Modelling Multivariate Conditional Volatility

Michael McAleer

Econometric Institute
Erasmus School of Economics
Erasmus University Rotterdam
and
Tinbergen Institute
The Netherlands
and
Institute of Economic Research
Kyoto University
Japan

Updated: 22 October 2010

Outline: This course is concerned with modelling financial volatility, and provides an econometric analysis of alternative models and techniques for analysing high frequency financial data. In the first of the three primary areas of volatility modelling, namely Conditional Volatility or Generalised Autoregressive Conditional Heteroskedasticity (GARCH), Stochastic Volatility (SV) and Realized Volatility (RV), univariate volatility models of individual financial assets and multivariate volatility models of portfolios of assets, will be examined critically, the mathematical structural properties of the models will be established, the associated estimation algorithms will be developed, the statistical properties of the estimators will be derived, and the forecasting performance will be evaluated.

Time allocation: Each session will analyse in detail some recent theoretical papers in the financial volatility literature. The course will consist of an examination of recent papers in the GARCH literature. The remaining lectures will suggest some graduate research topics associated with the conditional volatility literature.

Applications: Financial econometrics, agricultural finance, environmental finance, and tourism finance.

Overview: Modelling risk and volatility are crucial ingredients for purposes of forecasting Value-at-Risk (VaR) and minimizing daily capital charges. Sessions on forecasting VaR and minimizing daily capital charges will examine the following papers.
Modeling volatility, or predictable changes over time and space in a variable, is crucial in the natural and social sciences. Life can be volatile, and anything that matters, and which changes over time and space, involves volatility. Without volatility, many temporal and spatial variables would simply be constants. Our purpose is to propose a scientific classification of the alternative volatility models and approaches that are available in the literature, following the Linnaean taxonomy. This scientific classification is used because the literature has evolved as a living organism, with the birth of numerous new species of models.

Credit risk is the most important type of risk in terms of monetary value. Another key risk measure is market risk, which is typically concerned with stocks and bonds, and related financial derivatives, as well as exchange rates and interest rates. This paper examines market risk management and monitoring under the Basel II Accord, and presents Ten Commandments for optimizing Value-at-Risk (VaR) and daily capital charges, based on choosing wisely from: (1) conditional, stochastic and realized volatility; (2) symmetry, asymmetry and leverage; (3) dynamic correlations and dynamic covariances; (4) single index and portfolio models; (5) parametric, semiparametric and nonparametric models; (6) estimation, simulation and calibration of parameters; (7) assumptions, regularity conditions and statistical properties; (8) accuracy in calculating moments and forecasts; (9) optimizing threshold violations and economic benefits; and (10) optimizing private and public benefits of risk management. The Basel II Accord would seem to encourage risk taking at the expense of providing accurate measures and forecasts of risk and VaR.

(A) Conditional Volatility – GARCH

Alternative univariate and multivariate, symmetric and asymmetric, GARCH models will be examined, the mathematical structural properties of the models will be established, the associated estimation algorithms will be developed, and the statistical properties of the quasi-maximum likelihood estimators (QMLE) will be derived.

The sessions on Conditional Volatility will examine the following papers.

Stress and distress are unavoidable aspects of dealing with the vagaries of financial markets and financial advisers. The purpose of this paper is to try to reduce the discomfort in dealing with investment advisers, and to make the journey up and down the financial mountain a little less stressful and more satisfying. The commandments deal with defining investment policies precisely, diversifying asset classes, choosing a consistent benchmark for investment policies, structuring precisely the asset allocation process, defining risk and risk management procedures, monitoring the portfolio carefully, matching the allocation and investment horizons, being active or passive according to investment policies, being agnostic about model forecasts, and being aware that, while buy low and sell high is a truism, investors and financial advisers are only human, and therefore make mistakes.


Under the Basel II Accord, banks and other Authorized Deposit-taking Institutions (ADIs) are required to communicate their daily market risk estimates to the relevant national monetary authority at the beginning of each trading day, using one of a variety of Value-at-Risk (VaR) models to measure risk. The purpose of this paper is to provide a simple explanation and a set of prescriptions for managing VaR under the Basel II Accord. The commandments deal with understanding the Basel II colours, understanding the risk model before choosing, varying the choice of risk model, avoiding the green zone and being willing to violate, incurring large violations, stopping before the red zone, avoiding frequent violations, avoiding the estimation of large portfolios, aggregating portfolios into a single index, and interpreting commandments sensibly as guidelines.


The paper reviews the recent theoretical literature on univariate and multivariate GARCH models.


The paper develops the mathematical structural properties of a family of univariate GARCH processes.
(6) Ling, S. and M. McAleer (2002b), “Necessary and sufficient moment conditions for the GARCH(r,s) and asymmetric power GARCH(r,s) models”, Econometric Theory, 18, 722-729 (according to Essential Science Indicators, ISI Web of Knowledge, this paper has 85 citations).

The paper develops the mathematical structural properties of some univariate GARCH and asymmetric power GARCH processes.

(7) Ling, S. and M. McAleer (2003), “Asymptotic theory for a vector ARMA-GARCH model”, Econometric Theory, 19, 278-308 (according to Essential Science Indicators, ISI Web of Knowledge, this paper has 89 citations).

The paper develops the mathematical structural and asymptotic properties of a new multivariate GARCH process, VARMA-GARCH. This model has been programmed in the RATS econometric software package.


The paper reviews a wide range of univariate and multivariate conditional volatility models, and provides a novel automated method for modelling univariate and multivariate conditional volatility.


The paper develops the mathematical structural and asymptotic properties of a new multivariate asymmetric GARCH process, VARMA-AGARCH. This model has been programmed in the RATS econometric software package.


The paper develops the mathematical structural and asymptotic properties of the most popular univariate asymmetric GARCH process, the GJR threshold model.


The paper develops the mathematical structural and asymptotic statistical properties of a new dynamic univariate asymmetric GARCH process with multiple thresholds.

The paper develops three new models of non-trading day (or holiday) effects in asymmetric and exponential conditional volatility models.


The paper derives the scalar special case of the well known BEKK multivariate GARCH model using a Vector Random Coefficient Autoregressive model, and establishes the structural and asymptotic properties of the scalar BEKK model.


The paper develops a parsimonious portfolio spillover GARCH model to forecast Value-at-Risk (VaR) thresholds.


Using univariate and multivariate conditional volatility models, the paper evaluates the performance of the single index and portfolio models in forecasting Value-at-Risk (VaR) thresholds of a portfolio.


The paper develops the structure of a parsimonious portfolio index GARCH (PI-GARCH) model which specifies the volatility of a portfolio directly, and examines the effects of symmetric and asymmetric shocks. A portfolio index BEKK model is also developed and evaluated.


The paper develops a generalized autoregressive conditional correlation (GARCC) model when the standardized residuals follow a multivariate random coefficient autoregressive process. The GARCC model provides a motivation for dynamic conditional correlations.

The paper develops the mathematical structural and asymptotic properties of a new dynamic multivariate asymmetric GARCH (DAM-GARCH) process with multiple thresholds, and presents a novel news impact surface to measure the impact of news on volatility deriving from a portfolio of assets.


Large and very large portfolios of financial assets are routine for many individuals and organizations. The two most widely used models of conditional covariances and correlations are BEKK and DCC. BEKK suffers from the archetypal “curse of dimensionality” whereas DCC does not. This is a misleading interpretation of the suitability of the two models to be used in practice. The primary purposes of the paper are to define targeting as an aid in estimating matrices associated with large numbers of financial assets, analyze the similarities and dissimilarities between BEKK and DCC, both with and without targeting, on the basis of structural derivation, the analytical forms of the sufficient conditions for the existence of moments, and the sufficient conditions for consistency and asymptotic normality, and computational tractability for very large (that is, ultra high) numbers of financial assets, to present a consistent two step estimation method for the DCC model, and to determine whether BEKK or DCC should be preferred in practical applications.

(F) Applications to Financial Econometrics, Agricultural Finance, Commodities and Tourism Research

(a) Financial econometrics and empirical finance


(b) Agricultural finance


(c) Oil spot, forward and futures prices


(d) Tourism research


