Maxima of Gaussian processes and applications

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Regular Gaussian random processes and random fields (the last case corresponds to spatial processes) appear naturally in lot of applications as econometrics, statistics, genetics, complexity.

We first present basic definitions and properties of such processes and address, in particular, the problems of continuity, differentiability and boundedness of the paths. Then we present the Rice formula for the moment of crossings that will be our main tool.

The elementary bounds for the tail of the maxima are obtained and two applications are presented : change points detection and quantitative gene detection.

A other direction is to replace the continuous time approach be a discrete approach in high dimension. We use results by Genz on computation of high dimension Gaussian integral using Monte-Carlo Quasi-Monte-Carlo methods (MCQMC). An application to the construction of simultaneous bands for the prediction of curves will be presented with an application to the prediction of electrical load curves of french firm EDF.

Spatial statistics uses spatial random models often called "random fields". We extend the Rice method to such random fields. Two applications will be presented. 1) The mean number of specular points of random sea-waves. Specular points are points that appear in bright on a photo of the sea. 2)The mean number of real roots of random polynomials systems.

Validation : take-home exam.

References

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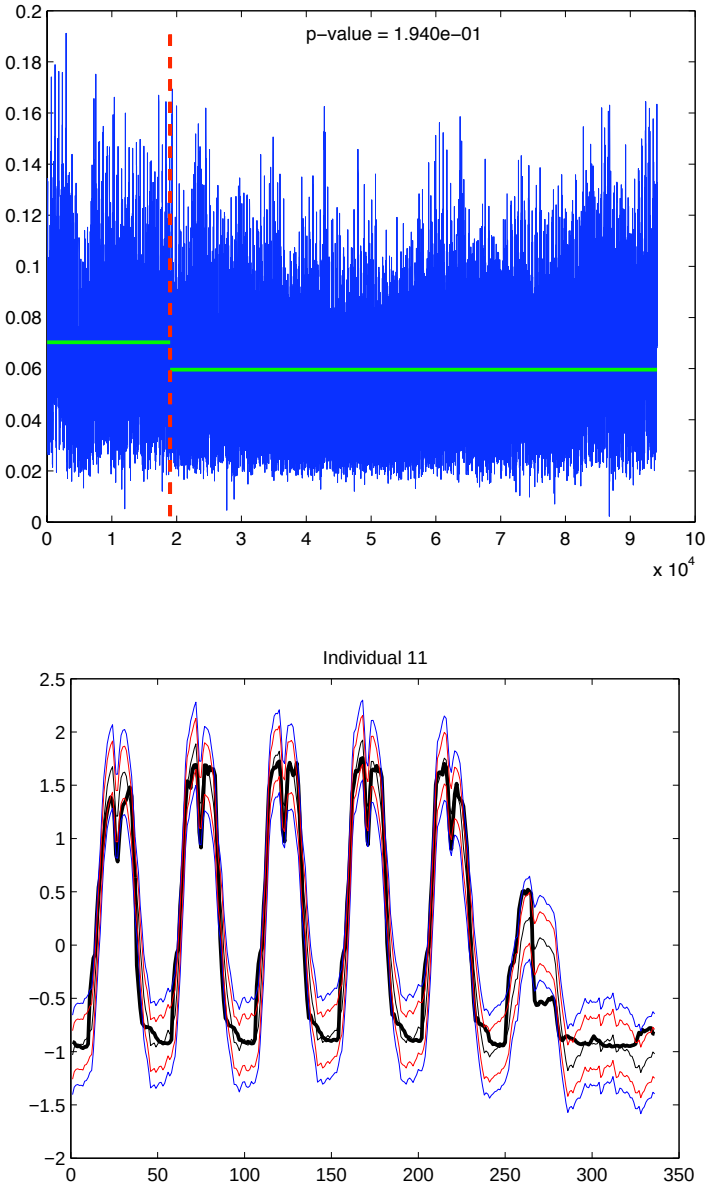

Figure 1: Change point detection for GPS error. Confidence bounds for load curve prediction