An Equilibrium Model of Asset Pricing and Moral Hazard

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May 29, 2006
Introduction

• The principal-agent models developed so far, such as Holmström (1982), Baiman and Demski (1980) and Diamond and Verrecchia (1982), have found that relative performance evaluation (RPE) can improve welfare because it can be used to filter out common risk from agent’s compensation, however, the results of empirical tests have been mixed, as summarized in Abowd and Kaplan (1999) and Prendergast (1999).

• In the Capital Asset Pricing (CAPM) model of Sharpe (1964), Linter (1965) and Mossin (1966), managerial incentives and certain firm characteristics, such as idiosyncratic risk, play no role in the determination of expected asset returns. The model has conflict with the reality.

• The objective of this article is to develop an integrated model of asset pricing and moral hazard.
1. The Model

- The following assumptions characterize our economy:
  
  - **Assumption 1**: There are $N$ risk assets and one riskless asset available for continuous trading. The cash flow process of firm $i$ is given by for $i = 1, 2, \ldots N$:
    
    $$dD_i = (A_{it} + \Pi_i D_{it})dt + \sigma_i dB_{ct} + \sigma_B dB_{it}$$
    
    $$= (A_{it} + \Pi_i D_{it})dt + b_t dB_t$$

  - **Assumption 2**: The manager of firm $i$ incurs a cumulative cost of associated with managing the firm form time $0$ to time $T$. The cost rate is given by a convex function:
    
    $$c_i(t, A_{it}) = \frac{1}{2} k_i(t) A_{it}^2$$
1. The Model (cont.)

- **Assumption 3:** There are N managers, one for each of the N firms. Each manager has a negative exponential utility function with a constant risk aversion coefficient $R_a$. Given the investor’s contract, the manager expends effort to maximize his own expected utility

$$\sup_{\{A_i\}} E_0 \left( -\frac{1}{R_a} \exp \left[ -R_a \left( S_T^i - \int_0^T c_i(t, A_{ii}) dt \right) \right] \right)$$

- **Assumption 4:** There are N identical investors in our economy. Each investor decides on the number of shares to invest in the risk less bond and the risky stocks, and designs incentive contracts for managers to maximize her expected utility over her terminal wealth:

$$\sup_{\{\mu_i, A_i, S_T^i\}} E_0 \left( -\frac{1}{R_p} \exp \left[ -R_p \left( W_T - \sum_{i=1}^N S_T^i (\{\mu_i\}) \right) \right] \right)$$

subject to the manager’s participation and incentive compatibility constraints as well as the investor’s budget constraint.
1. The Model (cont.)

- **Assumption 5**: Assume that the equilibrium pricing function is of the following linear form:

\[ P_{it} = \lambda_{i0}(t) + \sum_{j=1}^{N} \lambda_{ij}(t)D_{jt} \]

where the coefficients \( \lambda(t) \)s are time-dependent deterministic continuous functions. The stock price vector can be written in a compact as:

\[ dP_t = a_p dt + b_p dB_t \]

- **Assumption 6**: The compensations to the managers are described by:

\[ S_T^i = q_i(T, P_T) + \int_0^T g_i(t, P_t) dt + \int_0^T h_i(t, P_t) dP_t \equiv \int_0^T dS_t^i \]
2. An Exchange Economy in the Absence of Managers

- We first consider a benchmark case in which the $A_{it}$’s in the cash flow processes are exogenously given or in which there are no managers.
- The excess dollar return for holding asset $i$ within time interval $dt$ is defined as
  \[ dQ_{it} = dP_{it} - rP_{it}dt = a^i_{Q_t}dt + b^i_{Q_t}dB_t \]
- The wealth process of investor $i$’s portfolio is given by:
  \[ dW_{it} = rW_{it}dt + \mu_{it}dQ_t \]
- The objective of the investor is choose the number of shares to invest in the risky assets to maximize her expected utility over the terminal wealth:
  \[
  \sup_{\{\mu_i\}} E_0 \left[- \frac{1}{R_p} \exp \left( R_pW_T \right) \right] \\
  s.t. dW_{it} = rW_{it}dt + \mu_{it}dQ_t
  \]
2. An Exchange Economy in the Absence of Managers (cont.)

- **Lemma 1** \( \text{In equilibrium, the expected excess dollar return } a_{Q_i}^i \text{ on stock } i \text{ is a linear function of the expected excess dollar return } a_{Q_t}^M \text{ on the market portfolio:} \)
  \[
  a_{Q_t}^i = \frac{\text{cov}(dP_{it},dP_{Mt})}{\text{var}(dP_{Mt})} a_{Q_t}^M
  \]
  where the market portfolio is defined as \( P_{Mt} = P_{1t} + P_{2t} + \ldots + P_{Nt} \). This is the CAPM relation in terms of the expected excess dollar returns. The expected excess rates of returns on stock \( i \) and the market portfolio, \( R_{Q_t}^i = \frac{1}{P_t} a_{Q_t}^i \) and \( R_{Q_t}^M = \frac{1}{P_t} a_{Q_t}^M \), also satisfy the CAPM equation:
  \[
  R_{Q_t}^i = \frac{\text{cov}(R_{it},R_{Mt})}{\text{var}(R_{Mt})} R_{Q_t}^M = \beta R_{Q_t}^M
  \]
  where \( R_{it} = \frac{dP_{it}}{P_{it}} \) and \( R_{Mt} = \frac{dP_{Mt}}{P_{Mt}} \)
3. A Principal-Agent Economy

- We now extend the current asset literature by partially endogenizing the cash flow processes. The key point here is to first impose the agent’s participation constraint and his incentive compatibility constraint on the original contract form in assumption 6 and then solve the unconstrained investor’s problem under her budget constraint.

- Given the manager’s compensation in assumption 6, we define a value function process $V^i(t, P_t)$ for manager $i$’s maximization problem as

$$V^i(t, P_t) = \sup_{\{\mu_u\}} E_t \left\{ -\frac{1}{R_a} \exp \left[ -R_a \left( q_i(T, P_T) + \int_t^T [g_i(u, P_u) - c_i(u, A_{iu})] du + \int_t^T h_i(u, P_u) dP_u \right) \right] \right\}$$

- The Bellman-type equation for the manager’s maximization problem is then given by:

$$0 = \sup_{\{A_u\}} \left[ -R_a V^i(t, P) \left\{ g_i(t, P) + h_i(t, P) a_p - c_i(t, A_{iu}) - \frac{1}{2} R_a h_i(t, P) b_p b^T_i h_i(t, P) \right\} + V^i_t + V^i_t \left[ a_p - R_a b_p b^T_i h_i(t, P) \right] + \frac{1}{2} tr \left( V^i_{pp} b_p b^T_i \right) \right]$$
3. A Principal-Agent Economy (cont.)

- **Lemma 2**: Using the manager’s participation constraint at time 0, the above PDE, Itô’s lemma and a transformation, we can arrive at an expression for the equilibrium compensation:

$$
\overline{S}_T^i = \varepsilon_{i0} + \int_0^T c_i(t, A_{it}) dt + \frac{R_a}{2} \int_0^T \overline{h}_i b_p b_p^T \overline{h}_i^T dt + \int_0^T \overline{h}_i b_p dB_i
$$

where $\varepsilon_{i0}$ denotes manager i’s certainty equivalent wealth at time 0, and where $c_i(t, A_{it}) = \frac{1}{2} k_i(t) A_{it}^2$ and $\overline{h}_i = \overline{h}_i(t, P_t) = h_i(t, P_t) - \frac{v_{it}^T}{V_{Ra}}$. 
3. A Principal-Agent Economy (cont.)

- We consider the impact of moral hazard on expected asset returns and relative performance evaluation.

- Imposing the FOCs of the manager's Bellman equations, we obtain:

\[
\bar{h}_{ii} = \frac{k_i(t)A_{it} - \sum_{j \neq i}^{N} \bar{h}_{ij}\lambda_{ji}(t)}{\lambda_{ii}(t)}
\]

where \(A_{it}\) and \(\bar{h}_{ij}\) shall be determined from the investor’s maximization problem.

- The investor’s problem is now given by:

\[
\sup_{\{A_{ij}, \bar{h}_{ij}, \mu_j, i \neq j\}} E_0 \left[ \frac{1}{R_P} e^{-R_P \left( W_T - \int_0^T \mu_i dS_t \right)} \right]
\]

subject to her wealth process
3.1 Optimal Contracts and RPE

- When the investors are risk neutral, the RPE with respect to the market portfolio reduces to:
  
  $$-k_i(t)A_i^* \frac{\sigma_{ic}}{\sigma_M} = -\frac{1}{1+R_u k_i \sigma_{ii}^2 \sigma_M}$$

- When investors are risk averse, however, they would like to share the common risk with managers, resulting in a positive component in the RPE term.

- In equilibrium, the manager’s compensation is equal to:
  
  $$S_T^i = constant + k_i A_i^* \sigma_{ii} B_{iT} + \frac{1}{N} \frac{R_p}{R_p + R_a} \sum_{j=1}^{N} P_{jT}$$

- In the model, a small PPS can induce a very large effort. Suppose that $k_i A_i^* = 1.0\%$, $R_u = 1.0\%$ and $\sigma_{ii}^2 = 1.0\%$. From the equilibrium effort equation, we obtain $k_i = 10^{-12}$ and thus $A_i^* = 10^{10}$.
3.2 Expected Returns, Idiosyncratic Risk and Managerial Incentives

- Suppose that the number of firms in the economy approaches infinity;

- Firm-specific risk has no impact on the expected excess dollar return, however, the investor’s cost of providing incentives increases with it;

- The risk premium on an asset $i$, defined as $E^i(t)/P_{it}$ where $P_{it} > 0$, decreases with managerial incentive or pay-performance sensitivity (PPS), and so does the expected rate of return on the asset;

- Individual firm characteristics such as idiosyncratic risk associated with agency considerations do not serve as independent risk factors, they affect expected asset returns only through systematic risk.
4. Further Discussion

• The result regarding RPE also holds under general assumptions rather than the normal cash flow processes and negative exponential utility functions;
4.1 Log-Normal Cash Flow Processes and General Utility Functions

- Suppose that the cash flow generating processes are log-normal:

\[
\frac{dD_{it}}{D_{it}} = A_{it} dt + \sigma_{it} dB_{it} + \sigma_{ct} dB_{ct} = A_{it} dt + \sigma_i dB_t, \ i = 1, 2, \ldots, N,
\]

- The investors and managers possess power utility functions, \( X_{i}^{\gamma_l} / \gamma_l, l = p, a \), where \( X_p \) and \( X_a \) denote the investors’ and managers’ terminal wealth or consumption;

- For simplicity, we assume that manager and investors share the terminal cash flow \( D_{iT} \) at time \( T \) without intertemporal payments or consumption;

- Consider the case in the absence of managers, and denote by \( \delta_t \) the cost of capital for the cash flow of a firm and is a deterministic function of time \( t \). The stock price process is given by:

\[
\frac{dP_t}{P_t} = \delta_t dt + \sigma dB_t,
\]
4.1 Log-Normal Cash Flow Processes and General Utility Functions

• Introduce moral hazard into the problem and consider a simple linear contract form \( a + bD_T \);

• The optimal contracts and the equilibrium prices are intractable;

• Suppose that \( \delta_t \) is the time dependent discount rate of return, and the rate of return process can be written as:

\[
\frac{dP_t}{P_t} = \delta_t dt + \left( 1 + \frac{a \exp(-\int_u^T \delta_u du)}{P_t} \right) \sigma dB_t
\]

nonlinear contract form will render more terms in the expression for both the stock price \( P_t \) and more nonlinear functions of \( P_t \) in the diffusion term of the stock price process.
4.2 Endogenous Interest Rates

• To enhance the generality of the current model, we relax the assumptions such that the interest rate is endogenously determined and that the bond is zero-net supply;

• In a one-period model, assume the cash flow of firm $i$ is given by:

$$D_i = A_i + \sigma_{ii} \varepsilon_i + \sigma_{ic} \varepsilon_c, i = 1, 2, \ldots, N,$$

and the contract for manager $i$ is of the linear form given as

$$S_i = g_i + \sum_{j=1}^{N} h_{ij} D_j$$

and all other assumptions are similar to those in the previous section;

• Our results still hold under the general conditions that the interest rate is endogenously determined and that the bond is zero-net supply;
4.3 Agency Models and Their Empirical Tests

• Consequently, any empirical test of agency models is perhaps flawed in the absence of an asset pricing consideration and without distinguishing between the properties of cash flows and those of market prices. A thorough empirical test must incorporate an asset pricing model that clearly defines systematic risk and idiosyncratic risk in terms of both market prices and cash flows.