Investors' Optimism: A Hidden Source of High Markups in the Mutual Fund Industry *

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Abstract

Why are investors buying underperforming mutual funds? To address this problem, we develop a principal-agent model based on a sequential game played by a representative investor and a fund manager in an asymmetric information framework. The model shows that investors' perceptions of the fund market play the key role in the fund's fee-setting mechanism. The managers' true ability to deliver performance is not relevant. Along with a simple relation between fees and funds' performance, empirical evidence suggests that most U.S. domestic equity mutual funds have added high markups in recent years. For these fees to be justified, we show that investors would have expected the fund managers to be able to deliver an overall annual excess-return of around 3% over the S&P 500, net of fees, irrespective of the investment style and of the risk level of the funds. Therefore, we interpret these high markups as resulting from the investors' optimism bias whose root can be found in their lack of financial literacy as well as in funds' marketing effort. We demonstrate that investors' over-optimism and their misperception about the fund market drive them into buying underperforming mutual funds, which then allows mutual fund managers to charge high markups.

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1 Introduction

The lack of performance of the mutual fund industry and the lack of investors' reaction to these poor performances is a widely reported phenomenon[1]. However, despite its apparent underperformance, the total net assets managed by U.S. mutual funds has increased from 6.39 trillion dollars in 2002 to 12.02 trillion dollars in 2007, according to the Investment Company Institute (2008). Why do investors keep buying these obviously underperforming investment vehicles? At the same time, how can one explain the long-standing puzzle of high markups in the mutual fund industry? These questions are even more crucial in recent years, given the emergence of new investment vehicles such as low-cost index funds[2] and exchange-traded funds (ETFs). With growing competition and increasing disclosure and transparency in the fund market, and given their relatively poor performance, one could expect that mutual funds would reduce their fees, but a recent report by the U.S. Security and Exchange Commission shows clearly that mutual fund total expense ratios (TER) have been overall on the rise since the late 1970s (SEC 2001).

The present paper proposes a simple solution for these two related puzzles in terms of a principal-agent model in an asymmetric information framework. Our model provides a theoretical framework to account for the investors' limited abilities and to test some of its observable consequences. We find that the optimal fee level, determined by the fund managers in their own interest, depends only on the information and preferences of the investor. Because investors have only partial information on the fund managers' true abilities and limited knowledge of the financial markets, they may hold a biased view of the fund's true performance. Fund managers can then take advantage of the investors' optimism bias, attracting their investment and charging them an additional premium significantly above the competitive level of fund fees.

Our model assumes that the origin of the misperception of investors about the fund performance is not due to their failure to update their priors but lies only in their limited or misguided information[3]. Given this limitation, investors make rational decisions. This assumption has the advantage of reducing the problem to a simple one-period set-up. In contrast, taking into account the process of information updates would require a multi-period model with learning.

In addition, our model suggests two alternative fee-setting mechanisms. First, when the fund provides diversification benefits from the perspective of an investor's global portfolio, demand-insensitive investors have to pay higher fees to get access to this benefit. This scenario has been described by Gil-Bazo and Ruiz-Verdu (2007). Second, when a leveraged fund does not provide diversification benefits but actually adds additional risks to the investor's global portfolio, the fund manager will lower fund fees to attract more money inflow from less performance-sensitive investors.

[1]See Palmiter and Taha (2008), Berk and Green (2004), Nanigian, Finke, and Waller (2008), and Glode (2008).

[2]Some index funds track S&P 500 indices with an annual expenses of 10 basis point and has no load fees.

[3]The two following perspectives about the rationality of investors can not be distinguished within our framework: either investors are rational but take their decisions based on limited information or they have only bounded rationality.

The existing literature attributes the investors' puzzling investment behavior either to the existence of redemption fees[4], or to the performance of funds in bad economic times[5]. In contrast, Berk and Green (2004) suggested that investor's irrational behavior and information asymmetry should play a significant role to explain the two puzzles mentioned above. Whether fund fees are excessive and whether market competition truly works in the mutual fund industry is a long-standing debate among academics. Several studies such as Coates and Hubbard (2007) and Grinblatt, Ikheimo, and Keloharju (2008) argue that there is an adequate level of competition in the mutual fund industry and fees in the fund market are thus competitive. However, the result of our empirical study contradicts this view. More specifically, we show that after accounting for the returns on funds, diversification benefits and fees, most US domestic equity mutual funds, both actively and passively managed, have added markups in the past years. In addition, these mutual funds demonstrate competitive disadvantage to low-cost index funds or index ETFs. In this respect, our study provides additional evidence to previous works (School 1982, Freeman 2007, Freeman and Brown 2001). These studies found that mutual fund advisers charge significantly higher fees than free-market prices would suggest. As a consequence, they call for more market competition and regulation in the mutual fund industry. In an in-depth study, Palmiter and Taha (2008) discussed fund investor profile studies released by the SEC. They argued that, due to the fact that most investors are financially unsophisticated, the SEC's current policy of imposing adequate disclosure is not sufficient because it fails to account for investor's own limitations, i.e. lack of financial knowledge.

Previous works have identified investors' optimism bias towards equities issued in their domestic market[6]. In particular, academic research on mutual funds has focused on investor's lack of financial literacy. Barber, Odean, and Zheng (2005) and Choi, Laibson, and Madrian (2008) provide empirical evidence that investors are more sensitive to salient fees such as front-load fees. Surprisingly, investors appear to be unaware of the existence of mutual funds' expense ratios. Capon, Fitzsimons, and Prince (1996) and Alexander, Jones, and Nigro (1998) demonstrated in their respective studies that investors are not familiar with many basic facts about mutual funds such as the level of fees they are paying to their funds. These empirical findings of investor's deviation from rationality are in line with our model's emphasis on investor's limited financial knowledge of the mutual fund industry.

Investors' optimism bias can be closely related to their lack of knowledge of the fund market, leading them to choose sub-optimal benchmarks such as bank savings instead of low-cost index funds or ETFs. Besides, investors' optimism bias is probably influenced and reinforced by the marketing practices of mutual funds, which promote the sale of fund shares. Since 1980, after approval of the SEC Rule 12b-1, mutual funds have been allowed to charge marketing and distribution fees to their investors by adopting a 12b-1 plan[7]. Khorana and Servaes (2004) and Barber, Odean, and Zheng (2005) have identified a positive impact of 12b-1 fees on funds'

[4]See Alves and Mendes (2007) and Nanigian, Finke, and Waller (2008).

[5]See Glode (2008).

[6]This well-documented fact is called "equity home bias". Investments tend to be too concentrated in the home equities, failing to reap the benefit of international diversification. See French and Poterba (1991), Cooper and Kaplanis (1994), Tesar and Werner (1995) and Strong and Xu (2003). Brennan and Cao (1997) attributed this bias to asymmetric information between domestic and foreign investors.

[7]See Malhotra and McLeod (1997)

money flows. Murray (1991) explains various marketing strategies in promoting fund sales.

Another important empirical fact about the mutual fund industry is the dispersion in fees across various funds. Empirical evidence shows that funds with higher expense ratios deliver lower before-fee returns, as demonstrated by Elton, Gruber, and Das (1993), Gruber (1996) and Chevalier and Ellison (2002). Gil-Bazo and Ruiz-Verdu (2007) interpret this observation as a selection bias: underperforming funds target the pool of investors who are least sensitive to fund performance, while better performing funds charge lower fees to compete for and to attract performance sensitive investors. Christoffersen and Musto (2002) studied money market funds with a similar argument and showed that demand-curve variations explain fee variations.

The construction of our model follows the literature on the theory of optimal contracts between managers and investors, which considers a variety of factors and mechanisms to explain the fee structure of portfolio management services. For instance, Das and Sundaram (2002) compared a symmetric fulcrum contract enforced by law[8] with an asymmetric incentive contract, using a model where good managers choose the appropriate contract structure to signal their ability. First, they found that incentive fees usually lead to more risky portfolios. Comparing incentive fees with fulcrum fees, they also determined that the former (respectively later) provides increasing investor welfare under imperfect competition (respectively under competitive market conditions). Stracca (2006) provides a good survey of the relevant literature on this topic.

In this context, our model is based on the observation that the relationship between a manager and a representative investor constitutes an example of the general principal-agent problem. Following the seminal work of Ross (1973) and Holmstrom (1979), numerous studies have applied the principal-agent model to various situations of economic exchange between two parties. In a nutshell, an investor "hires" a mutual fund manager and the fee structure of the mutual fund is the mechanism used to attempt to align their interest, under prevailing conditions of incomplete and asymmetric information between them. In the language of the principal-agent problem, the manager sells her service, presented as information gathering ability and managerial efforts, to the investor in return for a compensation represented as the management fees (Starks 1987, Golec 1992, Heinkel and Stoughton 1994). We assume that managers with better information have full access to an investor's private information, whereas investors have no access to a manager's private information.

We derive the demand function of the representative investor and the optimal level of management fees charged by the manager. The information on the manager's skills is revealed to the investor by the return history of the managed fund. For a specified level of management fees, the demand function of the representative investor is determined by the composition of her optimal portfolio. This portfolio is defined as a mixture of the mutual fund investment and of other vehicles that she picks up herself. The fund manager uses her private knowledge of her own management skills and the full understanding of the investor's decision process to determine the optimal fee level, which maximizes her expected utility. Our approach generalizes in

[8]The "fulcrum" fee is a form of incentive fee specified for mutual funds by law, according to a 1970 amendment to the Investment Company Act of 1940. A typical fulcrum fee must be centered around an index of reference, with increases in fees for performance above that index matched by decreases in fees for performance below the index.

three directions the work of Golec (1992), who also studied a one-period principal-agent model in which mutual fund managers trade their information-gathering abilities with investors: (i) we solve the principal-agent problem by fully considering the informational disadvantage of the investor; (ii) we focus our attention on the fixed-fee compensation scheme as it is the most-used standard in the mutual fund industry; and (iii) our treatment is not restricted to the mean-variance utility function, even if we eventually show that it is often sufficient to provide reasonable results.

We obtain two main theoretical results. Firstly, the fee-setting mechanism including the fees at equilibrium is fully determined by the information available to the investor, while the manager's information is irrelevant. The investor's information includes her choice of the benchmark portfolio[9] and her anticipation of the fund's relative performance in terms of both diversification benefits and returns, when compared to her benchmark. Secondly, we provide a simple formula to analyze a fund's risk-adjusted performance, when fees come into play. In addition, our results do not require any restrictive assumptions on the form of the fund's return distribution and the investor's utility function as long as it is an increasing and concave function.

Then, we test whether the returns delivered by US equity mutual funds can justify their fees in recent years, given the perspective offered by our model. For this, we use a data set of 3,875 U.S. domestic equity funds over the period from July 2003 to March 2007 from the CRSP Survivor-Bias-Free US Mutual Fund database. The results show that most funds have charged high markups to their investors. At the same time, we interpret the continued presence of high markups as an indication of investors' over-optimism about the funds' future performance when they make their investment decisions. The over-optimism of investors translates into decisions made on the basis of limited or misguided information, and leads to investments in underperforming mutual funds. Furthermore, we show that investor's over-optimism can be explained either by the fund's marketing efforts or by the investor's incorrect selection of benchmarks, or by a combination of both, possibly due to the lack of sufficient investment knowledge.

This paper is organized as follows. Section 2 describes the model, stressing its economic underpinning. In section 3, we present our main results, with the characterization of the equilibrium and its main properties. Section 4 presents the empirical framework and results. Section 5 summarizes our conclusions.

2 The Model

2.1 General Set-up

Let us consider a fund manager and a representative investor who play a one period game. Consistent with the literature on mutual fund fees (Herman 1963, Holmstrom 1979, Luo 2002, Freeman 2007), we focus on the situation in which the representative investor has no bargaining power. This setup implies a competitive supply of capital to mutual funds, as

[9] The benchmark portfolio represents the investor's standard choice of investment vehicles. This can be ETFs or bank savings, depending on the investor's financial knowledge and risk preference.

suggested by Berk and Green (2004). This assumption is quite realistic, given the large number of small investors and the relatively small number of mutual funds in today's mutual fund market.

The investor is a utility-maximizer with no bargaining power and incomplete information. This approach is consistent with the fact that, except for index funds or ETFs, the whole mutual funds market can be considered to be a monopoly (Chordia 1996). The investor chooses the optimal amount of her money to invest in the managed fund, on the basis of its perceived average return and risk profile as well as the management fees charged by the fund manager. This latter is assumed to be a utility-maximizer too, privy of her own personal information.

We consider exclusively a one period game between the fund manager and the representative investor. This convenient simplification is not restrictive, because it actually reflects the reality that most mutual fund investors buy for the long-term and redeem their shares infrequently. This buy-and-hold strategy is encouraged by the policy of most funds, which apply penalties for early redemption[10]. Although our model is one-period, this does not imply that the fund is statically managed. We do not make any assumption about the underlying management process, which can include any general dynamic strategy. This makes our model quite versatile and relevant for both static and active mutual fund strategies.

The game unfolds as follows. At the beginning of the period, the fund manager decides on the level of management fees as a percentage of her asset under management. The representative investor observes the proposed fee structure and builds up her portfolio accordingly. She can purchase shares from the manager's fund, which involves a cost specified by the management fees. She can also buy shares from a "benchmark portfolio" which is accessible at zero management cost. This benchmark asset can be the risk-free asset or any exchange-traded fund[11] (ETFs) that replicates a market index or a risk factor which is representative of the asset class used by the manager.

The initial endowment of the investor is equal to one monetary unit. She invests ω in the managed fund and the rest, $1 - \omega$, in the benchmark portfolio. The benchmark portfolio can be sold short[12] but only long positions are allowed for the managed fund, so that $\omega \geq 0$. At the end of the period, the manager extracts her fees and then redeems the remaining capital to the investor.

Following the prevalent habit in the mutual fund industry, our set-up assumes a fixed-fee compensation scheme for the mutual fund, i.e., the fee is a fixed percentage per period of the net asset under management. This fee structure is to be contrasted with the incentive fee that usually includes a proportional base fee plus a percentage of the return above a certain

[10] Alves and Mendes (2007) showed that back-end load fees do have an impact on investor's redemption activity. On average, the Investment Company Institute (2001) suggested a redemption rate of 15% for domestic equity funds over the period from 1992 to 1999. Redemption rate is calculated as fund's total outflows in a given year as a percentage of starting assets.

[11] Static index ETFs have no annual expenses. There is only a commission as low as 5-20 basis points when buying and selling the shares. Assuming 5 years holding period, this means 2-8 basis point per year. Poterba and Shoven (2002) compares the S&P 500 SPDR trust, the largest ETF to the Vanguard Index 500 and concludes that they both offer similar returns to investors.

[12] Unlike index-funds, short-selling ETFs is possible.

benchmark, which is the prevalent compensation scheme in the hedge fund industry. For the mutual fund industry, Golec (1992) reported that, in 1985, only 27 out of 476 U.S. equity funds used performance based compensation schemes. More recently, Elton, Gruber, and Blake (2003) documented that, in 1999, only 108 out of 6716 U.S. mutual funds specializing in bonds and stocks used incentive fees. This justifies our focus on the fixed-fee scheme.

According to the report on mutual funds fees and expenses (SEC 2001), mutual fund fees consist of both shareholder fees and annual fund operating expenses (TER). Shareholder fees are paid to the broker and/or to the fund itself, when investors purchase or redeem their shares. Most brokers charge both front-loads on purchase of shares and back-end loads on redemption of shares. Annual fund operating expenses include management fees, distribution (12b-1) fees, and other expenses. Some administrative fees that are not included in the "other expenses" category are also payable to the investment adviser. Distribution(12b-1) fees include fees paid for marketing and selling fund shares, and for advertising. Other expenses include legal expenses, accounting expenses and so on. SEC regulations impose a full disclosure of the annual expense ratio.

We denote by f_e the expense ratio for the one-period and by f the corresponding management fees. Both are expressed as a percentage of the investor wealth under management in the fund. The compensation of the manager only includes the management fee f. Denoting by f_0 the remaining part of the expense ratio, we have

$$f_e = f_0 + f \ , \tag{1}$$

where f_0 gathers all the costs that do not contribute to the manager's compensation, such as annualized shareholder costs[13], distribution(12b-1) costs, legal costs and so on. The fund manager cannot benefit directly from f_0, but the investor must pay this cost. Consequently, denoting by $\tilde{r}_m$ and $\tilde{r}_i$ the return on the mutual fund and on the benchmark portfolio respectively, the investor's terminal wealth $\tilde{W}_i$ reads

$$\tilde{W}_i = (1-\omega)(1+\tilde{r}_i) + \omega(1+\tilde{r}_m)(1-f_e), \tag{2}$$

while the manager's compensation $\tilde{W}_m$ is given by

$$\tilde{W}_m = \omega(1+\tilde{r}_m)f. \tag{3}$$

2.2 Description of the manager's and the investor's optimization problems

The game in our model is sequential. First, the fund manager announces the fee f she will charge. Then, the investor chooses the optimal amount ω of her initial wealth she wishes to invest in the mutual fund.

Definition 1. The *investor's demand function* is the mapping $\Omega : \ f_e \mapsto \omega = \Omega(f_e)$. It relates the expense ratio charged by the manager to the fraction of wealth invested by the investor in the mutual fund.

[13]When f_0 includes annualized shareholder costs, the expense ratio f_e is replaced by Total Shareholder Cost (TSC), denoted as f_{TSC}.

We assume that the investor is rational; her demand function $\Omega(f_e)$ is such that it maximizes the expected utility of her terminal wealth $\tilde{W}_i$, conditional on her information set $\mathcal{I}_i$

$$\begin{aligned} \Omega(f_e) = \arg\max_{\omega} \mathbf{E}\left[U_i\left(\tilde{W}_i\right)\middle|\mathcal{I}_i\right], \\ s.t. \quad \omega \geq 0. \end{aligned} \tag{4}$$

This problem makes sense if and only if a solution exists, which requires that the managed fund is not *undesirable*, i.e.

Hypothesis 1. In the absence of any management fees ($f_e = f_0$), the managed fund is not undesirable if

$$\exists \omega > 0, \text{ such that } \quad \mathrm{E}\left[U_i\left((1-\omega)(1+\tilde{r}_i) + \omega(1+\tilde{r}_m)(1-f_0)\right)|\mathcal{I}_i\right] \geq \mathrm{E}\left[U_i\left(1+\tilde{r}_i\right)|\mathcal{I}_i\right] . \tag{5}$$

The demand function is strictly decreasing with respect to f_e. Since we do not allow short-selling of fund shares, it is convenient to define the *reservation* fee as the upper limit for f such that the demand remains always positive.

Definition 2. The reservation fee, i.e. the maximum level of management fees, denoted by $f_{\max}$, is

$$f_{\max} = \min\left\{1 - f_0, \inf\{f|\Omega(f_0 + f) > 0\}\right\}. \tag{6}$$

We immediately get the following result:

Proposition 1. *Given a non-undesirable managed fund, the reservation fee the manager can charge is*

$$f_{\max} = (1 - f_0) - \frac{\mathrm{E}\left[(1+\tilde{r}_i) \cdot U'\left(1+\tilde{r}_i\right)|\mathcal{I}_i\right]}{\mathrm{E}\left[(1+\tilde{r}_m) \cdot U'\left(1+\tilde{r}_i\right)|\mathcal{I}_i\right]}. \tag{7}$$

It is the largest fee that makes the managed fund non-undesirable.

We assume that the fund manager knows that the investor is an utility-maximizer. The fund manager thus chooses the optimal level of fees in response to her *expected* investor's demand, conditional on her own information set $\mathcal{I}_m$. For simplicity,

Hypothesis 2. We assume that the manager knows the expression of the investor's demand function.

Such an assumption is rather strong and may seem both simplistic and unrealistic. On the contrary, as we will see later, this hypothesis is quite reasonable from a practical point of view. Indeed, we shall prove that, irrespective of the specific shape of the investor's utility function, her optimal demand function always remains quite close to a linear (affine) function of the expense ratio. Therefore, denoting by U_m the manager's utility function and by W_0 her initial personal wealth, her optimization problem reads

$$\begin{aligned} &\max_f \mathbf{E}\left[U_m\left(W_0 + \tilde{W}_m\right)\middle|\mathcal{I}_m\right], \\ &s.t. \begin{cases} \omega = \Omega(f_0 + f), \\ f \in [0, f_{\max}]. \end{cases} \end{aligned} \tag{8}$$

In our setting, the manager can only choose the percentage of the management fees f. She plays no role in the determination of f_0, which is exogenously set. The determination of the optimal value of f_0 is a subtle problem and is beyond the scope of this article. *A priori*, f_0 should be kept as small as possible in order to reduce the total fees and therefore attract the largest number of investors. But, among others, f_0 includes the advertisement costs which may increase the demand for the fund as argued by Sirri and Tufano (1998). We make the assumption that the optimal levels for f_0 and f can be determined independently and that f_0 has already been fixed by the various running costs and the commercial strategy of the fund. We can then state the following important result, whose proof is given in appendix A:

Proposition 2. *Let f^* be the solution to the manager's optimization problem (8). If a solution f^* exists, it solves the optimization problem*

$$\begin{array}{l} \max\limits_{f} f \cdot \Omega(f_0 + f) \\ s.t. \quad f \in [0, f_{\max}] \end{array} . \tag{9}$$

The optimal management fee f^* depend neither on the manager's preferences U_m, nor on the manager's perceptions about the distribution of asset returns $(\tilde{r}_i, \tilde{r}_m | \mathcal{I}_m)$.

A priori, since the investors have no bargaining power, the management fees should appear as a commitment from the manager, and therefore should depend on her own preferences. The fact that the optimal management fee does not depend on the manager's preferences is a result of the following assumptions: (i) Investors have no market power, they are price-takers and can only passively react to the fund manager's fee-setting strategy. (ii) The fund manager has a full knowledge of investor's preferences and therefore of her demand function.

It is worth noticing that this result is independent of (i) the distribution of both manager's and investor's portfolio, (ii) the form of the investor's utility function, as long as it is increasing and concave, (iii) the investor's rationality, as long as investors exhibit a decreasing demand function. In detail, as Capon, Fitzsimons, and Prince (1996) suggested, investors don't have to be utility-maximizers. They can exhibit some deviations from pure rationality in their decision process, leading to possibly nonlinear demand functions.

Proposition 2 suggests that all relevant information for the analysis of mutual fund fees is contained in the investor's information set on the fund and on the benchmark portfolio, which is a subset of all the public information available in the market, due to the existence of search-cost, investor's limited information processing and gathering ability[14] and ignorance of easily accessible public information. Proposition 2 is all the more interesting because it makes the fund manager's private information irrelevant to the determination of what should be the right fee level.

This proposition put emphasis on the role of investor's limited ability and knowledge in explaining fund's market power and potential mispricing of the fund services. First, investors may receive biased information about the fund's historical performance. While financial information disclosed by mutual funds have to comply with SEC rules, there is still room for funds to make their performance appear better within these legal constraints. The Standards

[14]This is referred to as bounded rationality, see Simon (1982)

of Practice Handbook of CFA Institute (2005) provide many examples of how funds may potentially beautify their performance. Second, unsophisticated investors may simply follow recommendations from their friends or from the funds themselves, instead of performing their own analysis. Third, it is difficult for most investors to assess correctly the fund's future performance based on publicly available information because of the lack of persistence in returns[15].

3 Characterization of the equilibrium, of the investor demand function and of the optimal management fee

Let us now search for the possible existence of an equilibrium, defined as follows.

Definition 3. An equilibrium solution $(\omega^*, f^*) \in \mathbb{R}_+ \times [0, f_{\max}]$ is a solution to the optimization problems (8) and (4) with $\omega^* = \Omega(f_0 + f^*)$.

We first focus on the case where the investor chooses the risk-free rate as her benchmark. Then, we investigate the consequences of her choice of a risk-portfolio as the benchmark.

3.1 Case where the benchmark portfolio is the risk-free asset

We first consider the case where the benchmark portfolio is the risk-free asset whose return is denoted r_f. We assume that, conditional on the investor's information set, the returns of the managed fund are distributed according to

$$\tilde{r}_m | \mathcal{I}_i \sim \mathcal{N}\left(\bar{r}_m, \sigma_m^2\right), \tag{10}$$

with $\bar{r}_m > r_f$. In order to get a closed form expression, we restrict our attention to the case where the investor is equipped with a CARA utility function. Denoting by a the coefficient of absolute risk aversion of the investor, hypothesis 1 is satisfied if and only if $(1 - f_0)(1 + \bar{r}_m) > (1 + r_f)$. This simply means that, in the absence of management fees, for the manager's fund to be non-undesirable, the expected return of the managed fund, net of operating costs, must be larger than the risk free rate. The demand function which solves the optimization problem (4) then reads

$$\Omega(f_e) = \frac{(1 + \bar{r}_m)(1 - f_e) - (1 + r_f)}{a \cdot \sigma_m^2 (1 - f_e)^2}\ . \tag{11}$$

This allows us to state the following

Proposition 3. *An equilibrium solution exists if and only if hypothesis 1 holds. It is characterized by the demand*

$$\omega^* = \frac{(1 - f_0)^2 \cdot (1 + \bar{r}_m)^2 - (1 + r_f)^2}{4a \cdot \sigma_m^2 (1 - f_0)^2 \cdot (1 + r_f)}, \tag{12}$$

[15]See Berk and Green (2004). Gruber (1996), Carhart (1997). Generally there is no persistence of fund's performance over the long term. However, there is some evidence of persistence over shorter quarterly horizons.

and by the optimal management fee charged by the fund manager

$$f^* = (1 - f_0) \cdot \frac{(1 + \bar{r}_m) \cdot (1 - f_0) - (1 + r_f)}{(1 + \bar{r}_m) \cdot (1 - f_0) + (1 + r_f)}, \tag{13}$$

provided that $f^* \in \left[0, \frac{\bar{r}_m - r_f}{1+\bar{r}_m} - f_0\right]$.

Hypothesis 1 ensures that the interval $\left[0, \frac{\bar{r}_m - r_f}{1+\bar{r}_m} - f_0\right]$ is non-empty, and therefore that an equilibrium solution exists. The proof of Proposition 3 is given in appendix B.

Expression (13) can be approximated by

$$f^* \approx \frac{1}{2}\left(\frac{\bar{r}_m - r_f}{1 + \bar{r}_m} - f_0\right) \equiv \frac{f_{max} - f_0}{2} \ , \tag{14}$$

where f_{max}, as given by definition 2, is the absolute maximum level of fees the manager can charge, as seen from expression (2). Formula (14) shows that the optimal management fee is close to one-half of this maximum value. We will show below that this result is quite general.

Further insight into this result can be obtained by remarking that, since $\bar{r}_m$ is usually much smaller than 1, the optimal management fee is approximately equal to one-half the excess return of the managed fund over the risk free rate minus all other fees. Thus, in equilibrium, the benefits of the fund management resulting in a non-zero excess return over the risk-free rate and the other costs should be equally shared between the investor and the manager. An alternative interpretation is that, given the fee, the investor expects a rate of return on the managed fund equal to

$$\bar{r}_m \approx r_f + 2 \cdot f + f_0, \tag{15}$$

so that her expected gain, net of fees, is

$$\bar{r}_m - f_e = r_f + f, \tag{16}$$

i.e, the risk free rate plus the management fees. Thus, higher management fees must be justified by higher expected returns, both before and after fees.

3.2 Case of a benchmark portfolio made of risky assets

We now assume that, conditional on the investor's information set, the joint distribution of returns of the benchmark portfolio and of the mutual fund is given by

$$\left(\begin{array}{c} \tilde{r}_i \\ \tilde{r}_m \end{array} \middle| \mathcal{I}_i \right) \sim \mathcal{N}\left(\left(\begin{array}{c} \bar{r}_i \\ \bar{r}_m \end{array} \right), \left(\begin{array}{cc} \sigma_i^2 & \rho\sigma_i\sigma_m \\ \rho\sigma_i\sigma_m & \sigma_m^2 \end{array} \right) \right). \tag{17}$$

We still restrict our attention to the case where the investor is equipped with a CARA utility function[16]. We find that Hypothesis 1 is satisfied if and only if $(1 - f_0) \cdot (1 + \bar{r}_m - a\rho\sigma_i\sigma_m) >$

[16] We have also solved this problem for CRRA utility functions. The setting of the optimization problem is given in Appendix D. In this case, the solution does not have a closed analytical form and requires numerical computations. The numerical results confirm the remarkably strong robustness of our analytical result derived for CARA utility functions. Calculations and Figures for CRRA utility functions are available from the authors upon request.

$1+r_f-a\sigma_i^2$. This inequality means that, in the absence of management fees, the risk-adjusted expected return on the managed fund, net of operating costs, must be larger than the risk-adjusted expected return on the benchmark portfolio. We then get

Proposition 4. *An equilibrium solution exists if and only if hypothesis 1 holds. It is then characterized by the demand function*

$$\Omega(f_e) = \frac{1}{a} \cdot \frac{(1+\bar{r}_m - a\rho\sigma_i\sigma_m)\cdot(1-f_e) - (1+\bar{r}_i - a\sigma_i^2)}{\sigma_m^2(1-f_e)^2 - 2\rho\sigma_i\sigma_m(1-f_e)+\sigma_i^2} \tag{18}$$

and by the optimal management fee charged by the fund manager

$$\begin{aligned} f^* &= \frac{R_m\left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right]}{R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2} \\ &- \frac{\sqrt{\left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right]\cdot\left[R_m^2\sigma_i^2 - 2R_mR_i\rho\sigma_i\sigma_m + R_i^2\sigma_m^2\right]}}{R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2}, \end{aligned} \tag{19}$$

where

$$R_m = 1+\bar{r}_m - a\rho\sigma_m\sigma_i \qquad \text{and} \qquad R_i = 1+\bar{r}_i - a\sigma_i^2, \tag{20}$$

provided that $f^* \in \left[0, \frac{R_m-R_i}{R_m} - f_0\right]$.

As stated previously, hypothesis 1 ensures that the interval $\left[0, \frac{R_m-R_i}{R_m} - f_0\right]$ is non-empty, and therefore that an equilibrium solution exists. The proof of Proposition 4 is given in appendix C.

To provide more insight, we expand the cumbersome expression (19) to the first order with respect to $\hat{r}_i = R_i - 1$, $\hat{r}_m = R_m - 1$ and f_0. The optimal management fee can then be simplified into

$$f^* \approx \frac{\hat{r}_m - \hat{r}_i}{2} - \frac{f_0}{2} \approx \frac{f_{\max} - f_0}{2}. \tag{21}$$

This equation has the same structure as (14), except that r_f is now replaced by $\hat{r}_i$. There is also an adjustment for risk, as r_m and r_f are replaced by $\hat{r}_m$ and $\hat{r}_i$ respectively. Again, the optimal management fee is approximately half of the maximum level of fees the manager can charge to the investor.

As proved in Appendix E, this rule is to a large extent independent of (i) the joint distribution of returns on the benchmark portfolio and the managed portfolio and (ii) the form of the investor's utility function, as long as it is increasing and concave. In fact, it holds as long as the investor's demand function is almost linear, which turns out to be a very good approximation for most practical situations.

Relation (21) links fees to the fund's performance conditional on the investor's information. It provides the amount of fees the investor is willing to pay for the fund's investment management service, in equilibrium. This fee is fully characterized by the investor's choice of the benchmark and her anticipation of the future performance of both the fund and the benchmark. More transparently, we have

$$f^* + f_e \approx (\bar{r}_m - \bar{r}_i) - a(\beta_m - 1)\sigma_i^2 \tag{22}$$

This general relation will be the cornerstone of our empirical analysis presented below.

Figure 1 plots the demand function (18) versus the total fee f_e for different values of the coefficient of absolute risk aversion a. The reservation fee $f_{\max}$ is the value corresponding to the intersection of the curves with the horizontal axis. As previously announced, the various demand functions are very close to straight lines. In addition, irrespective of the value of the coefficient of absolute risk aversion a, all the curves intersect at the point

$$f_p = \frac{\bar{r}_m - \bar{r}_i}{1 + \bar{r}_m} \ , \quad \Omega(f_p) := \Omega_p = \frac{\sigma_i^2 - \frac{1+\bar{r}_i}{1+\bar{r}_m}\rho\sigma_i\sigma_m}{\sigma_m^2 \left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2} . \tag{23}$$

The approximate linear dependence of the demand function observed in Figure 1 can be rationalized analytically by a first order expansion of (18) around this fixed point, yielding

$$\Omega(f_e) \approx \Omega_p + \left[\frac{\rho\sigma_i\sigma_m + \left(\frac{1+\bar{r}_i}{1+\bar{r}_m} \cdot {\sigma_m}^2 - \rho\sigma_i\sigma_m\right)\Omega_p - a^{-1}\left(1+\bar{r}_m\right)}{\sigma_m^2 \left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}\right] \cdot (f_e - f_p) \ . \tag{24}$$

The two leftmost terms in the numerator of the fraction are generally much smaller than the rightmost one. Thus, the slope of the demand function is almost inversely proportional to the investor's absolute risk aversion. This linearized expression (24) of the demand function shows that the absolute value of the slope, i.e. the elasticity of the demand, decreases when the coefficient of absolute risk aversion increases. Investors tend to be less sensitive to a change in fees when they are more risk averse.

Two distinct scenarios are presented in Figure 1. In the upper panel, the investor considers the fund to have diversification benefits in the context of her own portfolio strategy. This is the case when $\Omega_p > 0$. The fund's beta, defined as

$$\beta = \frac{\mathrm{Cov}(\tilde{r}_m, \tilde{r}_i)}{\mathrm{Var}(\tilde{r}_i)} \ , \tag{25}$$

is close to one[17]. In the lower panel of Figure 1, the fund is a leveraged fund. In this case, we have $\Omega_p < 0$ and the fund's beta is strictly larger than one.

In the first case, the equilibrium solutions (f^*, ω^*), given by the stars (*) on the different curves, show that the manager actually exploits the diversification value perceived by the investor by charging higher fees when the investor coefficient of absolute risk aversion is larger. This rationalizes the interpretation of Gil-Bazo and Ruiz-Verdu (2007) according to which fees tend to increase when the elasticity of the demand decreases. In the case of a leveraged fund, a reversed relationship is revealed: If funds are risky enough, they attract risk-averse investors by charging smaller fees.

Figure 2 depicts the changes of the optimal management fee (upper panel) and of the corresponding investor's demand (lower panel) as a function of the investor's anticipated correlation between the mutual fund and the benchmark portfolio, for different degrees of the

[17] Actually we have $\beta = \rho\frac{\sigma_m}{\sigma_i} < \frac{1+\bar{r}_m}{1+\bar{r}_i}$, $\bar{r}_m$ is larger than $\bar{r}_i$ and the right term is usually slightly bigger than 1.

investor's absolute risk aversion a. The upper panel predicts that the equilibrium fee decreases when the correlation between the two portfolios increases. This reflects the fact that investors put more value on those funds which provide a greater diversification potential with respect to their benchmark portfolio. In addition, this figure confirms that more risk averse investors accept higher fees when the fund provides diversification benefits.

The lower panel shows the dependence of the demand as a function of the correlation, for different types of investors. For the chosen parameters, we observe that the less risk averse investors put a larger fraction of their wealth in the managed portfolio when the correlation increases, while the more risk averse investors invest less in this portfolio for the same correlation. This reflects the fact that more risk averse investors are more eager to seek diversification.

Figure 3 plots the equilibrium fee (upper panel) and the investors' demand (lower panel) as a function of the investor's expected future volatility (standard deviation) of the benchmark portfolio. The correlation coefficient is set to $\rho = 0.7$. We recall that the expected future volatility of the mutual fund is $\sigma_m = 0.08$. The upper panel shows that, overall, investors accept higher fees when the expected volatility of the benchmark portfolio increases. This result is not surprising in so far as, everything else taken equal, the larger the benchmark volatility, the more attractive the managed portfolio. In addition, as previously, the more risk-averse investors are the more sensitive to an increase of the benchmark volatility and are thus more agreeable to paying higher fees. The lower panel confirms that the more risk-averse investors buy more mutual fund shares than the less risk-averse investors even if the number of shares they buy decreases, overall, when the benchmark volatility increases.

Figure 4 shows the equilibrium fee (upper panel) and the investor's demand (lower panel) as a function of the investor's expected return on the mutual fund. According to expression (21), the equilibrium fee increases with the expected return on the mutual fund. The upper panel shows that the change is almost insignificant with respect to the different levels of risk aversion. In contrast, the lower panel shows that the level of risk aversion affects the demand significantly. More risk-averse investors are more sensitive.

In the upper panel of figures 2, 3 and 4, all curves intersect at one single point. At this point, the optimal management fee is the same for investors with different value of the risk aversion coefficient a. In fact, the management fee in equation (19) depends on the coefficient of risk aversion a only through the ratio

$$\frac{R_i}{R_m} = \frac{1 + \bar{r}_i - a\sigma_i^2}{1 + \bar{r}_m - a\rho\sigma_m\sigma_i} \tag{26}$$

Therefore, if the relation

$$\frac{1 + \bar{r}_i}{\sigma_i} = \frac{1 + \bar{r}_m}{\rho\sigma_m} \tag{27}$$

holds, the optimal management f^* fee is independent from a and it is given by

$$\begin{aligned} f^* \quad = \quad & \frac{\rho\sigma_i^2 - 2(1-f_0)\rho^2\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\rho}{\rho(1-f_0)\sigma_m^2 - 2\rho^2\sigma_i\sigma_m + \sigma_i\sigma_m} \\ & - \frac{\sigma_i\sqrt{\left[(1-f_0)^2\sigma_m^2 - 2(1-f_0)\rho\sigma_i\sigma_m + \sigma_i^2\right] \cdot \left[1-\rho^2\right]}}{\rho(1-f_0)\sigma_m^2 - 2\rho^2\sigma_i\sigma_m + \sigma_i\sigma_m} \end{aligned} \tag{28}$$

This result rationalizes in a general way the two distinct scenarios that we show in figure 1. In one scenario we have

$$\frac{1+\bar{r}_i}{\sigma_i} \geq \frac{1+\bar{r}_m}{\rho\sigma_m} \ , \tag{29}$$

whereas the opposite inequality holds for the alternative scenario. The optimal management fee is either increasing or decreasing in the risk aversion coefficient a in these two scenarios.

4 Empirical Analysis

In the light of the theoretical results presented in the previous section, and particularly relation (22), we now analyze the management fees charged by fund managers between July 2003 and March 2007. For this, we use the CRSP Survivor Bias-Free US Mutual Fund Database.

4.1 Description of the Empirical Model

Our model shows that, in equilibrium, fees are determined solely by the investors' anticipations on the future performance of the mutual funds. Expression (22) quantifies how this anticipation is transformed into an equilibrium fee level that investors agree to pay.

The CRSP Mutual Fund Database only gives access to the *ex-post* performance of the funds through their historical returns. Our strategy is to infer the *ex-ante* expectations of the investors on the fund performance on the basis of the amount of fees they are willing to pay, by using relation (22). In this way, we test the following questions:

(i) Does the performance achieved by funds justify the fees they charge, given a rational choice of the benchmark asset?

(ii) Do funds possess a competitive advantage in terms of fees, realized returns and diversification benefits, when compared to the benchmark asset?

(iii) Do investors correctly anticipate funds relative performances, given their benchmark asset?

As the industry practice suggests, a natural choice of the benchmark portfolio that investors should use is an index portfolio for the market in which the fund operates. In the following empirical test, we impose the S&P500 total return index[18] to be the investor's benchmark portfolio for U.S. domestic equity mutual funds. This is in line with the fact that investors can buy Exchange Traded Funds(ETFs) or low-cost index funds to achieve index performance while paying a nearly-zero cost.

[18]We also use the Dow Jones Industrial Average total return index for a robustness check of our empirical results.

For convenience, we define both the *after-fees excess return*[19] and the *adjusted beta* for the fund j over a given period, as follows:

$$\text{after fees excess return}^j = (\bar{r}^j - \bar{r}_{index}) - (\bar{f}^j + \bar{f}^j_{TSC}) \ , \quad (30)$$

$$\text{adjusted beta}^j = (\beta^j - 1)(\sigma_{index})^2 \ , \quad (31)$$

where

$\bar{f}^j$ and $\bar{f}^j_{TSC}$	=	average management fee and average total shareholder cost,
$\bar{r}^j$ and β^j	=	fund's realized average return and beta,
$\bar{r}_{index}$ and σ_{index}	=	realized average return and volatility of the market index.

Similarly to Khorana, Servaes, and Tufano (2009), we define fund j's total shareholder cost (TSC) as a sum of both annual total expenses and annualized shareholder fees, given a five-year holding period[20] in our analysis:

$$\bar{f}^j_{TSC} = \bar{f}^j_e(\text{average TER}) + \text{front-load}/5 + \text{back-end load at five years}/5. \quad (32)$$

Then, expression (22) leads to the regression model:

$$\text{after fees excess return}^j = a \cdot \text{adjusted beta}^j + b + \varepsilon_j, \quad (33)$$

where a stands for the investor's risk aversion and b is an intercept that should equal zero if investor's ex-ante expectation matches exactly fund's ex-post performance. The value of the bias b reflects the deviation of the ex-post performance of the funds from the investors' ex-ante expectations of the fund performance. A positive (respectively negative) value of the bias b can be interpreted as the fact that investors underestimate (respectively overestimate) the funds relative performance.

When the investors' benchmark is the risk-free asset, equation (15) predicts that investors do not demand any excess return associated with the fund's risk. Investors expect ex-ante a minimal return of $\bar{r}_{risk-free} + f_{TSC} + f$. Given the fund's ex-post return $\bar{r}$, we define the bias b_* for the fund j as follows:

$$b^j_* = \bar{r}^j - \bar{r}^j_{risk-free} - f^j_{TSC} - f^j \ . \quad (34)$$

If our model was the whole story of what determines the strategic interactions between homogeneous investors and mutual fund managers, and what represents the risk-return performances of mutual funds and of the benchmark asset, then the regression model (33) should provide directly a unique estimation of investors' risk aversion for the whole mutual fund universe. However, this expectation is of course naive, given the heterogeneity of mutual funds and of investors. Notwithstanding the formidable problem of making sense of the extraordinary heterogeneity in fund performance and in their fee structure, we can nevertheless identify remarkably robust and meaningful regularities. The key insight was to organize our universe

[19] We have used $\bar{f}^j_{TSC}$ instead of $\bar{f}^j_e$ to calculate after-fees excess return, because the former includes shareholder fees that are not included in the TER. Shareholder cost include both front-loads and back-end loads. It is simple to show that these loads, if annualized, can be added directly to the annual total expense ratio.

[20] The annual redemption rate is equal to 20%.

of mutual funds into deciles of decreasing risk-adjusted return performance quantified by their Sharpe ratio. As we show in the following sections, remarkably good regressions with model (33) are found, which provide insightful economic interpretations. Particularly, we identify different groups of investors characterized by their specific risk aversion coefficient a. And we are able to relate these groups to distinct fund characteristics, such as their leverage level and their relative performance. Our model also allows us to identify several subclasses of "abnormal" funds, which either provide good diversification and after-fees over-performance or give lower diversification benefits and sub-performance. This classification is performed on the basis of clustering analysis and the values of the regression intercept b.

4.2 Description of the Data Sample

We obtained our sample from the CRSP Survivorship-Bias-Free US Mutual Fund Database. The CRSP mutual fund database contains monthly data for more than 25,000 U.S. open-end mutual fund from January 1, 1962 to March 2007. Data on total expense ratios (TER) for US mutual funds is available from January 1962 to March 2007. However, management fees, front-loads and back-end loads have only been reported since July 2003. Therefore, our initial sample is chosen to contain all open-ended mutual funds operating during the period form July 2003 to March 2007. From this initial sample, we excluded all non-domestic equity mutual funds. In selecting funds, we resort to the Standard & Poor's category codes and the geographic codes provided by the CRSP.

Furthermore, we removed funds that have no complete information on TER, management fees, loads, monthly total returns, or total net assets throughout the entire time interval from July 2003 to March 2007. Examination of the remaining sample showed that some funds have weighted average operating expenses that are anomalously large for mutual funds. For example, some funds have a weighted average operating expense over 40%. We also deleted these funds, as being outliers.

The sample under study contains therefore 3,875 US domestic equity mutual funds with 170,500 fund-month observations. Table 1 characterizes our sample. We present descriptive statistics of funds for their weighted average total net assets (TNA), their asset-weighted average turnover ratio, their Sharpe ratio and the various fees such as the average TER, average management fees, average total shareholder cost (TSC), both front-load and back-end load fees.

We include both passive and active funds in our studies, as shown in Table 1 which exhibits turnover ratios ranging from zero to 2582% per year[21]. We are able to do so because our model does not make any assumptions on the funds' underlying trading mechanism. This distinguishes our studies from several others studies[22].

[21]This later figure implies that the fund portfolio is totally rebalanced 26 times per year, or once every two weeks, on average.

[22]Several studies such as Gruber (1996), Wermers (2000) and Glode (2008) focus on actively managed mutual funds.

4.3 Descriptive Statistics and Characteristics of Sharpe Ratio-Sorted Domestic Equity Funds

In Table 2, we sort our selected set of 3,875 domestic equity funds into 10 deciles according to their Sharpe ratios. We present several descriptive statistics. For each decile, we provide the mean value of the Sharpe ratio, total returns, fees, turnover, and beta.

Table 2 shows that the total expense ratio (TER), total shareholder costs (TSC) and management fees tend to increase when the Sharpe ratio decreases. Funds from the bottom decile charge on average 14 basis points more in management fee and 49 basis point more in TSC than top decile funds. These differences are economically significant. This is in line with existing empirical evidence that worse performing funds tend to charge higher management fees and total expenses. Worse-performing funds tend to also have a higher beta and a higher turnover ratio.

Another interesting observation provided by Table 2 is that funds from the last two Sharpe ratio deciles charge the highest back-end loads and very low front-loads, compared to better performing funds. This could be rationalized as a strategic behavior of mutual fund managers, exploiting deficient information gathering or inattention on the part of investors: poor-performing funds attract investor's initial investment by lowering front-load fees. This strategy makes sense, given the observation by Barber, Odean, and Zheng (2005) that investors are more sensitive to salient fees such as front loads than to operating expenses. Then, by raising back-end load fees, bad-performing funds hinder investor's redemption activity. In support of this reasoning, Alves and Mendes (2007) and Nanigian, Finke, and Waller (2008) indeed suggested that the lack of reaction of investors to bad-performing funds is caused by the existence of back-end loads.

Furthermore, Table 2 shows that both 12b-1 fee and the maximal level of 12b-1 fees tend to increase when the fund's Sharpe ratio decreases. This suggest that bad-performing funds tend to spend more on marketing and distribution than better-performing funds. This is in line with previous reports that worse-performing funds tend to charge higher fees than better-performing funds[23]. Funds from the first four deciles have seized a share of around 60% of the fund market, and the funds from last two deciles still keep approximately 10% market share.

In each Sharpe-ratio based deciles, funds have various investment styles. Table 3 lists all available S&P investment styles and codes for US domestic equity funds. Empirical studies[24] have shown that funds with various styles exhibit different performances. These funds may target different groups of investors, with diverse degrees of risk aversion. It may thus be fruitful to distinguish distinct fund segments within a single Sharpe-ratio based decile. A first idea would be to classify funds according to their investment styles. This approach is however unreliable because funds often change their investment style and sometimes report misleading information [25]. Below, we will use a cluster analysis based on the formulation of our regression

[23]See Elton, Gruber, and Das (1993), Gruber (1996) and Chevalier and Ellison (2002).

[24]See Wermers (2000),Brown and Goetzmann (1997) , Chan, Chen, and Lakonishok (2002), Brown and Harlow (2002), Barberis and Shleifer (2003), Horan (1999).

[25]Brown and Goetzmann (1997) argued that reported investment styles are not satisfying in terms of reflecting a fund's true investment activities. Brown and Goetzmann (1997) and Donnelley (1992) have shown that some funds misclassify themselves.

model, that reveals striking regularities in the best- and worst-performing funds.

4.4 Empirical Tests of the Model

Figure 5 plots the after-fees excess returns for the whole sample of 3,875 US domestic equity mutual funds as a function of their adjusted beta's. The regression line obtained from relation (33) is also shown. The bias is significant and negative ($b = -1.2\%$) but the determination coefficient R^2 is very small, so that the model has nearly no explanatory power when applied to the whole sample of mutual funds. This result is not surprising given the huge heterogeneity of fund styles and investor preferences.

Given that investors have heterogeneous risk aversions and, as a consequence, look for different risk vs. return trade-offs, it is necessary to sort funds so as to isolate homogeneous sub-samples. As suggested by Hartman and Smith (1990), funds investors can be best segmented by their risk tolerance. Each market segment characterized by a given risk-return profile corresponds to a homogeneous class of investors with a common coefficient of risk aversion. We consider the simplest and most generally used measure of risk-return profile, namely the Sharpe ratio. By sorting and grouping funds by their Sharpe-ratios, our regression model identifies new regularities about mutual funds, which can be traced back to the risk aversion of their investors.

For each sub-sample, we use the mixed Gaussian clustering method (Press, Teukolsky, Vetterling, and Flannery 2007) for a refinement of the analysis of our universe of mutual funds. Brown and Goetzmann (1997) already developed a cluster classification scheme based on fund returns, that identified seven clusters. Our cluster analysis is however completely different because it uses the fund data organized according to the regression model (33). In other words, different clusters, if any, correspond in our approach to different pairs of after fees excess return and adjusted beta. This can be interpreted to correspond to different investors risk aversion a and different abnormal excess returns b.

Figure 6 is typical of the results for the different Sharpe ratio deciles. It shows the existence of two main clusters for the second decile and three clusters for the bottom decile. In each cluster, the linear regression (33) provides an excellent fit to the data, with R^2 values in the range from 0.45 to 0.84. This provides a strong support to our model, while at the same time offering a novel tool for identifying abnormal funds. We refer to the largest cluster as the "normal" set constituted of mutual funds with a well-defined risk-adjusted performance and a clear selection of investors with a specific risk-aversion level. The other smaller clusters identify anomalies, whose characteristics are described below.

Since the benchmark portfolio corresponds to the origin of these plots, distances from and positions with respect to the origin provide a natural classification of fund performance. It is convenient to label funds with a beta smaller (respectively larger) than one as "diversification funds" (respectively "leveraged funds"). The top-left side of the two panels of Figure 6 corresponds to the best funds which provide both diversification benefits and absolute benchmark-beating performances. Funds in the bottom-right side of the panels of Figure 6 are the worst funds, that are inferior to the index, even with leverage. Regardless of investors' risk aversion, funds in the top-left side provide superior relative performance and funds in the top-right side provide inferior relative performance, when compared with the benchmark portfolio. Funds in the top-right side of the panels provide index-beating performance, but at the price of higher risks (higher leverage). Funds in the bottom-left side of the panels provide diversification benefits but fails to beat the benchmark's return after fees. The comparison of fund performance between funds from the top-right side and bottom-left side and the benchmark depends on how investors value risks against returns, i.e. on the risk aversion coefficient in our model.

Consider first the upper panel of Figure 6. The two regressions on the two clusters with model (33) show that investors of the abnormal funds (upper cluster) have a higher risk aversion (slope a of the regression line) compared with investors of the normal funds. This is rational, given the fact that most of these funds are diversification funds. In the lower panel of figure 6, there is a cluster of under-performing funds, for which our model determines that the corresponding investors have quite low risk aversion, in line with the fact that most of them are leveraged funds. In addition, abnormal funds in the 2^{nd} decile generate on average much higher returns than normal funds. In contrast, abnormal funds generate much lower returns than normal funds in the last decile. The segmentation of the universe of funds according to the two variables, after fees excess return and adjusted beta, thus provides an intuitive explanation of the impact of the risk aversion on the fund's performance relative to the market index. The higher the absolute value of the slope a, the more risk averse are investors. This rationalizes the segmentation found in the ten different deciles.

Table 4 complements Figure 6 by providing data on the size of each different segments, according to the different investment styles. Specifically, for each decile of the distribution of Sharpe-ratios and for each segment identified by the cluster analysis, we report the number of funds of each investment style. There are total 20 investment styles for all US domestic equity funds. According to this table, abnormal funds are mainly funds that operate in special industries such as the Equity US Real Estate Sector (URE), the Equity Utilities Sector (UTI), the Equity Energy Sector (NRG) and the Equity Information Technology Sector (UTE). In the top decile, abnormal funds consist mainly of utilities funds and real estate funds. In the last decile, information technology funds dominate in the segments of abnormal funds. We confirm, as shown in many studies[26], that investment styles have a large impact on performance. For instance, we find that growth funds tend to underperform value funds. Moreover, we find that sector funds such as real estate funds and energy, utility funds have performed very well over the time period from July 2003 to March 2007. Real estate funds have benefited from a booming housing market, whereas energy and utility funds have benefited from soaring oil prices during this time. In contrast, information technology funds were consistently underperforming within this period. While these results are not new in themselves, the good correspondence with our robust cluster segmentation and our regression model provides additional support for our theory.

Table 5 presents the breakdown of the population of funds, classified according to the Sharpe ratio deciles, their leverage and excess returns, both for normal and abnormal funds. Unsurprisingly, diversification funds with positive after-fee excess return (AFER) are mostly concentrated in the top decile. At the other extreme, leveraged funds with negative after-fee excess return (AFER) are mostly concentrated in the bottom decile. The two other categories

[26] See Chan, Chen, and Lakonishok (2002), Brown and Harlow (2002), Horan (1999).

are more uniformly populated. One can also observe that most normal funds are leveraged funds with a negative after-fee excess return. In the last three deciles, more than 80% of the funds belong to this category. In contrast, diversification funds dominate in the top two deciles only: The top (respectively second) decile has about 60% (respectively 50%) of diversification funds. Table 5 shows that only 51% of funds in the top decile and 12% in the next decile beat the index-tracking funds, whereas over 60% of funds from the last four deciles underperform the benchmark funds. This result is an indication of the funds underperformance over the period under consideration.

Table 6 presents the results of the OLS regression of the funds' after fee excess returns against their adjusted beta according to (33), for both normal and abnormal funds within each Sharpe ratio decile. We also provide the mean value of the Sharpe ratio, total returns, fees, turnover, beta, and investors ex-ante expected returns. We find that, compared to normal funds, abnormal funds tend to charge higher fees as shown in their TSC and TER. They also have higher turnover and higher standard deviation.

For both the normal and the abnormal funds, we find that the coefficient a of risk aversion tend to be larger for the top decile (better-performing) funds than for the bottom deciles (worst-performing) funds. Interpreted within our model, this indicates investors' self-selection according to the inverse risk measure provided by the Sharpe ratio. This behavior is fully rational: the more risk-averse investors choose the less risky funds, i.e. those with the highest Sharpe ratio. In contrast, the less risk-averse investors choose the most risky funds, i.e., those with the smallest Sharpe ratio. In addition, the underperforming funds are mostly leveraged funds with an average beta of 1.30 for normal funds and an average beta of 2.3-2.5 for abnormal funds. Therefore, this situation is in accordance with the second scenario obtained from our theoretical model and exemplified on the lower panel of Figure 1: when funds are leveraged, managers tend to charge either higher fees to less risk averse investors or tend to offer lower fees to attract the more risk averse investors.

Furthermore, the coefficient b of the regression (33) is found statistically different from zero (see below) for all deciles. We find that only the funds in the top and the 2^{nd} deciles exhibit a positive value of the intercept b. These funds perform better than low-cost index-tracking investment vehicles, and outperform the investors' ex-ante expected performance. Within all abnormal funds, funds from the first four deciles have a very significantly positive value of the bias b. These funds have performed well in this period. However, the majority of U.S. equity mutual funds exhibit a negative value of b. More specifically, b is significantly smaller than zero for eight of ten deciles for normal funds and for five of the 10 deciles for abnormal funds. Within our regression model, this can be interpreted as an evidence that about 70-80% of US domestic equity funds have added markups over the period from July 2003 to March 2007. The rationalization is the following: In a market with perfect competition, if a fund underperforms the benchmark in terms of returns, diversification benefits and fees, for several years, it either has to lower down its fees to match its relative after-fees performance to the benchmark asset, or to exit the market. The continuing existence of these seriously underperforming funds is an indication for possible markups in the fund industry. Along with statistics on fees across all deciles in Table 6, our model provides a natural characterization for the well-known observation that worse-performing funds tend to charge higher fees than better-performing funds[27].

Using the OLS regression model (33), we estimate the investors ex-ante expected returns defined by

$$\text{Ex-ante expected return} = \text{Total return} - \text{TER} - b. \tag{35}$$

where TER refers to the total expense ratio. We find that the investors ex-ante expected return is remarkably well-behaved, when the S&P500 index is taken as the investor's benchmark. The mean value of the investors' expected annual return is approximatively constant across all Sharpe-ratio sorted deciles and is confined within a small range of 14 -15.7%. ANOVA tests for the mean (Hogg and Ledolte 1987) show that we cannot reject the hypothesis that the mean value of investors' expected return of funds from the 2nd, 3rd, 5th, 6th, 7th and 8th deciles are equal, at the 10% significance level. We come to the same conclusion for funds from the 1st, 4th and 9th deciles. This results suggest that investor's ex-ante expected return shows no upward or downward trend as a function of the Sharpe ratio[28]. Within our model, this suggests that, irrespective of their risk aversion, investors have homogeneous anticipations. They expect a typical mutual fund to deliver an excess-return after fees of approximately 3% above the benchmark. This constancy of the investors ex-ante expected returns is a strong point in favor of the explanatory power of our model, as it identifies one "universal" in an otherwise complex universe of mutual fund characteristics.

Investors' optimism bias is revealed by (i) the negative values of b and (ii) investors' high ex-ante expected returns. As negative b's are found for all deciles except the top two, this suggests that most investors overestimate the performance of funds in terms of fee-adjusted excess returns and of diversification potential. A possible origin of this optimism bias may be ascribed to the lack of financial literacy, as reported in Capon, Fitzsimons, and Prince (1996) and Alexander, Jones, and Nigro (1998). Funds' marketing efforts, as reflected by their 12b-1 fees, could also play an important role. Table 6 shows that both normal and abnormal funds of the last two deciles tend to spend significantly more on marketing and distribution than funds from the first two deciles.

Finally, the last column of Table 6 presents the expected return biases b_* of various deciles sorted by their Sharpe ratios, calculated according to equation (34) derived from our model, assuming that the investors choose a risk-free asset, such as their bank savings or a T-Bill as their benchmark. We actually take the four-weeks treasury bills as the proxy for the risk-free benchmark. The last column of Table 6 shows that the biases b_* of normal funds in all deciles and of the abnormal funds in the top eight deciles are significantly positive. Nearly all US equity funds exhibit over-performance, when investors take the bank saving as their benchmark. This suggests that the investors' optimism bias may come from the choice of such a suboptimal benchmark, possibly due to lack of sufficient financial literacy.

[27] See Elton, Gruber, and Das (1993), Gruber (1996) and Chevalier and Ellison (2002).

[28] We used the standard boxplots, Kruskal-Wallis test (Siegel and Castellan 1988) and ANOVA test (Hogg and Ledolte 1987) to demonstrate the equality of the mean value of the expected returns on the funds from the same Sharpe-ratio sorted decile. Details and figures are available upon request from the authors.

5 Conclusion

In order to understand why investors are buying underperforming investment vehicles, we have proposed a one-period principal-agent model based on a sequential game played by a representative investor and a fund manager in an asymmetric information framework. Our first main result is that only investor preference and information set determines the fee level of mutual funds. The manager's true ability is irrelevant here. Second, we have derived an analytical formula and provided an empirical framework that can help investors to gauge their funds and their portfolios. Third, our model has identified two alternative fee-setting scenarios depending on the fund's possible diversification benefits. Leveraged funds tend to exploit demand insensitive investors by charging them higher fees while funds providing diversification benefits lower fees to attract more risk averse investors and charge higher fees to the less averse investors. A salient point of our model is that investor are making rational decisions, but these are based on limited, misguided or incorrect information as a result of their possible misperception about the fund returns and the overall market. This misperception is identified in the later empirical results as investors' over-optimism about funds' future returns, which suggests possible mismatch between information perceived by the investors and the reality.

Our empirical study of the U.S domestic equity fund market over the period from July 2003 to March 2007 has identified positive markups for around 80% of the funds in our database. This basically means that these funds underperform low-cost index funds or ETFs, after taking the returns, the diversification benefits and the fees into account. However, investors keep investing in these underperforming funds. Within the information asymmetry framework of our model, we have shown that this puzzling investment behavior can be interpreted as an optimism bias towards funds' future performances. We have been able to estimate that, on an *ex-ante* basis, investors expect the fund managers to deliver an overall annual excess-return of around 3% over the S&P 500, net of fees, irrespective of the investment style and of the risk level of the funds. Investor's optimistic expectations of the fund market leads to the high markups in today's fund market. The correlation between investors' overconfidence and the high management fees and distribution (12b-1) fees found in our analysis suggest that the later play a role in promoting the former. Another element for investors' optimism bias is their lack of financial knowledge. More specifically, we demonstrated that this optimism bias can be rationalized by assuming that investors choose a risk-free asset as their benchmark. Our empirical analysis suggests that both factors may explain investor's overconfidence.

Our one-period model provides a static view of investors' behavior whose main advantage is its simplicity and versatility. We did not need any assumption on the fund management strategy. We focused on the crucial effect of information asymmetry on the pricing of mutual funds in order to disentangle it from learning effects. Our results raise the intriguing question of why investors have been continuously overoptimistic over time, apparently failing to learn the lessons of past under-performance of their investments. To address this question, a dynamic framework that includes learning would be needed. A priori, both asymmetric information and lack of learning may contribute to higher pricing of funds. This paper demonstrated the role of the former ingredient. The study of the impact of learning and of its lack thereof is worthy of future research.

Table 1: Summary Statistics of US Domestic Equity Mutual Funds

This table summarizes the main features of the 3,875 US Domestic Equity Mutual Funds extracted from the CRSP Survivorship-Bias-Free US Mutual Fund Database over the period from July 2003 to March 2007. Sharpe ratios are calculated according to the definition given by Sharpe (1994). The risk free rate is approximated by the four-weeks U.S. treasury bills. Fees, returns and Sharpe ratios are calculated with monthly data and then annualized. TSC refers to total shareholder costs. 12b-1 fee is the annual fee that funds have charged for marketing and distribution. Max 12b-1 is the maximal level of 12b-1 fee that funds could have charged. Both front-load and back-end load are incurred when investors buy or sell fund's shares.

	Total Net Assets ($1M)	Expense Ratio (%/year)	Mgm Fee (%/year)	12b-1 Fee (%/year)	Max 12b-1 (%/year)	Front-load (%)	Back-End Load (%)	TSC (%/year)	Turnover (%/year)	Total Return (%/year)	Std. Dev. (%/year)	Beta	Sharpe Ratio
Mean	528.71	1.46	0.70	0.39	0.42	1.19	1.04	1.70	96	13.53	10.66	1.17	1.02
S.D.	2537.92	0.59	0.24	0.40	0.42	2.18	1.69	0.71	141	5.01	3.35	0.31	0.42
Median	76.74	1.39	0.72	0.25	0.25	0.00	0.00	1.83	71	13.13	9.93	1.13	1.04
min	0.10	0.02	0.00	0.00	0.00	0.00	0.00	0.02	0	-23.15	2.43	-2.36	-1.67
max	74094.09	7.69	2.50	1.00	1.00	8.50	6.00	8.84	2582	38.72	28.89	3.31	2.87

Table 2: Summary statistics for Sharpe Ratio-Sorted US Domestic Equity Funds

This table summarizes the main features of all ten Sharpe Ratio-sorted deciles of the 3,875 US Domestic Equity Mutual Funds extracted from the CRSP Survivorship-Bias-Free US Mutual Fund Database over the period from July 2003 to March 2007. TSC refers to total shareholder costs. 12b-1 fee is the annual fee that funds have charged for marketing and distribution. Max 12b-1 is the maximal level of 12b-1 fee that funds could have charged. Both front-load and back-end load are incurred when investors buy or sell fund's shares. TNA is the median of the decile and all other fees and ratios are the mean of the corresponding deciles. Market Share is all total net assets from a decile divided by total net assets of all domestic equity funds under consideration.

Fractile	Avg No	Sharpe Ratio	Total Return	Std. Dev. (%/year)	Expense Ratio (%/year)	Mgm Fee (%/year)	12b-1 Fee (%/year)	Max 12b-1 (%/year)	Front-load (%)	Back-End Load (%)	TSC (%/year)	Beta	Turnover (%/year)	TNA (million $)	Market Share (%)
Top 10%	387	1.76	18.97	8.54	1.27	0.66	0.32	0.34	1.33	0.84	1.54	0.90	69	165.47	13.5
2nd 10%	388	1.44	18.11	9.98	1.41	0.68	0.39	0.42	1.20	1.03	1.65	1.04	57	85.61	14.7
3rd 10%	387	1.29	16.43	9.96	1.28	0.63	0.34	0.37	1.22	0.93	1.52	1.09	68	112.22	16.0
4th 10%	388	1.18	14.54	9.47	1.22	0.57	0.32	0.34	1.04	0.81	1.43	1.11	75	104.575	14.1
5th 10%	387	1.09	14.59	10.36	1.44	0.67	0.41	0.44	1.22	1.08	1.68	1.19	87	84.86	8.1
6th 10%	388	0.99	13.58	10.63	1.51	0.72	0.41	0.44	1.15	1.11	1.74	1.20	95	78.935	6.7
7th 10%	387	0.86	12.32	10.96	1.52	0.73	0.40	0.43	1.20	1.09	1.76	1.21	116	68.8	8.6
8th 10%	388	0.72	11.06	11.59	1.53	0.75	0.38	0.41	1.34	1.00	1.80	1.28	128	53.74	8.4
9th 10%	387	0.58	9.47	12.08	1.64	0.76	0.45	0.48	1.13	1.26	1.86	1.29	136	44.8	5.1
Bottom 10%	388	0.31	6.25	13.02	1.81	0.80	0.49	0.52	1.10	1.26	2.03	1.33	131	41.11	4.9
S&P 500 TRI		1.24	12.37	7.36								1.00	0		

Table 3: S&P Investment Styles and Codes

Codes	Investment Style
ACG	Equity All Cap Growth
ACV	Equity All Cap Value
LCB	Equity Large Cap Blend
LCG	Equity Large Cap Growth
LCV	Equity Large Cap Value
MCB	Equity Mid Cap Blend
MCG	Equity Mid Cap Growth
MCV	Equity Mid Cap Value
NRG	Equity Energy Sector
NTR	Equity Materials Sector
SCB	Equity Small Cap Blend
SCG	Equity Small Cap Growth
SCV	Equity Small Cap Value
SEC	Equity Sector
UCO	Equity Telecommunications Sector
UFI	Equity Financial Sector
UHC	Equity Healthcare Sector
URE	Equity US Real Estate Sector
UTE	Equity Information Technology Sector
UTI	Equity Utilities Sector

Table 4: Cluster Analysis and Investment Styles Statistics

For each decile of the distribution of Shape ratio, this table reports the results of the cluster analysis and the number of mutual funds in each of the 20 S&P investment styles. We have a total of 3,875 US Domestic Equity Mutual Funds over the time period from July 2003 to March 2007. All deciles except the last decile can be classified into two fund segments. The method used for the cluster analysis is the mixed Gaussian method (Press, Teukolsky, Vetterling, and Flannery 2007). All the abbreviations are reported in Table 3.

Fractile	Cluster Analysis			Investment Styles																			
	No of Clusters	Rank	Avg No	ACG	ACV	LCB	LCG	LCV	MCB	MCG	MCV	NRG	NTR	SCB	SCG	SCV	SEC	UCO	UFI	UHC	URE	UTE	UTI
Top 10%	2	1	300	1	8	18	0	176	4	0	67	0	0	2	0	13	6	0	4	0	0	0	1
	2	2	87	0	2	0	0	3	0	0	4	2	0	0	0	2	0	0	0	2	25	0	47
2nd 10%	2	1	323	4	19	35	3	164	13	8	25	0	0	3	7	36	1	5	0	0	0	0	0
	2	2	65	0	0	0	0	0	0	0	0	16	0	0	0	0	0	0	0	2	47	0	0
3rd 10%	2	1	347	7	14	86	4	88	42	15	11	0	2	4	2	63	6	1	2	0	0	0	0
	2	2	40	0	0	0	1	0	0	0	0	12	0	0	0	1	0	0	1	5	20	0	0
4th 10%	2	1	375	8	12	155	10	49	28	22	14	0	3	15	11	45	0	0	3	0	0	0	0
	2	2	13	0	0	1	0	1	0	0	0	3	0	0	0	0	1	0	0	3	4	0	0
5th 10%	2	1	370	9	4	113	18	41	18	35	7	0	0	47	24	39	4	0	7	0	0	4	0
	2	2	17	0	1	1	0	2	0	1	2	2	0	4	2	0	1	0	0	0	1	0	0
6th 10%	2	1	376	17	6	66	57	26	4	54	10	0	0	39	32	41	5	0	13	1	0	5	0
	2	2	12	1	0	1	2	0	0	0	0	0	0	0	0	0	2	0	0	1	3	2	0
7th 10%	2	1	371	24	2	43	96	14	3	69	5	0	0	25	52	17	5	1	11	2	0	2	0
	2	2	16	0	0	0	0	0	0	2	0	0	0	1	0	0	4	0	2	6	1	0	0
8th 10%	2	1	353	12	2	26	135	2	0	66	4	0	0	9	80	2	6	1	3	1	0	4	0
	2	2	35	0	0	2	5	1	1	6	0	0	0	0	5	1	2	0	2	9	0	1	0
9th 10%	2	1	384	14	3	13	180	0	2	44	0	0	0	2	64	7	2	5	2	21	0	25	0
	2	2	3	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0
Bottom 10%	3	1	249	10	0	12	149	0	6	6	0	0	1	0	21	2	3	2	3	15	0	19	0
	3	2	112	2	0	0	26	0	0	0	0	0	0	3	20	0	0	0	0	4	0	57	0
	3	3	27	1	0	6	2	0	0	0	0	0	0	1	1	0	9	0	0	2	0	5	0

Table 5: Sharpe Ratio-Sorted U.S. Domestic Equity Funds Analysis I

For each decile of the distribution of Shape ratios, this Table reports the number of mutual funds in each of four following mutual fund categories: (i) leveraged funds with positive after-fee excess return (AFER), (ii) leveraged funds with negative AFER, (iii) diversification funds with positive AFER and (iv) diversification funds with negative AFER. We have a total of 3,875 US Domestic Equity Mutual Funds over the time period from July 2003 to March 2007. We perform count statistic for normal funds and abnormal funds separately.

Fractile	Number of funds	Leveraged Fund		Diversification Fund	
		Positive AFER (%)	Negative AFER (%)	Positive AFER (%)	Negative AFER (%)
Normal Funds					
Top 10%	300	35.67	0.00	51.00	13.33
2nd 10%	323	51.08	3.10	12.07	33.75
3rd 10%	347	54.76	19.88	0.58	24.78
4th 10%	375	40.80	28.80	0.00	30.40
5th 10%	370	43.24	36.49	0.00	20.27
6th 10%	376	33.78	46.28	0.00	19.95
7th 10%	371	16.44	62.26	0.00	21.29
8th 10%	353	1.13	89.52	0.00	9.35
9th 10%	384	0.26	86.72	0.00	13.02
Bottom 10%	249	0.00	78.31	0.00	21.69
Abnormal Funds					
Top 10%	87	10.34	0.00	87.36	2.30
2nd 10%	65	38.46	0.00	61.54	0.00
3rd 10%	40	22.50	0.00	75.00	2.50
4th 10%	13	23.08	0.00	46.15	30.77
5th 10%	17	76.47	0.00	11.76	11.76
6th 10%	12	50.00	8.33	16.67	25.00
7th 10%	16	25.00	6.25	18.75	50.00
8th 10%	35	8.57	42.86	0.00	48.57
9th 10%	3	0.00	0.00	0.00	100.00
Bottom 10%	112	0.00	98.21	0.00	1.79
Bottom 10%	27	0.00	33.33	0.00	66.67

Table 6: Sharpe Ratio-Sorted U.S. Domestic Equity Funds Analysis II

This Table presents the results of the OLS regression of the after fees excess return against its adjusted beta

$$\text{after fees excess return}_j = a \cdot \text{adjusted beta}_j + b + \epsilon_j$$

with

$$\text{after fees excess return}^j = (\bar{r}^j - \bar{r}_{index}) - (\bar{f}^j + \bar{f}^j_{TSC}) \ ,$$
$$\text{adjusted beta}^j = (\beta^j - 1) \cdot \sigma^2_{index} \ ,$$

for 3,875 US Domestic Equity Mutual Funds during the period from July 2003 to March 2007. For each decile of the distribution of Sharpe ratios, we perform the OLS regression on both normal funds and abnormal funds separately. The classification in normal and abnormal funds is based on the cluster analysis reported in Table 4. We report the t-statistics (in parenthesis) and the significance levels for the coefficients a and b, of the regression as well as the R^2. ***,** and * denote the significance at the 1%, 5% and 10% levels, respectively. Both after fees excess return and adjusted beta are calculated with monthly data and then annualized. Results with robust regression from Huber (1981) are very similar to the OLS regression and are available upon request from the authors. TSC refers to Total Shareholder costs, TRI refers to the total return index and TNA refers to total net assets. The Expected return reflects investors' ex-ante expectation and is defined as

$$\text{Expected Return} = \text{Total Return} - \text{TER} - b.$$

The investors' bias b_*, given the bank saving as the benchmark, is defined as

$$b_* = \bar{r} - \bar{r}_{risk-free} - f_{TSC} - f$$

We use the return on the four week treasury bill as the return on the bank saving. The average annual return on the four week treasury bill from July 2003 to March 2007 is 2.89%. The t-test is used to infer statistical significance. TNA is the median of the decile and all other fees and ratios are the mean of the corresponding deciles.

Fractile	Avg No	Sharpe Ratio	coeff a	coeff b (%/year)	R Square	Total Return (%/year)	Std. Dev. (%/year)	Expense Ratio (%/year)	Mgm Fee (%/year)	12b-1 Fee (%/year)	Max 12b-1 (%/year)	Front Load (%)	Back Load (%)	TSC (%/year)	Beta	Turnover (%/year)	TNA (million $)	Expected Return (%/year)	b_* (%/year)
The Whole Sample																			
	3875	1.02	-0.018(-0.038)	-1.24***(-13.25)	0	13.53	10.66	1.46	0.70	0.39	0.42	1.19	1.04	1.70	1.17	96	76.74		
Normal Funds																			
Top 10%	300	1.67	26.01*** (25.90)	3.22*** (32.88)	0.69	17.19	8.04	1.24	0.65	0.30	0.32	1.36	0.77	1.51	0.96	64	175.97	12.73	12.14***
2nd 10%	323	1.44	29.91*** (40.48)	0.72*** (10.78)	0.84	16.16	8.73	1.39	0.67	0.39	0.42	1.22	1.04	1.64	1.05	58	94.24	14.04	10.96***
3rd 10%	347	1.29	28.40*** (43.85)	-0.70*** (-10.18)	0.85	15.51	9.31	1.26	0.62	0.34	0.37	1.22	0.92	1.51	1.11	64	117.09	14.94	10.49***
4th 10%	375	1.18	25.00*** (43.72)	-1.61*** (-25.93)	0.84	14.36	9.32	1.21	0.57	0.31	0.33	1.03	0.79	1.41	1.12	70	105.79	14.76	9.49***
5th 10%	370	1.09	23.41*** (46.07)	-2.72*** (-36.69)	0.85	14.38	10.18	1.43	0.67	0.41	0.45	1.20	1.09	1.67	1.19	82	83.71	15.67	9.15***
6th 10%	376	0.99	20.30*** (47.29)	-3.57*** (-51.97)	0.86	13.51	10.56	1.50	0.71	0.42	0.44	1.14	1.13	1.73	1.21	92	78.25	15.58	8.18***
7th 10%	371	0.86	16.45*** (44.21)	-4.59*** (-68.23)	0.84	12.26	10.90	1.53	0.74	0.41	0.44	1.20	1.12	1.77	1.22	114	63.38	15.32	6.86***
8th 10%	353	0.72	12.86*** (37.06)	-6.02*** (-83.13)	0.80	11.09	11.67	1.57	0.76	0.40	0.44	1.41	1.08	1.85	1.30	119	53.09	15.54	5.59***
9th 10%	384	0.58	8.16*** (27.65)	-6.88*** (-97.00)	0.67	9.45	12.00	1.64	0.75	0.46	0.48	1.12	1.26	1.86	1.30	136	44.34	14.69	3.95***
Bottom 10%	249	0.38	8.72*** (26.37)	-9.16*** (-124.21)	0.74	6.87	11.34	1.66	0.73	0.49	0.52	0.97	1.27	1.86	1.23	107	46.20	14.36	1.39***
S&P 500 TRI		1.24				12.37	7.36								1.00				
Abnormal Funds																			
Top 10%	87	2.08	20.65*** (8.66)	13.75*** (25.71)	0.47	25.12	10.28	1.41	0.70	0.39	0.42	1.20	1.09	1.65	0.70	84	123.12	9.97	19.88***
2nd 10%	65	1.44	40.63*** (7.69)	13.01*** (36.29)	0.48	27.82	16.14	1.48	0.74	0.39	0.40	1.12	1.02	1.70	1.00	51	66.54	13.33	22.49***
3rd 10%	40	1.30	47.26*** (8.88)	10.66*** (21.62)	0.67	24.41	15.65	1.42	0.69	0.34	0.37	1.26	1.04	1.67	0.96	103	73.00	12.33	19.16***
4th 10%	13	1.18	65.97*** (8.71)	8.04*** (9.58)	0.87	19.76	13.74	1.71	0.80	0.48	0.50	1.13	1.38	1.94	0.91	240	97.80	10.00	14.13***
5th 10%	17	1.10	9.94 (1.65)	2.40 (1.74)	0.15	19.14	14.31	1.57	0.84	0.28	0.28	1.66	0.82	1.90	1.30	202	90.50	15.17	13.51***
6th 10%	12	0.98	13.74*** (6.56)	-0.47 (-0.72)	0.81	15.82	12.96	1.65	0.88	0.24	0.24	1.65	0.65	1.98	1.14	190	85.94	14.64	10.07***
7th 10%	16	0.87	6.95** (2.55)	-0.29 (-0.34)	0.32	13.77	12.37	1.34	0.69	0.25	0.25	1.23	0.39	1.58	0.84	162	287.35	12.72	8.61***
8th 10%	35	0.74	11.17*** (14.12)	-3.73*** (-21.20)	0.86	10.77	10.76	1.18	0.66	0.15	0.18	0.59	0.21	1.30	1.03	225	62.63	13.32	5.92***
Bottom 10%	112	0.26	7.53*** (9.40)	-12.40*** (-36.30)	0.45	6.09	16.53	2.02	0.89	0.48	0.53	1.43	1.29	2.30	1.72	150	32.72	16.47	0.00
Bottom 10%	27	-0.23	3.27* (1.98)	-14.26*** (-11.20)	0.14	1.15	13.96	2.37	1.02	0.50	0.50	0.86	0.97	2.54	0.70	278	28.41	13.03	-5.30***
S&P 500 TRI		1.24				12.37	7.36								1.00				

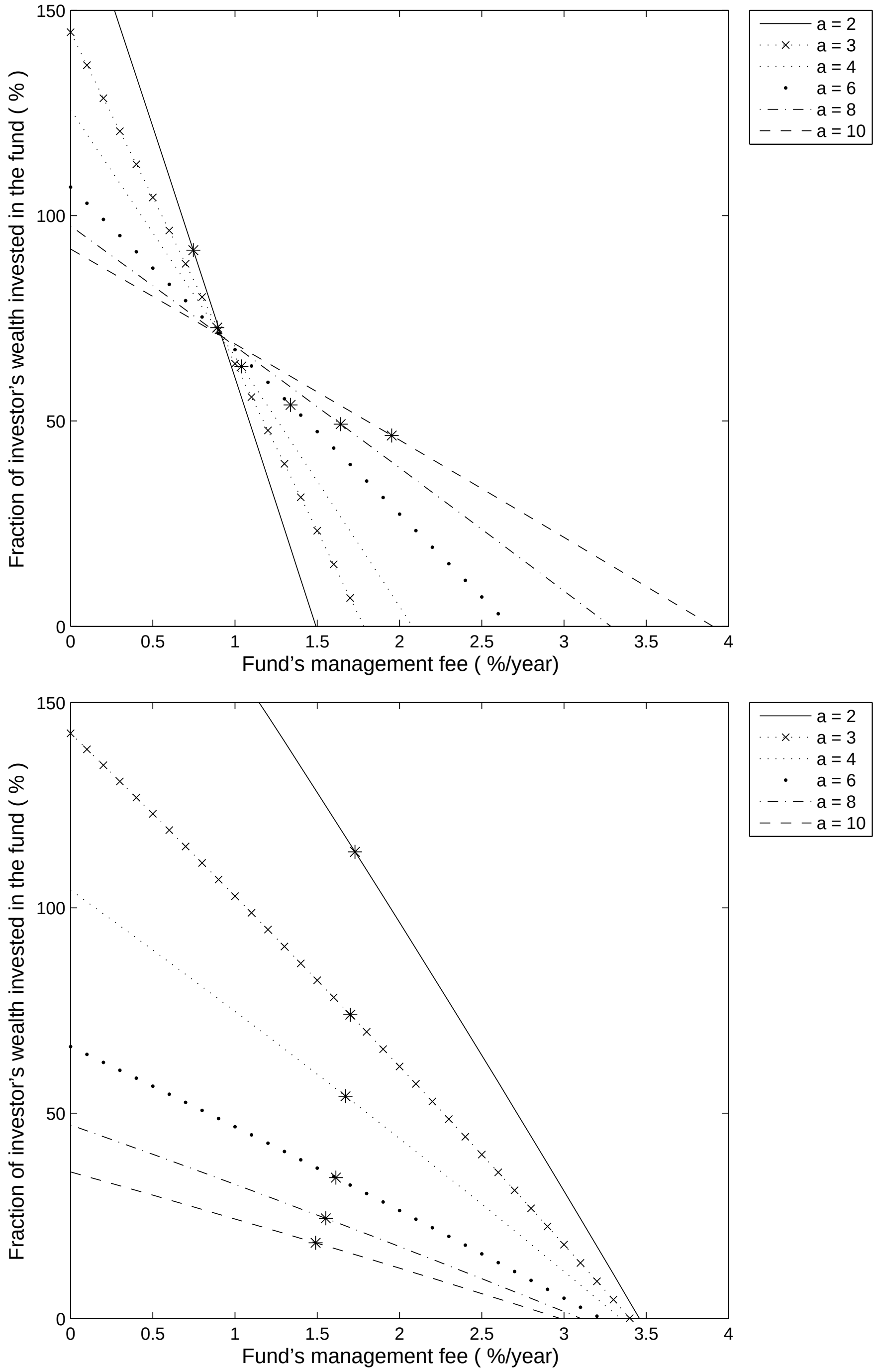

Figure 1: Investor's demand function for different values of the risk aversion coefficient a. The investor's invested amount is plotted against the fund's management fee. The equilibrium solutions $(f_p, \omega(f_p))$ for each value a has been marked with an asterisk. For diversification funds with a beta smaller than 1 (upper panel), the equilibrium weight is decreasing and the equilibrium fee is increasing when investors becomes more risk averse. For leveraged funds with a beta larger than 1 (bottom panel), both the equilibrium weight and the equilibrium fee are decreasing. The parameters for top figure are: $\bar{r}_i = 0.08$, $\bar{r}_m = 0.09$, $\sigma_i = 0.09$ and $\sigma_m = 0.08$; the parameters for bottom figure are: $\bar{r}_i = 0.08$, $\bar{r}_m = 0.12$, $\sigma_i = 0.08$, and $\sigma_m = 0.13$. Both have $\rho = 0.7$ and $f_0 = 0$.

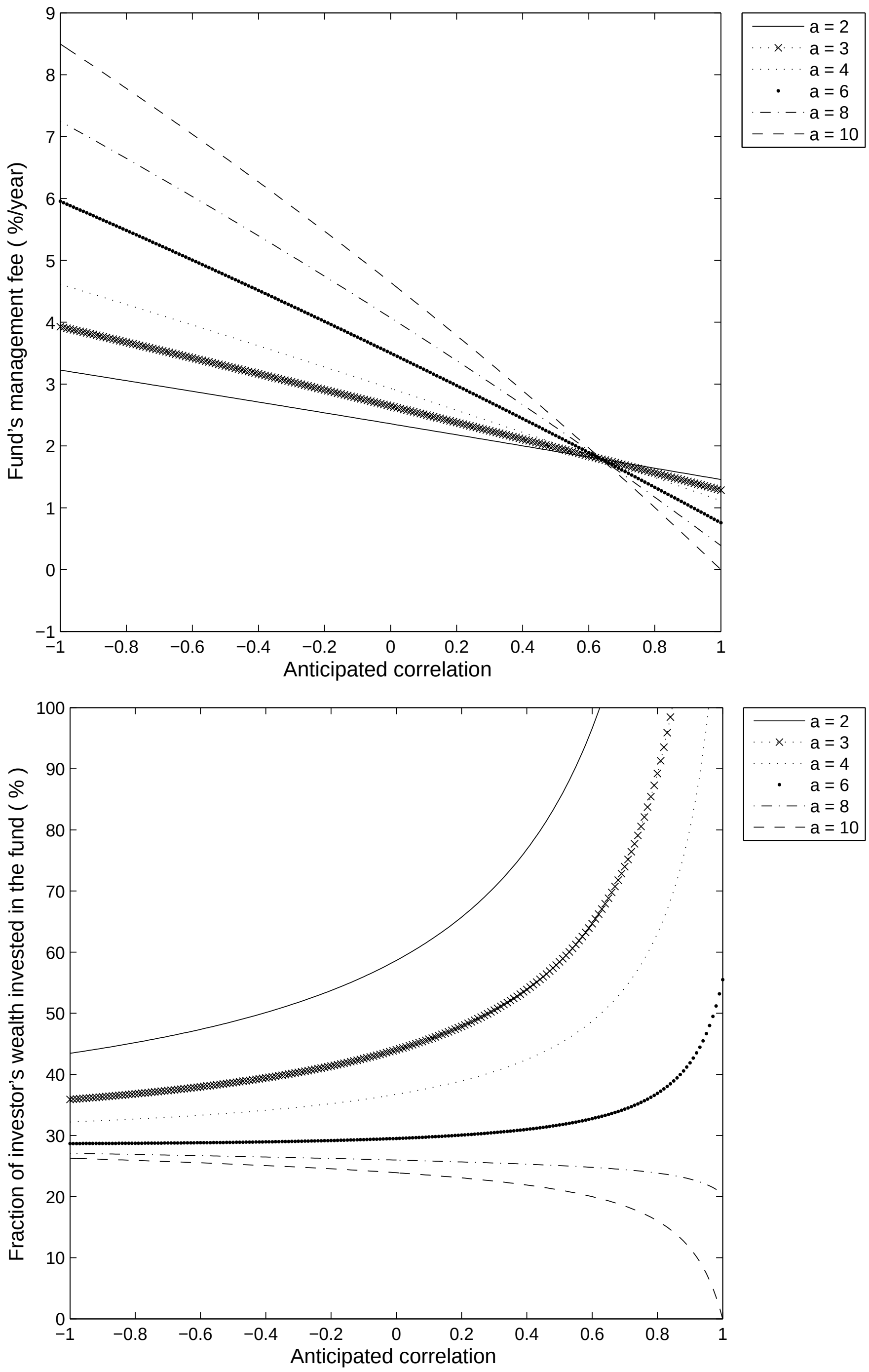

Figure 2: Fund's management fee and investor's invested amount in the mutual fund as a function of the anticipated correlation between the fund's portfolio and investor's benchmark portfolio, with all the other parameters being kept fixed. When the two portfolios are perfectly correlated, it should not be surprising to find that, depending on their risk aversion and taste for mean return, investors may invest fully in the benchmark portfolio or the mutual fund, or partly in the mutual fund and partly in the benchmark portfolio. The parameters are: $\bar{r}_i = 0.08$, $\bar{r}_m = 0.12$, $\sigma_m = 0.13$, $\sigma_i = 0.08$ and $f_0 = 0$.

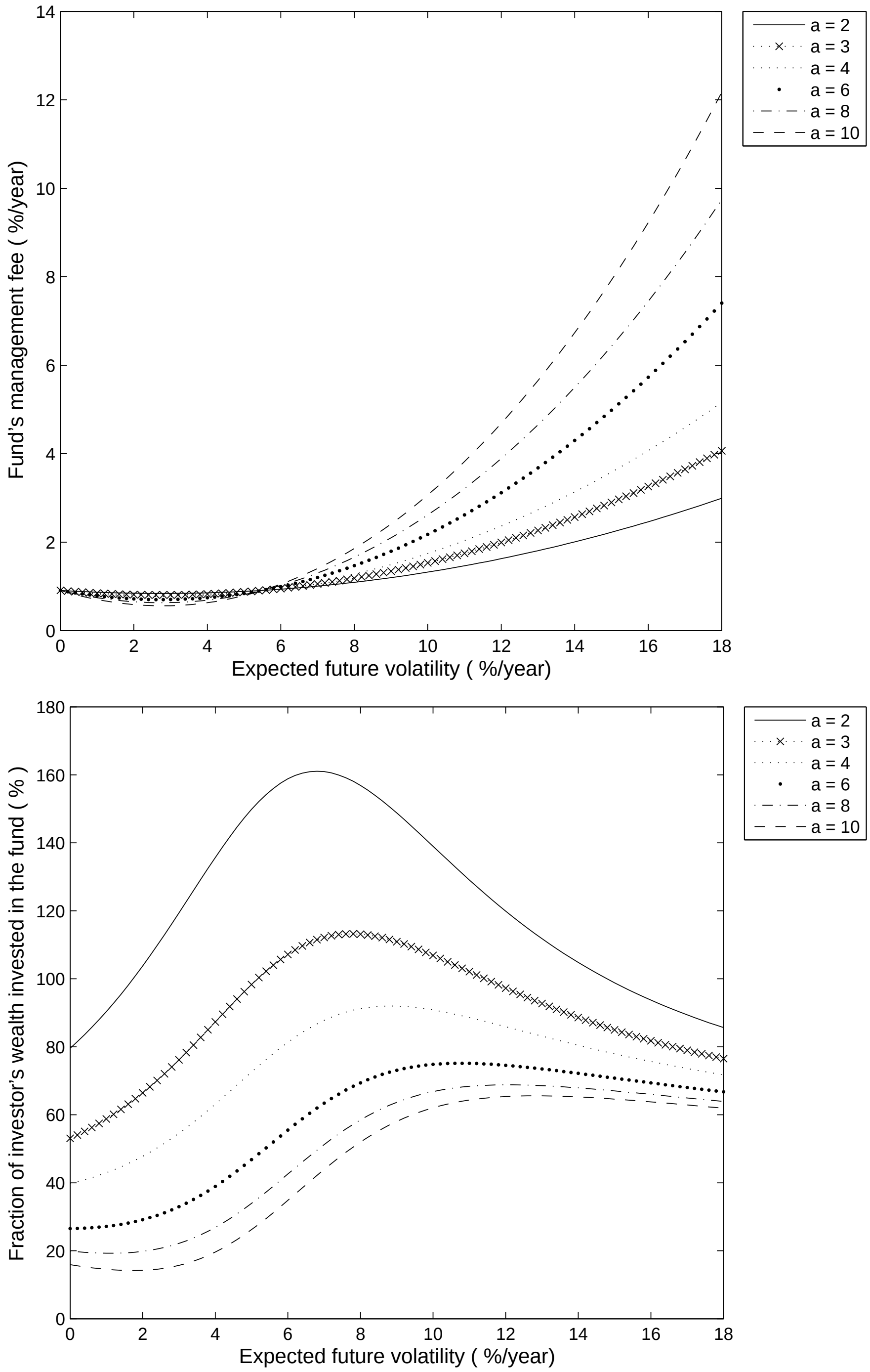

Figure 3: Fund's management fee and investor's invested amount in the mutual fund as a function of the expected future volatility (standard deviation) of the investor's benchmark portfolio, with all the other parameters being kept fixed. The parameters are: $\bar{r}_i = 0.08$,$\bar{r}_m = 0.10$, $\sigma_m = 0.08$, $\rho = 0.7$ and $f_0 = 0$

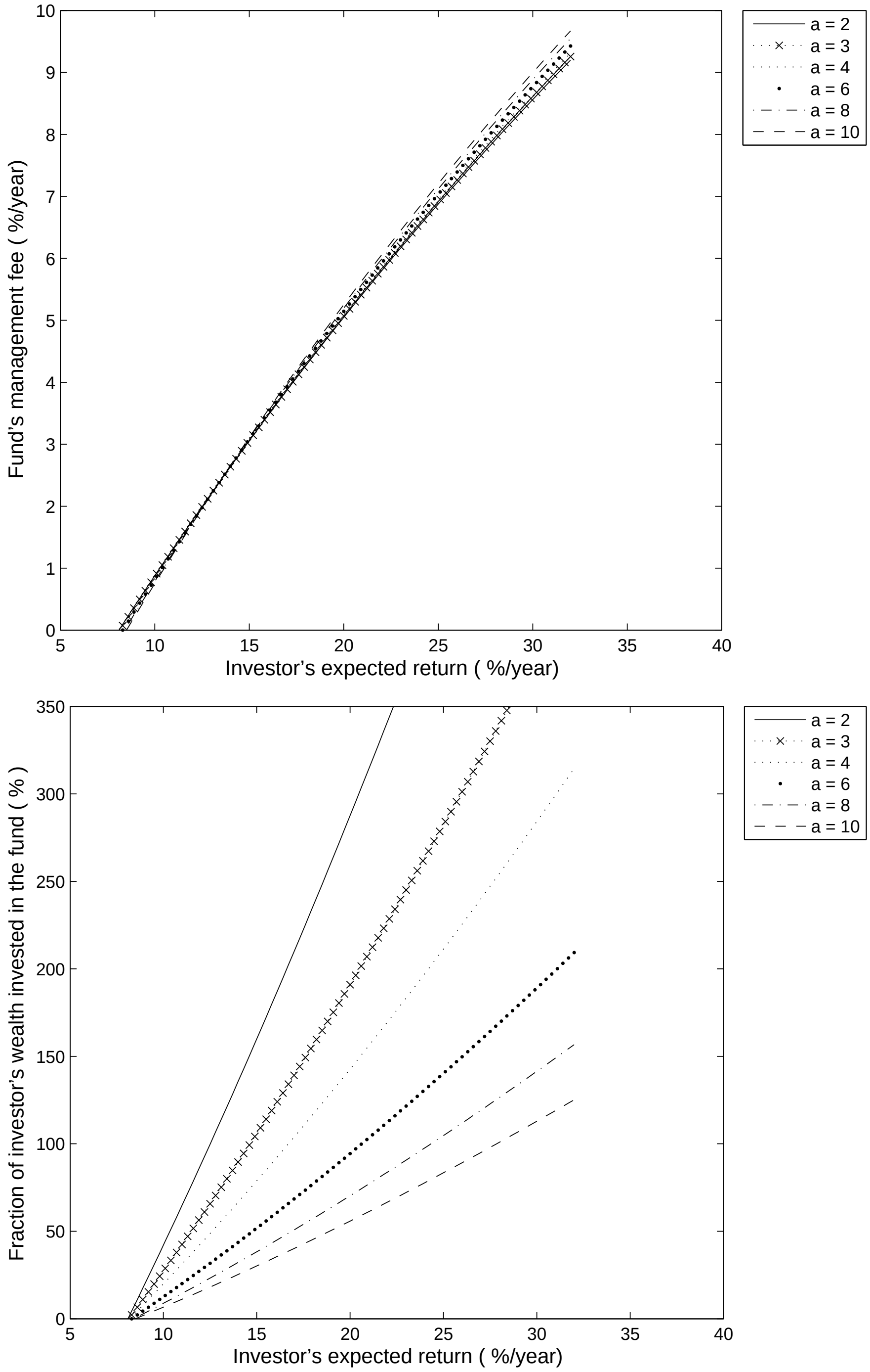

Figure 4: Fund's management fee and investor's invested amount in the mutual fund as a function of the expected return on the mutual fund, with all other parameters being kept constant. The parameters are: $\bar{r}_i = 0.08$, $\sigma_i = 0.10$, $\sigma_m = 0.15$, $\rho = 0.7$ and $f_0 = 0$.

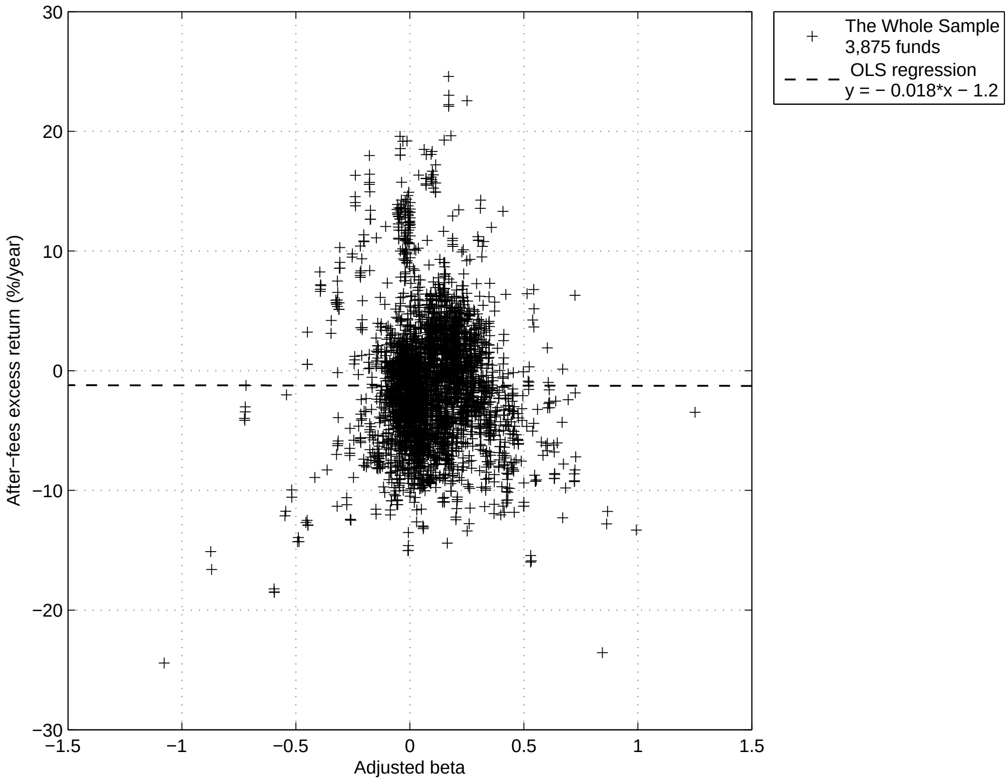

Figure 5: Plot of the after-fees excess returns for the whole sample of 3,875 US domestic equity mutual funds versus their adjusted betas. The dotted line shows the regression obtained with relation (33).

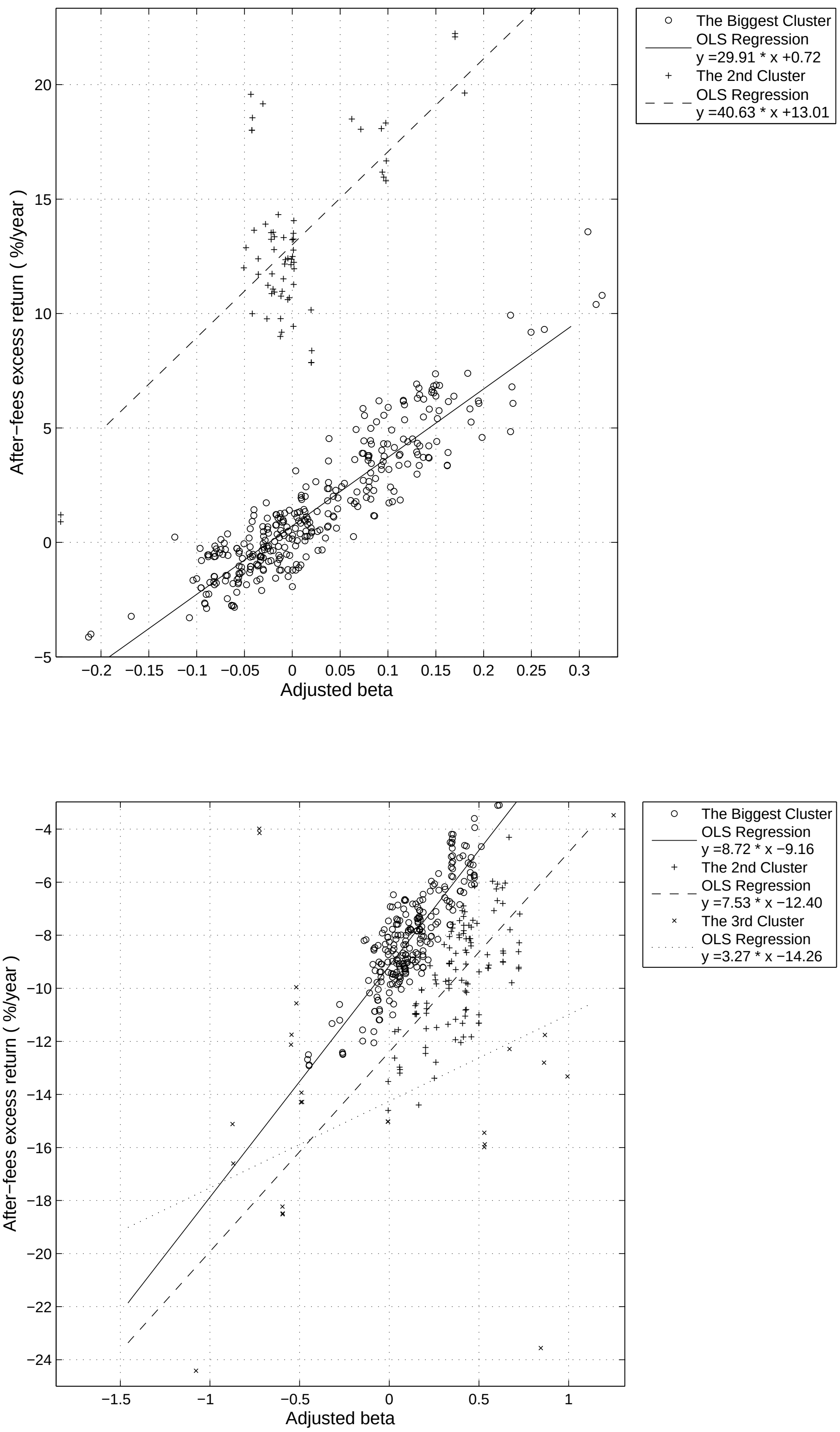

Figure 6: Cluster Analysis of the 2^{nd} decile (top panel) and of the last decile (bottom panel) of the distribution of Sharpe ratios for the U.S. Domestic Equity Mutual funds from July 2003 to March 2007.

Appendices

A Proof of Proposition 2

The necessary condition for the existence of solutions to the problem

$$\max_{f|\omega(f)} \mathbf{E}[U_m(\tilde{W}_m)|\mathcal{I}_m] \tag{36}$$

is

$$\frac{\partial \mathbf{E}\left[U_m(W_0 + \Omega(f)(1+\tilde{r}_m)f)|\mathcal{I}_m\right]}{\partial f} = 0 \ , \tag{37}$$

namely

$$\mathbf{E}\left[U_m^{'}(W_0 + \Omega(f)(1+\tilde{r}_m)f)(1+\tilde{r}_m)(\Omega(f) + f\frac{\partial \Omega(f)}{\partial f})\middle| \mathcal{I}_m\right] = 0 \ . \tag{38}$$

The demand function $\Omega(f)$ is a deterministic function of f, therefore,

$$\mathbf{E}[U_m^{'}(W_0 + \Omega(f)(1+\tilde{r}_m)f)(1+\tilde{r}_m)|\mathcal{I}_m](\Omega(f) + f\frac{\partial \Omega(f)}{\partial f}) = 0 \ . \tag{39}$$

We have in reality $\tilde{r}_m > -1$ and because utility functions are increasing with wealth, namely $U_m^{'}(x) > 0$, we get

$$\mathbf{E}[U_m^{'}(W_0 + \Omega(f)(1+\tilde{r}_m)f)(1+\tilde{r}_m)|\mathcal{I}_m] > 0 \tag{40}$$

Therefore, the solution to (36) must be the solution to (9). The concavity of U_m ensures the sufficiency of the first order condition. We stress that we did not need to specify the form of the utility function U_m, nor that of the demand function $\Omega(f)$. Q.E.D.

B Proof of Proposition 3

Using equation (11), the investor's demand function reads

$$\Omega(f_e) = \frac{(1+\bar{r}_m)(1-f_e) - (1+r_f)}{a \cdot \sigma_m^2 (1-f_e)^2} \tag{41}$$

with $f_e = f + f_0$. Then the solution to the manager's optimization problem is given by proposition 2. The first order condition yields

$$f = (1-f_0) \cdot \frac{(1+\bar{r}_m)\cdot(1-f_0) - (1+r_f)}{(1+\bar{r}_m)\cdot(1-f_0) + (1+r_f)}, \tag{42}$$

while the second order condition

$$(1-f_0)(1+r_f) \geq 0 \tag{43}$$

always holds. Q.E.D.

C Proof of Proposition 4

The investor's demand function is solution to the problem

$$\max_{\omega|f_e} \mathrm{E}\left[-e^{-a((1-\omega)(1+\tilde{r}_i)+\omega(1+\tilde{r}_m)(1-f_e))}|\mathcal{I}_i\right] \tag{44}$$

with $\omega \geq 0$ and $(\tilde{r}_i, \tilde{r}_m)$ distributed according to (17). The expectation can be readily calculated

$$\mathrm{E}\left[-e^{-a((1-\omega)(1+\tilde{r}_i)+\omega(1+\tilde{r}_m)(1-f_e))}|\mathcal{I}_i\right] = -e^{\phi_\omega(-a)}, \tag{45}$$

where $\phi_\omega(-a)$ is the cumulant generating function of a Gaussian random variable at point $-a$, so that

$$\begin{aligned} \phi_\omega(-a) &= \frac{a^2}{2}\left[(1-\omega)^2\sigma_i^2 + 2\omega(1-\omega)(1-f_e)\rho\sigma_i\sigma_m + \omega^2(1-f_e)^2\sigma_m^2\right] \\ &- a\left[(1-\omega)(1+\bar{r}_i) + \omega(1-f_e)(1+\bar{r}_m)\right] \ . \end{aligned} \tag{46}$$

Maximizing the expectation in (44) is equivalent to minimize ϕ and therefore the first order condition yields

$$\Omega(f_e) = \frac{1}{a} \cdot \frac{(1+\bar{r}_m - a\rho\sigma_i\sigma_m)\cdot(1-f_e) - (1+\bar{r}_i - a\sigma_i^2)}{\sigma_m^2(1-f_e)^2 - 2\rho\sigma_i\sigma_m(1-f_e) + \sigma_i^2} \tag{47}$$

while the second order condition

$$a^2\left({\sigma_i}^2 - 2\left(1-f_e\right)\rho\sigma_i\sigma_m + \left(1-f_e\right)^2{\sigma_m}^2\right) \geq 0 \tag{48}$$

always holds.

Since f_e is defined within the range that satisfies $\omega \geq 0$, it is easy to check that

$$f_{max} = \frac{\bar{r}_m - \bar{r}_i + a\sigma_i^2 - a\rho\sigma_i\sigma_m}{1+\bar{r}_m - a\rho\sigma_i\sigma_m} - f_0. \tag{49}$$

As for the manager's optimization problem, proposition 2 shows that the optimal fee is solution to the first order condition

$$\Omega(f_0+f) + f\cdot\partial_f\Omega(f_0+f) = 0 \ , \tag{50}$$

namely, with the notations of proposition 4

$$\begin{aligned} &\left[R_m(1-f_0)\sigma_m - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2\right]f^2 \\ -\ &2R_m\left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right]f \\ +\ &\left[R_m(1-f_0) - R_i\right]\cdot\left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right] = 0 \end{aligned} \tag{51}$$

whose solutions are

$$\begin{aligned} f_\pm &= \frac{R_m\left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right]}{R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2} + \\ &\pm \frac{\sqrt{\left[(1-f_0)^2\sigma_m^2 - 2(1-f_0)\rho\sigma_i\sigma_m + \sigma_i^2\right]\cdot\left[R_m^2\sigma_i^2 - 2R_mR_i\rho\sigma_i\sigma_m + R_i^2\sigma_m^2\right]}}{R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2} \ . \end{aligned} \tag{52}$$

Since

$$\partial_f^2 \left(f \cdot \Omega(f_0 + f) \right)_{f=f_\pm} = \pm \sqrt{ \begin{array}{l} \left[(1-f_0)^2 \sigma_m^2 - 2(1-f_0)\rho\sigma_i\sigma_m + \sigma_i^2 \right] \\ \times \left[R_m^2 \sigma_i^2 - 2R_m R_i \rho \sigma_i \sigma_m + R_i^2 \sigma_m^2 \right] \end{array} } \ , \tag{53}$$

the second order condition leads us to choose f_-.

However, this solution is admissible if and only if $f_- \geq 0$, which requires

$$R_m^2 \left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2 \right] \gtreqless R_m^2\sigma_i^2 - 2R_m R_i \rho\sigma_i\sigma_m + R_i^2\sigma_m^2 \tag{54}$$

and

$$R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2 \gtreqless 0 \ . \tag{55}$$

Factorizing (54), we get

$$\left[(1-f_0)R_m - R_i \right] \cdot \left[R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2 \right] \gtreqless 0, \tag{56}$$

so that, according to (54) and (55)

$$f_- \geq 0 \qquad \Longleftrightarrow \qquad (1-f_0)R_m \geq R_i \tag{57}$$

which holds by the assumption made in proposition 4. Q.E.D.

D Robustness Check: CRRA utility function with Log-Prices

The dependence of the results of our model is tested for a larger class of utility functions:

$$U_i(x) = \frac{x^{1-a}}{1-a} \tag{58}$$

$$U_i(x) = \frac{x^{1-b}}{1-b} \tag{59}$$

with $a, b > 0$ and $a, b \neq 1$, a, b represent respectively the constant relative risk aversion level of the investors and managers.

To avoid negative prices, we use log-returns. The wealth of the investor at period 1 is

$$\tilde{W}_i = (1-\omega)e^{\tilde{r}_i} + \omega e^{\tilde{r}_m}(1-f_e) \ . \tag{60}$$

The wealth of the manager at period 1 is

$$\tilde{W}_m = \omega e^{\tilde{r}_m} f \tag{61}$$

The log-returns $\tilde{r}_i$ and $\tilde{r}_m$ are Gaussian distributed as

$$\tilde{r}_i | i \sim N(\bar{r}_i, \sigma_i^2) \tag{62}$$

$$\tilde{r}_m | i \sim N(\bar{r}_{m,i}, \sigma_{m,i}^2) \tag{63}$$

Their correlation perceived by the investor is denoted ρ.

The investor's optimization problem is formulated as:

$$\max_{\omega|f_e} \mathbf{E}[\frac{((1-\omega)e^{\tilde{r}_i} + \omega e^{\tilde{r}_m}(1-f_e))^{1-a}}{1-a}|i] \tag{64}$$

with $\omega > 0$. The manager' optimization problem is formulated as

$$\max_{f|\omega^*} \mathbf{E}[\frac{(\omega e^{\tilde{r}_m} f)^{1-b}}{1-b}|m] \tag{65}$$

with $0 < f + f_0 < 1$ given

$$\tilde{r}_m|m \sim N(\bar{r}_{m,m}, \sigma^2_{m,m}) \tag{66}$$

In this setup, a closed form solution is difficult to find and we resort to numerical methods.

For the investor's optimization problem, we have

$$\mathbf{E}[((1-\omega)e^{\tilde{r}_i} + \omega e^{\tilde{r}_m}(1-f_e))^{1-a}|i] = \frac{1}{2\pi\sigma_i\sigma_{m,i}\sqrt{1-\rho^2}(1-a)}\mathbf{I} \tag{67}$$

with

$$\mathbf{I} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} ((1-\omega)e^{r_i} + \omega e^{r_m}(1-f_e))^{1-a} e^{-\frac{1}{2(1-\rho^2)}(\frac{(r_i-\bar{r}_i)^2}{\sigma_i^2} + \frac{(r_{m,i}-\bar{r}_{m,i})^2}{\sigma^2_{m,i}} - \frac{2\rho(r_i-\bar{r}_i)(r_{m,i}-\bar{r}_{m,i})}{\sigma_i\sigma_{m,i}})} \mathbf{d}r_i\mathbf{d}r_{m,i} \tag{68}$$

We numerically calculated this double integral and solved the optimization problem.

For the managers' optimization problem, we have (notice $\omega^*(f)$ is the same as $\omega^*(f_e)$ here)

$$\mathbf{E}[\frac{(\omega e^{\tilde{r}_m} f)^{1-b}}{1-b}|m] = \frac{1}{\sqrt{2\pi}\sigma_{m,m}(1-b)}(\omega^*(f)f)^{1-b} I(b, r_{m,m}, \sigma_{m,m}) \tag{69}$$

with

$$I(b, r_{m,m}, \sigma_{m,m}) = \int_{-\infty}^{+\infty} e^{r_m(1-b)} e^{-\frac{1}{2}(\frac{(r_m-\bar{r}_{m,m})^2}{\sigma^2_{m,m}})} \mathbf{d}r_m \tag{70}$$

Notice that only $(\omega^*(f)f)^{1-b}$ depends on f, therefore, the manager's optimization problem (65) is equivalent to

$$\max_{f|\omega^*} (\omega^*(f)f)^{1-b} \tag{71}$$

The numerical results confirm the remarkably strong robustness of our analytical result derived for CARA utility functions. Figures are available from the authors upon request.

E Generalization

We now consider the general case where the investor's utility function can be any increasing and concave function. We do not make any assumption on the joint distribution of returns on the benchmark portfolio and on the managed portfolio. We just assume that the funds are not too risky and that the investors are not too risk averse to justify a second order expansion of the investor's utility function. Then, as proved in Appendix F, the following result holds

Lemma 1. *Up to second order terms in an expansion in powers of the returns and fees, the investor's demand function reads*

$$\Omega = \frac{\left[\sigma_i^2 - (1-f_e)\rho\sigma_i\sigma_m\right] + a(\Omega)^{-1} \cdot \left[(1+\bar{r}_i) + (1-f_e)(1+\bar{r}_m)\right]}{\sigma_i^2 - 2(1-f_e)\rho\sigma_i\sigma_m + (1-f_e)^2\sigma_m^2}, \tag{72}$$

where $a(\Omega)$ denotes the investor's absolute risk aversion at the point $(1-\Omega)(1+\bar{r}_i) + \Omega(1-f_e)(1+\bar{r}_m)$.

As previously, the demand is independent of the risk aversion for $f_e = f_p = \frac{\bar{r}_m - \bar{r}_i}{1+\bar{r}_m}$. Thus, up to the first order in $f_e - f_p$, we get

$$\Omega(f_e) \approx \Omega_p + \left[\frac{\rho\sigma_i\sigma_m + \left(\frac{1+\bar{r}_i}{1+\bar{r}_m} \cdot {\sigma_m}^2 - \rho\sigma_i\sigma_m\right)\Omega_p - a^{-1}\left(1+\bar{r}_m\right)}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}\right] \cdot \left(f_e - f_p\right). \tag{73}$$

As checked in for the case of CRRA utility functions, the linear approximation of the demand functionis quite good. This justifies hypothesis 2 according to which the fund manager knows the investors' demand function.

Without loss of precision, (73) can be simplified by replacing $a(\Omega)$ with $a(\Omega_p)$ in (72). Then, performing the same calculation as in Appendix C, we can generalize the result of proposition 4. More importantly, we can state

Proposition 5. *Within the limits of the hypothesis of this section, irrespective of the investor's utility function and of the distributions of returns on the benchmark portfolio and on the managed portfolio, the optimal management fee the manager should charge is approximately half the fee that starts to make the managed fund undesirable to the investor:*

$$f^* \approx \frac{\hat{r}_m - \hat{r}_i}{2} - \frac{f_0}{2}, \tag{74}$$

where

$$\hat{r}_m = \bar{r}_m - a\left(\Omega_p\right)\rho\sigma_i\sigma_m \qquad \text{and} \qquad \hat{r}_i = \bar{r}_i - a\left(\Omega_p\right)\sigma_i^2 \tag{75}$$

are the risk-adjusted expected returns on the managed and on the benchmark portfolio.

As a corollary to this proposition, we can generalize the results derived when the benchmark portfolio is the risk-free asset.

Corollary 1. After adjustment for the level of risk, the expected return on the managed fund, net of all fees, is equal to the expected return on the benchmark plus the management fee

$$\hat{r}_m - f_e = \hat{r}_i + f^*. \tag{76}$$

F Proof of lemma 1

Let us denote by U the investor's utility function. It is assumed increasing and concave. The demand function is solution to

$$\max_\omega \mathrm{E}\left[U\left((1-\omega)(1+\tilde{r}_i) + \omega(1-f_e)(1+\tilde{r}_m)\right)\right] \tag{77}$$

We denote by $1+\bar{r}(\omega)$ the average gross rate of return and by $\sigma(\omega)^2$ the variance of the return on the investor's portfolio. Then expending the utility function in the neighborhood of $\bar{r}$ up to the second order, the optimal demand solves

$$\partial_\omega \bar{r}(\omega) = -\frac{U''(\bar{r})}{U'(\bar{r})} \cdot \frac{\partial_\omega \sigma(\omega)^2}{2}, \tag{78}$$

where we recognize the coefficient of absolute risk aversion $-U''/U'$. The higher order term $\partial_\omega \bar{r}(\omega) U'''(\bar{r}) \sigma(\omega)^2$ has been neglected.

Expressing $\bar{r}(\omega)$ and $\sigma(\omega)^2$ and substituting in the equation above, we get

$$-(1+\bar{r}_i) + (1-f_e)(1+\bar{r}_m) = -\frac{U''(\bar{r})}{U'(\bar{r})} \left[\omega(1-f_e)^2\sigma_m^2 + (1-2\omega)(1-f_e)\rho\sigma_i\sigma_m - (1-\omega)\sigma_i^2\right]. \tag{79}$$

To simplify notations, we define

$$a(\omega) := -\frac{U''(\bar{r}(\omega))}{U'(\bar{r}(\omega))}, \tag{80}$$

so that the optimal demand function reads

$$\Omega(f_e) = \frac{\left[\sigma_i^2 - (1-f_e)\rho\sigma_i\sigma_m\right] - a(\Omega)^{-1} \cdot \left[(1+\bar{r}_i) - (1-f_e)(1+\bar{r}_m)\right]}{\sigma_i^2 - 2(1-f_e)\rho\sigma_i\sigma_m + (1-f_e)^2\sigma_m^2}. \tag{81}$$

This equation remains implicit since the level of the demand appears in the right-hand side to set the level of absolute risk aversion. But, if it does not vary too fast, it is reasonable to make the approximation that it is locally constant.

Expanding this relation around $f_e = f_p$, we first evaluate

$$\Omega(f_p) = \Omega_p = \frac{\sigma_i^2 - \frac{1+\bar{r}_i}{1+\bar{r}_m}\rho\sigma_i\sigma_m}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}, \tag{82}$$

and

$$\Omega'(f_p) = \frac{\rho\sigma_i\sigma_m + \left(\frac{1+\bar{r}_i}{1+\bar{r}_m} \cdot {\sigma_m}^2 - \rho\sigma_i\sigma_m\right)\Omega_p - a(\Omega_p)^{-1}\left(1+\bar{r}_m\right)}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}, \tag{83}$$

so that

$$\Omega(f_e) \simeq \Omega_p + \left[\frac{\rho\sigma_i\sigma_m + \left(\frac{1+\bar{r}_i}{1+\bar{r}_m} \cdot {\sigma_m}^2 - \rho\sigma_i\sigma_m\right)\Omega_p - a(\Omega_p)^{-1}\left(1+\bar{r}_m\right)}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}\right] \cdot (f_e - f_p) \tag{84}$$

References

Alexander, G. J., J. D. Jones, and P. J. Nigro (1998): "Mutual fund shareholders: characteristics, investor knowledge, and sources of information," *Financial Services Review*, 7(4), 301–316.

Alves, C. F., and V. Mendes (2007): "Are Mutual Fund Investors in Jail?," *Applied Financial Economics*, 17.

Barber, B. M., T. Odean, and L. Zheng (2005): "Out of Sight, Out of Mind: The Effects of Expenses on Mutual Fund Flows," *Journal of Business*, 78(6), 2095–2120.

Barberis, N., and A. Shleifer (2003): "Style investing," *Journal of Financial Economics*, 68(2), 161–199.

Berk, J. B., and R. C. Green (2004): "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy*, 112(6), 1269–1295.

Brennan, M. J., and H. H. Cao (1997): "International Portfolio Investment Flows," *The Journal of Finance*, 52, 1851–1880.

Brown, K. C., and W. V. Harlow (2002): "Staying the Course: The Impact of Investment Style Consistency on Mutual Fund Performance," *Working Papers*.

Brown, S. J., and W. N. Goetzmann (1997): "Mutual fund styles," *Journal of Financial Economics*, 43(3), 373–399.

Capon, N., G. J. Fitzsimons, and R. A. Prince (1996): "An individual level analysis of the mutual fund investment decision," *Journal of Financial Services Research*, 10(1), 59–82.

Carhart, M. M. (1997): "On Persistence in Mutual Fund Performance," *The Journal of Finance*, 52(1), 57–82.

Chan, L. K. C., H.-L. Chen, and J. Lakonishok (2002): "On Mutual Fund Investment Styles," *The Review of Financial Studies*, 15(5), 1407–1437.

Chevalier, J., and G. Ellison (2002): "Are Some Mutual Fund Managers Better Than Others? Cross-Sectional Patterns in Behavior and Performance," *The Journal of Finance*, 54(3), 875 – 899.

Choi, J. J., D. Laibson, and B. C. Madrian (2008): "Why Does the Law of One Price Fail? An Experiment on Index Mutual Funds," *Working Papers*.

Chordia, T. (1996): "The structure of mutual fund charges," *Journal of Financial Economics*, 41(1), 3–39.

Christoffersen, S. E. K., and D. K. Musto (2002): "Demand Curves and the Pricing of Money Management," *The Review of Financial Studies*, 15(5), 1499–1524.

Coates, J. C., and R. G. Hubbard (2007): "Competition in the Mutual Fund Industry: Evidence and Implications for Policy," *Journal of Corporation Law*, 33(1), 151–222.

Cooper, I., and E. Kaplanis (1994): “Home bias in equity portfolios, inflation hedging, and international capital market equilibrium,” *The Review of Financial Studies*, 7, 45–60.

Das, S. R., and R. K. Sundaram (2002): “Fee Speech: Signaling, Risk-Sharing, and the Impact of Fee Structures on Investor Welfare,” *The Review of Financial Studies*, 15(5), 1465–1497.

Donnelley, B. (1992): “What’s in a name? Some mutual funds make it difficult to judge,” *Wall Street Journal*, May 5, Section C.

Elton, E., M. J. Gruber, and S. Das (1993): “Efficiency with Costly Information: A Reinterpretation of Evidence from Managed Portfolios,” *The Review of Financial Studies*, 6(1), 1–22.

Elton, E. J., M. J. Gruber, and C. R. Blake (2003): “Incentive Fees and Mutual Funds,” *The Journal of Finance*, 58(2), 779–804.

Freeman, J. P. (2007): “John Freeman’s Working Paper Responding to the Advisory Fee Analysis in AEI Working Paper 127, June 2006, ’Competition and Shareholder Fees in the Mutual Fund Industry: Evidence and Implications for Policy’ by John C. Coates, IV, and R. Glenn Hubbard,” *Working Papers*.

Freeman, J. P., and S. L. Brown (2001): “Mutual Fund Advisory Fees: The Cost of Conflicts of Interest,” *Journal of Corporate Law*, pp. 610–673.

French, K., and J. Poterba (1991): “Investor diversification and international equity markets,” *American Economic Review*, 81(2), 222–226.

Gil-Bazo, J., and P. Ruiz-Verdu (2007): “Yet Another Puzzle Relation between Price and Performance in the Mutual Fund Industry,” *Working Papers*.

Glode, V. (2008): “Why Mutual Funds ’Underperform’,” *Working Papers*.

Golec, J. H. (1992): “Empirical Tests of a Principal-Agent Model of the Investor-Investment Advisor Relationship,” *Journal of Financial and Quantitative Analysis*, 27(1), 81–95.

Grinblatt, M., S. Ikheimo, and M. Keloharju (2008): “Are Mutual Fund Fees Competitive? What IQ-Related Behavior Tells Us,” *Working Papers*.

Gruber, M. J. (1996): “Another Puzzle: The Growth in Actively Managed Mutual Funds,” *The Journal of Finance*, 51(3), 783–810.

Hartman, D. E., and D. K. J. Smith (1990): “Building a Competitive Advantage in Mutual Funds Sales,” *Journal of Retail Banking*, 12, 43–49.

Heinkel, R., and N. M. Stoughton (1994): “The Dynamics of Portfolio Management Contracts,” *The Review of Financial Studies*, 7(2), 351–387.

Herman, E. S. (1963): “Mutual Fund Management Fee Rates,” *The Journal of Finance*, 18(2), 360–376.

Hogg, R. V., and J. Ledolte (1987): *Engineering Statistics*. MacMillan.

HOLMSTROM, B. (1979): "Moral Hazard and Observability," *The Bell Journal of Economics*, 10(1), 74–91.

HORAN, S. H. (1999): "The Implications of Style Analysis for Mutual Fund Performance Evaluation," *CFA Digest*, 29(1), 80–81.

HUBER, P. J. (1981): *Robust Statistics*. John Wiley & Sons, Inc.

INSTITUTE (2008): *2008 Investment Company Fact Book*. Investment Company Institute.

INSTITUTE, C. (2005): *Standards of Practice Handbook ninth edition*. CFA Institute.

INSTITUTE, I. C. (2001): "Redemption Activity of Mutual Fund owners," *Fundamentals, Investment Company Institute Research in Brief*, 10(1).

KHORANA, A., AND H. SERVAES (2004): "Conflicts of Interest and Competition in the Mutual Fund Industry," *Working Paper*.

KHORANA, A., H. SERVAES, AND P. TUFANO (2009): "Mutual Fund Fees Around the World," *The Review of Financial Studies*, 22, 1279 – 1310.

LUO, R. G. Y. (2002): "Mutual Fund Fee-setting, Market Structure and Mark-ups," *Economica*, 69, 245–271.

MALHOTRA, D. K., AND R. MCLEOD (1997): "An empirical analysis of mutual fund expenses," *Journal of Financial Research*, 20(2), 175–190.

MURRAY, N. (1991): *Serious Money: The Art of Marketing Mutual Funds*. Robert a Stanger & Co.

NANIGIAN, D., M. S. FINKE, AND W. WALLER (2008): "Redemption Fees: Reward for Punishment," *Working Papers*.

PALMITER, A. R., AND A. E. TAHA (2008): "Mutual Fund Investors: Divergent Profiles," *Working Papers*.

POTERBA, J. M., AND J. B. SHOVEN (2002): "Exchange Traded Funds: A New Investment Option for Taxable Investors," *American Economic Review*, 92, 422–427.

PRESS, W. H., S. A. TEUKOLSKY, W. T. VETTERLING, AND B. P. FLANNERY (2007): *Numerical Recipes 3rd Edition: The Art of Scientific Computing*. Cambridge University Press.

ROSS, S. A. (1973): "The Economic Theory of Agency: The Principal's Problem," *The American Economic Review*, 63(2), 134–139.

SCHOOL, W. (1982): *Study of Mutual Funds: Report of the Committee on Interstate and Foreign Commerce, 87th Congress, 2nd Session, House Report No. 2274*. William S. Hein & Company.

SEC (2001): *Report on Mutual Fund Fees and Expenses*. U.S. Securities and Exchange Commission (SEC).

Sharpe, W. (1994): "The Sharpe Ratio," *Journal of Portfolio Management*, 21(1), 49–58.

Siegel, S., and N. J. Castellan (1988): *Nonparametric Statistics for the Behavioral Sciences (second edition)*. McGraw-Hill.

Simon, H. (1982): "Models of Bounded Rationality," *Economic Analysis and Public Policy*, 1, 235–244.

Sirri, E. R., and P. Tufano (1998): "Costly Search and Mutual Fund Flows," *The Journal of Finance*, 53(5), 1589–1622.

Starks, L. T. (1987): "Performance Incentive Fees: An Agency Theoretic Approach," *Journal of Financial and Quantitative Analysis*, 22(1), 17–32.

Stracca, L. (2006): "Delegated portfolio management: a survey of the theoretical literature," *Journal of Economic Surveys*, 20(5), 823–848.

Strong, N., and X. Xu (2003): "Understanding the Equity Home Bias: Evidence from Survey Data," *The Review of Economics and Statistics*, 85(2), 307–312.

Tesar, L. L., and I. M. Werner (1995): "Home bias and high turnover," *Journal of International Money and Finance*, 14(4), 467–492.

Wermers, R. (2000): "Mutual Fund Performance: An Empirical Decomposition into Stock-Picking Talent, Style, Transactions Costs, and Expenses," *The Journal of Finance*, 55(4), 1655–1695.