ is designed in a way that the investor invests iff she receives one of $(m, \tilde{l})$, $(h, \tilde{l})$ or $(h, \tilde{h})$. Now, the difference in expected payoffs for the entrepreneur is given by:

$$U^E(\{m,l\}, \tilde{\pi}_M) - U^E(\{m,l\}, \pi^*_M) = -(1-q)\int_{X_m}^{X_m+\eta_1} (\varepsilon + X - s(X))f(X)dX$$
$$+ q\int_{I-\eta_2}^{I} (\varepsilon + X - s(X))f(X)dX + (1-q)\int_{I}^{I+\eta_3} (\varepsilon + X - s(X))f(X)dX$$

Now consider the contrary, that the introduced experiment is an optimal experiment. Then $\eta_1^* = \eta_2^* = \eta_3^* = 0$ should satisfy the first order conditions for the following maximization problem:

$$\max_{\eta_1,\eta_2,\eta_3} \quad -(1-q)\int_{X_m}^{X_m+\eta_1} (\varepsilon + X - s(X))f(X)dX + q\int_{I-\eta_2}^{I} (\varepsilon + X - s(X))f(X)dX$$
$$+(1-q)\int_{I}^{I+\eta_3} (\varepsilon + X - s(X))f(X)dX$$

$$s.t. \quad \eta_1, \eta_2 \geq 0, \qquad q\int_{I-\eta_2}^{I} (s(X) - I)f(X)dX + (1-q)\int_{I}^{I+\eta_3} (s(X) - I)f(X)dX \geq 0, \quad \textit{and}$$
$$(1-q)\left[\int_{X_m}^{X_m+\eta_1} (s(X) - I)f(X)dX + \int_{I-\eta_2}^{I} (s(X) - I)f(X)dX\right] + q\int_{I}^{I+\eta_3} (s(X) - I)f(X)dX \leq 0$$

Let κ_1 and κ_2 be the multipliers for the first constraints, respectively. Since $s(I) = I$, the FOC for η_2 is positive at $\eta_2 = 0$: $[\eta_2]|_{\eta_2=0} : q\varepsilon f(I)$.

Similarly, the FOC for η_3 is positive at $\eta_3 = 0$. Therefore, the optimal values are non-zero. It shows that the experiment $(\{m,l\}, \pi^*_M)$ cannot be optimal for any $q \in (\frac{1}{2}, 1)$.

□

Lemma (B5). *No three-signal experiment in the form of $(\{l,m,h\}, \pi)$, whereby signals m and h are both sent with positive probability, is optimal.*

Proof. First note we are considering the parameter range $q \in (\frac{1}{2}, \bar{q}]$. Suppose the contrary and there exists a three-signal optimal experiment $(\{l,m,h\}, \pi^*_{HM})$, where the investor invests iff she receives one of $(m, \tilde{l})$, $(h, \tilde{l})$ and $(h, \tilde{h})$. Then $\pi^*_{HM}(m|X)$ and $\pi^*_{HM}(h|X)$ solve the following optimization problem.

$$\max_{\pi(h|X),\pi(m|X)} \int_0^I (\varepsilon + X - s(X))(\pi(h|X) + (1-q)\pi(m|X))f(X)dX$$
$$+ \int_I^1 (\varepsilon + X - s(X))(\pi(h|X) + q\pi(m|X))f(X)dX \tag{52}$$

$$s.t. \quad q\int_0^I (s(X) - I)\pi(h|X)f(X)dX + (1-q)\int_I^1 (s(X) - I)\pi(h|X)f(X)dX \geq 0$$
$$(1-q)\int_0^I (s(X) - I)\pi(m|X)f(X)dX + q\int_I^1 (s(X) - I)\pi(m|X)f(X)dX \geq 0$$
$$\pi(h|X), \pi(m|X) \in [0,1]$$

Let λ^h and λ^m be the multipliers for the first two restrictions, respectively. Define $c_m(X)$ and $c_h(X)$ as

follows:

$$c_h(X) = \begin{cases} \varepsilon + X - s(X) + q\lambda^h(s(X) - I) & 0 \le X < I \\ \varepsilon + X - s(X) + (1-q)\lambda^h(s(X) - I) & I \le X \le 1 \end{cases}$$

$$c_m(X) = \begin{cases} (1-q)(\varepsilon + X - s(X) + \lambda^m(s(X) - I)) & 0 \le X < I \\ q(\varepsilon + X - s(X) + \lambda^m(s(X) - I)) & I \le X \le 1 \end{cases}$$

Then $\pi(h|X)$ and $\pi(m|X)$ solves the following optimization problem subject to $0 \le \pi(h|X), \pi(m|X) \le 1$

$$\max_{\pi(h|X),\pi(m|X)\in[0,1]} \int_0^1 [c_h(X)\pi(h|X) + c_m(X)\pi(m|X)]f(X)dX \tag{53}$$

Now, we appeal to lemma A1. Note that the optimization problem (53) is linear in $\pi(h|X)$ and $\pi(m|X)$. Moreover, it is easy to see that their multipliers are equal at most in a measure-zero subset of $[0,1]$. Therefore, $\pi(h|X), \pi(m|X) \in \{0,1\}$ almost surely. Therefore, there are two subsets $M, H \in [0,1]$ such that the signals m and h are sent for the members in M and H, respectively. Moreover, define $M^1 = M \cap [0, I)$, $M^2 = [I,1] \cap [I,1]$ and define H^1 and H^2, correspondingly.

If M_1 is empty, then signal m is just sent for a subset of $X \in [I,1]$. In this case, the investor invests even if she receives $(m, \tilde{l})$, which is in contrast with the definition of signal m. Therefore, suppose M_1 is non-empty. For $X \in M_1$ we have $c_m(X) \ge \max\{c_h(X), 0\}$. Rearranging the expressions of $c_h(X)$ and $c_m(X)$, we have

$$\frac{q\lambda^h - (1-q)\lambda^m}{q} \ge \frac{\varepsilon + X - s(X)}{I - s(X)} \ge \lambda^m \Rightarrow q\lambda^h \ge \lambda^m \tag{54}$$

Moreover, note that if M_2 is empty, then the investor does not ever invest when she receives m, which is a contradiction with the definition of m. Therefore, M_2 is not empty and there exists $X \in M_2$. For X, we have:

$$\begin{aligned} c_m(X) \ge c_h(X) \quad &\Rightarrow \quad (q\lambda^m - (1-q)\lambda^h)(s(X) - I) \ge (1-q)(\varepsilon + X - s(X)) \\ &\Rightarrow \quad q\lambda^m - (1-q)\lambda^h > 0 \end{aligned} \tag{55}$$

Combining (54) and (55), we get $q\lambda^h > \frac{1-q}{q}\lambda^h \Rightarrow q(1+q) > 1$, which contradicts our assumption about the value of q.

□

Lemmas B4 and B5 imply that the a two-signal experiment with $\{h, l\}$ must be optimal. In this experiment, the investor completely disregards her own signal. It is easy to see that the optimal two-signal experiment has a threshold scheme, where the threshold $\bar{X}(q)$ should satisfy (49). The rest of the results for this part are similar to Lemma 1.

Proof for Part (b) We can rewrite $\bar{K}_C^\varepsilon(\mu; q) = \mu\mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \ge \max\{\bar{X}(q), \hat{X}(\mu)\}\}}]$, and define

$$\begin{aligned} \mu^l &= \sup\left\{\mu | \bar{K}_S^\varepsilon \text{ is increasing over } [0,\mu]\right\} \\ \mu^h &= \inf\left\{\mu | \bar{K}_S^\varepsilon \text{ is decreasing over } \left[\mu, 1 - \frac{\varepsilon}{I - \bar{X}}\right]\right\} \end{aligned} \tag{56}$$

where $\bar{K}_S^\varepsilon(\mu) = \mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \ge \hat{X}(\mu)\}}\right]$. Since $\bar{X}(q) < I - \varepsilon$, there exists $\hat{\mu}(q)$ such that $\bar{X}(q) = \hat{X}(\hat{mu}(q))$. Because $\bar{X}(q)$ is strictly increasing in q in $[\frac{1}{2}, \bar{q}]$, $\hat{\mu}(q)$ is strictly decreasing in q. If $\hat{\mu}(q) < \mu^l$, then $\bar{K}_C^\varepsilon(\mu; q)$ is increasing over $[0,1]$. If that is not the case, then the function is U-shape over $[\mu^h, 1]$.

Proof for Part (c) Note that as ε goes to zero, $\bar{K}_S^{\varepsilon}(\mu)$ becomes strictly increasing in $[0,1]$. Therefore, μ^l converges to one as ε goes to zero.

□

In Proposition 13, the assumption $q \leq \bar{q}$ substantially simplifies the analysis, as the entrepreneur still follows a threshold scheme for disclosure. It is not generally the case, as we show in Lemma B6. In particular, the equilibrium experimentation has a nested-interval structure, consistent with Guo and Shmaya (2017) that finds similar structures under more general settings. For illustration, we take $\mu = 1$ in the next lemma and corollary, the case with interim competition or when $\mu < 1$ are similar.

Lemma (B6). *For $q > \bar{q}$, there exists $q^* > \bar{q}$ such that for $q \geq q^*$, the entrepreneur optimally uses three signals $\{l, m, h\}$. The investor continues the project iff she observes $(m, \tilde{h})$, $(h, \tilde{l})$ or $(h, \tilde{h})$. Specifically, there are $X_m^L(q), X_h^L(q), X_h^H(q) \in (0,1)$ such that the entrepreneur sends h in $[X_h^L(q), X_h^H(q)]$ and m in $[X_m^L(q), X_h^L(q)) \cup (X_h^H(q), 1]$.*

Proof. First, we show that the two-signal experiment introduced in Proposition 13(a) is not optimal for big enough values of q. Then, we show that when a three-signal experiment is optimal, a nested interval structure is used for providing endogenous information.

Non-optimality of two-signal experiments. Suppose the contrary holds that a threshold scheme, with threshold $\bar{X}(q)$ (as introduced in (49)) is optimal. Then Lemma A1 says it is the optimal experiment among all two-signal experiments that the high signal always induces investment. Now consider η_1, η_2 and η_3 that satisfy the following conditions:

$$q \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + (1-q) \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \leq 0$$

$$(1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + q \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \geq 0$$

If the two-signal experiment is optimal, then the following three-signal experiment should implement a suboptimal investment function for the entrepreneur.

$$\tilde{\pi}(h|X) = \begin{cases} 1 & X \in [\bar{X}(q) + \eta_2, 1 - \eta_3) \\ 0 & X \in [0, \bar{X}(q) + \eta_2) \cup [I - \eta_3, 1] \end{cases} \qquad \tilde{\pi}(m|X) = \begin{cases} 1 & X \in [\bar{X}(q) - \eta_1, \bar{X}(q) + \eta_2) \cup [1 - \eta_3, 1] \\ 0 & X \in [0, \bar{X}(q) - \eta_1) \cup [\bar{X}(q) + \eta_2, 1 - \eta_3) \end{cases}$$

Therefore, $\eta_1 = \eta_2 = \eta_3 = 0$ should be the solution to the following optimization problem:

$$\max_{\eta_1, \eta_2, \eta_3} (1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)} (\varepsilon + X - s(X)) f(X) dX - (1-q) \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (\varepsilon + X - s(X)) f(X) dX - q \int_{1-\eta_3}^{1} (\varepsilon + X - s(X)) f(X) dX$$

$$s.t. \quad q \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + (1-q) \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \leq 0$$

$$(1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + q \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \geq 0$$

Suppose κ_1 and κ_2 are the Lagrange multipliers for the above constraints. The FOCs at $\eta_1 = \eta_2 = \eta_3 = 0$ are as follows:

$$\begin{aligned}[\eta_1]|_{\eta_1=0} = 0 \Rightarrow & f(\bar{X}(q))[(1-q)(\varepsilon + \bar{X}(q) - s(\bar{X}(q)) + \kappa_2(s(\bar{X}(q)) - I))] = 0 \\ \Rightarrow & \kappa_2 = \frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))}\end{aligned} \tag{57}$$

$$\begin{aligned}[\eta_2]|_{\eta_2=0} = 0 \Rightarrow & f(\bar{X}(q))[-q(\varepsilon + \bar{X}(q) - s(\bar{X}(q))) + q\kappa_1(s(\bar{X}(q)) - I) + (1-q)\kappa_2(s(\bar{X}(q)) - I)] = 0 \\ \Rightarrow & -\kappa_1 = \frac{2q-1}{q}\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} = \frac{2q-1}{q}\kappa_2\end{aligned} \tag{58}$$

$$\begin{aligned}[\eta_3]|_{\eta_3=0} = 0 \Rightarrow & f(1)[-(1-q)(\varepsilon + 1 - s(1)) + (\kappa_1(1-q) + \kappa_2 q)(s(1) - I)] = 0 \\ \Rightarrow & \frac{\varepsilon + 1 - s(1)}{s(1) - I} = \frac{\kappa_1(1-q) + \kappa_2 q}{1-q} = \frac{3q^2 - 3q + 1}{q(1-q)}\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))}\end{aligned} \tag{59}$$

Note that:

$$\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} \geq \frac{\varepsilon}{I}$$

Therefore, (59) implies that for every $q \in [\frac{1}{2}, 1]$, the following inequality holds:

$$\frac{I}{\varepsilon}\frac{\varepsilon + 1 - s(1)}{s(1) - I} \geq \frac{3q^2 - 3q + 1}{q(1-q)} \tag{60}$$

The RHS in (60) goes to infinity as $q \to 1$, while the LHS is constant. It is the contradiction with the earlier assumption that the two-signal experiment is optimal. Therefore, a three-signal experiment is optimal for large enough values of q.

Nested interval structure. Guo and Shmaya (2017) prove the second part of the lemma in their Theorem 3.1 and Discussion 6.3, for securities $s(X)$ such that $\frac{s(X)-I}{\varepsilon+X-s(X)}$ is increasing in X. Note that this condition holds for $s_i(X) = X$.

□

Corollary 5. *For $q \geq q^*$, $\mathcal{I}^q(X) < 1$ for a positive measure of $X \in [I, 1]$, implying a positive probability of inefficient termination. Moreover the investor's interim rent, $\bar{K}_C^{\varepsilon}(q, 1)$, is non-monotone over the region $(\bar{q}, 1]$.*

Corollary 5 provides a counter-intuitive result that the investor's payoff is not globally increasing in investor sophistication. In fact, if the information the investor receives is very accurate, then the entrepreneur adopts a less informative experiment to increase the chance of inefficient continuations with the cost of positive probability of inefficient terminations. Similar to Proposition 5 in Kolotilin (2017), Lemma B6 highlights a "crowding-out effect" of higher levels of the investor's sophistication on the entrepreneur's information production. This effect implies that the investor might be better off by committing to ignoring some part of her independent information, which constitutes an interesting topic for future research.

In the following proposition, we show that while interim competition ($\mu < 1$) can improve the efficiency of investment decisions, it monotonically decreases the insider's interim rent for reasonably sophisticated investors, consistent with Petersen and Rajan (1995).

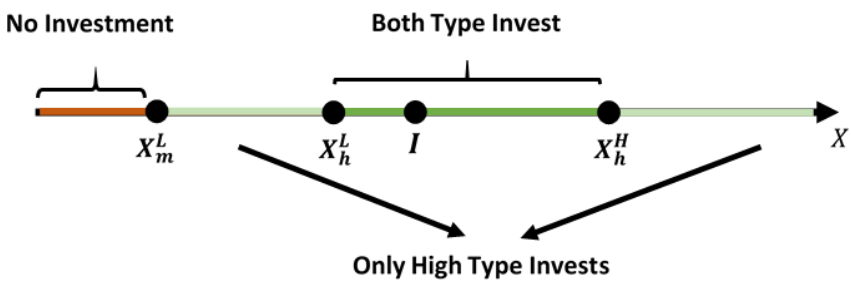

Figure 6: The nested interval structure of the entrepreneur's experimentation

Proposition 14 (Impact of Competition (extended))**.**

(a) For a given security with interim renegotiation, the equilibrium investment function becomes more socially optimal as interim competition increases (μ decreases).

(b) There exists $q^H < 1$ such that for $q \in (q^H, 1)$, the insider's interim rent is monotone in μ. Therefore, the insider never benefits from interim competition for very high levels of sophistication.

Proof. **Proof for Part (a)**

Suppose $\mu_2 > \mu_1$ and the equilibrium investment functions are respectively $\mathcal{I}_2(X)$ and $\mathcal{I}_1(X)$. We show that:

$$\mathbb{E}[(\varepsilon + X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(\varepsilon + X - I)\mathcal{I}_2(X)] \tag{61}$$

Suppose the contrary. Note that both $\mathcal{I}_1(X)$ and $\mathcal{I}_2(X)$ are implementable for the entrepreneur tThere are experiments that implement these investment functions), since q is fixed. The optimality of the investment functions implies:

$$\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] \tag{62}$$

Therefore, if (61) does not hold, (62) implies:

$$\mathbb{E}[(X - I)\mathcal{I}_2(X)] > \mathbb{E}[(X - I)\mathcal{I}_1(X)] \tag{63}$$

Furthermore, the fact that the investor receives positive interim payoff for $q > \frac{1}{2}$ and optimality of $\mathcal{I}_2(X)$ for μ_2 implies:

$$\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] > \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_2(X)] \geq \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_1(X)] \tag{64}$$

Therefore, (62) and (64) result in:

$$\begin{aligned} &\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_1(X)] - \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_1(X)] \\ \geq &\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] - \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_2(X)] \\ \Rightarrow &\mathbb{E}[(X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(X - I)\mathcal{I}_2(X)] \end{aligned} \tag{65}$$

which contradicts (63). Therefore, (61) holds. It shows the equilibrium investment function become more socially efficient as μ decreases.

Proof for Part (b)

As it is shown in Lemma B6, there exists q^* such that for $q > q^*$, a three-signal experiment is optimal. Fix a level of sophistication in the range q. Therefore, there are functions $M_l(\mu)$, $H_l(\mu)$ and $H_h(\mu)$ such that the entrepreneur sends a high signal for $X \in [H_l(\mu), H_h(\mu)]$ and a medium signal in $[M_l(\mu), H_l(\mu)] \cup [H_h(\mu), 1]$. Moreover, interim competition does not affect the entrepreneur's experimentation if $\varepsilon+(1-\mu)(M_l(1)-I) \geq 0$. Therefore, without loss of generality, we assume μ is small enough that $M_l(\mu) = I - \frac{\varepsilon}{1-\mu}$.

Moreover, as discussed earlier, the insider does not get any interim rent conditional on realization of a medium signal. Therefore, the insider's interim rent is as follows:

$$K_C(\mu; q) = \mu \int_{H_l(\mu)}^{H_h(\mu)} (X - I) f(X) dX = \frac{(2q-1)\mu}{q} \int_I^{H_h(\mu)} (X - I) f(X) dX \tag{66}$$

where the last equality comes from the fact that the constraint for sending a high signal is binding. The following lemma characterizes the derivative with respect to μ when $M_l(\mu) = I - \frac{\varepsilon}{1-\mu}$.

Lemma (B7). *If $\varepsilon + (1-\mu)(M_l(1) - I) < 0$, then*

$$\frac{\partial}{\partial \mu} K_C(\mu; q) = \frac{2q-1}{q} \left[\int_I^{H_h} (X - I) f(X) dX - \frac{q(1-q)}{2q-1} \frac{\mu \varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu}) \right] \tag{67}$$

Proof. By taking derivative from (66) with respect to μ, we get:

$$\frac{\partial}{\partial \mu} K_C(\mu; q) = \frac{(2q-1)}{q} \int_I^{H_h(\mu)} (X - I) f(X) dX + H_h'(\mu) \frac{(2q-1)\mu}{q} (H_h(\mu) - I) f(H_h(\mu)) \tag{68}$$

We know that the conditions for sending high and medium signal binds for all values of μ. Therefore:

$$\begin{aligned} &\frac{\partial}{\partial \mu} \left\{ (1-q) \int_{I - \frac{\varepsilon}{1-\mu}}^{H_l} (X - I) f(X) dX + q \int_{H_h}^{1} (X - I) f(X) dX \right\} = 0 \\ &\Rightarrow -\frac{(1-q)\varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu}) + (1-q) H_l'(H_l - I) f(H_l) - q H_h'(H_h - I) f(H_h) = 0 \end{aligned} \tag{69}$$

$$\begin{aligned} &\frac{\partial}{\partial \mu} \left\{ q \int_{H_l}^{I} (X - I) f(X) dX + (1-q) \int_I^{H_h} (X - I) f(X) dX \right\} = 0 \\ &\Rightarrow (1-q) H_h'(H_h - I) f(H_h) + q H_l'(H_l - I) f(H_l) = 0 \end{aligned} \tag{70}$$

By combining (69) and (70), we get:

$$H_h'(H_h - I) f(H_h) = -\frac{q(1-q)}{2q-1} \frac{\varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu})$$

We get (67) by substituting the last equality in (68). □

Note that H_h and H_l are the solution to the following maximization problem, where $M_l = I - \frac{\varepsilon}{1-\mu}$:

$$\begin{aligned} \max_{H_l, H_h} \quad & (1-q) \textstyle\int_{M_l}^{H_l} (\varepsilon + (1-\mu)(X - I)) f(X) dX + \int_{H_l}^{H_h} (\varepsilon + (1-\mu)(X - I)) f(X) dX \\ & + q \textstyle\int_{H_h}^{1} (\varepsilon + (1-\mu)(X - I)) f(X) dX \\ s.t. \quad & (1-q) \textstyle\int_{M_l}^{H_l} (X - I) f(X) dX + q \int_{H_h}^{1} (X - I) f(X) dX \geq 0 \\ & q \textstyle\int_{H_l}^{I} (X - I) f(X) dX + (1-q) \int_I^{H_h} (X - I) f(X) dX \geq 0 \end{aligned}$$

According to Lemma B6, we know that both constraints bind. It implies $H_l \to I$ and $H_h \to 1$, as $q \to 1$. Since $f(.)$ and $\frac{\mu}{(1-\mu)^3}$ are bounded ($\mu < 1 - \frac{\varepsilon}{I}$), then the result follows from Lemma B7.

□

B3. Contracting under IPH

When λ is close to 1, payoff channel is not playing a big role, and we can compare IPH to traditional moral hazards of effort provision. The "efficient effort" in our case is to produce continuation only when $X + \varepsilon > I$, the cost of that effort is the loss of private benefit when we terminate projects. The "action" in our setup is the experimentation, and the outcomes of the experiments correspond to noisy signals of the action.

It is a well-known result that for risk-neutral agents, the optimal security is debt (Innes (1990)). Moreover, even when we can contract on noisy signals of an agent's action, the outcome is generally not first best (e.g., Holmstrom (1979)). Yet the optimal design is partially indeterminate in our model and restores social efficiency. This result only relies on the contractibility of actions the interim information leads to, namely continuation or termination. Our findings point to the key difference between MIP and those in conventional models.

This contrast derives from two subtle differences between our setting and the ones in Holmstrom (1979) and Innes (1990). First, the principal takes an action based on the information produced by the agent, which implies that the agent's effort can affect his final payoff through affecting the principal's continuation decision. In fact, we show in Section B1 that by designing a security that makes the principal's continuation decision and thus the agent's payoff more sensitive to the agent's effort, we can also align the agent's incentives. Second, which is more important, the agent's effort in our model affects information production, but not the final output X, thus allowing the principle to know exactly how the entrepreneur's action has affected the final payoff. In other words, upon seeing the cash flow X, everyone knows if the continuation decision is socially optimal or not, hence the entrepreneur ex ante can design the security and contract contingent on the continuation payoff to perfectly incentivize the entrepreneur during the interim to take the right action. In some sense, the noise in the entrepreneur's action — signal to continue or terminate – is orthogonal to the continuation payoff, and making the agent's payoff contingent on both fully solves the agency problem. It is also worth pointing out that unlike the literature on costly experimentation and learning (e.g., Bergemann and Hege (1998) and Hörner and Samuelson (2013)) with hidden effort and hidden information, the principal in our setting observes perfectly the signal the agent produces, which is important for attaining the first-best outcome with the optimal contract.

B4. Conventional Effort Distortion

We have emphasized the entrepreneur's role as an information provider, whereas earlier studies concern the entrepreneur's costly effort to improve project cash flows. We now discuss how these two actions interact.

To model effort, we assume the entrepreneur can choose from a set of conditional distributions $f(X|e)$, where $e \in \mathcal{E}$ is the set of available levels of effort. Function $c(\cdot) : \mathcal{E} \to \mathcal{R}$ shows the cost associated with each level of effort. First, suppose $\mathcal{I}(X; e)$ is the equilibrium investment function when effort level e is chosen. Then, $e^* \in \mathcal{E}$ is *constrained first best* if it solves the following maximization problem:

$$e^* \in \quad argmax \;\; \mathbb{E}[(X - I + \varepsilon)\mathcal{I}(X; e)|e] - c(e) \tag{71}$$

It is straight-forward to show that absent investor sophistication and interim competition, there is no effort distortion: given the equilibrium information structure for each level of effort, the entrepreneur chooses the one that is socially preferred. Neither the insider's information monopoly nor IPH distorts entrepreneurial

effort. The reason is that when the insider investor gets no rent, the entrepreneur fully internalizes the benefit and the cost of effort. Naturally, in presence of a sophisticated investor and interim competition, the investor may get positive interim rent that distorts effort provision, but still to a lesser extent compared to the case where information production is exogenous and the insider enjoys full monopoly rent.