A Theory of Debt Market Illiquidity and Leverage Cyclicality*

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Abstract

We analyze determinants of secondary debt market liquidity, identifying conditions under which a large investor can profitably acquire large stakes and offer ex post efficient debt relief. Secondary debt markets can have multiple equilibria, with anticipation of low sales by small investors inducing the large investor to curtail buying, and vice-versa. Further, debt markets are particularly prone to low-liquidity equilibria during recessions, when small investors face relatively severe adverse selection. Debt sells at a discount in primary markets to compensate investors for adverse selection and illiquidity. In contrast to debt, equity is always fairly-priced and perfectly liquid in our framework, even with private trading by the large investor. Anticipation of low liquidity equilibria in secondary debt markets results in higher yields and expected bankruptcy costs, inducing discrete shifts into equity financing absent any change in economic fundamentals.

1 Introduction

As argued by Shleifer (2003) the sale of debt to a large number of small lenders deters borrowers from strategically requesting debt relief despite having sufficient funds to deliver promised payments. However, it has also been argued that widely-held debt interferes with the provision of debt relief

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for legitimately distressed borrowers. For example, Gilson, John and Lang (1990) find that broad ownership inhibits restructuring corporate debt. Bolton and Jeanne (2007) argue that dispersed ownership makes it more difficult to achieve sovereign debt relief. Finally, a large number of commentators, e.g. Eggert (2007), have pointed to the diffuse ownership of mortgages arising from securitization as a principle factor preventing debt relief for distressed homeowners.\footnote{Piskorski, Seru and Vig (2008) find that loans held on a banks’ balance sheet have lower foreclosure rates.}

The economic costs of the failure to grant debt relief can be significant. For example, Weiss (1990) estimates direct costs of formal bankruptcy for U.S. corporations at 3% of asset value. Posner and Zingales (2009) estimate the deadweight cost due to failed mortgage renegotiations in the US to be $300 billion.

In this paper, we address two closely related questions: If debt relief is efficient, will trading in competitive secondary markets result in an ex post efficient shift to concentrated ownership, facilitating voluntary debt relief by a large bondholder? And if not, why do debt markets freeze? These are not simply academic questions. During the height of the credit crisis, trading volume in debt and debt-linked securities fell to zero. At the same time, policymakers argued that voluntary debt relief was essential to avoiding large deadweight bankruptcy costs. The discussion fell into a theoretical void given the absence of a benchmark model for understanding the liquidity of secondary debt markets and the interplay between those markets and the debt relief process. Worse still, as argued by Spiegel (2008), existing theories of security market liquidity generally predict that debt markets should have higher liquidity (trading volumes) than equity markets since debt has relatively low “informational sensitivity” as traditionally measured.

We show that the puzzle of debt market illiquidity can be understood as arising naturally from the link between feasible debt relief and debt ownership structure. This is because the prospect of a large investor granting debt relief creates the potential for severe mispricing (adverse selection) in secondary debt markets, even when the large debt investor has no informational advantage about underlying parameters.\footnote{In the same way, if a large investor could influence operating cash flow, then he would introduce adverse selection into stock markets.} In turn, adverse selection can lead to a breakdown of trading in debt
markets. However, we show that the prospect of debt relief need not lead to low trading volumes (illiquidity). Rather, secondary debt markets are prone to multiple equilibria with varying degrees of liquidity. In some equilibria there is a virtuous cycle, with high selling volume by small investors leading to a high probability of debt purchases by the large investor, and vice-versa. But there are also inefficient equilibria in which the anticipation of lower selling volume by small investors induces the large investor to buy with lower probability.

To place these results in context, we sketch the broad model details. The model opens with an agent selling debt in a primary market. Following Hart and Moore (1994), the agent has the ability to withdraw essential human capital from the project at an implementation stage. The prospect of strategic default induces dispersed debt ownership in the primary debt market, since the large investor is vulnerable to opportunistic requests for debt relief at the implementation stage.\footnote{Alternatively, one may motivate dispersed ownership by introducing risk aversion and diversification motives.} The focus of the analysis is on equilibrium in the secondary market where the atomistic investors have the option to trade with each other, as well as an anonymous large investor. Importantly, the large investor does not have any private information regarding operating cash flows, nor can he alter operating cash flows through intervention in the firm’s operations. Rather, the large investor shares with all other investors the ability to forgive a portion of any debt due to him, with the aim of avoiding costly formal bankruptcy proceedings.

The model of the secondary debt market is in the spirit of the seminal work by Kyle (1985), making three critical departures. First, we analyze a concave debt claim. Second, there is a feedback effect from ownership structure to fundamental debt value since the former influences incentives for debt relief. Finally, we depart from the pure noise-trader assumption of Kyle and consider the trading incentives of small investors hit by preference shocks that bias them towards selling. The atomistic investors rationally trade-off liquidity preference against adverse selection costs in making their trading decisions. As shown below, each model feature plays an important part in explaining patterns of debt market liquidity.

The model delivers a number of important insights regarding factors conducive to trade in...
secondary debt markets, concentrated ownership, and debt relief. The feasibility of debt relief ex post depends on interim economic conditions. During a recession, the probability of concentrated debt ownership is reduced by two factors. First, the large investor bears high free-riding costs ex post and at the time he places his buy order, since the possibility of debt relief by a large investor is partially capitalized into secondary market prices. Second, small investors are reluctant to sell their debt holdings fearing underpricing, with the magnitude of underpricing being more severe during recessions. In this way, the model can explain the freezing of secondary debt markets at precisely the points in time when trading of debt from small to large investors would have high social value.

In our model, the multiplicity of equilibria is due to the fact that small investors perceive adverse selection as being a non-monotone function of the large investor’s buying probability. Intuitively, if the large investor grants debt relief with probability zero, there is no chance for debt being under-priced in the secondary market. Conversely, if the large investor grants debt relief with probability one, there is also no potential for mispricing since the market makers will then correctly price the debt to reflect the certainty of debt relief.

In the high-liquidity equilibrium, high selling volume by small investors induces the large investor to buy/restructure with a high probability. In turn, the high probability of debt relief alleviates adverse selection and promotes selling by the small investors. Conversely, there are also socially inefficient equilibria in which low anticipated liquidations by small investors result in low buying/restructuring intensity by the large investor. In turn, over some ranges reductions in the probability of the large investor buying debt actually exacerbate the adverse selection problem perceived by small investors, deterring them from selling. In this sense, low liquidity equilibria are self-fulfilling prophecies.

The model also sheds light on corporate financing patterns. After characterizing the set of equilibria in secondary debt markets, we examine the firm’s financing choice in the presence of a tax deduction for interest expense. Our model is based upon the assumption that the large investor has no private information regarding operating cash flow, and cannot influence operating cash flow. This
assumption is a reasonable approximation of many large investors, e.g. pension and mutual funds. It follows that if the firm finances with equity, this security will always be priced at fundamental value, resulting in perfect risk-sharing (high liquidity). In contrast, debt trades at a discount in the primary market to compensate small investors for underpricing in the secondary market, as well as illiquidity.

The model delivers a liquidity-augmented trade-off theory, in which bankruptcy costs are the result of endogenous shifts in debt ownership via secondary market trading. However, in contrast to standard trade-off theory, in our model the multiplicity of equilibria in secondary debt markets results in sharp swings in financing policies absent any change in economic fundamentals. When the primary debt markets anticipate high-liquidity equilibria in the secondary markets, expected bankruptcy costs fall, yields fall, and firms finance with debt. Conversely, anticipation of the low-liquidity equilibria leads to a sharp increase in yields and a dramatic shift away from debt finance ex ante, absent any change in underlying economic parameters. Thus, the model can be viewed as offering a liquidity-based explanation for leverage cycles, one in which large fluctuations in aggregate leverage require nothing more than sunspots inducing agents to anticipate high or low trading volumes in secondary debt markets.

We turn now to related literature. A variant of the free-rider problem at the heart of our model was first analyzed by Grossman and Hart (1980) in the context of hostile takeovers. Closer to our model is that developed by Shleifer and Vishny (1986), who analyze the interplay between free-ridership and the endogenous ownership structure of equity. They show that with public trading, the large investor never finds it profitable to acquire a stake consistent with takeover. Aside from considering debt, our model differs in that preference shocks create the possibility for profitable trading by the large investor.

Our model is also similar to those of Maug (1998) and Mello and Repullo (2004), who develop pure noise-trader models to analyze the incentive to implement value-enhancing takeovers with endogenous equity ownership structure. The key difference between the models is that we analyze
debt, debt relief, and consider endogenous trading decisions by uninformed investors.

Dooley (2000), Shleifer (2003) and Bolton and Jeanne (2007) share our argument that dispersed debt may arise as an ex ante efficient response to borrower moral hazard. Gertner and Scharfstein (1991) analyze the use of exchange offers to overcome free-riding by atomistic lenders. They show a necessary condition for a successful exchange offer is that the new claim have higher priority. We do not explicitly analyze exchange offers, although one can view the default costs in our model as being a reduced-form representation of the costs of making an exchange offer. As argued by Gilson (1991), exchange offers are not costless, although they are less costly for corporations than the Chapter 11 process. Another reason for ruling out exchange offers is that it might be optimal for bond covenants to prohibit them given that exchange offers can also be used to expropriate bondholders.

There is a voluminous literature building upon the pioneering work of Kyle (1985) and Glosten and Milgrom (1985) which analyzes trading in markets occupied by informed and noise-traders. However, most of this literature is concerned with linear equity claims and does not consider the role of adverse selection in deterring trade by uninformed investors. Further, there is no feedback from ownership structure to fundamental value in most models.

With its emphasis on endogenous trading by small investors concerned with adverse selection, the model is similar in spirit to papers by Gorton and Pennachi (1990) and DeMarzo and Duffie (1999). However, in those papers debt is the most liquid monotone security. This is because each of these models features a setting in which some party has private information regarding the distribution of operating cash flows. In contrast, we consider a setting in which no agent has superior information regarding operating cash flows, implying that equity is liquid, while debt becomes illiquid due to potential debt relief by a large investor.

Our paper also contributes to a recent literature that attempts to explain bouts of extreme illiquidity. The liquidity preference in our model can be viewed as a reduced-form representation of the recent margin-constrained investor model of Brunnermeier and Pedersen (2008), for example. A key differentiating factor is that our model predicts that forced liquidations can actually be beneficial
in that an increase in liquidations by small investors serves to promote concentrated ownership and efficient debt relief.

2 The Economic Setting

There are four dates $t \in \{t_0, t_1, t_2, t_3\}$ and two investor classes. There is a large investor $I$ and a continuum of ex ante identical investors $N$ with generic member $n$. All investors are risk-neutral and have access to a riskless asset with interest rate zero. There is a penniless agent $A$, who must finance a project by tapping primary security markets at $t_0$. Optimal financing is discussed in Section 6. For the moment, assume $A$ finances the project with debt having a face value of 1 due at $t_3$. The debt is risky since the project generates a single observable and verifiable cash flow $Y \in \{L, M, H\}$ at $t_3$ where

$$0 < L < M < 1 < H.$$  

In order for the project to actually generate the cash flow $Y$, the agent $A$ must guide the project through an implementation stage at $t_1$. If $A$ quits at this point in time, the project is worthless. Once the implementation stage has been completed, $A$ is no longer necessary and investors need only wait for $Y$ to be realized at $t_3$. Although stylized, this feature of the model captures the idea that a founding manager-entrepreneur is likely to have skills and knowledge that are critical for early-stage success. After that stage, firms can and do often turn to the pool of professional managers.

The ownership structure of debt is common knowledge and $F_j$ denotes the face value of debt outstanding at the start of period $t_j$. At the start of period $t_1$, $A$ has the ability to exploit the fact that his skills are essential for project implementation. In particular, he can make a take-it or leave-it offer to bondholders requesting debt relief: “If the total face value of debt is not voluntarily reduced to $F_1 < 1$, I will quit.” In this way, the model captures the strategic default problem stemming from inalienable human capital as described by Hart and Moore (1994).

The debt claim can be traded in a secondary market at $t_2$. The interim economic state, denoted $\omega \in \{b, g\}$, is common knowledge at the start of $t_2$. If the interim state is bad ($b$) there is a one-half
chance of the project yielding $H$ ("success" below) and a one-half chance of the project yielding $L$ ("failure" below). In contrast, if the interim state is good ($g$) there is a one-half chance of the project yielding $H$ and a one-half chance of the project yielding $M$. One can motivate this simple setup by thinking of the firm as finding it optimal to sell (some) assets if the project fails, with equilibrium asset values being lower during recessions.

At the start of $t_2$, a fraction $\gamma \leq 1/2$ of the investors $N$ (randomly selected) face a transactions cost. The transaction cost is equal to a fraction $c \in (0,1]$ of the gross proceeds from the sale of the financial asset. The transaction cost: is effective at either $t_2$ or $t_3$; is common to all $\gamma$ small investors; is private information to them; and the timing is equally likely. If the small investors face a transaction cost at time $t_3$, they will be labeled impatient since the cost biases them towards selling at $t_2$ rather than holding until $t_3$. Borrowing common terminology, we say that impatient investors face a “liquidity shock.” Conversely, if the small investors face a transaction cost at $t_2$, they will be labeled patient, since they are biased towards holding until maturity. The model subsumes pure noise-trading as a special case if one sets $c = 1$. For $c < 1$, impatient bondholders will only sell if perceived underpricing is not too severe.

Given the possibility of strategic default at the implementation stage, investor $I$ will not buy any debt in the primary market, regardless of price. To see this, suppose to the contrary $I$ buys a fraction $s_0 \in (0,1]$ of the debt at $t_0$. Then $A$ will propose $F_1 = 1 - (s_0 - \varepsilon)$ where $\varepsilon$ is arbitrarily small. For the atomistic investors, it is a weakly dominant strategy to reject requests for debt relief.\footnote{Consequently, the equilibrium in our model differs from that in the takeover model of Bagnoli and Lipman (1988) featuring any-and-all tender offers. In their model, it is optimal for an atom to accept if all other investors reject. In our model, an atom is indifferent between accept and reject if other investors reject, since he gets zero regardless.} However, investor $I$ is pivotal and will forgive $s_0 - \varepsilon$ of his debt. Anticipating such an outcome, $I$ will never buy any debt in the primary market. Thus, the free-rider problem in debt relief makes atomistic lenders tough, which is precisely what is necessary when confronting the threat of strategic default. However, the same free-rider problem also causes atomistic lenders to refuse to grant debt relief even when confronted with the prospect of costly formal bankruptcy proceedings.
In equilibrium, the strategic default problem early in the project’s life causes all the debt to be owned by the atomistic investors $N$ at the start of $t_2$. Our primary focus is on trading in the secondary market. We solve for the perfect Bayesian equilibrium (PBE) of a trading game commencing at $t_2$. The game starts with nature publicly drawing the economic state $\omega$, with small investors then facing private preference (transactions cost) shocks. Given their respective information sets, investors simultaneously submit market orders to a continuum of competitive market makers ($MM$), with no short sales allowed. The aggregate orders of the large investor and atomistic investors are denoted $x^I$ and $x^N$, respectively. Following Kyle (1985), $MM$ observe total order flow $X \equiv x^I + x^N$.

Given that they have no private information whatsoever, there is no incentive for the fraction $1 - \gamma$ of $N$ with zero transactions costs to submit any orders. Thus, these investors hold an aggregate inventory of $1 - \gamma$, which is sufficient to meet any aggregate order flow on the equilibrium path. Thus, in our model the market makers are simply the members of $N$ who face zero transactions costs, since they are willing and able to meet any aggregate demand.

We let $\sigma^*_\omega$ denote the equilibrium probability of investor $I$ buying debt in the secondary market when the interim state is $\omega$. Since the large investor buys debt in order to restructure debt, $\sigma^*_\omega$ is equivalent to the probability of the large investor granting debt relief.

After the secondary market clears, $I$ enters period $t_3$ with an endogenous stake $s$ in the debt claim. At this same point in time, the cash flow $Y$ is publicly observed. If $Y = H$, lenders are paid 1 and the agent $A$ keeps $H - 1$. If $Y \in \{L, M\}$, lenders receive $Y(1 - \alpha)$ if there is no voluntary debt relief, where $\alpha \in (0, 1)$ captures deadweight costs of formal bankruptcy proceedings. Alternatively, the pivotal investor $I$ can voluntarily forgive some debt. If $I$ forgives a sufficient amount of debt such that the resulting face value $F_3 = Y$, formal bankruptcy costs are avoided.\(^5\) The measure zero bondholders would then be paid in full, receiving $1 - s$, leaving investor $I$ to collect $Y - (1 - s)$.

Figure 1 provides a timeline of events.

\(^5\)It is never optimal for the large investor to forgive more debt than needed to avoid default costs.
3 Debt Market Equilibrium

The objective of this section is to characterize the set of PBE for each interim economic state \( \omega \in \{b, g\} \). The only difference between the bad and good interim state is that the former results in a very low cash flow \( L \) if the project fails while the latter results in a higher cash flow of \( M \) if the project fails.

Conveniently, one can express the set of PBE for either intermediate state in terms of that state’s cash flow in the event of project failure. To this end, let \( y_\omega \) denote project cash flow in the event of failure, as a function of the interim state. Thus, \( y_b = L \) and \( y_g = M \). With this notation in hand we solve for the PBE in each interim state via backward induction.

3.1 The final period

Suppose it is time \( t_3 \) and the project has failed so that cash flow is \( y_\omega \), implying debt relief is necessary to avoid formal bankruptcy costs. A large investor holding a debt stake \( s \) is willing to grant debt relief iff

\[
y_\omega - (1 - s) \geq s(1 - \alpha)y_\omega.
\]

(1)

The following Lemma is a useful summary of the implications of the inequality above.

**Lemma 1.** The large investor is willing to grant debt relief at \( t_3 \) iff he holds a stake at least as large as

\[
s(y_\omega, \alpha) \equiv \frac{1 - y_\omega}{1 - y_\omega + \alpha y_\omega}.
\]

(2)

It is readily verified that the minimum stake \( s \) is decreasing in both \( y_\omega \) and \( \alpha \), with

\[
\lim_{\alpha \downarrow 0} s(y_\omega, \alpha) = 1
\]

\[
\lim_{\alpha \uparrow 1} s(y_\omega, \alpha) = 1 - y_\omega.
\]

Lemma 1 tells us that the large investor must have a higher debt stake if he is to grant debt relief during a recession since \( s(L, \alpha) > s(M, \alpha) \). Intuitively, during a recession the large investor
must write-down the face value of his own debt substantially if the firm is to avoid bankruptcy. Thus, the free-riding problem is more severe during recessions.

It is also apparent that the prospect of incurring high default costs encourages debt relief by the large investor. This incentive channel has the potential to mitigate deadweight bankruptcy costs, especially for firms facing the prospect of very costly formal bankruptcy proceedings.

### 3.2 Large investor incentives

Suppose it is the start of $t_2$ and consider an arbitrary interim state $\omega$. With probability one-half, debt relief will be necessary to avoid costly formal bankruptcy proceedings. From Lemma 1 the necessary condition for successful debt relief is that the large investor hold a stake of at least $s(y_\omega, \alpha)$. Under what conditions will $I$ obtain such a stake via secondary market trading? To address this question consider the trading game taking place at time $t_2$.

We are interested in PBE such that debt relief occurs and compute prices consistent with that conjecture. The $\gamma$ investors in $N$ who face preference shocks (transaction costs) are labeled atoms below. We conjecture these investors have a simple trading rule at time $t_2$. In each PBE, atoms sell a block of aggregate size $\gamma^* \in (0, \gamma]$ if impatient, and none sell if patient. Thus, the equilibrium value of $\gamma^*$ determines the volume of trade.

Each atom is a price-taker since no single atom can push aggregate order flow to an off-equilibrium quantity via unilateral deviation. To sustain any conjectured PBE we must verify that each impatient (patient) atom weakly prefers to sell (hold) given the pricing rule for order flows occurring on the equilibrium path. If impatient atoms strictly prefer to sell then in equilibrium $\gamma^* = \gamma$. In contrast, if impatient atoms are indifferent between selling and holding, then it is possible to support PBE in which only a proper subset of them liquidates, with $\gamma^* \in (0, \gamma)$.

In equilibrium, the only way for $I$ to make weakly positive trading gains is to mask his trades by purchasing a block of size $\gamma^*$ whenever he buys debt in the secondary market. Further, he must play a mixed strategy since his trading gains will be negative if he buys with probability one. Let
\(\sigma^*\) denote the equilibrium probability of \(I\) placing a buy order, which is equal to the probability of him granting debt relief at time \(t_3\). Since the large investor has positive measure, he has the ability to move order flow to an off-equilibrium quantity via unilateral deviation. Typically, we support the PBE by having the market makers set prices equal to 1 in response to off-equilibrium path order flows caused by deviations by the large investor. Appendix A contains a discussion of market maker beliefs in response to order flow off the equilibrium path.

Table 1 depicts possible outcomes of the trading game. Only three order flows occur on the equilibrium path, with

\[
X \in \{-\gamma^*, 0, \gamma^*\}. \tag{3}
\]

When there is positive net order flow, the equilibrium is fully revealing and the \(MM\) know the large investor is buying stake \(\gamma^*\). The market makers then set the price (per unit of face value) equal to 1. At the opposite extreme, negative net order flow reveals that the large investor is not buying debt. In this case the fair price of debt is

\[
P_{-\omega}^\omega \equiv \frac{1 + (1 - \alpha)y_\omega}{2}. \tag{4}
\]

Zero net order flow is non-revealing, forcing the market makers to set a pooling price. Using Bayesian updating, the \(MM\) respond to zero net order flow by setting the price:

\[
P_{0\omega}^\omega \equiv \sigma^* + (1 - \sigma^*)P_{-\omega}^\omega. \tag{5}
\]

Consider Table 1 from the perspective of a single impatient atom. Conditional upon the other impatient atoms liquidating, the atom knows that if he too liquidates he will receive a fair price if the large investor is not in the market (bottom row), but faces underpricing if the large investor is in the market (second row) since the fundamental value of debt is then equal to 1. Given this risk of underpricing, an impatient atom will only sell if his preference for liquidity \((c)\) is sufficiently large. Consider next the order flow table from the perspective of a patient atom. Conditional upon the other impatient atoms holding, the atom knows price is equal to fundamental value in the top row, but that debt is overpriced in the penultimate row. This overpricing biases him towards selling
today, but he still prefers to hold his debt if facing a sufficiently high transaction cost today. Finally, consider the order flow table from the perspective of the large investor. He makes a trading loss in the top row, since he buys debt for 1 despite his valuing the debt at less than this amount due to his making debt concessions at $t_3$. Conversely, the large investor makes a trading gain in the second row when he buys at the pooling price.

Since $I$ obtains a debt stake of $\gamma^* \leq \gamma$ when he buys debt, a necessary condition for debt relief is:

$$\gamma \geq z(y_\omega, \alpha).$$

Intuitively, the large investor relies upon broad liquidity shocks to mask his buy orders. If the liquidity shocks only hit a small fraction of atomistic bondholders, the large investor cannot acquire a stake sufficiently large such that he will be willing to grant debt relief at time $t_3$.

We first conjecture a PBE in which all impatient atoms sell, with $\gamma^* = \gamma$. In order to confirm that the resulting trading outcomes are indeed a PBE we must verify that: the large investor is playing an optimal strategy; impatient atoms weakly prefer to sell; and patient atoms prefer to hold.

Let $G(\sigma, \tilde{\gamma}, y_\omega, \alpha)$ denote the expected trading gain perceived by the large investor in the event that with probability $\sigma$ he places a buy order of size $\tilde{\gamma}$, where $\tilde{\gamma}$ is the measure of liquidating impatient atoms, and $y_\omega$ measures the cash flow if the project fails. The gain is equal to his expected payoff at time $t_3$ net of the expected price paid to acquire the stake $\tilde{\gamma}$:

$$G(\sigma, \tilde{\gamma}, y_\omega, \alpha) = \frac{1}{2} \tilde{\gamma} + \frac{1}{2} [y_\omega - (1 - \tilde{\gamma})] - \frac{\tilde{\gamma}}{2} (1 + P_{\omega}^0)$$

$$= \frac{1}{2} \left[ \tilde{\gamma}(1 - \sigma)(1 - y_\omega + \alpha y_\omega) - (1 - y_\omega) \right].$$

Differentiating $G$ one finds

$$G_\sigma = -\frac{\tilde{\gamma}(1 - y_\omega + \alpha y_\omega)}{4} < 0$$

$$G_{\tilde{\gamma}} = \frac{(1 - \sigma)(1 - y_\omega + \alpha y_\omega)}{4} > 0$$

$$G_{y_\omega} = 1 - \frac{\tilde{\gamma}(1 - \sigma)(1 - \alpha)}{2} > 0$$

$$G_\alpha = \frac{\tilde{\gamma}(1 - \sigma)y_\omega}{4} > 0.$$
The intuition for each comparative static in (8) is as follows. First, the expected gain to buying is decreasing in the buying intensity \( \sigma \) since a higher \( \sigma \) results in a higher pooling price \( P_0^\omega \). Second, we see that higher selling volume by small investors serves to increase the gain to buying. Intuitively, higher selling by small investors serves to increase the equilibrium stake of the large investor and reduce free-riding costs. Third, high cash flow in the event of project failure raises the gain to buying, since higher cash flow reduces the size of the required debt write-down. Finally, consider the effect of bankruptcy costs on large investor incentives. Returning to Table 1, one can see that the large investor only makes profits when order flow is not fully revealing \( (X = 0) \) and the market makers set the pooling price of \( P_0^\omega \). This pooling price is lower, and trading gains for the large investor are higher, when bankruptcy costs are high.

Since the function \( G \) is strictly decreasing in its first argument and increasing in its second argument, a necessary condition for the large investor to enter the secondary market is that \( G \) be strictly positive in the limit as \( \sigma \) converges to zero for \( \hat{\gamma} = \gamma \). From this one obtains the following necessary condition for the large investor to enter the secondary debt market with positive probability

\[
\gamma > \gamma_\omega \equiv \frac{2(1 - y_\omega)}{1 - y_\omega + \alpha y_\omega} = 2s(y_\omega, \alpha).
\]  

Once again, we see that broad liquidity shocks (high \( \gamma \)) are necessary for debt relief. In fact, the necessary condition for large investor entry into the secondary debt market at time \( t_2 \) (specified in condition (9)) is twice as stringent as the necessary condition for voluntary debt relief at \( t_3 \) (condition (6)). The intuition is as follows. The latter condition simply ensures that a large investor standing at time \( t_3 \) would be willing to grant debt relief, despite the free-riding costs he would bear at that point in time. The former condition ensures that the large investor can cover all free-riding costs, only a portion of which is incurred at time \( t_3 \), with additional free-riding costs capitalized into debt prices at time \( t_2 \).

If \( \gamma \leq \gamma_\omega \), the large investor cannot possibly make non-negative trading gains from implementing the buy/relief strategy and the only possible PBE entails \( \sigma_\omega^* = 0 \) and there is zero probability of
debt relief. Since there is no “informed” trading in this case, the patient atoms always hold and the impatient atoms always liquidate, with \(\gamma^* = \gamma\). The market makers then take the opposite side of the trade.

The rest of this subsection considers the remaining case where \(\gamma > \gamma^*\). In this case, there is a unique \(\bar{\sigma}_\omega \in (0, 1)\) satisfying:

\[
G(\bar{\sigma}, \gamma, y_\omega, \alpha) = 0. \tag{10}
\]

In particular,

\[
\bar{\sigma}_\omega = 1 - \frac{2(1 - y_\omega)}{\gamma(1 - y_\omega + \alpha y_\omega)} = 1 - \frac{\gamma_\omega}{\gamma} \in (0, 1). \tag{11}
\]

The variable \(\bar{\sigma}_\omega\) measures the buying intensity of the large investor provisional upon him actually facing maximum debt market liquidity, with all impatient atoms liquidating \((\gamma^* = \gamma)\). For this reason, we refer to \(\bar{\sigma}_\omega\) as the large investor’s provisional trading intensity.

Differentiating equation (11) reveals:

\[
\frac{\partial \bar{\sigma}_\omega}{\partial \gamma} = \frac{1 - \bar{\sigma}_\omega}{\gamma} > 0 \tag{12}
\]

\[
\frac{\partial \bar{\sigma}_\omega}{\partial \alpha} = \frac{(1 - \bar{\sigma}_\omega)y}{1 - y + \alpha y} > 0
\]

\[
\frac{\partial \bar{\sigma}_\omega}{\partial y_\omega} = \frac{2}{\gamma(1 - y + \alpha y)} \left(1 - \frac{\gamma(1 - \bar{\sigma}_\omega)(1 - \alpha)}{2}\right) > 0.
\]

The intuition for these comparative statics is identical to that provided for those in (8).

Figure 2 plots the large investor’s provisional trading intensities \((\bar{\sigma}_b, \bar{\sigma}_g)\) as functions of underlying parameters. Panel A shows the effect of broader liquidity shocks, as measured by \(\gamma\). If the breadth of liquidity shocks is too narrow, the large investor does not enter the secondary debt market. With sufficient breadth, he does enter, and his provisional trading intensity then increases monotonically in \(\gamma\). If the interim state is \(b\), the large investor buys with lower probability, since he must undertake a larger debt write-down if the project fails.

Panel B of Figure 2 shows the effect of bankruptcy costs on the large investor’s provisional trading intensity. As was the case in Panel A, we see that the large investor trades with higher intensity if the interim state is \(g\). Further, the large investor can buy at a lower pooling price \(P^0\).
if bankruptcy costs are high, so his provisional trading intensity is increasing in $\alpha$. Thus, enhanced trading by the large investor has the potential to mitigate the costs of formal bankruptcy. For example, as $\alpha$ goes to one, the large investor buys debt and provides debt relief with probability $9/10$ if the interim state is $g$. Thus, in this case there is only a 10% probability of the firm incurring formal bankruptcy costs.

### 3.3 Small investor incentives

There exists a PBE with $\sigma^*_\omega = \bar{\sigma}_\omega$ iff patient atoms prefer to hold their debt and impatient atoms prefer to sell their debt. To evaluate the viability of such an equilibrium we now analyze the trading incentives of individual atoms. An individual patient atom strictly prefers to hold his debt if the expected proceeds from selling at $t_2$, net of transactions cost, is less than the expected proceeds from selling at $t_3$. Taking into account his ability to condition his expectation on the fact that the other atoms are not selling, an individual patient atom will also choose not to sell if:

$$
(1 - c)[\sigma + (1 - \sigma)P^0_\omega] < P^0_\omega \iff c > \xi(\sigma, P^-_\omega) \equiv \frac{\sigma(1 - \sigma)(1 - P^-_\omega)}{P^-_\omega + \sigma(2 - \sigma)(1 - P^-_\omega)}.
$$

(13)

From inspection of the order flow table it is clear that the trading decision of a patient atom reflects competing forces. On one hand, the transaction cost at $t_2$ encourages him to defer selling until time $t_3$. However, each patient atom knows the bond is overpriced on average. Equation (13) provides a sufficient condition such that the transactions cost dominates.

Consider next the trading decision of an individual impatient atom. Such an investor will liquidate if the expected value captured by selling today is larger than the payoff to unilaterally deviating by holding onto the debt and facing the proportional cost $c$ at $t_3$. Given his conditioning information that other impatient atoms are selling, an individual impatient atom weakly prefers to sell if:

$$
\sigma P^0_\omega + (1 - \sigma)P^-_\omega \geq (1 - c)P^0_\omega \iff c \geq \xi(\sigma, P^-_\omega) \equiv \frac{\sigma(1 - \sigma)(1 - P^-_\omega)}{P^-_\omega + \sigma(1 - P^-_\omega)}.
$$

(14)

It is readily verified that

$$
\xi(\sigma, P^-_\omega) < \xi(\sigma, P^-_\omega) \quad \forall \quad \sigma \in (0, 1).
$$
This leads us to a useful lemma.

**Lemma 2.** Impatient atoms weakly prefer to liquidate at $t_2$ if $c \geq \zeta(\sigma, P^-)$. If $(c, P^-, \sigma)$ are such that impatient atoms weakly prefer to liquidate at $t_2$, then patient atoms strictly prefer not to sell at $t_2$.

Based upon Lemma 2 for the remainder of the analysis we shall only check that impatient atoms weakly prefer to liquidate at a conjectured PBE, since the patient atoms will strictly prefer not selling, consistent with the conjectured order flow scenarios in Table 1.

In our model, impatient atoms face a special form of adverse selection. Although the large investor has no private information regarding the firm’s operating cash flow $Y$, or any other parameter, knowledge of his own trading decision gives him an advantage over the small bondholders. The function $\zeta$ captures the degree of adverse selection facing impatient atoms. This gives rise to a simple trading rule: sell iff the liquidity preference, as captured by the parameter $c$, is larger than the cost of adverse selection $\zeta$.

Importantly, the impatient atoms are less willing to sell if the interim state is $b$, since

$$\zeta(\sigma, P^-_b) > \zeta(\sigma, P^-_g) \forall \sigma \in (0, 1).$$

The intuition for (15) is as follows. The impatient atoms face underpricing of their debt in the second row, when the market makers set the pooling price $P^0_\omega$ despite the fundamental value of the debt being equal to 1. The gap between the fundamental value and the pooling price is larger in the state $b$ since $P^0_b < P^0_g$.

Adverse selection costs are non-monotone in the trading intensity of the large investor. To see this, note that impatient atoms are always willing to sell for limiting values of $\sigma$ since

$$\lim_{\sigma \downarrow 0} \zeta(\sigma, P^-_\omega) = \lim_{\sigma \uparrow 1} \zeta(\sigma, P^-) = 0.$$

Intuitively, impatient atoms are reluctant to sell due to fear of selling at too low a price. In particular, price is below fundamental value in the second row of Table 1 in the sense that a hold-out atom
would capture a payoff of 1 if he deviated, which is greater than the pooling price $P_0^\omega$. If $\sigma = 0$, there is zero possibility of reaching that row. If $\sigma = 1$, the second row can be reached, but the market makers then set price equal to the fundamental value of 1. That is, high values of $\sigma$ do not entail large mispricing risk for the small bondholders because the market makers will set a high debt price in response to high $\sigma$.

Intuitively, the adverse selection problem as perceived by small investors is most severe for intermediate values of $\sigma$, where market makers are very much confounded when they see an order flow of zero. Consistent with this intuition, the function $c(\cdot, P_\omega^-)$ reaches a unique maximum at an interior point denoted $\sigma_{\max}^\omega$, where

$$
\sigma_{\max}^\omega(P_\omega^-) \equiv \arg\max_{\sigma} c(\sigma, P_\omega^-) \tag{16}
$$

$$
\Rightarrow \sigma_{\max}^\omega(P_\omega^-) = \frac{\sqrt{P_\omega^- - P_\omega^-}}{1 - P_\omega^-} \in (0, 1)
$$

$$
\Rightarrow c(\sigma_{\max}^\omega, P_\omega^-) = \frac{(1 - \sqrt{P_\omega^-})^2}{1 - P_\omega^-} \tag{17}
$$

Figure 3 plots the functions $c(\cdot, P_b^-)$ and $c(\cdot, P_g^-)$, with the three panels considering alternative levels for the impatience parameter $c$. Consistent with the inequality in equation (15), in each figure the impatient atoms are less willing to sell if the interim state is $b$, reflecting the greater potential for debt underpricing in that state. Panel A considers a case where the liquidity preference parameter $c$ is high with $c > c(\sigma_{\max}^b, P_b^-)$. In this case, each impatient atom strictly prefers to sell regardless of the trading intensity of the large investor, and regardless of the interim economic state. Panel B depicts an intermediate value for the liquidity preference parameter. In that panel, impatient atoms strictly prefer to sell if the interim state is $g$, but may prefer no-trade if the interim state is $b$. Panel C depicts low values of the liquidity preference parameter, such that the impatient investors may opt for no-trade regardless of the interim state.
3.4 The equilibrium set

Having discussed the large investor’s trading incentives and the impatient atoms’ liquidation incentives, we can pin down the set of PBE for an arbitrary interim state $\omega \in \{b, g\}$. The critical variables determining the nature and efficiency of any PBE are the pair $(\sigma^*, \gamma^*)$ with the first element measuring the probability of the large investor placing a buy order (and granting debt relief) and the second representing the measure of impatient atoms selling. Since the large investor places a buy order of size $\gamma^*$ to mask his trades, one can properly interpret $\gamma^*$ as measuring trading volume.

Recall that if the liquidity shocks are too narrow, with $\gamma \leq \gamma_\omega$, the large investor cannot possibly make non-negative trading gains from implementing the buy/relief strategy and the only possible PBE entails $\sigma_\omega^* = 0$. Since there is no informed trading in this case, the patient atoms always hold and the impatient atoms always liquidate, with $\gamma_\omega^* = \gamma$. The market makers then take the opposite side of orders and set the price equal to $P_\omega^-$ for all order flows occurring on the equilibrium path, with $X \in \{-\gamma, 0\}$ in equilibrium.

The rest of this subsection considers the remaining case where $\gamma > \gamma_\omega$. To understand the nature of the construction, first note that in any PBE the large investor plays a mixed strategy, so the following indifference condition is always satisfied:

$$G(\sigma^*_\omega, \gamma^*_\omega, y_\omega, \alpha) = 0.$$  \hspace{1cm} (18)

Further, there are two possibilities regarding the actions of the impatient atoms. First, they may strictly prefer to liquidate, in which case $\gamma_\omega^* = \gamma$. Alternatively, it may be possible to support PBE in which only a proper subset of the impatient atoms liquidate, but in order for this to be the case each impatient atom must be just indifferent between selling and not selling. More formally, we have:

$$c(\sigma^*, P^-_\omega) > c \Rightarrow \gamma^* = \gamma$$  \hspace{1cm} (19)

and

$$\gamma^* < \gamma \Rightarrow c(\sigma^*, P^-_\omega) = c.$$
From here the construction is most easily followed by referring back to Figure 3. Consider Panel A where \( c > c(\sigma_{w}^{\max}, P_{w}^{-}) \). In this case, impatient atoms strictly prefer to sell regardless of the buying intensity of the large investor. It follows that \( \gamma^{*} = \gamma \) in any PBE. Further, only the provisional trading intensity \( \tilde{\sigma} \), as defined in equation (11), satisfies the large investor’s indifference condition (18) when he anticipates all impatient atoms selling. Thus, the unique PBE consists of \((\sigma_{w}^{*}, \gamma_{w}^{*}) = (\tilde{\sigma}_{w}, \gamma)\).

Consider next an arbitrary case where \( c < c(\sigma_{w}^{\max}, P_{w}^{-}) \). As shown in Figure 3, in such cases there is a pair \((\sigma_{w}^{1}, \sigma_{w}^{2})\) at which each impatient atom is just indifferent between selling and holding. These points of indifference are defined by the equation \( c(\sigma, P_{w}^{-}) = c \). Solving this equation we find

\[
\begin{align*}
\sigma_{w}^{1} &= \frac{1 - c - \sqrt{(1 - c)^2 - 4cP^{-}/(1 - P^{-})}}{2} \\
\sigma_{w}^{2} &= \frac{1 - c + \sqrt{(1 - c)^2 - 4cP^{-}/(1 - P^{-})}}{2}
\end{align*}
\]

It is readily verified that \( \tilde{\sigma}_{w} \in (0, \sigma_{w}^{1}) \Rightarrow (\sigma_{w}^{*}, \gamma_{w}^{*}) = (\tilde{\sigma}_{w}, \gamma) \) \( (21) \) And further, the PBE is unique. The uniqueness of equilibrium under the maintained condition follows from the following argument. Any decrease in \( \sigma \) would result in \( G > 0 \), and the only way to restore \( G = 0 \) would be to reduce the measure of atoms liquidating. But for such \( \sigma \) all impatient atoms strictly prefer to sell. Conversely, an increase in \( \sigma \) is also not possible in a PBE since this would result in \( G < 0 \), with further increases in the measure of liquidating atoms being impossible.

It is also readily verified that

\[
\begin{align*}
\sigma_{w} \in (\sigma_{w}^{1}, \sigma_{w}^{2}) \Rightarrow (\sigma_{w}, \gamma_{w}) = (\sigma_{w}^{*}, \gamma_{w}^{*}) \\
\gamma_{w}^{1} &= \left( \frac{1 - \tilde{\sigma}_{w}}{1 - \sigma_{w}^{1}} \right) \gamma < \gamma.
\end{align*}
\]

This PBE is also unique, with the reasoning being as follows. Under the maintained condition it is clear that \( \tilde{\sigma}_{w} \) cannot occur in a PBE because the impatient atoms are unwilling to sell. Clearly, one can induce the impatient atoms to sell with \( \sigma \in (0, \sigma_{w}^{1}] \). However, \( \sigma < \sigma_{w}^{1} \) cannot be a PBE in
the present case since then all impatient atoms strictly prefer to liquidate and one obtains $G > 0$. In contrast, it is possible to maintain a PBE with $\sigma^* = \sigma_1^\omega$ in which a proper subset of the atoms liquidate. By construction $\gamma_1^\omega$ maintains the large investor’s indifference condition given $\sigma^* = \sigma_1^\omega$. Finally, under the maintained condition one cannot support a PBE at $\sigma > \sigma_1^\omega$ since any such $\sigma$ would result in $G < 0$.

The equilibrium described in (22) well illustrates one of our central messages: Small investor concern over adverse selection can significantly reduce trading volumes and the probability of debt relief. To see this, note that a model with pure noise-trading ($c = 1$) would predict that the unique equilibrium in the present example is $(\sigma_\omega, \gamma)$. However, in our model adverse selection induces the impatient atoms to sell a smaller block of size $\gamma_1^\omega < \gamma$. In turn, the decline in liquidity induces the large investor to curtail the intensity of his buying.

The PBE is not necessarily unique. For example:

$$\sigma_\omega \in (\sigma_2^\omega, 1) \Rightarrow (\sigma_\omega^*, \gamma_\omega^*) \in \{(\sigma_\omega, \gamma), (\sigma_2^\omega, \gamma_2^\omega), (\sigma_1^\omega, \gamma_1^\omega)\}$$

$$\gamma_2^\omega \equiv \left(\frac{1 - \sigma_\omega}{1 - \sigma_2^\omega}\right) \gamma \in (\gamma_1^\omega, \gamma).$$

The pair $(\sigma_\omega, \gamma)$ Pareto-dominates the other PBE, with anticipation of high liquidity ($\gamma_\omega^* = \gamma$) inducing the large investor to buy/restructure with high probability. However, the large investor’s indifference condition $G = 0$ can be maintained at lower $\sigma$ values by reducing the measure of impatient atoms liquidating. Since only a proper subset of atoms sell, it must be the case that each atom is indifferent between holding and liquidating, which is only possible for $\sigma \in \{\sigma_1^\omega, \sigma_2^\omega\}$. Thus, there are only two points at which indifference can be maintained for both the large investor and impatient atoms: $(\sigma_2^\omega, \gamma_2^\omega)$ and $(\sigma_1^\omega, \gamma_1^\omega)$.

The less efficient equilibria at $(\sigma_2^\omega, \gamma_2^\omega)$ and $(\sigma_1^\omega, \gamma_1^\omega)$ are consistent with the casual intuition that illiquidity is a self-fulfilling prophecy. At each of these pairs, anticipation of low liquidity induces the large investor to reduce his buying/restructuring intensity. In turn, the reduction in his trading intensity from $\sigma_\omega$ to $\sigma \in \{\sigma_1^\omega, \sigma_2^\omega\}$ is sufficient to tilt the impatient atoms from a strict preference for liquidating to indifference.
The only remaining case to consider is \( \bar{\sigma}_\omega = \sigma^*_2 \). In this case, one PBE consists of \((\sigma^*_2, \gamma)\). However, another equilibrium can be supported at \((\sigma^*_1, \gamma^*_1)\).

Proposition 1 summarizes: 6

**Proposition 1 [Equilibrium].** If \( \gamma \leq \gamma_\omega \), then \( (\sigma^*_\omega, \gamma^*_\omega) = (0, \gamma) \). If \( \gamma > \gamma_\omega \) and \( c > c(\sigma^\text{max}_\omega, P^-) \), then \( (\sigma^*_\omega, \gamma^*_\omega) = (\bar{\sigma}_\omega, \gamma) \). If \( \gamma > \gamma_\omega \) and \( c < c(\sigma^\text{max}_\omega, P^-) \), then

\[
\bar{\sigma}_\omega \in (0, \sigma^*_1] \Rightarrow (\sigma^*_\omega, \gamma^*_\omega) = (\bar{\sigma}_\omega, \gamma)
\]

\[
\bar{\sigma}_\omega \in (\sigma^*_1, \sigma^*_2) \Rightarrow (\sigma^*_\omega, \gamma^*_\omega) = (\sigma^*_1, \gamma^*_1)
\]

\[
\bar{\sigma}_\omega = \sigma^*_2 \Rightarrow (\sigma^*_\omega, \gamma^*_\omega) \in \{(\sigma^*_2, \gamma^*_2), (\sigma^*_1, \gamma^*_1)\}
\]

and

\[
\bar{\sigma}_\omega > \sigma^*_2 \Rightarrow (\sigma^*_\omega, \gamma^*_\omega) \in \{(\bar{\sigma}_\omega, \gamma), (\sigma^*_2, \gamma^*_2), (\sigma^*_1, \gamma^*_1)\}.
\]

4 **Bond Market Liquidity and Macroeconomic Conditions**

Proposition 1 describes the set of PBE for an arbitrary interim economic state \( \omega \in \{b, g\} \). We interpret the former state as a recession and the latter state as an expansion. During the credit crisis of 2007/2008 it appeared that debt markets became illiquid at precisely the time when the macroeconomy weakened. This section examines the ability of the model to explain the apparent positive relationship between the macroeconomy and debt market liquidity. This section adopts the working assumption that \( \gamma > \gamma_b \) which ensures that the large investor would always be willing to enter the secondary debt market if he knew that all impatient atoms would in fact liquidate. In the interest of brevity, this section discusses two liquidity demand scenarios, high and medium, which correspond to the first two panels in Figure 3. The remaining scenario is discussed in Appendix B.

---

6If \( c \) is just tangent to \( c \) then \( \bar{\sigma} \) is always a PBE. Further, if \( \bar{\sigma} > \sigma^\text{max}_\omega \) then the tangency point \( \sigma^\text{max}_\omega \) can also be supported as a PBE.
4.1 High liquidity demand

Suppose first that $c$ is

$$ High : c > c(\sigma^\text{max}_b, P^-_b). $$

This case is depicted in Panel A of Figure 3. For such high values of $c$ the impatient atoms’ preference for liquidity is so strong that each strictly prefers to sell. From Proposition 1 it follows that in the scenario under consideration

$$ \sigma^*_g = \tilde{\sigma}_g > \tilde{\sigma}_b = \sigma^*_b. $$

Thus, the model predicts that if the liquidity preference is high, there will be a unique $\omega$-contingent PBE. Further, the probability of the large investor granting debt relief is higher during expansions than recessions. Intuitively, when the liquidity preference is high, the equilibrium trading intensity is determined exclusively by the incentives of the large investor. During an expansion, the large investor has a stronger incentive to enter the secondary debt market since the free-riding problem is less severe. In particular, the large investor realizes that during an expansion he will only need to undertake a relatively small write-down on his debt should the project fail.

In part, this causal mechanism may explain the apparent absence of large investors willing to invest in distressed debts during the recent credit crunch. However, we argue that this particular scenario, with high $c$, misses an important stylized fact: low trading volumes in debt markets. To see this note that when $c$ is high, all impatient atoms strictly prefer to sell, with the model predicting high debt trading volume regardless of the state of the macroeconomy: $\gamma^*_b = \gamma^*_g = \gamma$. For this reason, our favored model scenario entails medium levels of liquidity preference as considered in the next subsection.

4.2 Medium liquidity demand

Consider next the case depicted in Panel B of Figure 3, in which $c$ is

$$ Medium : c(\sigma^\text{max}_g, P^-_g) < c < c(\sigma^\text{max}_b, P^-_b). $$

23
In this particular scenario, each impatient atom always prefers to sell during the expansion, but may not be willing to sell during the recession given high potential mispricing. From Proposition 1 we know that:

\[ \sigma_g^* = \bar{\sigma}_g > \sigma_b^* \in \{\bar{\sigma}_b, \sigma_2^b, \sigma_1^b\} \]

\[ \gamma_g^* = \gamma \geq \gamma_b^* \in \{\gamma, \gamma_2^b, \gamma_1^b\}. \]

Thus, with intermediate liquidity demand we see that during an expansion the large investor trades with higher intensity and trading volume is weakly higher. Intuitively, the model predicts that the volume of trade in debt markets should fall during recessions because the adverse selection problem as perceived by small investors becomes more severe as the economy cools. In turn, the decline in trade by small uninformed investors reduces the incentive of a large investor to enter the debt market.

Let us now consider a particularly salient example regarding the state-contingency of debt market liquidity. Suppose \( \bar{\sigma}_b > \sigma_2^b \). In this case, we know that during the recession there is an equilibrium in which the large investor trades with very low intensity (\( \sigma_1^b \)) relative to his equilibrium trading intensity during an expansion (\( \bar{\sigma}_g > \bar{\sigma}_b > \sigma_2^b > \sigma_1^b \)). Further, in that same recessionary equilibrium, trading volume falls to \( \gamma_1^b \), which is significantly lower than trading volume during the expansion (\( \gamma \)).

Thus, multiple equilibria in debt markets, resulting from the adverse selection problem perceived by small uninformed bondholders, can explain large declines in trading volumes as the economy cools. It is here worth emphasizing that a pure noise-trading model (\( c = 1 \)) would not generate such a prediction since in such models impatient investors always sell, failing to capture the macro-contingent liquidity of debt markets.
5 Primary Market Debt Pricing

This section derives the price of debt when it is sold at time $t_0$ and explores how it is affected by expected conditions in the secondary debt market. Recall that the threat of strategic default at $t_1$ causes the large investor to boycott the primary debt market at $t_0$. Thus, the bond is sold to atomistic investors in the primary market. These investors set the price so that they are just indifferent between the riskless asset and purchasing the risky bond.

Let $v_\omega$ denote the value that a small bondholder attaches to owning a bond at the start of period $t_2$ after observing the aggregate economic state $\omega$ but before he knows whether he is vulnerable to any transaction cost (preference shock). With probability $(1 - \gamma/2)$ he will not be impatient, and so values the security at fundamental value. With probability $\gamma/2$ he will be impatient and weakly prefers to sell. When an impatient atom is indifferent regarding selling, we may without loss of generality compute his payoff as if he were one of the sellers since in this case the adverse selection discount borne by the selling atoms is just equal to the illiquidity discount ($t_3$ transaction cost) borne by impatient atoms who do not sell. The resulting expected value is

$$v_\omega = \left(1 - \frac{\gamma}{2}\right)\left[\sigma^*_\omega + (1 - \sigma^*_\omega)P^-\right] + \frac{\gamma}{2}\left[\sigma^*_\omega P^0 + (1 - \sigma^*_\omega)P^-\right] \quad (24)$$

The second line in equation (24) provides an intuitive expression for the value a small bondholder attaches to his ownership rights. The term in square brackets is equal to the fundamental value of the bond, accounting for the fact that the bond will be rendered safe with probability $\sigma^*_\omega$. The last term captures the second row in Table 1, where small bondholders are exposed adverse selection. As shown there, with probability $\sigma^*_\omega/2$ impatient bondholders sell their debt at the pooling price of $P^0$ which is less than the fundamental value of 1. When the PBE is such that only a proper subset of impatient atoms liquidate, the loss due to adverse selection borne by those who do sell is just equal to the illiquidity loss borne by the impatient atoms who do not sell. Thus, in our model illiquidity discounts and adverse selections discounts are equal.
Since the interim states $b$ and $g$ are equally likely and the discount rate is zero, the debt will sell in the primary market at price

$$V = \frac{1}{2}(v_b + v_g).$$

(25)

In the remainder of this section we present quantitative illustrations of the model, focusing on the determinants of the primary market debt price $V$. Recall that Proposition 1 provides analytical characterizations of all possible PBE so quantitative solutions are easily obtained. We first compute $(\sigma^*_b, \sigma^*_g)$ using Proposition 1. The primary debt market value is then computed by substituting the large investor’s buying intensities into equation (24).

Figure 4A shows how debt value varies with the breadth of the preference shocks ($\gamma$) and with the particular PBE selected. The numerical example assumes $M = .9688$, $L = .9375$, $\alpha = .70$, and $c = .1091$. The parameter $c$ was chosen such that the impatient atoms will always sell if the state is $g$ but may not sell if the state is $b$. Panel B of Figure 3 captures this scenario schematically. Panel B of Figure 4 shows the buying intensity of the large investor, while Panel C plots the fraction of impatient atoms selling their debt.

For low $\gamma$ values the large investor does not enter the secondary debt market and the debt trades at a deeply discounted price of 0.643. As $\gamma$ increases, the large investor increases his buying intensity in a weakly monotone fashion, reflecting the fact that his trading profits are increasing in the measure of impatient atoms selling. As shown in Panel B of Figure 4, once the large investor enters the market at $\gamma = .088$, his buying intensity in the good state $\sigma^*_g$ increases in a strictly monotone fashion. This is because in the present case, with intermediate liquidity demand $c$, all impatient atoms sell in the good state and the large investor ends up buying a block of corresponding size $\gamma$. Therefore, each increase in $\gamma$ reduces the severity of the free-rider problem and encourages the large investor to buy debt.

Continuing to focus on Panel B, we see that if the interim economic state is $b$, the large investor will only enter the secondary debt market if $\gamma$ exceeds 0.17. In contrast to the state $g$, the trading intensity $\sigma^*_b$ does not increase in a strictly monotone fashion once this entry threshold has been
exceeded. This reflects the fact that the small investors may not be willing to sell at the large investor’s provisional trading intensity $\bar{\sigma}_b$. For $\gamma \in [.285, .35]$, $\bar{\sigma}_b \in (\sigma_1^b, \sigma_2^b)$ and the unique PBE entails $\sigma_\ast^b = \sigma_1^b$. For all $\gamma \in (.35, .50]$, $\bar{\sigma}_b > \sigma_2^b$ and Proposition 1 indicates there are three possible equilibria with $\sigma_\ast^b \in \{\bar{\sigma}_b, \sigma_2^b, \sigma_1^b\}$. As shown in Panel C of Figure 4, the trading intensities $(\sigma_2^b, \sigma_1^b)$ arise when the large investor (rationally) anticipates low selling volumes by atomistic investors.

Returning to Panel A of Figure 4, one sees that for high $\gamma$ values the primary market value of debt hinges upon the nature of the conjectured secondary market equilibrium. For example, when $\gamma = .50$, debt sells for 0.89 if the market conjectures the high liquidity PBE and trades for only 0.84 if the market conjectures the low liquidity PBE. The respective debt yields are 12.4% versus 18.9% in these two PBE. In this way, the model predicts that debt prices and yields can jump by large amounts without any change in economic fundamentals (i.e. holding all parameters fixed).

The three panels of Figure 5 analyze the effect of changes in formal bankruptcy costs ($\alpha$) on debt price, the large investor’s trading intensity, and the propensity of small investors to sell. The numerical example assumes $M = .975$, $L = .50$, $\gamma = .50$, and $c = .06$. These numerical values imply that the large investor never enters the debt market if the state is $b$, allowing us to confine attention to the PBE for state $g$.

As shown in Panel A of Figure 5, the sensitivity of debt price to bankruptcy cost depends on how the change in bankruptcy cost influences the trading intensity of the large investor. Recall, increases in bankruptcy costs tend to stimulate debt purchases by the large investor since this results in a lower pooled debt price $P^d$. Thus, endogenous increases in debt purchases by the large investor mitigate the effect of bankruptcy costs on debt value. For example, when attention is confined to the best PBE at each point, debt value is convex in bankruptcy cost.

Panel A of Figure 5 shows once again that the primary market price of debt is highly sensitive to which secondary market PBE is conjectured. For example, if $\alpha = 1$ then debt price is 0.7201 in the best PBE and only 0.5132 in the worst PBE. The respective bond yields are 39% and 95%. What explains this sharp difference in prices? Panel B of Figure 5 provides the intuition. Once
α reaches 0.40, \( \tilde{\sigma}_g > \sigma_g^2 \) and Proposition 1 tells us that \( \sigma^*_g \in \{ \tilde{\sigma}_g, \sigma_g^2, \sigma_g^1 \} \). Of course, increases in \( \alpha \) result in higher values of \( \tilde{\sigma}_g \), which explains why the probability of debt relief increases in the best PBE. Now note that increases in \( \alpha \) increase the informational sensitivity of debt in the sense of raising the schedule \( \xi \). Thus, increases in \( \alpha \) on the present region cause \( \sigma_g^2 \) to increase, while \( \sigma_g^1 \) actually decreases. Consequently, the worst PBE gets progressively worse with increases in \( \alpha \).

Examination of Panels B and C reveals the symbiotic relationship between the large investor’s trading intensity and anticipated selling by small investors. When anticipated liquidations drop, so too does the trading intensity of the large bondholder. In turn, low trading intensity by the large investor can induce low trading volumes since, as shown in Figure 3, the small investors perceive that adverse selection is non-monotone in \( \sigma \).

6 Debt versus Equity Finance

We now close the model by examining the agent’s financing decision at date \( t_0 \). For now, assume the fixed cost of issuing any given security are sufficiently large such that \( A \) will only issue one security in order to fund his project. Assume also there is a tax advantage of debt due to the existence of a linear corporate income tax at rate \( \tau > 0 \), with no loss-limitations. Finally, in order to avoid re-solving the model for each different parameter vector considered, assume that the debt obligation with face value of 1, considered throughout the paper, is always just sufficient to fund the project.\(^7\)

The value obtained by the agent if he finances the project with debt is just equal to the value of the debt tax shield plus the expected value of the dividend he receives in the event the project succeeds. Interest expense on the corporate tax return is computed as bond yield (\( \psi \)) times initial loan principal (\( V \)). Using the fact that \( V = 1/(1 + \psi) \) it follows that interest expense is just equal to \( 1 - V \). Thus, under debt finance the ex ante payoff to the agent is equal to:

\[
Debt \Rightarrow \tau(1 - V(\alpha)) + \frac{1}{2}(H - 1).
\] (26)

\(^7\)Technically, we assume there is a tax advantage to debt over external equity, but that there is no tax incentive to float debt to fund ex ante payouts.
Suppose instead the agent finances the project with equity. To preserve symmetry, one may assume that the agent has the ability at time $t_1$ to request concessions of shareholders. Suppose he can make a take-it or leave-it offer to shareholders: “If the fractional claim to the final dividend held by outside equity is not reduced to $z$, I will quit.” As with debt, this will result in the equity being sold to the atomistic investors at date zero. However, the market value of total shareholder equity is here just equal to firm cash flow, since any purchases of equity in the secondary market by the large investor have no effect on its fundamental value. That is, equity will always be fairly priced, and hence perfectly liquid, since the large investor has no ability to influence and no private information regarding operating cash flows.

Recalling our working assumption that the debt obligation is just sufficient to cover the cost of the project, under equity finance the agent receives a payoff equal to expected cash flow less project cost, or:

$$Equity = \frac{1}{2} + \frac{L}{4} + \frac{M}{4} - V(\alpha). \quad (27)$$

Comparing (26) and (27) we see that debt finance is optimal iff:

$$V(\alpha) \geq \frac{1}{2} + \frac{L}{4} + \frac{M}{4} - \tau.$$ \hspace{1cm} (28)

However, we recall from Figure 5 that the primary market price of debt is not unique, with debt value being lower if the market anticipates the low-liquidity equilibrium. Therefore, whether condition (28) is satisfied depends upon which secondary market equilibrium is anticipated. For example, if the best (worst) PBE is anticipated, then debt (equity) is more likely to be the optimal source of external funds. To reinforce this intuition, we may compare equations (26) and (27), concluding that debt dominates equity iff:

$$\tau \geq \tau^*(\alpha) = \frac{1}{2} + \frac{L}{4} + \frac{M}{4} - V(\alpha).$$ \hspace{1cm} (29)

Figure 6 plots this critical value $\tau^*$ such that debt dominates equity for the same parameter vector considered in Figure 5. Importantly, the critical value depends upon the nature of the conjectured PBE. For example, at $\alpha = .50$, if the market conjectures the best PBE then debt dominates equity.
if the corporate tax rate is 40%. However, if the market conjectures the worst PBE, then debt dominates equity only if the corporate tax rate exceeds 60%. To put this another way, all firms occupying the parameter region between the solid line and the short dotted line in Figure 6 would switch from debt to equity finance if the market were to move to the low-liquidity PBE conjecture. In this way, the model has the potential to explain sharp changes in financing patterns absent any change in economic fundamentals (which are here captured by the fixed underlying model parameters).

As the discussion above illustrates, since equity is fairly priced and fully liquid in this model, it weakly dominates all non-debt claims. So confining attention to ordinary equity was without loss of generality when considering non-debt claims. However, if we move away from the working assumption that fixed costs induce the firm to finance with one security, the issue of optimal debt structure becomes nontrivial and rather complex (due to inherent nonlinearities and multiple PBE). For example, if there are no fixed costs, it may be optimal for the firm to obtain its debt tax shields through multiple debt classes. For example, the firm could carve out a safe senior claim which would be fully liquid. However, junior debt would have high informational sensitivity and could become illiquid. A complete characterization of optimal debt structure is beyond the scope of this paper, although we view this as a promising direction for future research.\footnote{Gorton and Pennachi (1990) explore this theme, but in a rather different setting where a large trader has private information regarding operating cash flow, and without the prospect for debt relief.}

7 Conclusion

The toughness of dispersed lenders serves to deter strategic default. However, when the likelihood of bona fide insolvency increases, a more concentrated ownership structure becomes optimal, since concentration facilitates efficient debt relief. This paper has identified conditions under which trading of debt in secondary markets can bring about an efficient shift from dispersed to concentrated ownership. Specifically, we have developed a theoretical framework to analyze a secondary market

\footnote{Gorton and Pennachi (1990) explore this theme, but in a rather different setting where a large trader has private information regarding operating cash flow, and without the prospect for debt relief.}
where endogenously determined ownership structure affects the value of a traded debt security. A novel feature of our model, one which is critical for our findings, is that we allow small investors to consider potential mispricing before submitting their market orders.

A main obstacle inhibiting ex post efficient ownership concentration is that small bondholders free-ride off the debt relief granted by the large investor, with free-ridership capitalized into secondary market debt prices. Further, the prospect of a large investor granting debt relief also reduces the willingness of small bondholders to sell, even when they are impatient. We show that the free-ridership problem and small investor concerns regarding adverse selection are both more severe during recessions. Consequently, trading volumes in secondary debt markets, as well as the prospect of debt relief, will fall during downturns.

The most important prediction generated by the model is that small investor concern over adverse selection creates the potential for multiple equilibria in secondary debt markets. In contrast to equity markets, we show that debt markets are vulnerable to multiple equilibria, with differing volumes of trade and differing probabilities of debt relief. For example, there exist low-liquidity equilibria in which the large investor buys debt with low probability because he anticipates that only a low percentage of small bondholders will sell. Conversely, a low buying intensity by the large investor can actually discourage small investors from selling their debt claims due to the fact that adverse selection is non-monotone in the trading intensity of the large investor. We also show that the equilibrium set is contingent on the macroeconomic state. Since small investors face more severe adverse selection during recessions, the secondary debt market is then more likely to be in a low-liquidity equilibrium.

The model offers a novel explanation for leverage cycles, one which is based upon the multiplicity of secondary market equilibria. Anticipation of low-liquidity equilibria in secondary debt markets leads to sharp increases in expected bankruptcy costs and required debt yields, inducing firms to shift from debt to equity financing. Significantly, the model predicts that sharp changes in financing patterns can occur absent any change in economic fundamentals such as tax rates and bankruptcy
cost parameters. Rather, large fluctuations in aggregate leverage require nothing more than sunspots
inducing agents to anticipate high or low trading volumes in secondary debt markets.
Appendix A: Beliefs Off the Equilibrium Path

We here briefly discuss market maker beliefs off the equilibrium path. Beliefs and prices in response to order flow $X$ off the equilibrium path are only relevant to our analysis of the large investor’s incentive to deviate, since small investors have measure zero and are thus powerless to push order flow off the equilibrium path. To deter deviations by the large investor we assume the market makers form the least favorable beliefs, in the following sense. Upon observing any order flow $X$ off the equilibrium path, the market makers assume that all impatient atoms are liquidating and that the large investor is placing an order to buy $x = X + \gamma$. If the implied $x \geq s$ the market makers set $P = 1$ and if not they set the price to $P^-$. Facing such beliefs, the large investor cannot gain if he were to ever deviate by placing a buy order of any size other than $\gamma^*$. In our setting, when the market makers see a non-equilibrium order flow, they know the deviator is the large investor. However, the Cho-Kreps refinement has no bite in our setting since the only party capable of pushing order flow off the equilibrium path does not even have a “type.”

Appendix B: The Low Liquidity Demand Scenario

Consider the case depicted in Panel C of Figure 3, in which $c$ is

$$Low : c < \xi(\sigma^g_{\max}, P^-_g) < \xi(\sigma^b_{\max}, P^-_b).$$

In this particular scenario, impatient atoms may be unwilling to liquidate even if the interim economic state is $g$.

In this particular case, one generally anticipates $\sigma^*_g > \sigma^*_b$. To see this, note that if $\tilde{\sigma}_\omega \in (0, \sigma^*_1]$ for both states then $\sigma^*_g = \tilde{\sigma}_g > \tilde{\sigma}_b = \sigma^*_b$. Further, if $\tilde{\sigma}_\omega \in (\sigma^*_1, \sigma^*_2)$ for both states, then $\sigma^*_g = \sigma^1_g > \sigma^1_b = \sigma^*_b$. However, it is not necessarily the case that the trading intensity of the large investor will be higher in state $g$. To see this, suppose $\tilde{\sigma}_\omega > \sigma^*_2$ for both states. Then it is possible that $\sigma^*_\omega = \sigma^*_2$, for example. But then the large investor buys with higher intensity in the bad state since $\sigma^2_b > \sigma^2_2$. More generally, due the multiplicity of PBE, the ranking of the trading intensities becomes indeterminate when $\tilde{\sigma}_\omega > \sigma^*_2$ for both interim economic states.
References


Figure 1: Timeline

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<thead>
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<th>$t_2$</th>
<th>$t_3$</th>
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<td>State: b or g</td>
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<td>Primary market</td>
<td>Implementation</td>
<td>Preference shock</td>
<td>Relief/Not</td>
</tr>
<tr>
<td>Market orders</td>
<td>Secondary market</td>
<td>Net orders observed</td>
<td>Bankruptcy/Not</td>
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Table 1: Order Flows and Price Setting

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<th>Buy</th>
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<th>$x^1$</th>
<th>$x^N$</th>
<th>$X$</th>
<th>Price</th>
<th>Probability</th>
<th>True Value</th>
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<tr>
<td>Y</td>
<td>N</td>
<td>$\gamma^*$</td>
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<td>$\gamma^*$</td>
<td>1</td>
<td>$\sigma/2$</td>
<td>1</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>$\gamma^*$</td>
<td>$-\gamma^*$</td>
<td>0</td>
<td>$\sigma+(1-\sigma)P^-$</td>
<td>$\sigma/2$</td>
<td>1</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\sigma+(1-\sigma)P^-$</td>
<td>$(1-\sigma)/2$</td>
<td>$P^-$</td>
</tr>
<tr>
<td>N</td>
<td>Y</td>
<td>0</td>
<td>$-\gamma^*$</td>
<td>$-\gamma^*$</td>
<td>$P^-$</td>
<td>$(1-\sigma)/2$</td>
<td>$P^-$</td>
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</table>
Figure 2A: Provisional Buying Intensity and Breadth of Shocks

Figure 2B: Provisional Buying Intensity and Bankruptcy Costs
Figure 6: The Debt v. Equity Decision

- **Best PBE**
- **Med PBE**
- **Worst PBE**