Follow the Market!
Co-monotonicity of Optimal Investments and the Design of Structured Financial Products

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Outline

1 Introduction
   - No Arbitrage Condition and the Covariance Formula
   - Co-monotonicity
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   • Co-monotonicity

2 Following the market
   • A general co-monotonicity result for optimal products
   • The case of the “private investor”
   • The case of the “fund manager”
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3 Optimal financial products
   - Existence of optimal financial products
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4. Applications to Barrier Products
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5 Conclusions

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Structured Products

Basic model

We study a two-step model of a complete financial market. A **structured product** can be described by its return distribution $p$. State space can be finite or infinite. We study only products with fixed price. $R$ is the risk-free rate, $m$ is the inverted state price density.
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A classical derivation gives the following price constraint for $p$:

$$\mathbb{E}(p) - R = \frac{\text{cov}(p, m)}{\text{var}(m)}(\mathbb{E}(m) - R).$$
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A classical derivation gives the following price constraint for $p$:

$$E(p) - R = \frac{\text{cov}(p, m)}{\text{var}(m)} (E(m) - R).$$

Special case: CAPM
Then $m$ equals the return distribution of the market portfolio.
Conjoint probability measure

\[ \text{cov}(p, m) \] depends on the conjoint probability measure \( T \) of \( p \) and \( m \):

\[ T(x, y) > 0 \]

Fundamental question of this talk:

What can we learn about this conjoint probability measure?
What is co-monotonicity?

Idea: two random variables $A$ and $B$ are co-monotone if whenever $A$ becomes larger or smaller, so does $B$ (or is at least constant).
Co-monotone conjoint probability measures

What is co-monotonicity?

Idea: two random variables $A$ and $B$ are co-monotone if whenever $A$ becomes larger or smaller, so does $B$ (or is at least constant).

Precise definition

$T$ is co-monotone if, for all $(x_1, y_1), (x_2, y_2) \in \text{supp } T$ with $x_2 > x_1$, we have $y_2 \geq y_1$.

Illustration:

Co-monotone conjoint probability measure $T(x,y) > 0$
2. Following the market

What is an optimal financial product?

An optimal financial product is a conjoint probability measure $T$ such that:

- $T$ is the conjoint probability of $p$ and $m$, where $m$ is the inverted state-price density.
- $T$ satisfies the price constraint (covariance formula).
- $T$ maximizes the investor’s utility.

Precise definition:

Let $T$ be the conjoint probability measure with marginals $p_{1T}$ and $p_{2T}$. We say that $T$ is optimal if it maximizes the utility $U: P(R \times R) \rightarrow \mathbb{R}$ subject to $p_{1T} = p$, $p_{2T} = m$ and $E(p) - R = \text{cov}(p, m) \cdot \text{var}(m) (E(m) - R)$. 

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Precise definition:

Let $T$ be the conjoint probability measure with marginals $\text{pr}_1 T$ and $\text{pr}_2 T$. We say that $T$ is optimal if it maximizes the utility $U : \mathcal{P}(\mathbb{R} \times \mathbb{R}) \to \mathbb{R}$ subject to $\text{pr}_1 T = p$, $\text{pr}_2 T = m$ and

$$\mathbb{E}(p) - R = \frac{\text{cov}(p, m)}{\text{var}(m)}(\mathbb{E}(m) - R).$$
Implicit assumptions

We always assume:

- Common beliefs. (Return distribution is known.)
- No background risk. (Investment of total wealth.)
The investor’s utility

Assumptions on the utility

We assume that $U : \mathcal{P}(\mathbb{R} \times \mathbb{R}) \to \mathbb{R}$ satisfies:

(i) $U(T) \leq U(T(\cdot, \cdot - c))$ for all $c > 0$,

(ii) There is a non-decreasing function $h : \mathbb{R} \to \mathbb{R}$ and a function $\tilde{U} : \mathcal{P}(\mathbb{R}) \to \mathbb{R}$ such that $U(T) = \tilde{U}(p_h)$, where $p_h(y) := \text{pr}_1 T(x, y + h(x))$.

What do these assumptions mean?

(i) “More money is better”: investor prefers a risky asset \textit{plus cash} over the same risky asset \textit{without cash}.

(ii) Will be clearer when we study two important special cases.
Theorem (Main result)

Under the above assumption on the utility, every optimal financial product is co-monotone with the inverted state-price density.
General co-monotonicity result

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Theorem (CAPM-version)
Under the above assumption on the utility, every optimal financial product in a CAPM market is co-monotone with the market.

Remarks
In the CAPM case, we say that every optimal product "follows the market": if the market yields higher returns, so does the product. Even in the CAPM case, the investor's preferences do not have to follow the Mean-Variance framework. Instead of the main result, we will later only prove a special case.
General co-monotonicity result

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The case of the “private investor”

Assumptions on the utility functional:

- “People only care about the result”: The utility only depends on the outcome distribution. – No dependence on the market performance!
- “More money is better”: If an outcome distribution $p$ is shifted to the right, its utility grows.

Assumptions satisfied by:
- Expected utility theory
- Mean-variance theory
- Prospect theory

Definition

We call an agent satisfying these assumptions a **private investor**.
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Result for private investors

Theorem
Every optimal financial product for a private investor is co-monotone with the inverted state-price density.
Result for private investors

**Theorem**

Every optimal financial product for a private investor is co-monotone with the inverted state-price density.

**Practical consequence on CAPM-market:**

A private investor should never speculate on a decline of the market.

**Previously known:**

Proof.

Let $p$ be given. If we maximize $\text{cov}(p, m)$, then the conjoint probability measure must be monotone (see below).
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3. Let $\tilde{T}$ be the (monotone) conjoint probability that maximizes $\text{cov}(p, m)$. Then $d := R + \text{cov} \tilde{T}(p, m)(\mathbb{E}(m) - R) - \mathbb{E}(p) > 0$. 

4. Now shift the returns of the product by $d$ and call the result $\tilde{p}$. 

5. By assumption this shift increases the utility.
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$$\mathbb{E}(\tilde{p}) = R + \text{cov}_{\tilde{T}}(\tilde{p}, m)(\mathbb{E}(m) - R),$$

since $\text{cov}_{\tilde{T}}(\tilde{p}, m) = \text{cov}_{\tilde{T}}(p, m)$. 

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How to prove the auxiliary result

Conjoint probability measures are a special case of transport plans, as introduced by Kantorivich in 1942:

**Definition (Transport problem, simple case)**

Let \( \mu, \nu \in \mathcal{P}(\mathbb{R}) \) and let \( c \colon \mathbb{R} \times \mathbb{R} \to \mathbb{R} \).

Find a probability measure \( T \in \mathcal{P}(\mathbb{R} \times \mathbb{R}) \) minimizing

\[
C(T) := \int_{\mathbb{R}} \int_{\mathbb{R}} c(x, y) dT(x, y),
\]

such that

\[
\pi_1 T = \mu, \quad \pi_2 T = \nu.
\]
Co-monotonicity of transport plans

Theorem

Let $c$ be a continuous function satisfying for all $x_1 < x_2$ and $y_1 < y_2$

$$c(x_1, y_1) + c(x_2, y_2) > c(x_1, y_2) + c(x_2, y_1),$$

then the transport problem admits a unique minimizer which is co-monotone. If only $\geq$ holds, there still exists a co-monotone minimizer.
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Idea of proof
1. Suppose \( T \) is non-co-monotone minimizer.
2. Approximate \( T \) by Dirac measures.
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**Illustration**

Non-monotone conjoint probability measure

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Co-monotonicity of transport plans

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**Illustration**

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Performance relative to market index. (Benchmarking)

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Definition

We call an agent satisfying these assumptions a fund manager.
Theorem

Optimal structured products for fund managers follow the market.
Result for fund manager

**Theorem**

Optimal structured products for fund managers follow the market.

**Conclusion:**

Changing the viewpoint (frame) does not change this general property! In fact, private investors and fund managers are just special instances for the general result.
BENCHMARKING LEADS TO RISKY PRODUCTS

THEOREM (RISKY FINANCIAL PRODUCTS FOR GENERAL UTILITY)

An optimal product for a fund manager in a CAPM market is at least as risky as the market portfolio, i.e. the difference between the return distribution and the market return is a non-decreasing function of the return.
Benchmarking leads to risky products

**Theorem (Risky financial products for general utility)**

An optimal product for a fund manager in a CAPM market is *at least as risky* as the market portfolio, i.e. the difference between the return distribution and the market return is a non-decreasing function of the return.

**Theorem (Expected Utility case)**

If a fund manager has preferences described by a smooth Expected Utility function, then his optimal product in a CAPM market is *riskier* than the market portfolio, i.e. the difference between the return distribution and the market return is a non-decreasing and non-constant function of the return.
3. Optimal financial products

Preliminary observation

Using the results on co-monotonicity and the fact that co-monotone conjoint probability measures are unique, we can describe every optimal financial product by its return distribution \( p \) instead of \( T \).
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Using the results on co-monotonicity and the fact that co-monotone conjoint probability measures are unique, we can describe every optimal financial product by its return distribution $p$ instead of $T$.

Existence

Question: Under what conditions exist optimal products?
Existence of optimal financial products

Assumptions

- Preferences described by Expected Utility Theory. (In terms of final wealth.)
- Utility function $u$ is sublinear, i.e. $u(x)/x \to 0$ as $x \to \infty$.
- Support of state price density is bounded. ($\text{supp } m \in [0, M]$.)
- Market variance large enough: $\text{var}(m) > M(\mathbb{E}(m) - R)$.

Theorem (Existence)

Under the above assumptions there exists an optimal product $p$. This product satisfies $\mathbb{E}(p) < \infty$. 
Sketch of proof

1. There is a $C < \infty$ such that: If $p \in \mathcal{P}$ satisfies the price constraint, then $\mathbb{E}(p) \leq C$. 

Remark: To prove the existence of $p$, we need to show that $p_n$ is tight (where we need the technical conditions on $m$) and apply Prokhorov’s Theorem. The third step uses Jensen’s Inequality. Sublinearity of $u$ is used in the fourth step. The last step requires a delicate approximation argument.
Sketch of proof

1. There is a $C < \infty$ such that: If $p \in \mathcal{P}$ satisfies the price constraint, then $\mathbb{E}(p) \leq C$.

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3. The supremum of $U(p)$ over all $p \in \mathcal{P}$ satisfying the price constraint, is finite.
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4. Applications to Barrier Products

Financial product defined by:

- If the barrier (e.g. 80% of start price) is not hit: capital protection.
- If the barrier has been hit: no capital protection anymore.

Payoff diagram:
Why barrier products are not optimal

Conjoint probability:

Not monotone!
Optimizing barrier products

Barrier product:

- return of product
- return of underlying

Optimized product:

- return of product
- return of underlying

Monotonization optimizes variant to have the same return distribution but lower price. Here direct computation is possible (see working paper).

General situation: use numerical approximation.

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Optimizing barrier products

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**Monotonization**

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Why are barrier products popular?

We have seen: barrier products could be improved. However, we have overlooked behavioral factors...

Comparison of hit-probabilities

Data: Dow Jones 1984-2004, one year maturity.
Red line: probability to hit barrier during life-time of product.
Blue line: probability to hit barrier at maturity.
Relative difference of hit-probabilities

Results for the Dow Jones Index (one year maturity, starting date between 1984 and 2004):

The relative difference increases sharply for low barrier levels.
Misestimation resolves the puzzle

Experimental results

This sharp increase is not reflected in experiments performed with economics students:
Out of 109 students, only 58% estimated a larger relative difference for lower barrier levels.
Median relative difference was 2.0 for barrier levels of 80% and 90%.
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In other words:
Probability to hit barrier during lifetime of product is underestimated, probability to be below barrier at maturity is overestimated.
This makes the “monotonized product” less attractive compared to the barrier product.
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We conclude:

Systematic misestimation can explain the popularity of barrier options.
5. Conclusions

Results

- Co-monotonicity of optimal financial products is a very general property.
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What remains to be done:
- Generalization of existence results.
- Numerical computation of optimal financial products.

Thanking you for your attention!

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