A Theory of Mergers in a Network Context

(Very preliminary, please do not quote)

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Abstract

We propose a theory of mergers in a network model. Mergers change the risk structure of cash flows and generate externalities which the firms can exploit by creating internal capital markets. We use Maskin (2003) result on coalitional formation with externalities as solution concept. The results show that firm values in a network should not be considered in isolation. “Maskin’s corrections” increase the equity value of the firms. Recession (prosperity), as measured by the average D/A ratio, decrease (increase) the number of coalitions in a network. Network dependence increases the externality values and the number of coalitions.
1 Introduction

The existence and the scope of firms has long puzzled economic theory. When profit generating motive is the only motive of the firms, only a single firm should exist. The question of a multitude of firms is not addressed in general equilibrium. In the finance literature, mergers were long a rather unexplained phenomena. Segall (1968) stated, that there is nothing known about mergers, no single hypothesis is general and useful enough and there are no generalizations for the merger activity explanation. Recently, the papers by Morellec and Zhdanov (2005), Lambrecht and Myers (2006) and Lambrecht (2004) question this statement and propose a theoretical framework for merger activity. They model the firm’s merger decision depending on the underlying real activity of the individual and the merged firm. The feature of all of these models is that eventually only a single firm would exist¹.

The first to provide a coherent theory for the reasons of firm integration (i.e. a takeover of a supplier firm by its main buyer) were the articles by Grossman and Hart (1986) and Hart and Moore (1990). They explain the firm scope and the reasons for mergers in an incomplete contract/property rights paradigm. They provide a rationale for merger when either one firm’s incentives or investment decisions rely sufficiently on the property rights of the other firm’s production.

From the finance perspective, the analysis of Grossman and Hart (1986) ignores the possibility, that the acquired firm, while producing benefits long-term, can end in default in a short run. This opposing effect is what the paper investigates. More specifically, we look at the merger as a creation of internal capital markets. Internalization of capital markets is an important part of firm’s strategy and the functioning of capitalist markets in general. It is the essence of the existence of firms itself, as emphasized by Williamson (1987). Internal capital markets change the risk structure of the firm. Consider a two firm example. Firm one is the buyer of goods and firm two is the supplier of them. In addition to the buy-supply relationship of the two firms, there are external cash flows, the nature of which we do not explain explicitly and model them statistically. From the perspective of firm one, the buy relationship decreases

¹This is the property of the exponential Brownian motion process which drives the asset values/prices in their models. Other processes governing the real economic activity would most probably generate different merger predictions.
asset value. In turn, it can also decrease the overall asset volatility, acting as a reducer of volatility to the external cash flows. The same is true for firm two with only difference, that the buy orders are replaced with supply orders. Both firms are therefore exposed to their buy-supply relationship. Now let us suppose that the two firms merge. What was before a classical market for goods is now an internal capital market. Neither firm one nor firm two is exposed to the fluctuations in the asset dynamics originating from buy-supply orders. The risk structure of asset fluctuations has changed. The costs of a merger is the vulnerability of the merged firm to the aggregation of external cash flows.

The problem is even harder to understand when there are more than two firms. In this case, firms can form coalitions. The in-between firm markets become coalitional internal capital markets. As before, the risk structure of cash flows changes. Similarly, but more emphasized, the coalitions exert externalities on other coalitions, due to the change in risk structure of cash flows.

The risk structure of cash flows are not the only important factor in determining the merger coalitions. Others include financial structure of the firms (such as debt/equity ratio) and total asset size.

We propose a model of firm merger in a network of firms, where economic dependencies between firms are given at the begining and can not be changed thereafter\(^2\). The asset dynamics are taken from the paper by Brumen and Vanini (2006), see Proposition 3.1. We assume that the equity value maximization (untill a specific date) of a single or a merged firm is the firms’ objective function. The firms than bargain and coalitions are formed. When firm coalitions are formed, the structure of cash flows changes and the equity of the merged firm is calculated.

The value of the equity is dependent on a multitude of factors. The analysis of factors that influence the

The results show that: **This part is not finished.**

Specifically, Hart (1995) in the exposition of property rights approach to firm scope makes the following statement (page 45): “If assets a1 and a2 [of firms 1 and 2] are independent, then non-integration is optimal”. Our analysis shows that this may be so, but is not necessarily the

\(^2\)An equivalent statement is that the cost of breaking an economic relationship is too high, see Grossman and Hart (1986).
case and the analysis shows that apart from firm business lines considered in Hart (1995) the 
 firms have several other business lines, not necessarily linked to the firm with which merger is 
in question. In the case when firms only businesses are linked to each other, the Hart result 
obtains.

The theory of coalition formation under externalities was analyzed, among others by 
Maskin (2003). The theory predicts the firm (coalition) values and coalition structure. The 
downside of the theory is that only the distribution over merger coalitions is produced.

Most literature on internal capital markets, such as Scharfstein and Stein (2000), Inderst 
and Laux (2006) and others focus mostly on the ...The paper by Habib and Mella-Barral 
(2006) investigates different type of firm-firm connections, such as partnerships and mergers.

Our article is closer to Jovanovic and Rousseau (2002) which predicts merger and acquisi-
tion activity in relation to firm’s (Tobin’s) Q value and technological changes. Contrary to the 
articles of Lambrecht (2004) and Morellec and Zhdanov (2005), our model does not predict 
the existence of a single firm and can explain the merger waves activity.

This paper is structured as follows. Section 2 illustrates the proposed merger theory in a 
two firm example. In Section 3 a general merger theory for a network of firms is presented. 
Section 4 concludes.

2 A simple example

We assume the setting of Brumen and Vanini (2006) of two dependent firms, a buyer and a 
supplier, see Figure 1(a). The buyer firm 1 transfers an amount $E_{12} \cdot P_1 = V_{12}$ of its total 
assets to the supplier firm 2 at jump times of the Poisson process $N_1$. $E_{12}$ is the number of 
links between firms 1 and 2 and denotes the strenght of business relationship between the 
firms. $P_1$ is the proportionality factor of asset transfers. In addition to network dependencies, 
the two firms have external cash flows $B$ with drift $B_0$ and proportional to the asset values, 
depicted as dashed arrows. The dynamics of the asset values of both firms is therefore

\[
\begin{align*}
    dA_1 &= B_{01}A_1dt + (B_{11} - V_{12})A_1dN_1 + B_{11}A_1dN_1 + B_{12}A_1dN_2 \\
    dA_2 &= B_{02}A_2dt + (B_{21} + V_{12})A_1dN_1 + B_{22}A_2dN_2
\end{align*}
\]

(1) (2)
We assume that both firms have issued equity and zero-coupon debt with principals $D_1$, $D_2$ and maturity $T$. We denote by $E_1$, $E_2$ the equity values. According to Proposition 4.2. of Brumen and Vanini (2006) we can express the value of the firm’s equity as given in the following Proposition.

**Proposition 2.1.** The price of equity on a network firm $i$ with principal value $D_i$ and maturity $T$ is given by

$$E_i = A_i(0)N(d_i^1) - D_i e^{-rT}N(d_i^2),$$

where $d_i^1 = \log \frac{A_i(0)D_i + (r + \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}$, $d_i^2 = d_i^1 - \sigma_i\sqrt{T}$ and

$$\sigma_1 = \sqrt{(B_{11} + V_{11})^2 \lambda_1 + B_{12}^2 \lambda_2}$$

$$\sigma_2 = \sqrt{(B_{21} + \frac{V_{21}}{A_2(0)} A_1(0))^2 \lambda_1 + B_{22}^2 \lambda_2}$$

where $V_{11} = -P_1E_{12}$, $V_{21} = -V_{11}$.

![Diagram](attachment:network_diagram.png)

**Figure 1:** Example of firms and cash flows in a two firm buyer - supplier network - 1(a). The network cash flow directions are denoted in solid and the external cash flows in dashed lines. A merged firm 1(b) possesses only external cash flows. Cash flows in a network 1(a) as seen by individual firms 1 and 2 are presented respectively in figures 1(c) and 1(d).

Now let us assume that the two firms merge. The merged firm operates the individual units as independent entities, with the difference that the transfer of goods between the firms is now an internal capital market and the only risks that the firm faces are those of external
cash flows, see Figure 1(b). The network cash flows have no effect on the merged firm. The only risk that the joint firm faces is that of external cash flows \((B_1, B_2)\). The dynamics of the merged company is then given as \(A_{12} = A_1 + A_2\), where \(A_1\) and \(A_2\) are defined as in (1)-(2) and the equity value of a merged company is (Proposition 2.1)

\[
E_{12} = A_{12}(0)N(d_{12}^{12}) - D_{12}e^{-rT}N(d_{22}^{12}),
\]

where \(A_{12}(0) = A_1(0) + A_2(0)\), \(D_{12} = D_1 + D_2\), \(\sigma_{12} = \sqrt{(B_{11} + B_{21})^2 \lambda_1 + (B_{12} + B_{22})^2 \lambda_2^2}\) and \(d_{12}^{12}, d_{22}^{12}\) are defined as above.

The process of firm mergers involves bargaining between them. We employ the solution criterium of Maskin (2003) to determine, when does the merger occur. The reasoning in Maskin is simple. Let us assume that firms 1 and 2 exist as autonomous entities. If the firm 2 was to be taken over by firm 1, firm 1 has to compensate the shareholders of firm 2 with sufficient amount. The amount certainly has to be greater than firm 2 can earn on its own. Similarly, from the joint firm, the shareholders of firm 1 will approve the merger if the revenues of the merger to them is higher than they can receive in isolation. Intuitively, the merger will only occur if \(E_1 + E_2 \leq E_{12}\). Similar reasoning, with the same condition on equities can be employed by firm 2. There is no disagreements about the condition of the merger. There is no agency conflicts between the shareholders of both firms. The next question is how to split the profits of the joint company. The reasoning is according to the marginal contributions of both firms. Let us assume that the merger conditions is fulfilled, and that the firm 1 makes the merger bid. The marginal contribution of firm 2 to join firm 1 (accept the bid) is therefore \(E_{12} - E_1\). This is what the firm 2 should receive. In the reverse situation when firm 2 makes the bid, firm 1 receives \(E_{12} - E_2\) and firm 2 receives 2. The average of this two values is the marginal contribution of firm 2. The following Propositions makes the statements precise and gives coalitional justification.

**Proposition 2.2.** In a two firm example the firms merge if and only if

\[
E_{12} \geq E_1 + E_2.
\]
In this case, the equities of firms 1 and 2 is worth $\frac{1}{2}(E_1 + E_{12} - E_2)$ and $\frac{1}{2}(E_2 + E_{12} - E_1)$.

![Diagram of a merger indicator with respect to the dependency value of the network.](image)

**Figure 2:** A representation of a merger indicator with respect to the dependency value of the network. The dependency value of the network was defined in Brumen and Vanini (2006), Section 4 and reduces to the number of connections between firms one and two. Proposition 2.2 indicates that the firms merge when $E_1 + E_2 \leq E_{12}$.

Figure 2 shows a number of features of a merger model. Firstly, in a two firm environment the merged firm equity does not depend on the network dependency value. Secondly, the merger is rejected at levels of high or low network dependency. At low levels, the merger does not generate sufficient surplus to the supplier firm. At high levels, the same happens with the buyer. The intermediate levels of dependence can be beneficial to both firms and it is here where the merger occurs. Different values of network parameters move/scale the merger indicator function but the shape remains fairly stable. The position of cut-off values with respect to the debt/total asset values are presented in Proposition 2.3

### 2.1 The effect of networks on merger waves

We show that the merger interval of network dependencies relies highly on debt/total assets (D/A) ratio. Figures 3(a)-3(f) indicate the incentives to merge in a two firm example when the D/A ratio changes. The lower the D/A ratio, the less incentives the firms have to merge. The higher it is, the more prone the firms are to fluctuations in asset process dynamics and

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3The statement is not entirely correct. The coalitional theory does not predict why the merger was rejected and the merger condition in Proposition 2.2 is the same for both firms.
therefore the higher the range of network values where firms merge. The dependence of merger

Figure 3: The dependence of firm equity values (and merger indicator) with respect to the average D/A ratio of the firms. The D/A ratio increases from left to right and up to down.

Proposition 2.3. Let $E^*$ be, for fixed asset value $A$, the solution of $E_{12}(A, E) - E_1(A, E) - E_2(A, E) = 0$, the “merger cut-off value”. Then the following comparative statics results hold

$$\frac{\partial E^*}{\partial A} = -A\sqrt{T}\lambda_1 p_1 \frac{N'(d^1_1) (B_{11} + V_{11})}{\sigma_1} + N'(d^2_1) \frac{(B_{21} + V_{21}) A_1(0)}{\sigma_2} \frac{A_2(0)}{N(d^2_1)} \frac{N(d^1_1) - N(d^1_1) - N(d^2_1)}{N(d^2_1)}. \quad (6)$$

The proposition follows from the implicit function theorem and the calculation of greeks.

3 A general model

We now turn to the case of $N$ firms in a general buyer-supplier network. The notation and the network structure is inherited from Brumen and Vanini (2006). For coherence we rephrase some of the notation and the main theorem (Proposition 3.1.) of that paper. A network
is characterized by its adjacency matrix $E$, payout intensities $(\lambda)$ and ratios $(P)$ and the structure of externally generated cash flows $B$. Firms issue buy orders, which are modeled as jumps of independent Poisson processes $N_i$. The assets of firms are gathered in vector $A$.

**Proposition 3.1.** Let $\alpha_i = (B_{ij} + \frac{V_{ij}A_{ij}(0)}{(1 + B_{ij}A_{ij}(0))}, B_{ii} + V_{ii})$ and $\alpha = (\alpha_1, \ldots, \alpha_N)'$. Under the condition of absence of arbitrage, the process $A^Q$ follows

$$\frac{dA_i^Q}{A_i^Q} = r dt + \alpha \lambda^{1/2} d\tilde{W}, \quad (7)$$

where $\tilde{W}$ is a $Q$ Brownian motion and the solution to (7) is given by

$$A_i^Q = \exp \left( \left( r - \frac{1}{2} \| \lambda^{1/2} \alpha_i \| ^2 \right) t + \alpha_i' \lambda^{1/2} \tilde{W}(t) \right). \quad (8)$$

We assume that the objective function of the firm is equity value maximization and there are no agency conflicts within the firm. Let us assume that the firms $1, \ldots, N$ merger into coalitions $C_1, C_2, \ldots, C_d$. Coalitions form a partition of $\{1, \ldots, N\}$. We assume that inside the coalition individual firms now operate as subdivisions.

Additionally it effects other coalitions as well as the other coalitions affect the coalition forming. In this section we do not assume the dynamic aspects of merging. All mergers are done at the beginning of period. We employ Maskin (2003) solution of merger formation, which can be implemented algorithmically. The assumptions and the main result are stated in the Appendix$^4$.

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*Table 1:* Model parameters for the network of five firms.

$^4$An important assumption on the value function used in Maskin’s algorithm is subadditivity. In the network framework, this is generally not the case. We have checked for subadditivity for parameter values in Table 3.
The effect of network dependencies on coalition formation is shown in Table 3. We notice a

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Table 2: The dependence of the firm values and number of coalitions with respect to the network dependency value of the network. The first column does not reflect the real dependency value, but only the increase in the dependency value of the whole network. The network matrix $E$ is computed from it as $E = M \cdot E(0)$, where $E(0)$ is given in Table 3. The first five columns represent the value of firms 1-5 incorporating the synergies/externalities effect of mergers. The next five columns are the individual firm values, not incorporating the merger effects. The last column gives the average number of coalitions with the selected parameter values. Other parameters are given in Table 3.

3.1 The dynamics of mergers and merger waves

The next thing is that we produce the comparative statics results for firm values and merging as the debt/total assets (D/A) ratio changes. D/A ratio can be considered a proxy for a booming or a recession economy. The results are shown in Table 3.1. Table 3.1 indicates that the D/A ratio increases the number of coalitions formed in a network. The firms merge in times of recession and split in times of prosperity. The volatility of cash flows can be exploited when the distance to default is large, but a much more careful firm policies (such as merger) should be considered in the opposite case. Not finished.
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<td>146.2</td>
<td>112.1</td>
<td>112.4</td>
<td>114.4</td>
<td>113.0</td>
<td>113.3</td>
<td>2.8</td>
</tr>
<tr>
<td>9.0</td>
<td>157.1</td>
<td>148.7</td>
<td>149.2</td>
<td>155.0</td>
<td>157.0</td>
<td>122.1</td>
<td>122.4</td>
<td>124.1</td>
<td>123.0</td>
<td>123.3</td>
<td>3.0</td>
</tr>
<tr>
<td>10.0</td>
<td>169.8</td>
<td>158.5</td>
<td>159.0</td>
<td>154.8</td>
<td>169.7</td>
<td>132.1</td>
<td>132.4</td>
<td>133.9</td>
<td>133.0</td>
<td>133.3</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 3: The dependence of the firm values and number of coalitions with respect to the debt/total assets (D/A) ratio of the firms. The first column does not reflect the real D/A ratio, but only the increase in the D/A ratio of all firms. The D/A ratio of firm \( i \) is given by the formula \( A_i(0) + \frac{D}{A} \times 10 \), and where \( A_i(0) \) is given in Table 3. The first five columns represent the value of firms 1-5 incorporating the synergies/externalities effect of mergers. The next five columns are the individual firm values, not incorporating the merger effects. The last column gives the average number of coalitions with the selected parameter values.

4 Conclusions

The paper examines the effect of network dependencies on firm equity values and merger formation process. We use the results of Maskin (2003) in order to determine the equity values in a context, where merger externalities are present. We focus on the fact that the internalization of capital markets acts as an externality on firm(s) in a network. Size and sign of this externality are determined by the interaction of network and externally generated cash flows. The results indicate that firm values in a network should not be considered in isolation. “Maskin’s corrections” increase the equity value of the firms. Recession (prosperity), as measured by the average D/A ratio, decrease (increase) the number of coalitions in a network. Network dependence increases the externality values and the number of coalitions. A possible extension would be a dynamic coalition formation process, c.f. Konishi and Ray (2003) and endogenous network formation.
A Appendix

A.1 Maskin’s Theory of Coalition formation with externalities

For coherence we restate the axioms of Maskin (2003) on which the computations are made. We denote by \( \mathbb{N} = \{1, \ldots, n\} \) the set of integers smaller than \( n + 1 \). A \( N \)-player transferable utility game with externalities is given by

- A set of players \( \overline{N} \).
- A function \( v \), which assigns a worth \( v(S, \mathcal{P}) \) to a coalition \( S \in \mathcal{P} \), given a partition \( \mathcal{P} \) of \( \overline{N} \).

We focus on coalitional games that are normalized to \( v(\emptyset, \mathcal{P}) = 0 \) and where \( v \) is subadditive, i.e. for all \( S_1, S_2 \in \mathcal{P} \)

\[
v(S_1, \mathcal{P}) + v(S_2, \mathcal{P}) \leq v(S_1 \cup S_2, \mathcal{P}_{12}),
\]

where \( \mathcal{P}_{12} \) is the partition of \( \overline{N} \) where \( S_1 \) and \( S_2 \) are merged. Let \( \varphi \) and \( \psi \) be as in Maskin the predictions of payoff \( \varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n) \) and coalitions \( \psi \) of players \( \overline{N} \). Let \( i \) be given and \( \mathcal{P} \) the partition of \( i-1 \). For any \( S \in \mathcal{P} \cup \emptyset \) and \( S' \in \mathcal{P} \cup \emptyset \) let

\[
\Phi^i(S, S') = \begin{cases} 
\sum_{j \in S \cup i} \varphi_j(\mathcal{P}, (i, S)) & S' = S \\
\sum_{j \in S} \varphi_j(\mathcal{P}, (i, S')) & S' \neq S
\end{cases}
\]

We assume the following axioms hold.

1. Non-negotiation commitment (NA): Given \( j, k \in i-1 \) and partial partition \( \mathcal{P} \), for every \( j \in S \in \mathcal{P} \) and \( k \notin S \). Then if \( j \in S^* \in \psi(\mathcal{P}) \) we also have \( k \notin S^* \).

2. Binding coalitions (BC): Given \( i-1 \) and partial partition \( \mathcal{P} \). If \( S \in \mathcal{P} \) then there exists \( S' \in \psi(\mathcal{P}) \) such that \( S \subset S' \).
(3) **Coalitional Pareto Optimality:** For all partial partitions $P$ of $\overline{N}$ and all $S \in \psi(P)$ we have

$$\sum_{j \in S} \varphi_j(P) = v(S, \psi(P))$$

(4) **Limited efficiency:** For all $i$, all partitions $P$ of $\overline{i-1}$ and all $S^o \in P \cup \emptyset$ if $i \in S^*$, where $S^* \in \psi(P)$ and $S^o \subset S^*$, then

$$\Phi^i(S^o, S^o) - \Phi^i(S^o, S^{oo}) \geq \Phi^i(S, S) - \Phi^i(S, S^o)$$

(11)

for all $S \in P \cup \emptyset$ such that $S \neq S^o$ where

$$S^{oo} \in \arg \max_{S \neq S^o, S \in P} \Phi^i(S, S) - \Phi^i(S, S^o).$$

(12)

(5) **Opportunity wages:** For all $i$ and all partitions $P$ of $\overline{i}$, if player $i$ is allocated to $S \in P$ he receives his opportunity wage

$$\varphi_i(P) = \Phi^i(S^{oo}, S^{oo}) - \Phi^i(S^{oo}, S^o).$$

(13)

(6) **Consistency:** For all $i$ and all partitions $P$ of $\overline{i}$ if player $i$ is allocated to $S^o \in P \cup \emptyset$ then

$$\varphi(P) = \varphi(P, (S^o, i))$$

$$\psi(P) = \psi(P, (S^o, i))$$

If axioms (1)-(6) hold, then there exists a payoff predictor $\varphi$ and coalitional predictor $\psi$ that satisfy the axioms.

**Theorem A.1.** For every positive number $N$ the coalitional game with externalities and value function $v$ satisfying (9) there exist $\varphi$ and $\psi$, which satisfy conditions (1)-(6) above.

Maskin in his June 2004 Toulouse Lectures changed the axioms under which the same theorem is obtained, but are easier to implement. Non-negotiation commitment and binding
coalitions (axioms (1) and (2)) above) are retained. New axioms are

Let \( S, S' \in \mathcal{P} \cup \emptyset \). Then

\[
\Phi^i(S, \hat{S}) = \begin{cases} 
    v(S^*, \psi(\mathcal{P}, S \cup i)) - \sum_{j > i} t_j(S|\mathcal{P}, S \cup i) & S = \hat{S} \\
    v(S^*, \psi(\mathcal{P}, \hat{S} \cup i)) - \sum_{j > i} t_j(S|\mathcal{P}, \hat{S} \cup i) & S \neq \hat{S}
\end{cases}
\]

(3') \textit{Competitive allocation (CA):} \( i \) is allocated to \( S^o \) such that

\[
\arg \max_{S \in \mathcal{P}} \sum_{s \in S} \Phi^i(S, \hat{S})
\]

(4') \textit{Competitive wages (CW)}:

\[
\varphi_i(\mathcal{P}) = \max_{S' \neq S^o} \sum_{s \in \mathcal{P}} (\Phi^i(S, S') - \Phi^i(S, S^o))_+
\]

(5') \textit{Vickrey payments (VP)}: If \( i \) is allocated to \( S^o \) then for all \( S, \hat{S} \in \mathcal{P} \) the following holds:

\[
t_i(S|\mathcal{P}, S^o \cup i) = \left\{ \max_{S \in \mathcal{P}} \left[ \sum_{\hat{S} \neq S} \left( \Phi^i(\hat{S}, S') - \Phi^i(\hat{S}, S^o) \right) \right] \right\}_+
\]

A.2 Proofs of Theorems

\textit{Proof.} (of Proposition 2.2) The only possible coalition structure after firm 1 has entered is \( \mathcal{P} = \{\{1\}, \emptyset\} \). Since in a network of two firms firm 2 enters last, we can compute the Maskin’s \( \Phi^2 \) for different coalitions in \( \mathcal{P} \):

\[
\Phi^2(\{1\}, \emptyset) = E_1 \\
\Phi^2(\{1\}, \{1\}) = E_{12} \\
\Phi^2(\emptyset, \{1\}) = 0 \\
\Phi^2(\emptyset, \emptyset) = E_2
\]
Therefore the optimal allocation of firm 2 is to a coalition according to CA axiom is the $S^o$ which maximizes the following

\[
S^o = \arg \max_S \left[ \Phi^2(\{1\}, \{1\}) + \Phi^2(\emptyset, \{1\}), \Phi^2(\{1\}, \emptyset) + \Phi^2(\emptyset, \emptyset) \right]
\]

\[
= \arg \max_S [E_{12} + 0, E_1 + E_2]
\]

This is exactly what the condition in Proposition 2.2 states. If the order of firms is reversed - first firm 2 and then firm 1 enters, the logic and the conclusions are exactly the same.

In the case $E_1 + E_2 > E_{12}$, the Maskin’s theory cannot be used. Therefore we just describe the payoff to the firms predictions in the case when the merger does occur, i.e. $E_1 + E_2 \leq E_{12}$. This case also satisfies the Maskin’s merger conditions.

We get

\[
\varphi_2(\{1\}, \emptyset) = (\Phi^2(\{1\}, \{1\}) - \Phi^2(\emptyset, \{1\}))_+ + (\Phi^2(\emptyset, \{1\}) - \Phi^2(\emptyset, \emptyset))_+
\]

\[
= E_{12} - E_1.
\]

Similar things happen when the order of firms is reversed, i.e.

\[
\varphi_1(\{2\}, \emptyset) = (\Phi^1(\{2\}, \{2\}) - \Phi^1(\emptyset, \{2\}))_+ + (\Phi^1(\emptyset, \{2\}) - \Phi^1(\emptyset, \emptyset))_+
\]

\[
= E_{12} - E_2.
\]

Maskin then predicts, that the payments to the firms 1 and 2 should be $\frac{1}{2}(E_1 + E_{12} - E_2)$ and $\frac{1}{2}(E_2 + E_{12} - E_1)$. This coincides with the Shapley values for firms 1 and 2. The calculation is similar for both firms and we only show it for firm 1:

\[
Sh(1) = \frac{(2 - 2)! (2 - 1)!}{2!} (E_{12} - E_2) + \frac{(2 - 1)! (2 - 2)!}{2!} (E_1 - 0),
\]

which is exactly as above after rearranging.

□
References


