Modelling Multivariate Conditional Volatility

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Outline: This course is concerned with modelling financial volatility, and provides an econometric analysis of alternative models and techniques for analysing high frequency financial data. In the first of the three primary areas of volatility modelling, namely Conditional Volatility or Generalised Autoregressive Conditional Heteroskedasticity (GARCH), Stochastic Volatility (SV) and Realized Volatility (RV), univariate volatility models of individual financial assets and multivariate volatility models of portfolios of assets, will be examined critically, the mathematical structural properties of the models will be established, the associated estimation algorithms will be developed, the statistical properties of the estimators will be derived, and the forecasting performance will be evaluated.

Time allocation: Each session will analyse in detail some recent theoretical papers in the financial volatility literature. The course will consist of an examination of recent papers in the GARCH literature. The remaining lectures will suggest some graduate research topics associated with the conditional volatility literature.

Applications: Financial econometrics, agricultural finance, environmental finance, and tourism finance.

Overview: Modelling risk and volatility are crucial ingredients for purposes of forecasting Value-at-Risk (VaR) and minimizing daily capital charges. Sessions on forecasting VaR and minimizing daily capital charges will examine the following papers.

(1) Caporin, M. and M. McAleer (2010), "A scientific classification of volatility models", *Journal of Economic Surveys*, 24(1), 192-195, Available at SSRN: http://ssrn.com/abstract=1314231.

Modeling volatility, or predictable changes over time and space in a variable, is crucial in the natural and social sciences. Life can be volatile, and anything that matters, and which changes over time and space, involves volatility. Without volatility, many temporal and spatial variables would simply be constants. Our purpose is to propose a scientific classification of the alternative volatility models and approaches that are available in the literature, following the Linnaean taxonomy. This scientific classification is used because the literature has evolved as a living organism, with the birth of numerous new species of models.

(2) McAleer, M. (2009), "The Ten Commandments for optimizing value-at-risk and daily capital charges", *Journal of Economic Surveys*, 23(5), 831-849, Available at SSRN: http://ssrn.com/abstract=1354686.

Credit risk is the most important type of risk in terms of monetary value. Another key risk measure is market risk, which is typically concerned with stocks and bonds, and related financial derivatives, as well as exchange rates and interest rates. This paper examines market risk management and monitoring under the Basel II Accord, and presents Ten Commandments for optimizing Value-at-Risk (VaR) and daily capital charges, based on choosing wisely from: (1) conditional, stochastic and realized volatility; (2) symmetry, asymmetry and leverage; (3) dynamic correlations and dynamic covariances; (4) single index and portfolio models; (5) parametric, semiparametric and nonparametric models; (6) estimation, simulation and calibration of parameters; (7) assumptions, regularity conditions and statistical properties; (8) accuracy in calculating moments and forecasts; (9) optimizing threshold violations and economic benefits; and (10) optimizing private and public benefits of risk management. The Basel II Accord would seem to encourage risk taking at the expense of providing accurate measures and forecasts of risk and VaR.

(A) Conditional Volatility – GARCH

Alternative univariate and multivariate, symmetric and asymmetric, GARCH models will be examined, the mathematical structural properties of the models will be established, the associated estimation algorithms will be developed, and the statistical properties of the quasi-maximum likelihood estimators (QMLE) will be derived.

The sessions on Conditional Volatility will examine the following papers.

(3a) Caporin, M. and M. McAleer (2010), "The Ten Commandments for managing investments", *Journal of Economic Surveys*, 24(1), 196-200, Available at SSRN: http://ssrn.com/abstract=1342265.

Stress and distress are unavoidable aspects of dealing with the vagaries of financial markets and financial advisers. The purpose of this paper is to try to reduce the discomfort in dealing with investment advisers, and to make the journey up and down the financial mountain a little less stressful and more satisfying. The commandments deal with defining investment policies precisely, diversifying asset classes, choosing a consistent benchmark for investment policies, structuring precisely the asset allocation process, defining risk and risk management procedures, monitoring the portfolio carefully, matching the allocation and investment horizons, being active or passive according to investment policies, being agnostic about model forecasts, and being aware that, while buy low and sell high is a truism, investors and financial advisers are only human, and therefore make mistakes.

(3b) Jiménez-Martin, J.-A., M. McAleer and T. Perez Amaral (2009), "The Ten Commandments for managing value-at-risk under the Basel II Accord", *Journal of Economic Surveys*, 23(5), 850-855, Available at SSRN: http://ssrn.com/abstract=1356803.

Under the Basel II Accord, banks and other Authorized Deposit-taking Institutions (ADIs) are required to communicate their daily market risk estimates to the relevant national monetary authority at the beginning of each trading day, using one of a variety of Value-at-Risk (VaR) models to measure risk. The purpose of this paper is to provide a simple explanation and a set of prescriptions for managing VaR under the Basel II Accord. The commandments deal with understanding the Basel II colours, understanding the risk model before choosing, varying the choice of risk model, avoiding the green zone and being willing to violate, incurring large violations, stopping before the red zone, avoiding frequent violations, avoiding the estimation of large portfolios, aggregating portfolios into a single index, and interpreting commandments sensibly as guidelines.

(4) Li, W.K., S. Ling and M. McAleer (2002), "Recent theoretical results for time series models with GARCH errors", *Journal of Economic Surveys*, 16, 245-269. Reprinted in M. McAleer and L. Oxley (eds.), *Contributions to Financial Econometrics: Theoretical and Practical Issues*, Blackwell, Oxford, 2002, pp. 9-33 (according to *Essential Science Indicators*, ISI Web of Knowledge, this paper has 75 citations).

The paper reviews the recent theoretical literature on univariate and multivariate GARCH models.

(5) Ling, S. and M. McAleer (2002a), "Stationarity and the existence of moments of a family of GARCH processes", *Journal of Econometrics*, 106, 109-117 (according to *Essential Science Indicators*, ISI Web of Knowledge, this paper has 72 citations).

The paper develops the mathematical structural properties of a family of univariate GARCH processes.

(6) Ling, S. and M. McAleer (2002b), "Necessary and sufficient moment conditions for the GARCH(r,s) and asymmetric power GARCH(r,s) models", *Econometric Theory*, 18, 722-729 (according to *Essential Science Indicators*, ISI Web of Knowledge, this paper has 85 citations).

The paper develops the mathematical structural properties of some univariate GARCH and asymmetric power GARCH processes.

(7) Ling, S. and M. McAleer (2003), "Asymptotic theory for a vector ARMA-GARCH model", *Econometric Theory*, 19, 278-308 (according to *Essential Science Indicators*, ISI Web of Knowledge, this paper has 89 citations).

The paper develops the mathematical structural and asymptotic properties of a new multivariate GARCH process, VARMA-GARCH. This model has been programmed in the RATS econometric software package.

(8) McAleer, M. (2005), "Automated inference and learning in modeling financial volatility", *Econometric Theory*, 21, 232-261 (according to *Essential Science Indicators*, ISI Web of Knowledge, this paper has 77 citations).

The paper reviews a wide range of univariate and multivariate conditional volatility models, and provides a novel automated method for modelling univariate and multivariate conditional volatility.

(9) McAleer, M., S. Hoti and F. Chan (2009), "Structure and asymptotic theory for multivariate asymmetric conditional volatility", *Econometric Reviews*, 28, 422-440.

The paper develops the mathematical structural and asymptotic properties of a new multivariate asymmetric GARCH process, VARMA-AGARCH. This model has been programmed in the RATS econometric software package.

(10) McAleer, M., F. Chan and D. Marinova (2007), "An econometric analysis of asymmetric volatility: theory and application to patents", *Journal of Econometrics*, 139, 259-284 (according to *Essential Science Indicators*, ISI Web of Knowledge, this paper has 29 citations).

The paper develops the mathematical structural and asymptotic properties of the most popular univariate asymmetric GARCH process, the GJR threshold model.

(11) Caporin, M., and M. McAleer (2006), "Dynamic asymmetric GARCH", *Journal of Financial Econometrics*, 4, 385-412.

The paper develops the mathematical structural and asymptotic statistical properties of a new dynamic univariate asymmetric GARCH process with multiple thresholds.

(12) Asai, M. and M. McAleer (2007), "Non-trading day effects in asymmetric conditional and stochastic volatility models", *Econometrics Journal*, 10, 113-123.

The paper develops three new models of non-trading day (or holiday) effects in asymmetric and exponential conditional volatility models.

(13) Caporin, M., and M. McAleer (2008), "Scalar BEKK and indirect DCC", *Journal of Forecasting*, 27, 537-549.

The paper derives the scalar special case of the well known BEKK multivariate GARCH model using a Vector Random Coefficient Autoregressive model, and establishes the structural and asymptotic properties of the scalar BEKK model.

(14) McAleer, M. and B. da Veiga (2008a), "Forecasting Value-at-Risk with a parsimonious portfolio spillover GARCH (PS-GARCH) model", *Journal of Forecasting*, 27, 1-19.

The paper develops a parsimonious portfolio spillover GARCH model to forecast Value-at-Risk (VaR) thresholds.

(15) McAleer, M. and B. da Veiga (2008b), "Single index and portfolio models for forecasting Value-at-Risk thresholds", *Journal of Forecasting*, 27, 217-235.

Using univariate and multivariate conditional volatility models, the paper evaluates the performance of the single index and portfolio models in forecasting Value-at-Risk (VaR) thresholds of a portfolio.

(16) Asai, M. and M. McAleer (2008), "A portfolio index GARCH model", *International Journal of Forecasting*, 24, 449-461.

The paper develops the structure of a parsimonious portfolio index GARCH (PI-GARCH) model which specifies the volatility of a portfolio directly, and examines the effects of symmetric and asymmetric shocks. A portfolio index BEKK model is also developed and evaluated.

(17) McAleer, M., F. Chan. S. Hoti and O. Lieberman (2008), "Generalized autoregressive conditional correlation", *Econometric Theory*, 24, 1554-1583.

The paper develops a generalized autoregressive conditional correlation (GARCC) model when the standardized residuals follow a multivariate random coefficient autoregressive process. The GARCC model provides a motivation for dynamic conditional correlations.

(18) Caporin, M., and M. McAleer (2009), "Thresholds, news impact surfaces, and dynamic asymmetric multivariate GARCH", Available at SSRN: http://ssrn.com/abstract=1198702.

The paper develops the mathematical structural and asymptotic properties of a new dynamic multivariate asymmetric GARCH (DAM-GARCH) process with multiple thresholds, and presents a novel news impact surface to measure the impact of news on volatility deriving from a portfolio of assets.

(19) Caporin, M. and M. McAleer (2009), "Do we really need both BEKK and DCC? A tale of two covariance models", Available at SSRN: http://ssrn.com/abstract=1338190.

Large and very large portfolios of financial assets are routine for many individuals and organizations. The two most widely used models of conditional covariances and correlations are BEKK and DCC. BEKK suffers from the archetypal "curse of dimensionality" whereas DCC does not. This is a misleading interpretation of the suitability of the two models to be used in practice. The primary purposes of the paper are to define targeting as an aid in estimating matrices associated with large numbers of financial assets, analyze the similarities and dissimilarities between BEKK and DCC, both with and without targeting, on the basis of structural derivation, the analytical forms of the sufficient conditions for the existence of moments, and the sufficient conditions for consistency and asymptotic normality, and computational tractability for very large (that is, ultra high) numbers of financial assets, to present a consistent two step estimation method for the DCC model, and to determine whether BEKK or DCC should be preferred in practical applications.

(F) Applications to Financial Econometrics, Agricultural Finance, Commodities and Tourism Research

(a) Financial econometrics and empirical finance

(1) Allen, D.E., M. McAleer and B. da Veiga (2009), "Modelling and forecasting dynamic VaR thresholds for risk management and regulation", Available at SSRN: http://ssrn.com/abstract=926270.

(2) Asai, M. and M. McAleer (2009), "Alternative asymmetric stochastic volatility models", to appear in *Econometric Reviews*, Available at SSRN: http://ssrn.com/abstract=1464329.

(3) Asai, M. and M. McAleer (2009), "Dynamic conditional correlations for asymmetric processes", Available at SSRN: http://ssrn.com/abstract=1464325.

(4) Hammoudeh, S.M., Y. Yuan and M. McAleer (2009), "Exchange rate and industrial commodity volatility transmissions and hedging strategies", Available at SSRN: http://ssrn.com/abstract=1473939.

(5) Hammoudeh, S.M., Y. Yuan, M.A. Thompson and M. McAleer (2009), "Precious metals-exchange rate volatility transmissions and hedging strategies", to appear in *Review of Economics and Finance*, Available at SSRN: http://ssrn.com/abstract=1495748.

(6) McAleer, M., B. da Veiga and S. Hoti (2009), "Value-at-risk for country risk ratings", to appear in *Mathematics and Computers in Simulation*, Available at SSRN: http://ssrn.com/abstract=1468524.

(7) McAleer, M., J.-A. Jiménez-Martin and T. Perez Amaral (2009), "Has the Basel II Accord encouraged risk management during the 2008-09 financial crisis?", Available at SSRN: http://ssrn.com/abstract=1397239.

(8) McAleer, M., J.-A. Jiménez-Martin and T. Perez Amaral (2009), "What happened to risk management during the 2008-09 financial crisis?", to appear in R.W. Kolb (ed.), *Lessons from the Financial Crisis: Causes, Consequences, and Our Economic Future*, Wiley, New York, 2010, Available at SSRN: http://ssrn.com/abstract=1442034.

(9) McAleer, M., J.-A. Jiménez-Martin and T. Perez Amaral (2009), "Optimal risk management before, during and after the 2008-09 financial crisis", Available at SSRN: http://ssrn.com/abstract=1473191.

(10) McAleer, M. and M. Medeiros (2009), "Forecasting realized volatility with linear and nonlinear models", Available at SSRN: http://ssrn.com/abstract=1496861.

(11) McAleer, M., T. Perez Amaral and J.-A. Jiménez-Martin (2010), "A decision rule to minimize daily capital charges in forecasting value-at-risk", *Journal of Forecasting*, 29, 617-634, Available at SSRN: http://ssrn.com/abstract=1349844.

(12) da Veiga, B., F. Chan and M. McAleer (2009), "It pays to violate: How effective are the Basel Accord penalties?", Available at SSRN: http://ssrn.com/abstract=1494840.

(13) Wiphatthanananthakul, C. and M. McAleer (2009), "A simple expected volatility (SEV) index: application to SET50 index options", Available at SSRN: http://ssrn.com/abstract=1361906.

(14) Wong, W.-K. and M. McAleer (2009), "Financial astrology: mapping the Presidential Election Cycle in US stock markets", *Mathematics and Computers in Simulation*, 79, 3267-3277, Available at SSRN: http://ssrn.com/abstract=1307643.

(b) Agricultural finance

(1) Chang, C.-L., M. McAleer, M.-G. Chen and B.-W. Huang (2009), "Modelling the asymmetric volatility in hog prices in Taiwan: the impact of joining the WTO", Available at SSRN: http://ssrn.com/abstract=1355869.

(2) Huang, B.-W., M.-G. Chen, C.-L. Chang and M. McAleer (2009), "Modelling risk in agricultural finance: application to the poultry industry in Taiwan", *Mathematics and Computers in Simulation*, 79, 1472-1487.

(3) Khamkaew, T., R. Tansuchat, C.-L. Chang, and M. McAleer (2009), "Modelling conditional correlations in the volatility of Asian rubber spot and futures returns", Available at SSRN: http://ssrn.com/abstract=1496309.

(4) Tansuchat, R., C.-L. Chang and M. McAleer (2009), "Modelling long memory volatility in agricultural commodity futures returns", Available at SSRN: http://ssrn.com/abstract=1491890.

(c) Oil spot, forward and futures prices

(1) Chang, C.-L., M. McAleer and R. Tansuchat (2009), "Modelling conditional correlations for risk diversification in crude oil markets", *Journal of Energy Markets*, 2(4), 2009/10, 29-51, Available at SSRN: http://ssrn.com/abstract=1401331.

(2) Chang, C.-L., M. McAleer and R. Tansuchat (2009), "Forecasting volatility and spillovers in crude oil spot, forward and futures markets", Available at SSRN: http://ssrn.com/abstract=1402164.

(3) Chang, C.-L., M. McAleer and R. Tansuchat (2009), "Volatility spillovers between returns on crude oil futures and oil company stocks", Available at SSRN: http://ssrn.com/abstract=1406983.

(4) Lean, H.-H., W.-K. Wong and M. McAleer (2006), "Stochastic dominance test for risk seekers: an application to oil spot and futures markets", Available at SSRN: http://ssrn.com/abstract=916383.

(5) Lean, H.-H., M. McAleer and W.-K. Wong (2010), "Market efficiency of oil spot and futures: A mean-variance and stochastic dominance approach", Available at SSRN: http://ssrn.com/abstract=1537103.

(6) Manera, M., A. Lanza and M. McAleer (2006), "Modelling dynamic conditional correlations in WTI oil forward and futures returns", *Finance Research Letters*, 3(2), 114-132, Available at SSRN: http://ssrn.com/abstract=546484.

(7) Manera, M., M. McAleer and M. Grasso (2006), "Modelling time-varying conditional correlations in the volatility of Tapis oil spot and forward returns", *Applied Financial Economics*, 16, 525-533.

(8) Tansuchat, R., C.-L. Chang and M. McAleer (2010), "Conditional correlations and volatility spillovers between crude oil and stock index returns", Available at SSRN: http://ssrn.com/abstract=1534043.

(9) Tansuchat, R., C.-L. Chang and M. McAleer (2010), "Crude oil hedging strategies using dynamic multivariate GARCH", Available at SSRN: http://ssrn.com/abstract=1531187.

(d) Tourism research

(1) Bartolome, A., M. McAleer, V. Ramos and J. Rey-Maquieira (2009), "A risk map of international tourist regions in Spain", *Mathematics and Computers in Simulation*, 79(9), 2009, 2741-2758.

(2) Bartolome, A., M. McAleer, V. Ramos and J. Rey-Maquieira (2009), "Modelling air passenger arrivals to the Balearic and Canary Islands", *Tourism Economics*, 15(3), 481-500.

(3) Chang, C.-L., T. Khamkaew and M. McAleer (2009), "A panel threshold model of tourism specialization and economic development", Available at SSRN: http://ssrn.com/abstract=1496324.

(4) Chang, C.-L., T. Khamkaew, M. McAleer and R. Tansuchat (2009), "Interdependence of international tourism demand and volatility in leading ASEAN destinations", Available at SSRN: http://ssrn.com/abstract=1498414.

(5) Chang, C.-L. and M. McAleer (2009), "Daily tourist arrivals, exchange rates and volatility for Korea and Taiwan", *Korean Economic Review*, 25, 241-267, Available at SSRN: http://ssrn.com/abstract=1504651.

(6) Chang, C.-L. and M. McAleer (2010), "Aggregation, heterogeneous autoregression and volatility of daily international tourist arrivals and exchange rates", Available at SSRN: http://ssrn.com/abstract=1553377.

(7) Chang, C.-L., M. McAleer and C. Lim (2009), "Modelling short and long haul volatility in Japanese tourist arrivals to New Zealand and Taiwan", Available at SSRN: http://ssrn.com/abstract=1356567.

(8) Chang, C.-L., M. McAleer and D.J. Slottje (2009), "Modelling international tourist arrivals and volatility: an application to Taiwan", in D. Slottje (ed.), *Quantifying Consumer Preferences*, Contributions to Economic Analysis Series, Volume 288, Emerald Group Publishing, pp. 303-320, Available at SSRN: http://ssrn.com/abstract=1355108.

(9) Divino, J.A. and M. McAleer (2009), "Modelling the growth and volatility in daily international mass tourism to Peru", to appear in *Tourism Management*, Available at SSRN: http://ssrn.com/abstract=1361807.

(10) Divino, J.A. and M. McAleer (2009), "Modelling sustainable international tourism demand to the Brazilian Amazon", *Environmental Modelling and Software*, 24, 1411-1419, Available at SSRN: http://ssrn.com/abstract=1361816.

(11) Kuo, H.-I, C.-L. Chang, C.-C. Chen, B.-W. Huang and M. McAleer (2009), "Estimating the impact of Avian Flu on international tourism demand using panel data", *Tourism Economics*, 15(3), 501-511, Available at SSRN: http://ssrn.com/abstract=1365228.

(12) Kuo, H.-I, C.-C. Chen and M. McAleer (2009), "Estimating the impact of whaling on global whale watching", Available at SSRN: http://ssrn.com/abstract=1442444.

(13) McAleer, M., B.-W. Huang, H.-I Kuo, C.-C. Chen and C.-L. Chang (2010), "An econometric analysis of SARS and Avian Flu on international tourist arrivals to Asia", *Environmental Modelling and Software*, 25, 100-106, Available at SSRN: http://ssrn.com/abstract=1355109.

(14) McAleer, M., R. Shareef and B. da Veiga (2009), "Risk management of daily tourist tax revenues for the Maldives", Available at SSRN: http://ssrn.com/abstract=855924.

(15) Medeiros, M., M. McAleer, D. Slottje, V. Ramos and J. Rey-Maquieira (2008), "An alternative approach to estimating demand: neural network regression with conditional volatility for high frequency air passenger arrivals", *Journal of Econometrics*, 147(2), 372-383.

(16) Shareef, R. and M. McAleer (2007), "Modelling the uncertainty in international tourist arrivals to the Maldives", *Tourism Management*, 28, 23-45.