Financial Effects of External Auditing
(First, Incomplete Draft)

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Abstract

The paper develops a model of optimal auditing behavior when the economic environment adds a noise term to the firm’s cash flows, which can be reduced by employing an external auditor. The paper connects the optimal auditing policy and (i) share prices of the firm and (ii) auditor’s compensation. Results show that the optimal auditing amount is below the amount needed to totally eliminate the noise in the economy. Moreover there exist a cut-off point for the auditing costs (economic noise) above (below) which auditing is not anymore optimal. In the presence of incomplete information setting, the theory of global games is used to determine the optimal auditing behavior. EMPirical analysis to follow.

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JEL Classification: M42, G12, C71.
1 Introduction

The efficient market hypothesis, in its various forms, postulates that investors can get the information they need to get an appropriate valuation of the companies in their portfolio. The way investors actually acquire this information is usually ignored. Ignoring this delicate aspect is far from being innocuous, however, as shown for instance in the seminal paper of Grossman and Stiglitz (1980). Information is not free, and market participants have to decide how much they want to invest in obtaining or conveying it in a meaningful way.

Previous papers - such as Grossman and Stiglitz (1980) - have focused on the incentives (or benefits) investors have to acquire information at an individual level. They have usually assumed that (perfect) information on the true value of the firm can always be obtained at a fixed cost by any individual. We take a different tack in our paper. We abstract from coordination failures between investors and take instead a deeper look into the “technology” that allows information to reach shareholders.

We use information theory to analyze the investment a firm should make in conveying the optimal amount of information to current and potential investors. While reducing uncertainty about the true value of the firm can increase share prices, it does not come for free. Firms (and their owners) have to balance the benefits and costs of assembling and conveying information.

As stated above, we look at the costs borne by the firm in providing accurate financial information to investors. The costs will be ultimately affect shareholders - and shareholders will also be the final beneficiaries of a higher equity prices as a result of reduced uncertainty. The decisions are however made in a centralized way by management. We abstract from conflicts of interest between management and shareholders and look instead at the “pure” costs of conveying information, a problem interesting enough by itself. As in previous papers, individual investors can choose to expend their own resources in order to obtain additional, private information that can bring them trading profits. The starting point - having accurate financial reports - is no less important, however.

A look at “real-life” firms shows that the costs of gathering and transmitting financial information are far from negligible. For instance, the four biggest external auditing firms,
PricewaterhouseCoopers, Deloitte, Ernst & Young and KPMG have posted total global revenues of more than 70 billion US dollars in 2005.

Financial information is produced, assembled and confirmed by both internal accounting departments and external auditors. We use the term “auditing” for the entire set of activities involved in conveying useful information about the value of the firm. Since we do not look at conflicts of interest and incentives to distort information, the distinction between information production inside or outside the firm is of secondary importance. Since there is a correlation between the complexity of the information produced internally by company accountants and the effort of outside auditors that have to confirm the numbers, the recent data series on external auditors’ fees allow us to get a rough proxy of the cost of building an appropriate information channel for investors.

The paper answers the following three questions. What is the appropriate extent (quality) of auditing for a firm maximizing its share price? Do shareholders and debtholders disagree about the optimal auditing level and finally, what is the appropriate amount of auditor’s compensation?

Our approach generates theoretical predictions that are distinct from and complement the implications of models based on agency problems. It allows us to provide a more accurate picture of the various reasons for firms’ investment in information production. We try to explore the prediction of our model empirically in the final section of the paper.

We assume that even if the firms have perfect knowledge of the future cash flows, they can not communicate this information costlessly to the financial market investors. The overall economic environment adds a noise component, such as investors’ inability to verifyably deduce the financial statement (see Trojani and Vanini (2004)), the extent of agency conflicts within the firm or implicit firm contracting issues. The economic noise component can be reduced by appropriate choice of auditing, as identified by Gibson (1999), i.e. auditing provides a verifiable signal to the outside investors.
We model the optimal amount of auditing services by the firms as a choice of the size of an information channel. The size the information channel can refer to the number of words in a newspaper add or the extent of an auditing report. The establishment of an information channel of certain size has two opposing effects. On one side it reduces the perceived volatility of firm’s cash flows (and therefore firm value) by external auditors. In other words, let us assume that outside investors value the company somewhere in the interval \( V \in [V, \bar{V}] \). Auditing reduces the interval where \( V \) can lay to \( V \in [V_d, V^u] \), where \( \underline{V} \leq V_d \leq V^u \leq \bar{V} \) - auditing provides validation (certification) of information. In an economy where the representative investor is risk-averse, auditing raises stock prices.

On the other hand, auditing is costly. Apart from auditor’s fees, Gibson (1999) identifies other indirect costs of auditing: threat of product market competition, tax avoidance considerations, agency problems among different classes of shareholders. The strength of both of these effects determines the optimal auditing amount. Information theory, as developed by Shannon (1948) gives a quantitative answer as to how much the perceived volatility of the firm cash flows can be reduced with the establishment of the information channel.

We realize that there are other functions of auditing. Auditing can be viewed as an extreme case of inspection games (Avenhaus, von Stangel, and Zamir (2002) and Kaplan (1993)), i.e. a disciplinary device of the internal accounting department. Gibson (1999) names others: enhanced auditor’s report lower the cost of capital since they facilitate the moral hazard and adverse selection problems, auditing acts as a signaling device for firms in industries where products are close substitutes, or as a reputation device.

We show that the firm chooses the level of external auditing strictly below the level to convey the firm’s value perfectly. If the costs of conveying the information are too high, or the noise component volatility and risk-aversion are too low, no auditing is chosen at all. Firm’s share price reflects exactly this tradeoff. The auditor’s compensation in this case has a put option payoff with respect to auditing costs and a call option payoff structure with respect to the volatility of the economic environment, that is there exists a level of auditing costs (noise volatility) above (below) which the firm does not choose any external auditing.
We prove that in a multi-period setting, the per-period optimal information channel size has the same dependence on model parameters as in a one-period model.

We then analyze the optimal level of auditing chosen from the perspective of debtholders and shareholders. We prove that under general parameter conditions the level of auditing chosen by the debtholders is higher as the one chosen by the shareholders, especially if auditing is cheap. If auditing is very expensive, the level of auditing chosen by both types of agents is the same. We can therefore identify auditing as a means to protect debtholders.

Once the firm’s share price increase is obtained, we can compute the auditor’s compensation for a firm composed of multiple units. Using the Shapley value as a solution concept, we obtain an explicit formula for auditor’s compensation, relying on the number of firm units (proxied by the firm size) and the mean and volatility of economic activity, among others. It is a surprising result that the firm’s choice of auditing in a large firm is significantly smaller than the proportional amount of a smaller firm.

We further show that reporting quality in highly competitive auditing markets does not need to be imposed by an external institution or judicially imposed as by the SEC or the Sarbanes-Oxley act. In a competitive auditors’ market, share value maximization is sufficient for the firms to choose the optimal reporting quality. Our results therefore support the Coasian view.

The paper of Sims (2003) was the first to impose information capacity constraints on economic agents, based on the psychological results on the scarcity of attention in individuals’ decision making process. The papers by Peng (2005) and Peng and Xiong (2006) extended his analysis to intertemporal financial decisions and price formation under capacity constraints. Our model also precisely quantifies the game theoretic approach in Hermalin (2005) and Hermalin and Weisbach (2006).

The paper is structured as follows. Section 2 introduces the information model of optimal auditing behavior. Section 3 then develops the theory of the auditing compensation in and environment of a firm with multiple business units. Section 4 develops the empirically testable model of auditing compensation. Section 5 sums up the results.
2 A simple model of auditing

The problem of optimal coding is concerned with the best description of the random variable using the information of size $R$. The size of conveyed information can refer to the number of bits in an electronic mail message, the number of words in a newspaper add or the extent of an auditing procedure by an external auditor. The answer to the question of minimum information size that describes the random variable perfectly was answered by Shannon (1948) - $R$ has to be greater than the entropy of that random variable. But the uncertainty reduction in an economic setting is costly - in many cases to describe all the operations of a company credibly to the investors one would need to spend prohibitively large amounts. We are then in the environment when $R$ is usually smaller than the entropy of firms’ cash flows. In this case the uncertainty of the firm's cash flows can not anymore be communicated to the investors perfectly, but the uncertainty of the cash flows is nevertheless reduced.

The economic setting is as follows. We consider a firm, which lives for one period and generates cash flows $X$ at the end of that period. The firm estimates $X \sim N(\mu, \sigma^2)$, i.e. the cash flows are distributed normally with mean $\mu$ and intrinsic volatility $\sigma_{I}$. The overall economic uncertainty adds an independent noise component $Z$ to $X$ to form the distorted cash flows $\tilde{X} = X + Z$, as observed by the outside investors. We assume that $Z$ is distributed normally with mean 0 and variance $\sigma^2_{N}$. Therefore in this case, the outsiders observe $\tilde{X} \sim N(\mu, \sigma^2 = \sigma^2_{I} + \sigma^2_{N})$, i.e. the estimation of the cash flows by the market investors is unbiased.

The firm has a possibility to hire an external auditor. If it chooses to do so, the noise volatility $\sigma^2_{N}$ of cash flows to the outside investors is reduced. If the auditor establishes a channel of size $R$, it faces a cost $c(R)$, where we assume that the costs of implementing the channel are related to its size. At the same time, the noise variance $\sigma^2_{N}$ is reduced to $\sigma^2_{N} \cdot 2^{-2R}$.

Putting both effects together, the information channel of size $R$ provides investors with cash

$\footnote{This is the result of distortion theory for normally distributed random variables, see Appendix, Theorem A.1}$

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flows distribution

\[ \tilde{X}_r \sim N \left( \mu - c(R), \sigma^2 + \sigma_N^2 \omega^{-2R} \right), \]

The mechanism is depicted in Figure 1.

![Figure 1: Addition of the economic noise to the intrinsic cash flow volatility.](image)

The objective function of the firm is to maximize its stock price by choosing the channel capacity \( R \). They solve the following optimization problem

\[ \max_{R \geq 0} p(\tilde{X}_r). \]

There are two opposing effects of channel capacity increase. On one side it reduces the cash flows by the costs of establishing the channel \( c(R) \). On the other hand it also reduces the volatility of cash flows - risk averse investors in the financial market will prefer to hold shares of the low volatility firm. The tradeoff between these two effects is what determines the optimal channel capacity by the auditors.

We now make the following additional assumptions. The costs associated with establishing a channel of size \( R \) are linear\(^2\) in \( R \), i.e. \( c(R) = C \cdot R \). We assume that the market is composed of a single representative agent with CARA utility of absolute risk aversion \( \alpha \). The firm’s shares are in unit supply. The riskless rate in the economy is normalized to 0. The following theorem characterizes the price formation in this setting.

**Proposition 2.1.** In an economy described above, the auditor establishes a channel of size

\(^2\)A general form of the cost function will be treated in the empirical section of the paper.
\( R^* > 0 \) if and only if \( K = \frac{\alpha \sigma_N^2 \log(4)}{C} > 1 \). The price of the shares is given by

\[
p = \mu - CR^* - \alpha \sigma_I^2 - \alpha \sigma_N^2 2^{-2R^*},
\]

where \( R^* \) is given by

\[
R^* = \begin{cases} 
\frac{1}{2} \log_2 K & K > 1 \\
0 & K \leq 1 
\end{cases}
\]

The price of company shares has the same structure as under full information but with a changed cash flows’ expectation \( \mu' \) (instead of \( \mu \)) and volatility \( \sigma'^2 \) (instead of \( \sigma_I^2 + \sigma_N^2 \)). The CARA investors weigh the marginal costs \( C \) of establishing a credibility channel and the marginal benefits of volatility reduction \( \alpha \sigma_N^2 \log(4) \). The size of the information channel is positively related to the coefficient of investors’ risk aversion \( \alpha \) and the noise of the economy \( \sigma_N \). In the risk neutral economy, the price optimal level of channel capacity is \( R = 0 \), i.e. no auditing is necessary. The investors value only expected stock returns, which are highest when no auditing is performed. Stock price (14) can be decomposed into the classical component \( \mu - \alpha \sigma_I^2 \) not influenced by auditing and a reduced economy-wide noise component \( \alpha \sigma_N^2 2^{-2R^*} \) together with the auditing costs \( CR^* \). The results for the general cost function \( c \) are given in Appendix A.2, Proposition A.4.

The firm’s benefits from auditing \( B \) are

\[
B = -CR^* + \alpha \sigma_N^2 (1 - 2^{-2R^*}) = \begin{cases} 
\alpha \sigma_N^2 \left(1 - \frac{1}{K}\right) - \frac{C}{2} \log_2 K & K > 1 \\
0 & K \leq 1 
\end{cases}
\]

where \( R^* \) and \( K \) are given as in Proposition 2.1. We can compute the following comparative
statics results ($K > 1$, $\frac{\partial B}{\partial \alpha} = \frac{\partial B}{\partial \sigma_N^2} = \frac{\partial B}{\partial C} = 0$ otherwise).

$$
\begin{align*}
\frac{\partial B}{\partial \alpha} &= \sigma_N^2 - C \\
\frac{\partial B}{\partial \sigma_N^2} &= \alpha - \frac{C}{2\sigma_N^2} \\
\frac{\partial B}{\partial C} &= \frac{1}{2} - \frac{K}{\log 4} - \frac{1}{2} \log_2 K
\end{align*}
$$

(Graphical representation of auditing benefits and auditor’s compensation is presented in Figure 2. The firm’s auditing benefits $B$ has a call-option like structure with respect to the overall noise in the economy $\sigma_N^2$ and a put-option like payoff with respect to the costs associated with the formation of the information channel $C$. The intrinsic cash flow volatility $\sigma_I$ does not influence firm’s auditing benefits. For both parameters $C$ and $\sigma_N^2$ there exists a cut-off value $C^*$ and $(\sigma_N^2)^*$ such that for all $C > C^*$ and $\sigma_N^2 < (\sigma_N^2)^*$ the benefits are 0. Additionally, $B$ becomes linear in $\sigma_N^2$ as $\sigma_N^2 \to \infty$, i.e. $\frac{\partial B}{\partial \alpha} \to \alpha$. The same happens for $\frac{\partial B}{\partial \alpha} \to \sigma_N^2$ as $\alpha \to \infty$. The payoff with respect to average risk-aversion parameter $\alpha$ is similar to that of $\sigma_N^2$.

![Graphical representation of auditing benefits and auditor’s compensation](image)

**Figure 2:** Auditor’s compensation $L$ and auditing benefits to the firm $B$ drawn with respect to the cost of auditing $C$ and the overall noise in the economy $\sigma^2$.

The model also addresses the welfare implications of corporate governance structures. Certain requirements of the Sarbanes-Oxley act, such as the main auditor rotation and the independence of external auditor, as well as higher criminal penalties in the event of auditing mischief, all reduce the economic noise $\sigma_N^2$ in the economy. The results are in contradiction to the Coase theorem, where market mechanism produces welfare first-best auditing scheme.)
2.1 A multi-period model

We now consider a multiperiod model of auditing behavior of a representative investor. The model setting is similar to Vayanos (1999) with few changes to fit our setting.

Representative investor

Activity takes place at discrete times $t = 0, 1, \ldots$. There is one consumption good and two investment opportunities - a riskless one yielding a zero return and a risky technology yielding $X(t)$ at time $t$. The cash flows to the firm are known perfectly to the performing auditors but are noisy to the investors. We assume that in the presence of auditing $X(t)$ follows a dynamics

$$X(t) = \rho X(t-1) + S(t),$$

where $S(t)$ is a white noise process with mean $\mu(r) = \mu - Cr$ and variance $\sigma^2(r) = \sigma^2 + (1 - \rho)\sigma^2 2^{-2r}$. The utility function of the representative investor is given by

$$\mathbb{E} \left[ -\sum_{t=1}^{\infty} \beta^t \exp(-\alpha c(t)) \right]$$

where $c(t)$ is the consumption of the agent in period $t$. The agent is endowed with $M$ units of consumption good at time 0. The supply of stocks of the firm is constant and normalized to 1.

The firm

The firm sets the auditing quality $R$ so as to maximize long-run average firm share price, i.e. it maximizes

$$\max R \geq 0 \lim_{t \to \infty} \frac{1}{t} \mathbb{E} \left[ \sum_{s=1}^{t} p(s) \right]$$

It is a surprising result that the level of auditing investment per period coincides with the same level in a one-period model.

\footnote{The same analysis follows if the riskless return is non-zero. For this more general setting we refer to the Vayanos (1999) paper.}
Proposition 2.2. Under technical conditions, $r^\ast(t)$ is constant and has the same structural form as in (2) with $R^\ast$ replaced by $r^\ast$.

Proposition 2.2 states that the level of $r^\ast$ - per period investment in the establishment of a credible information channel - is equal in a one-period case as well as in the multiple period case. Therefore we can restrict the study of auditor’s compensation to the one period case with multiple units.

2.2 Auditing in view of the Debtholder-Shareholder conflict

In this section we analyze the shareholder/debtholder conflict in a firm from the perspective of auditing. We ask ourselves, if auditing provides protection to the debtholders and whether the optimal auditing quality levels differ for shareholders and debtholders?

Let us assume that the total value of the firm is distributed as before, i.e. if the agent (debtholder or shareholder) imposes the level of auditing in the amount $R$, the initial value of the company is $A(0) - c(R)$ and the firm volatility is reduced to $\sigma 2^{-R}$. Part of the company is financed with debt with principal amount $P < A(0)$ and the rest is financed by equity. We use the Leland and Toft (1996) model, see Appendix A.2, to determine the value of debt and equity under auditing. The following pictures shows one behavior of the debt and equity prices depending on the quality of external auditing. The results indicate

![Graphs showing the relationship between debt and equity prices in the presence of auditing quality $R$. The parameters of the model are given in Table STATE WHICH TABLE.]

Figure 3: The relationship between debt and equity prices in the presence of auditing quality $R$. The parameters of the model are given in Table STATE WHICH TABLE.

We emphasize that the shape of the figures changes with different parameter values. It
may happen, that both debtholders and stockholders demand zero auditing. The control of auditing costs is performed in the empirical section where a general cost function is assumed.

We next investigate the behavior of the optimal auditing quality $R$ with respect to leverage ratio and asset noise volatility $\sigma$. The results are presented in Figure 4. The optimal level of auditing from the perspective of shareholders and debtholders is different. The debtholders require auditing protection at low levels of leverage $L$ ($R_D^1, R_D^2$). The shareholders require it only for high leverage levels. The explanation for this puzzle is that auditing costs are born by shareholders for low leverage and by debtholders for high levels. This relationship does not change even when varying the volatility levels $\sigma$.

2.3 Strategic considerations regarding auditing

In the previous sections we assumed that both $\sigma_I, \sigma_N$ are known, that both the firm and the auditor agree on the amount of auditing (there is no negotiating power on neither side) and that there is only one auditor. In this section we relax these assumptions and examine the auditing decision under these new conditions.

We first address the issue of negotiating power, that is which party offers the contract. We distinguish between two extreme cases. In the first one, the firm offers an auditing contract
with specific quality. The auditor can either accept or decline the contract. In the second example, we consider two auditors, which simultaneously submit audit offers. The firm then chooses the one closest to the optimal level. If both auditors submit the same auditing offer, they share the auditing profits. The following proposition shows that the first best is achieved when the firm offers an auditing contract whereas the market breaks down in the case when negotiating power is with the auditor.

**Proposition 2.3.** Let $R^*$ be the first best auditing quality as defined in (2), Proposition 2.1. The following relationships hold:

(a) If the firm offers the auditing contract and the auditor can only accept or decline it, then the first best $R^*$ is accepted.

(b) If at least two auditors simultaneously submit audit offers, then the optimal auditing quality is 0, which is also trembling hand perfect equilibrium of the offer game.

Result (a) of Proposition 2.3 states that even though the auditor and the firm have different optimizing functions ((3) as opposed to (14)), the auditor accepts the auditing quality contract proposed by the firm. Result (b) shows two things. First, competition pushes the auditing quality down. Secondly, the auditing quality under laissez-faire conditions is below the first best level, i.e. there is a reason for regulation of auditing. This supports the provisions in the Sarbanes-Oxley act. We note that a repeated offer game in (b) could potentially solve the underauditing problem.

We next address the issue of $\sigma_N$. When employing an external auditor, the shareholders do not know if the increase in firm value comes from auditing procedure by reducing $\sigma_N^2$ or through the reduction of the intrinsic cash flow volatility $\sigma_I^2$, see (14). We model this behavior in a framework of global games as presented in Frankel, Morris, and Pauzner (2000). The game matrix for this global game is given in Table 1. The auditor chooses whether he accepts or declines the audit offer. The firm chooses auditing quality $r$. The firm and the auditor observe only a noisy signal of $\sigma_N$, $S_1$ and $S_2$ respectively. We use the theory in Morris and
Table 1: The game matrix for the global game between the firm and the auditor. The first entry in the parenthesis is the payoff to the firm and the second is the payoff to the auditor for both actions of the auditor. The allowed values for $r \in \mathbb{R}^+$. 

Shin (2001) generalized to a continuous action set of the firm to characterize the behavior of both players in this situation.

**Proposition 2.4.** In the setting above the firm proposes a contract with the level of auditing $R^*(S_1)$, where $R^*$ is defined in (2) and $S_1$ is the signal the firm receives about $\sigma_2^2$. The auditor decision to accept the auditing procedure has a cutoff value $S_2^*$, with signals above $S_2^*$ eliciting an acceptance of the auditing procedure.

The Proposition identifies that the auditing level chosen by the firm on average coincides with the first best outcome in (2).

### 3 A model of auditors’ compensation

We now apply the theory developed in the previous chapter to the compensation of external auditors. We assume that the firm is composed of $n$ identical business units, each of which produces cash flows as in Section 2. The firm has the possibility to invest resources into the reduction of the overall volatility of its business units. An interesting consequence from the information theory is that the more business units there are, the smaller proportion of money invested (per business unit) has to be spent in order to reduce the perceived volatility of that business unit to a desired level. In plain terms, larger firms have an auditing advantage over smaller ones. Shannon’s entropy therefore gives concrete quantitative insights into the economies of scope in auditing. For precise statement, see Appendix, Theorem A.2. We use Shapley value as a solution concept to determine the value added by the auditing department.

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4 This is a mere computational assumption and does not alter the overall results significantly.
There are two types of agents in the model - the external auditor, which we denote by $A$ and $n$ individual firm business units. We denote by $v(k)$ the value of $k$ firm’s business units without auditing and $v(A,k)$ the value of $k$ units under external auditing. Therefore, the following relationships hold:

\[
v(k) = Fk(\mu - \alpha \sigma_i^2 - \alpha \sigma_N^2) \quad (6)
\]
\[
v(A,k) = Fk(\mu - Cr^* - \alpha \sigma_i^2 - \frac{1}{k} \alpha \sigma_N^2 2^{-2r^*}) \quad (7)
\]

The value of $k$ firms’ business units is $Fk$ multiplied by the contribution of each business unit to the total firm share price (equation (14)). We interpret $F$ as the returns to scale function\(^5\).

The following proposition computes the value of external auditior to the firm with $n$ business units and constant returns to scale.

**Proposition 3.1.** Under the conditions stated above, the value of the auditor’s compensation is

\[
\varphi_A = \begin{cases} 
\frac{1}{n+1} F \left[ \frac{C}{2} \zeta(K) - \frac{C}{2} \log K \frac{K(K-1)}{2} + \alpha \sigma^2 \left( K \frac{K(K-1)}{2} + \log \left( \frac{K!}{K!} \right) \right) \right] & K > 1 \\
0 & K \leq 1
\end{cases} \quad (9)
\]

with $K$ as in Proposition 2.1, $K = K \land n$ and $\zeta(K) = \sum_{k=1}^{K} k \log k$. The total market firm capitalization is then

\[
V = F n \begin{cases} 
\mu - \alpha \sigma_i^2 - \alpha \sigma_N^2 & n \geq K \\
\left( \mu - Cr^* - \alpha \sigma_i^2 - \frac{1}{n} \alpha \sigma_N^2 2^{-2r^*} \right) & n < K
\end{cases} \quad (10)
\]

\(^5\)If we assume that for all $x, y \in \mathbb{R}$ the value $v(k) = f(k)(\mu - \alpha \sigma^2)$ (and correspondingly $v(A,k)$), where the function $f$ satisfies the Cauchy condition

\[
f(x) + f(y) \leq f(x + y), \quad (8)
\]

we account for business-unit synergies. It is shown in Theorem A.5 of the Appendix that the growth rate of function $f$ satisfying (8) is $f(k) \sim O(k^\alpha)$, where $\alpha \geq 1$. 

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where \( r^* \) is given as in equation (2). The last formula can also be expressed as

\[
\mu - \frac{C}{2} \log \frac{K}{n} \geq \frac{C}{2} \log \frac{K}{n}.
\]

Equation (9) reveals that the size of the managerial compensation does not depend on the mean production level \( \mu \) of firm’s business units and is proportional to the marginal contribution \( F \) of each business unit. What is interesting is that it is not proportional to the added auditing value to every business unit. This parallels the results in information theory. The description of a set of independent random variables demands lower channel capacity than the multiplication of capacities for every individual random variables. Auditing therefore exhibits intrinsic economies of scale.

4 Empirical estimation of the model

TO BE COMPLETED.

5 Conclusions

The paper develops a model of optimal auditing behavior when cash flows to the firm are observed perfectly by the firm but only imperfectly by the outside investors. In order to reduce this uncertainty, the firm has a possiblitiy to choose an external auditor which acts as a cash flows noise reducer. The optimal amount of external auditing has a cut-off value depending on the average level of risk-aversion in the economy, the costs of auditing and the overall noise in the economy. We then establish the optimal choice of external auditing by the firm and its implications on firm share prices and auditor’s compensation. In a competitive auditing market, imposing external auditing restrictions does not necessarily imply increased financial stability.
The model makes a series of simplifying assumptions, among which the most relevant are:
that the costs of information channel size increases linearly with channel size and that the firm has perfect information about its future cash flows. Considering a market equilibrium in the auditors’ market and closing the model would be a natural extension.
A Appendix

A.1 Basic results of Information Theory

**Theorem A.1** (adapted from Theorem 13.3.2 in Cover and Thomas (1991)). *In a noisy Gaussian channel where the noise variable $Z$ is distributed normally with mean zero and variance $\sigma^2$, the size $R$ of the channel necessary to reduce the variance of the input $\tilde{X}_r$ to $D$ is

$$R(D) = \begin{cases} \frac{1}{2} \log \frac{\sigma^2}{D} & 0 < D \leq \sigma^2 \\ 0 & D > \sigma^2 \end{cases}$$ \hspace{1cm} (11)$$

Theorem A.1 gives the size of the channel $R$ if all normally distributed random variables with variance $D$ or less are to be communicated without error. By inverting (11) and solving for $D$ we obtain that the normally distributed random variables with variance $D = \sigma^2 2^{-2R}$ can be communicated perfectly over the channel of size $R$.

It is a surprising result of the information theory that a normally distributed random vector of size $k$, where the variance of every component is equal, can be described effectively with size less than $kR$. This is the essence of the following Theorem.

**Theorem A.2** (adapted from Theorem 13.3.3 in Cover and Thomas (1991)). *In a noisy Gaussian channel for a vector $X$ of dimension $k$ of independent, identically distributed random variables with variance $\sigma^2$, the size of the channel necessary to reduce the variance of the input to $D$ is

$$R(D) = \frac{k}{2} \log \frac{\sigma^2}{D \cdot k},$$ \hspace{1cm} (12)$$

where we assume that $D < \sigma^2$. We call $R$ the *rate distortion* parameter. From the theorem we again establish the reduction
in variance as a result of channel size $R = kr$. We get that

$$D = \frac{1}{k} \sigma^2 e^{-2r/k} = \frac{1}{k} \sigma^2 e^{-2r}.$$ 

The description of $k$ independent random variables can be described by only a fraction $1/k$ of the original channel. The last result concerns the sequence of random variables which are not i.i.d. but serially correlated. This is described in the following Proposition.

**Proposition A.3.** Assume that $R \geq \log(1 + \rho)$. The rate distortion of a Gauss-Markov source process $X(t) = \rho X(t-1) + S(t)$, where $0 < \rho < 1$ and $S$ is an i.i.d. Gaussian $N(0, \sigma^2)$ sequence is given by

$$D(R) = (1 - \rho^2)\sigma^2 2^{-2R}.$$ 

### A.2 Proofs of Theorems

**Proof.** (of Proposition 2.1.) Since $\lim_{R \to \infty} \sigma^2(R) = \sigma_I^2$ and $\lim_{R \to \infty} c(R) = 0$ we have that the maximum is achieved either when $R = 0$ or in the interior of the positive real axis, when $\frac{\partial p}{\partial R} = 0$. The first order condition for this is

$$\frac{\partial p}{\partial R} = -C - \alpha \sigma_N^2 2^{-2R} \log(2^{-2})$$

This gives us the desired condition. If $R^* > 0$ then

$$\frac{\partial^2 p}{\partial R^2} = -\alpha \sigma_N^2 2^{-2R^*} \left( \log \frac{1}{4} \right)^2 < 0,$$

which guarantees that $p$ at $R^*$ obtains a maximum.

**Proposition A.4.** In an economy of Section 2, the auditor establishes a channel of size $R^* > 0$ if and only if there exists a solution to

$$c'(R) = \alpha \sigma_N^2 \log 42^{-2R}.$$  (13)
The price of the shares is given by

\[ p = \mu - c(R^*) - \alpha \sigma_R^2 - \alpha \sigma_N^2 2^{-2R^*}. \] (14)

**Proof.** (of Proposition 2.2.) The proof and the problem setting resembles closely that of Vayanos (1999). Let \( \mathbb{F}_t \) be \( \sigma \)-algebra generated by \( \{S(1), \ldots, S(t)\} \). We write \( \mathbb{E}[\cdot|\mathbb{F}_t] = \mathbb{E}_t[\cdot] \).

The investor is faced with the following maximization problem

\[
\max_{\{c(t), x(t)\}_{t \geq 0}} \mathbb{E} \left[ -\sum_{t=0}^{\infty} \beta^t e^{-\alpha c(t)} \right]
\] (15)

subject to the wealth and equity holdings' dynamics

\[
W(t) = W(t-1) - c(t-1) + e(t-1)X(t) - p(t)x(t-1)
\] (16)

\[
e(t) = e(t-1) + x(t-1)
\] (17)

where \( W(t) \) is wealth, \( c(t) \) consumption, \( e(t) \) stock holdings, \( x(t) \) demand for stocks, \( X(t) \) payoff from holding stocks, \( p(t) \) prices, all at time \( t \). We conjecture that the demand \( x(t) \) for the risky asset at time \( t \) is given by

\[
x(t) = AX(t) - Bp(t) + Ce(t) + D
\] (18)

with \( A, B, C \) and \( D \) constants and assume the following functional form for \( p \)

\[
p(t) = aX(t) + ce(t) + d,
\] (19)

where \( a, c \) and \( d \) are constants. From there we have that

\[
p(t + 1) - p(t) = a(X(t + 1) - X(t)) + c(e(t + 1) - e(t))
\]

\[
= a((\rho - 1)X(t) + S(t + 1)) + cx(t).
\]
Let $V$ be the representative's agent value function, i.e.

\[
V(W, X, e, t) = \max_{\{c(s), x(s)\}_{s \geq t}} \mathbb{E} \left[ -\sum_{s=t}^{\infty} \beta^s e^{-\alpha c(s)} \right]
\]

subject to constraints (16)-(17). We conjecture that the value function is given by

\[
V(W, X, e, t) = -\exp(-\alpha(HX(t)e(t) + Fe(t)^2 + GW(t) + L)).
\]

We proceed to compute $\mathbb{E}_t[V(W, X, e, t + 1)]$. We divide this into two parts.

\[
\mathbb{E}_t[e^{-\alpha GW(t+1)}] = e^{-\alpha G(W(t)-c(t))} \mathbb{E}_t[e^{-\alpha Ge(t)X(t+1)-\alpha Gp(t+1)z(t)}] \\
= e^{-\alpha G(W(t)-c(t)-e(t)pX(t))} \mathbb{E}_t[e^{-\alpha G(x(t)(p(t)+a(\rho-1)X(t)+aS(t+1)+cx(t))+e(t)S(t+1))}] \\
= e^{-\alpha G(W(t)-c(t)-e(t)pX(t)+x(t)(p(t)+a(\rho-1)X(t)+cx(t)))} \mathbb{E}_t[e^{-\alpha G(ax(t)+e(t))S(t+1)}]
\]

We next have

\[
\mathbb{E}_t[e^{-\alpha HX(t+1)e(t+1)}] = e^{-\alpha H\rho X(t)e(t+1)} \mathbb{E}_t[e^{-\alpha HS(t+1)e(t+1)}]
\]

Putting it all together we get

\[
EV_t = \mathbb{E}_t[V(W, X, e, t + 1)] = \exp(-\alpha G[W(t)-c(t)-e(t)pX(t)] + x(t)(p(t)+a(\rho-1)X(t)+cx(t)]) - \alpha H\rho X(t)e(t+1) \\
+ \frac{1}{2} \left(-\alpha G(ax(t)+e(t)) - \alpha He(t+1)\right)^2 \sigma^2 \quad (20)
\]

\[
-\alpha \mu(G(ax(t)+e(t)) + H(x(t)+e(t))) \quad (21)
\]

The Bellman equation can now be written as

\[
V(W, X, e, t) = \max_{c(t), x(t)} \left\{ e^{-\alpha c(t)} + \beta \mathbb{E}_t[V(W, X, e, t + 1)] \right\} \quad (22)
\]
Differentiating with respect to \( x(t) \) gives us the following equations

\[
0 = -\alpha G[p(t) + a(1 - \rho)X(t) + cx(t)] - \alpha Gcx(t) - \alpha H\rho X(t) + \sigma^2 \alpha^2 (Ga + H)(G(ax(t) + e(t)) + H(e(t) + x(t))) - \alpha \mu(Ga + H)
\]

The first equation can be written as

\[
L_0 p(t) + L_1 x(t) + L_2 e(t) + L_3 X(t) + L_4 = 0,
\]

where the coefficients \( L_i, i = 0, \ldots, 3 \) are as follows:

\[
\begin{align*}
L_0 & = -\alpha G \\
L_1 & = -2\alpha Gc + \alpha^2 \sigma^2 (Ga + H)^2 \\
L_2 & = \alpha^2 \sigma^2 (Ga + H)(G + H) \\
L_3 & = -\alpha Ga(1 - \rho) - \alpha H\rho \\
L_4 & = -\alpha \mu(Ga + H)
\end{align*}
\]

This confirms the structural form (19). Since our interest was only to obtain the functional form of \( p(t) \) putting emphasis on the entrance of \( \sigma^2 \) in \( p \), we

Since there is only a representative investor in the market, the equilibrium conditions require that \( e(t) = 1 \) and \( x(t) = 0 \) for all \( t \). We therefore get that \( p(t) = \vartheta_1 X(t) + \vartheta_2 \sigma^2(R^*) \).

The firms maximize the long-run average of stock prices, i.e. they choose per period auditing investment \( R \) as to solve (5). For simplicity\(^6\) we assume that \( \mathbb{E}(X(0)) = \mu_0 = \mu - CR^* \).

---

\(^6\)The general case is not much more difficult than this and it does not offer any significantly different economic insights.
Then from the recursive relation for $X(t)$ we get that $\mathbb{E}(X(t)) = (\mu - CR^*) \frac{1 - \rho^{t+1}}{1 - \rho}$. Therefore

$$
\lim_{t \to \infty} \frac{1}{t} \sum_{s=1}^{t} \mathbb{E}(p(s)) = \lim_{t \to \infty} \frac{1}{t} \sum_{s=1}^{t} [a \mathbb{E}(X(s) + c'(\sigma^2(R^*)))]
$$

$$
= \lim_{t \to \infty} \frac{1}{a} \frac{\mu - CR^*}{1 - \rho} \sum_{s=1}^{t} \mathbb{E}(X(s) + c'(\sigma^2(R^*)))
$$

$$
= \frac{a(\mu - CR^*)}{1 - \rho} \lim_{t \to \infty} \sum_{s=1}^{t} (1 - \rho^s) + c'(\sigma^2(R^*))
$$

$$
= \frac{a(\mu - CR^*)}{1 - \rho} + c'(\sigma^2(R^*))
$$

The optimal choice of $R$ by the firm is therefore the same as in the one-period case.

\[\square\]

**Theorem A.5.** Let $f$ be a function on the real line, which satisfies the Cauchy condition $f(x) + f(y) \leq f(x+y)$ for all $x,y \in \mathbb{R}$. Then the growth rate of $f \sim O(x^\alpha)$, where $\alpha \geq 1$.

**Proof.** Let us assume that $f(n) = \beta n^\alpha + o(n^\alpha)$ with $\alpha < 1$. Using the Cauchy inequality for $x = y = n$ we get

$$
2\beta n^\alpha + o(n^\alpha) \leq \beta (2n)^\alpha + o(n^\alpha).
$$

Canceling $\beta$ and dividing by $n^\alpha$ we arrive at

$$
2 \leq 2^\alpha + \frac{o(n^\alpha)}{n^\alpha}.
$$

Taking the limit $n \to \infty$ gives us a contradiction, since $0 < \alpha < 1$.

\[\square\]

**Proof.** (of Proposition 3.1.) We assume that the manager $M$ is labeled as last, i.e. $n + 1$-st. Equations (6) and (7) confirm that $v(M) + v(L_I) \leq v(M, L_I)$, $v(M, L_I) + v(L_J) \leq v(M, L_I \cup J)$, $v(L_I) + v(L_J) \leq v(L_I \cup J)$, where $I$ and $J$ are disjunct subsets of $\pi$. This proves that the superadditivity of function $v$.

We next compute the Shapley value of the accountant. The definition of the Shapley value
\[ \varphi_M = \frac{1}{(n+1)!} \sum_{\pi \in S^{n+1}} (v(S(\pi, n+1) \cup n+1) - v(S(\pi, n+1))), \]

where \( S(\pi, k) \) is the set of agents in \( \pi \) which are before \( n+1 \) in the ordering of \( \pi \). We have

\[ AD(\pi) = v(S(\pi, n+1) \cup n+1) - v(S(\pi, n+1)) \]
\[ = F \left( -Ckr^*(k) + \alpha \sigma^2 \left( k - 2^{-2r^*(k)} \right) \right) \]

where \( k = l(n, n+1) \) is the length of the ordering of agents in \( \pi \) before the manager \( M \).

Therefore costs are virtually increased to \( Ck \) and the variance is not increased \((k \text{ times the } \frac{\sigma^2}{k})\). Let \( K \) be as in Proposition 2.1, i.e. such that \( r^*(k) = 0 \) for \( k > K \) and non-zero otherwise. Then

\[ r^*(k) = \log_2 \left( \sqrt{\frac{\alpha \sigma^2 \log 4}{Ck}} \right) \]
\[ = \frac{1}{2} \log_2 \left( \frac{K}{k} \right). \]

Assuming that \( K \) is also an integer (otherwise we have to replace \( K \) by \( \lfloor K \rfloor \)) and denoting by \( \overline{K} = K \wedge n \), the minimum between \( K \) and \( n \)

\[ \sum_{\pi \in S^{n+1}} AD(\pi) = \sum_{M=\pi(1)} AD(\pi) + \sum_{M=\pi(2)} AD(\pi) + \ldots + \sum_{M=\pi(n+1)} AD(\pi) \]
\[ = n! F \sum_{k=1}^{K} \left( -Ckr^*(k) + \alpha \sigma^2 \left( k - 2^{-2r^*(k)} \right) \right) \]
\[ = n! F \sum_{k=1}^{\overline{K}} \left( -\frac{C}{2} k \log \frac{K}{k} + \alpha \sigma^2 \left( k - \log_2 \frac{k}{K} \right) \right) \]
\[ = n! F \left[ \frac{C}{2} \zeta(\overline{K}) - \frac{C}{2} \log K \frac{K(K-1)}{2} + \alpha \sigma^2 \left( \frac{K(K-1)}{2} + \log \left( \frac{K\overline{K}}{K!} \right) \right) \right] \]
where $\zeta(k) = \sum_{l=1}^{k} l \log l$. Putting everything together we get

$$
\varphi_M = \frac{1}{n+1} F \left[ \frac{C}{2} \zeta(K) - \frac{C}{2} \log K \frac{K(K-1)}{2} + \frac{\alpha \sigma^2}{2} \log \left( \frac{K}{K!} \right) \right]
$$

The proposition then follows. \hfill \Box

**Proof.** (of Proposition 2.3.) We define the zero-profit condition as the value $R_1$ which solves the equation

$$
CR = \alpha \sigma^2 (1 - 2^{-2R}),
$$

i.e. the auditing quality which generates zero profit for the auditor, see equation (3). We fiest establish the relationship between $R_1$ and $R^*$. In order to do so, we evaluate the left (LHS) and right hand side (RHS) of (23) for $R_1$ given in (2). We get

$$
LHS(R^*) = \frac{C}{2} \log K
$$

$$
RHS(R^*) = \alpha \sigma^2 (1 - 2^{-2R^*})
$$

$$
= \alpha \sigma^2 - \frac{C}{2 \log 2}
$$

$$
= \frac{C}{2} \left( \frac{2}{\log 4} - \frac{1}{\log 2} \right).
$$

Therefore $LHS(R^*) < RHS(R^*)$ from which it follows that $R_1 > R^*$. If the contract of quality $R^*$ is offered by the firm, we get that the auditor would accept it, since it generates positive payoff to the auditor.

Let us now consider the case when two auditors bid for a contract with a single firm which accepts the lower bid - the firm does not distinguish between the decrease in $\sigma_I$ and increased auditing quality $R$ which in turn reduces $\sigma_N$. It is obvious that the $(r_1, r_2) = (0, 0)$ is a Nash equilibrium. We further show that it is also trembling hand perfect. Let $\sigma_2$ be a mixed strategy of the second bidder. The payoff to the first bidder is then $1 - \frac{1}{2} \sigma_2(0)$. Now assume that the first bidder uses a strategy $\sigma_1(0) = 1 - \delta$, $\sigma_2(a) = \delta$, where $a > 0$. The payoff to the
first bidder is then
\[
(1 - \delta)(1 - \sigma_2(0)) + \frac{1}{2}(1 - \delta)\sigma_2(0) + \frac{1}{2}\delta \sigma_2(a) + \delta \int_{x>a} \sigma_2(dx) < 1 - \frac{1}{2}\sigma_2(0),
\]

since \( \frac{1}{2}(\sigma_2(a) + \sigma_2(0)) + \int_{x>a} \sigma_2(dx) < 1 \). It is easy to see that there do not exist any other pure strategy equilibria.

\[ \square \]

**Proof.** (of Proposition 2.4.) In line with Frankel, Morris, and Pauzner (2000) let the strategy of the firm be a function \( S_1 \mapsto s_1(S_1) \), where \( S_1 \) is the signal that the firm receives about the noise component \( \sigma_N^2 \). The strategy of the auditor is a cutoff decision, i.e. accept the auditing procedure if \( S_2 > b \), and decline it otherwise. Given this strategy profile we first analyze the decision of the firm. The firm sets \( s_1 \) so that it is a best response to the second player actions, i.e. it maximizes

\[
(1 - p_2(S_1, b))(\mu - \alpha \sigma_I^2 - \alpha S_1) + p_2(S_1, b) \left[ \int_R \{ \mu - C_1 s_1(r) - \alpha \sigma_I^2 - \alpha S_1 2^{-2s_1(r)} \} f_1(S_1, r) dr \right] \tag{23}
\]

where \( p_2(S_1, b) \) is the probability of the auditor to accept the auditing procedure given that the firm observed \( S_1 \) and that the perceived auditor’s cutoff level from the firm’s perspective is \( b \). The probability that the true value of the \( \sigma_N^2 \) is \( r \), given that the firm observed a signal \( S_1 \) is \( f_1(S_1, r) \). Since the strategy of the firm only affects the part of expression (23) in brackets, the optimal \( s_1 \) coincides with (2). The strategy is independent of the cutoff level \( b \). We next solve for the optimal cutoff level of the auditor. The auditor sets \( b \) so as to maximize

\[
1(S_2 > b) \cdot \int_R \{-Cs(y) + \alpha S_2(1 - 2^{-2s(y)})\} f_2(S_2, y) dy, \tag{24}
\]

where \( s \) is the optimal strategy of the firm as perceived by the auditor and \( f_2(S_2, y) \) is the probability that the true value of \( \sigma_N^2 \) is \( y \) given that the auditor observed a signal \( S_2 \). The function \( s \) is given by \( s(y) = \frac{1}{2} \log_2(Fy) \cdot 1(y \geq \frac{1}{F}) \), where \( F = \alpha \log_4 e \). Equation (24) simplifies
to

$$\mathbb{1}(S_2 > b) \cdot \left[ \alpha S_2 - \alpha S_2 \int_{-\infty}^{1/F} f_2(S_2, y) \, dy - \alpha S_2 \int_{1/F}^{\infty} \frac{1}{Fy} f_2(S_2, y) \, dy - C \int_{1/F}^{\infty} \frac{1}{2} \log(Fy) f_2(S_2, y) \, dy \right]$$

Since on the interval $[1/F, \infty)$ the function $y \mapsto \frac{1}{Fy} < 1$, we have that THE FIRST TWO TERMS ARE LESS THAN $\alpha S_2$.

THE DERIVATIVE WITH RESPECT TO $S_2$

$$\alpha - \alpha \Phi(1/F, S_2) - \alpha S_2 f_2(1/F, S_2) - \alpha \int_{1/F}^{\infty} \frac{1}{Fy} f_2(S_2, y) \, dy - \alpha S_2 \int_{1/F}^{\infty} \frac{1}{Fy} \frac{\partial f_2}{\partial S_2}(S_2, y) \, dy - C \int_{1/F}^{\infty} \frac{1}{2} \log(Fy) \frac{\partial f_2}{\partial S_2}(S_2, y) \, dy - C \int_{1/F}^{\infty} \frac{1}{2} \log(Fy) \frac{\partial^2 f_2}{\partial S_2^2}(S_2, y) \, dy$$

The following proposition is a restatement of the debt and equity pricing results in Leland and Toft (1996) and is repeated here for coherence.

**Proposition A.6** (Leland-Toft (1996)). Let the dynamics of firm assets be $\frac{dA}{A} = \mu dt + \sigma dW$. The firm issued zero-coupon debt with maturity $T$ and principal $P$ which is retired uniformly over the interval $[0, T]$. The firm defaults when $A$ falls below the default boundary $V_B$, determined below. The costs of bankruptcy are $\alpha V_B$. Then the value of the equity in this model, denoted by LT (mnemonic for Leland-Toft Equity value), is

$$LT(V, P, \sigma) = V - \alpha V_B \left( \frac{V}{V_B} \right)^x - D,$$

where the value of $D$ is given by

$$D = P \left( \frac{1 - e^{-rT}}{rT} - I(T) \right) + (1 - \alpha)V_B J(T),$$
where

\[ I(T) = \frac{1}{rT} (G(T) - e^{-rT} F(T)) \]

\[ J(T) = \frac{1}{z\sigma \sqrt{T}} \left( -\left( \frac{V}{V_B} \right)^{-a+z} N(q_1(T))q_1(T) + \left( \frac{V}{V_B} \right)^{-a-z} N(q_2(T))q_2(T) \right) \]

and the constants are given by

\[ F(t) = N(h_1(t)) + \left( \frac{V}{V_B} \right)^{-2a} N(h_2(t)) \]

\[ G(t) = \left( \frac{V}{V_B} \right)^{-a+z} N(q_1(t)) + \left( \frac{V}{V_B} \right)^{-a-z} N(q_2(t)) \]

\[ z = \frac{\sqrt{(a\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} \]

\[ q_1(t) = \frac{-b - \sigma^2 t}{\sigma \sqrt{t}} \quad h_1(t) = \frac{-b - a\sigma^2 t}{\sigma \sqrt{t}} \quad a = \frac{r - \delta - \sigma^2/2}{\sigma^2} \]

\[ q_2(t) = \frac{-b + \sigma^2 t}{\sigma \sqrt{t}} \quad h_2(t) = \frac{-b + a\sigma^2 t}{\sigma \sqrt{t}} \quad b = \log \left( \frac{V}{V_B} \right) \]

and the default boundary \( V_B \) is given by

\[ V_B = \frac{AP/(rT)}{1 + ax - (1 - a)B} \]

with

\[ A = 2ae^{-rT}N(a\sigma \sqrt{T}) - 2zN(z\sigma \sqrt{T}) - \frac{2}{\sigma \sqrt{T}} n(z\sigma \sqrt{T}) + \frac{2e^{-rT}}{\sigma \sqrt{T}} n(a\sigma \sqrt{T}) + z - a \]

\[ B = -\left( 2z + \frac{2}{z\sigma^2 T} \right) N(z\sigma \sqrt{T}) - \frac{2}{\sigma \sqrt{T}} n(z\sigma \sqrt{T}) + z - a + \frac{1}{z\sigma^2 T} \]
References


