Bansal, Zhou (2002): Term Structure of Interest Rates with Regime Shifts

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Outline

Term structure of interest rates with regime shifts

- Motivation and contribution of the paper
- Position in the literature
- A term structure model with regime shifts
- Empirical evidence
- Conclusion
Main idea: Regime shifts important to understand the behavior of the entire yield curve, since bond and bond options differ across economic regimes

Economic reason: Business cycle ‘regimes’ affect inflationary expectations, fiscal and monetary policy, nominal interest rate, jumps on investors’s confidence, ..., which in turn affect the entire term structure of interest rates

Relatively poor empirical performance of standard term structure models (CIR model and extensions, affine models)

Shortcoming of standard term structure models: regime shifts not incorporated

Shortcoming of regime shift term structure models so far: Regime shifts restricted to a particular part of the model (e.g. only to the short interest rate)
Contribution of the Paper, Preview of Main Results

- Main contribution of the paper: Introduction of a model in which regime shifts affect the entire term structure of interest rates
- Comprehensive empirical support: The proposed regime shift model performs better than standard term structure models
- Meaningful economic interpretation of the estimated parameters of the proposed regime shift model
Literature

Presented paper:

Related literature:
Model: Term Structure of Interest Rates with Regime Shifts

Benchmark model: 1-factor-CIR model
Additional assumption: Economy subject to discrete regime shifts
$s_t = 0, 1$ (two-state Markov process)
State variables $x_t$ with dynamics

$$x_{t+1} - x_t = \kappa_{s_{t+1}} (\theta_{s_{t+1}} - x_t) + \sigma_{s_{t+1}} \sqrt{x_t} u_{t+1}, \quad (1)$$

$k_{s_{t+1}}$ mean reversion, $\theta_{s_{t+1}}$ long-run mean, $\sigma_{s_{t+1}}$ volatility,
$u_t \sim N(0, 1)$ iid.
Pricing kernel

$$M_{t+1} = \exp \left\{ -r_{f,t} - \left( \frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}} \right)^2 \frac{x_t}{2} - \frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}} \sqrt{x_t} u_{t+1} \right\}, \quad (2)$$

$\lambda_{s_{t+1}}$ risk premium parameter
Pricing Formulae

Bond price with n periods to maturity at date t:

\[ P_{st}(t, n) = \exp \left\{ -A_{st}(n) - B_{st}(n)x_t \right\}, \quad (3) \]

\( A_0(n), A_1(n), B_0(n), B_1(n) \) constants

Analogously: 2-factor model (2 state variables, each dynamics like (1)).

Pricing formula:

\[ P_{st}(t, n) = \exp \left\{ -A_{st}(n) - \sum_{k=1}^{2} B_{k, st}(n)x_{k,t} \right\}, \quad (4) \]

\( A_0(n), A_1(n), B_{k,0}(n), B_{k,1}(n), \quad k = 1, 2 \) constants

Yield curve formula:

\[ Y_{st}(t, n) = -\frac{\log P_{st}(t, n)}{n} = \frac{A_{st}(n)}{n} + \sum_{k=1}^{2} B_{k, st}(n)x_{k,t} \quad (5) \]
Data and Tested Models

Data:
- 379 monthly observations of U.S. T-bond rates with nine maturities
- June 1964 - December 1995
- from the Center for Research in Security Prices (CRSP)
- replicate stylized facts of maturities six month and five years

Tested models:
- Regime Shift (RS) model (1, 2 factors; with and without restrictions on the risk premium parameter)
- CIR (1, 2, 3 factors)
- affine (AF) model (3 factors)
Methodology: Simulation-based EMM:

1st step: Estimate empirical conditional density of observed interest rates by a seminonparametric (SNP) series expansion

2nd step: Use score functions from the log-likelihood of the SNP density as moments to construct a GMM-type criterion function

Obtain: GMM-type estimator providing comparable measures for specification tests across models
Specification Test

Result:

- Only RS(2) cannot be rejected (p-value 0.1427, all others below 0.0001).

Confirmation by the t-values for fitted scores of the reprojected density.
Model Estimation of the RS(2) model

Estimated parameters and standard deviations of the RS(2) model:

<table>
<thead>
<tr>
<th></th>
<th>1, 0</th>
<th>1, 1</th>
<th>2, 0</th>
<th>2, 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{factor,regime}$</td>
<td>0.00437</td>
<td>0.00273</td>
<td><strong>0.00076</strong></td>
<td><strong>0.00540</strong></td>
</tr>
<tr>
<td></td>
<td>(0.00003)</td>
<td>(0.00001)</td>
<td>(0.00001)</td>
<td>(0.00144)</td>
</tr>
<tr>
<td>$\kappa_{factor,regime}$</td>
<td>0.03388</td>
<td>0.04375</td>
<td>0.01501</td>
<td>0.00344</td>
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<tr>
<td></td>
<td>(0.00199)</td>
<td>(0.00136)</td>
<td>(0.00019)</td>
<td>(0.00102)</td>
</tr>
<tr>
<td>$\sigma_{factor,regime}$</td>
<td>0.00421</td>
<td>0.00629</td>
<td>0.00392</td>
<td>0.00409</td>
</tr>
<tr>
<td></td>
<td>(0.00002)</td>
<td>(0.00002)</td>
<td>(0.00003)</td>
<td>(0.00004)</td>
</tr>
<tr>
<td>$\lambda_{factor,regime}$</td>
<td>0.01155</td>
<td>-0.03145</td>
<td><strong>-0.02079</strong></td>
<td><strong>-0.02287</strong></td>
</tr>
<tr>
<td></td>
<td>(0.00112)</td>
<td>(0.00086)</td>
<td>(0.00005)</td>
<td>(0.00075)</td>
</tr>
</tbody>
</table>

Transitional probabilities: $\Pi_{00} = 0.95(0.18)$, $\Pi_{11} = 0.92(0.22)$.

Further tests confirm that:

- Regime shifts do not seem to be an i.i.d. mixture
- Factor 1 has significant regime switches in level, mean reversion, volatility parameters, risk parameter
- Factor 2 has significant regime switches only in mean reversion and volatility
Pricing Error, Implied Regimes, Implied Yield Curve

Approach:

- Recover latent factors $X_t$ via the bond pricing function by $X_t = B^{-1}(Y_t - A)$

- Average pricing error $PE_t = \frac{\sum_{n=1}^{N} |\hat{Y}(n)_t - Y(n)_t|}{N}$, with

  $\hat{Y}(n)_t = \text{computed yield of maturity } n \text{ at date } t,$

  $Y(n)_t = \text{true yield of maturity } n \text{ at date } t$

- RS-models: Compute $PE_t$ for each regime, choose the one with minimal pricing error

- Find that factor 1 [2] tracks the short [long] yield

- Given $X_t$ and the estimated parameters, compute the yield curve for each date
Pricing, Error, Implied Regimes, Implied Yield Curve

Results: Average absolute pricing error (basis points):

<table>
<thead>
<tr>
<th></th>
<th>RS(1)</th>
<th>CIR(2)</th>
<th>RS(2)</th>
<th>RS(2)</th>
<th>CIR(3)</th>
<th>AF(3)</th>
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<tbody>
<tr>
<td>$\mu$</td>
<td>43</td>
<td>30</td>
<td>25</td>
<td>23</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>27</td>
<td>18</td>
<td>20</td>
<td>16</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>min</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>max</td>
<td>175</td>
<td>121</td>
<td>126</td>
<td>114</td>
<td>133</td>
<td>137</td>
</tr>
</tbody>
</table>

Results: Model-implied yield curves:

Figure 3. Point in time yield curve. $\bigcirc$: observed yield, $\cdots\cdots$: 2-Factor [CIR], $\cdots\cdots$: 2-Factor [RS(2)], $\cdots$: 2-Factor [RS], $\cdots\cdots$: 3-Factor [CIR], and $\square$: 3-Factor [AF].
Implied Regimes and Business Cycle

- 91-95: Almost identical average pricing errors (i.e. data not very informative regarding regime information)
- Suggests: Identify regime 0 [1] with economic recession [expansion]
## Implied Regimes and Business Cycle

Results: Characteristics across Regimes (Basis Points):

<table>
<thead>
<tr>
<th></th>
<th>expansion</th>
<th>reg. 1</th>
<th>recession</th>
<th>reg. 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>short rate level</td>
<td>652</td>
<td>691</td>
<td>936</td>
<td>721</td>
</tr>
<tr>
<td>yield spread</td>
<td>86</td>
<td>83</td>
<td>11</td>
<td>-3</td>
</tr>
<tr>
<td>convexity</td>
<td>-6</td>
<td>-8</td>
<td>0</td>
<td>16</td>
</tr>
</tbody>
</table>

This data also confirms that regime 0 [1] can be interpreted as economic recession [expansion].
Further empirical evidence

Expectation Hypothesis puzzle

- **Approach**: Regression analysis based on Monte-Carlo simulation
- **Result**: Only the RS(2) model can duplicate the violations of the expectations hypothesis at short and long horizons

Reprojected density and conditional second moments

- **Idea**: Characterize dynamics of observed variables conditional on its lags (here: obtain conditional density strictly in terms of observables)
- **Approach**: Estimate reprojected density by relying on simulated data for the yield series from the estimated structural model
- **Result**: RS(2) is the only model accounting for the conditional volatility, the conditional cross-correlation across yields, and conditional higher moments
Conclusion

- Introduction of regime shift term structure model
- Presentation of comprehensive empirical evidence supporting the RS(2) model and illustrating the improvement in the performance compared to traditional term structure models
- Interpretation of the RS(2) model: Identification of regimes with economic expansion/recession, interpretation of the factors