Globalization, Exchange Rate Regimes and Financial Contagion

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Abstract

The crisis of the Euro zone brought to the fore important questions including: what is the proper level of financial integration and what are the optimal exchange rate arrangements between countries that are part of tightly knit financial networks. Using a simple Diamond-Dybvig-style theoretical model we show that the effects of increased financial interconnectedness and different exchange rate regimes on financial stability should not be studied in isolation from one another. We demonstrate that a switch from fixed to flexible exchange rate regime in one country increases financial fragility of multi-currency networks under a complete system of financial links and decreases fragility if the set of links is incomplete. On the other hand, an increase in financial interconnectedness reduces fragility under fixed regime, but not necessarily does so under the flexible exchange rate regime.

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A profound paradigm shift took place in the global financial system over the past twenty years. From local deposit-funded operations confined within national boundaries, financial institutions transformed into large multinational conglomerates dependent on wholesale funding and interbank loans, often in foreign currency.[1] In Europe, creation of the Euro zone and enlargement of the European Union to Central and Eastern Europe facilitated establishment of large pan-European financial networks that incorporate countries both inside and outside of the Euro zone. Financial globalization helped improve quality, scope, and efficiency of financial services in Europe and many other regions of the world. At the same time, past two decades witnessed, also, a dramatic increase in frequency, severity and geographic reach of financial crises.[2] The recent financial and economic crisis exposed serious structural weaknesses of the global financial infrastructure[3] and showed that a global crisis can be triggered by a meltdown of the most developed markets, not just developing ones. The crisis in the Euro zone, in particular, raised important policy challenges for all countries that are part of closely interconnected financial networks (whether or not they are members of the Euro zone) since monetary (and fiscal) policy of one country can create an externality that adversely impacts all other countries of the network. For this reason it is important to understand what is the proper level of financial integration and what should be the exchange rate arrangements between countries that are part of tightly knit international financial networks.

For individual countries, a joint consideration of monetary policy and financial stability issues is increasingly becoming the paradigm of choice in policy making circles.[4] On the other hand, little is known about how different levels of financial interconnectedness and different exchange rate regimes within multi-country financial networks together influence fragility of these networks, i.e. how monetary policy and financial stability interact on multinational network level. To bridge this gap, we build a simple model that allows us to analyze: 1) How choice of the exchange rate regime influences the way in which a change in degree of financial interconnectedness impacts the fragility of a two-currency financial network and 2) How the degree of financial interconnectedness impacts the way in which a change in the exchange rate regime affects the fragility of such a network. To the best of our knowledge, this is the first theoretical contribution that analyzes joint effects of different degrees of network interconnectedness and exchange rate regimes on stability of

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[1] See International Monetary Fund report IMF (2010).

[2] For review of the empirical evidence on banking crises see Allen and Gale (2009), Chapter 1 and Degryse et al (2009), Chapter 7.

[3] On regulatory and political aspects of financial crises see Cao (2011) and Rochet (2011), among others.

[4] See, e.g., Freixas (2009) and the London School of Economics report LSE (2010).

multinational financial networks.

To keep the analysis simple, we connect and extend two benchmark models. The first is Allen and Gale (2000a). In that paper, the authors analyze the relationship between an increase in interconnectedness and possibility of financial contagion in a multi-region Diamond-Dybvig-style economy.[5] They consider a single-currency economy that consists of four regions with a representative bank in each of them. Banks attract deposits and invest them in liquid (short-term) and illiquid (long-term) assets. In order to mitigate exogenous liquidity risk, banks may hold deposits in banks of other regions. The system of interbank deposit links is taken as exogenous. The main finding of their model is that the higher degree of market completeness (higher interconnectedness) is likely to *reduce* the fragility of the banking system. In their model this is so since a higher number of links between regions provides a better insurance against exogenous liquidity shocks. We study whether their results generalize to situations when one part of the global network has a separate currency. Allen and Gale (2000a) note that this issue is of considerable interest but do not address it in their paper (see our research questions 1).

The second benchmark model that we use is Chang and Velasco (2000b). They analyze how different exchange rate regimes impact financial fragility of a small open economy and show that the flexible exchange rate regime may completely eliminate the possibility of both currency and banking crises in such an economy. On the other hand, in their model, under fixed exchange rate regime both types of crises are possible.[6] However, their setup does not allow for the analysis of fragility of a multi-currency financial network (see our research question 2).

We extend the analysis of financial contagion a la Allen and Gale (2000a) to the case of a two-country four-region economy with an open-economy monetary features of Chang and Velasco (2000b). In particular, we assume that in one of these countries there are three regions. The fourth region, in contrast to Allen and Gale, is a separate country with its own currency and a Central Bank. We refer to the former as the 'large' and to the latter as the 'small' country (but only in the sense that one of them has three and the other one just one, ex-ante identical, region). A possible

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[5]See Diamond and Dybvig (1983).

[6]Their work was motivated, in part, by the fact that in Mexico in 1994, in Asian countries in 1997 and in Russia in 1998, attempts to maintain the exchange rate peg invited speculative attacks that made the resulting devaluation deeper. As a result, it became a commonly shared view that emerging market economies should adopt 'corner solutions,' i.e. either a fully flexible exchange rate regime, or a complete dollarization (euroization). The collapse of the currency board in Argentina in 2001 was considered further evidence in favor of the floating exchange rate regime.

analogy is with the European banking system, with the Euro zone playing the role of the large country, and European countries outside of the Euro zone playing the role of the small country. Following Allen and Gale (2000a) we proxy for different levels of network interconnectedness between the four regions by considering two types of connected graphs: a network with a complete set of links in which banks of each region have interbank deposits in all other regions (see Figure 1), and a network with an incomplete set of links where banks from each region have deposits only in one adjacent region (see Figure 2).[7] Motivated by Chang and Velasco (2000b), we consider three exchange rate regimes in the small country: monetary union with the large country, fear of floating (where the central bank commits to intervene in order to prevent depreciation of the exchange rate in case of a crisis), and flexible exchange rate regimes. We show that the conditions for the global contagion, for either network type, are identical in case of the fear-of-floating and monetary union regimes. For this reason, we often refer to both regimes as the fixed exchange rate regime. From the policy perspective, switching from monetary union to the fear-of-floating exchange rate regime would not impact stability of the network.

Central banks play a pivotal role in preventing or allowing financial crises to spread. If it acts as a Lender of Last Resort (LOLR), a central bank can typically prevent contagion within its jurisdiction. As we have witnessed in recent times, central banks frequently do not allow commercial banks under their jurisdiction to fail, i.e. they often act as LOLR. For simplicity, we assume that the central bank of the small country always acts as LOLR. In contrast, we consider only situations when the central bank of the large country does not act as a LOLR. This assumption is essential for a global financial crisis to occur.[8] Under these assumptions a crisis in one region of the large country may spread, under certain conditions, to all other regions of the large country (a global contagion). These conditions depend, in general, on the region of origin of the crisis,

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[7]Four nodes is the minimum number of nodes needed to distinguish between connected networks with complete and incomplete set of links.

[8]Note that central banks do not always act as LOLR. The failure of the Federal Reserves to prevent the bankruptcy of Lehman Brothers in September 2008 provides a recent example. Also, the assumption of no LOLR in the large country makes our results comparable to the findings of Allen and Gale (2000a). They also assume away the existence of LOLR in the economy and argue that within their model contagion could be prevented if sufficient liquidity is injected into the system by the Central Bank. While we do not address the optimal bailout decisions of LOLR, Freixas (1999) shows that a mixed strategy may be optimal when LOLR decides whether to rescue a financial institution or not. Allen et al (2009) consider an economy with risky long-term asset. They show that when there is a lack of opportunities for banks to hedge aggregate and company-specific liquidity shocks, central bank can implement the constrained efficient allocation by using open market operations. In a recent extension, Allen et al (2011) show that when non-contingent deposit contracts are issued in nominal terms, the central bank can prevent financial crises and achieve the first best allocation. Cao and Illing (2011) caution, however, that interventions of the LOLR may increase the likelihood of future crises. We abstract, for simplicity, from these important issues.

network topology (degree of interconnectedness) as well as the exchange rate regime adopted by the small country. A combination of a given network topology and exchange rate regime is, in our terminology, more fragile than another combination if the set of conditions under which global contagion occurs is less restrictive (i.e. conditions are more likely to hold).

We generalize Allen and Gale (2000a) key insight to a multi-currency network and show that switching from an incomplete to a complete set of financial links unambiguously decreases financial fragility in a multi-currency network under fixed exchange rate regime in the small country. On the other hand, in case of flexible exchange rate regime in the small country the result can be reversed. In that case more interconnectedness may, under some circumstances, lead to greater network fragility.[9]

As for Chang and Velasco (2000b) insight about a single country, namely that flexible exchange rate regime reduces financial fragility in comparison to fixed or fear-of-floating regimes, we show that it holds for the network as a whole as well, as long as links between regions are incomplete. In contrast, when the set of links is complete, i.e. in case of higher interconnection between the regions, we find that switching from fixed exchanage rate regime to a flexible exchange rate regime unambiguously *increases* financial fragility. The result is quite robust, in that it does not depend on the shape of the utility function of the representative agent or the region of origin of the crisis.

From the policy perspective, when financial network is highly interconnected, breaking up of the monetary union and allowing currencies to float may be dangerous from the global financial stability point of view. If one is set to break up a monetary union, therefore, it may be important to first reduce financial links between the countries (anecdotal evidence exists that this is already happening in Europe).

Thus, high level of financial interconnectedness coupled with flexible exchange rate regime in part of the network may be particularly dangerous from the global financial stability point of view. This result is striking because it is obtained in a framework heavily tilted *in favor* of the floating exchange rate regime. First, our setup rules out a possibility that a financial crisis originates in the economy with the floating exchange rate. Second, when the set of links is complete, the exchange rate depreciation is *lower* under the flexible exchange rate regime than under the fear-of-floating regime when an external shock hits the economy. Last but not least, since we assume that all

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[9]We show that, under flexible exchange rate regime, whether or not network with a complete set of links is more or less fragile than the network with incomplete set of links, in principle, depends on the region of origin of the crisis and the shape of the utility function of the representative agent.

loans and deposits are made in local currencies, we abstract from important negative effects of the exchange-rate instability present in the 'real world' (including the real sector disruptions due to the balance sheet effects, the lack of nominal anchor, etc.). In many countries, especially in emerging economies, financial institutions suffer from the 'original sin' - i.e. they borrow in hard currency, but lend in domestic currency. In others, a significant fraction of commercial loans is pegged to or denominated in a foreign currency. In either case, this creates a strong reason to avoid currency fluctuations. [10] By assuming that both deposits and loans are made in the domestic currency, we rule out these important sources of instability related to foreign exchange fluctuations. This may be seen as an unrealistic assumption. But in fact, it strengthens our results: even under such extreme conditions flexible exchange rate regime (coupled with a complete system of financial links) makes the network more fragile.

Why is a combination of complete set of links and flexible exchange rate regime in part of the network particularly dangerous for network stability? Suppose that there is a bank run in one of the regions of the large country. When each region is financially linked to all other regions, the region with flexible exchange rate regime immediately 're-exports' negative shocks (including the shock of a bank run in one of the regions of the large country) to all other regions of the large economy via the exchange rate depreciation (instead of absorbing them). All of the regions of the large country are, therefore, under dual pressure as each of them loses a part of the value of the deposit in the region affected by the bank run, and a part of the value of the deposit in the small country. However, these regions cannot follow the small country and devalue, and so they are more likely to suffer from the financial meltdown.

We find that in the era of close financial ties between countries, their exchange rate policies are important determinant of global financial stability. In particular, what is good for an individual country (flexible exchange rate policy) may not necessarily be good for the stability of the network as a whole as long as financial links between the regions are strong enough. Our results, thus, rationalize 'fear of floating' in emerging markets, as well as worries about financial stability aspects of potential reshaping of the Euro zone.

Our paper is closely related to different streams of literature. The first one focuses on interbank

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[10]See Gale and Vives (2002) for an influential study of financial stability aspects of financial dollarization. Božović et al (2009) study a spillover of foreign currency risk into default risk for foreign currency-denominated loans, and show that an adverse move in the exchange rate may substantially increase the probability of a banking system meltdown.

linkages and contagion in the context of closed-economy Diamond-Dybvig-style models.[11]. The classic paper in this literature is Allen and Gale (2000a). Brusco and Castiglionesi (2007) extend the model of Allen and Gale (2000a) by allowing for the possibility of a moral hazard in the economy (but still considering a single currency). They show that in that case, in contrast to Allen and Gale (2000a), a connected network with incomplete set of links can be less fragile than network with a complete set of links. We show that by incorporating the second currency into the network, one may obtain qualitatively the same result as in Brusco and Castiglionesi (2007) without relying on the assumption of moral hazard in the economy.

The second stream of related literature extends the Diamond-Dybvig model to an open economy framework. In a series of papers Roberto Chang and Andres Velasco (Chang and Velasco, 2000a, 2000c, 2001) place the Diamond-Dybvig-type bank in a small open emerging market economy that suffers from potential international illiquidity. In Chang and Velasco (2000a, 2001) they study the interaction among access to external borrowing, depositors' and foreign creditors' panics. In Chang and Velasco (2000c) they study how the optimal contract changes if the probability of panic is given exogenously. These papers however, focus on a single small open economy and do not analyze the network aspects of financial crises. Furthermore, these are essentially 'non-monetary' models, as they abstract from domestic currency (i.e. all transactions are effectively done in units of internationally traded good). Chang and Velasco (2000b), in contrast, explicitly model demand for money balances and analyze the impact of different exchange rate regimes on financial fragility of a small emerging market country. They show that a switch from the fixed to flexible exchange rate regime makes a country less financially fragile. In particular, if the Central Bank of a country acts as a LOLR then under the fixed exchange rate regime only currency crisis is possible. If not, then the currency crisis is substituted by the banking crisis. Under the flexible exchange rate regime, the Central Bank has the opportunity to devalue the national currency. This allows the Central Bank to retain its reserves and eliminate the possibility of crises of any type.[12]

Allen and Gale (2000b) also study optimal monetary and exchange-rate policies in a small open

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[11]This literature, in contrast to our paper, typically assumes away effects that different exchange rate regimes play in financial contagion. For excellent reviews see Allen and Babus (2009) and Allen and Gale (2009).

[12]Kawamura (2007) presents a small open economy two-goods version of the Diamond-Dybvig model. He introduces the assumption of cash-in-advance constraints according to which both tradable and non-tradable goods can be purchased only with domestic currency issued by the Central Bank. He also establishes a comprehensive mechanism of exchange of domestic currency for tradeable and non-tradeable goods through the assumption of a time separation of market sessions. Kawamura demonstrates that under expansionary monetary policy a flexible exchange rate regime has multiple equilibria, one of which is a partial currency run.

Diamond-Dybvig-style monetary economy. They find that the central bank can achieve first-best allocation without runs if agents in this economy can borrow abroad in the domestic currency. The exchange rate should be state-dependent. If agents can borrow in the foreign currency only (which is the salient feature of most emerging market economies), then equilibria with bank runs and inefficient asset liquidation are possible. Note though, that Alen and Gale (2000b) rule out runs as coordination failures, i.e. they assume that in states with multiple equilibria only the no-run equilibrium is selected.

Finally, our paper complements the literature that studies the 'fear of floating' phenomenon. It has been argued that many emerging market economies mitigate exchange rate fluctuations, because they lack a developed financial system that would help them cope with the exchange rate variability and its adverse impact on balance sheets of firms and the government. In particular, many agents in developing economies have only limited ability to borrow long-term in their national currency and to hedge their exchange rate risk exposure. Hausmann et al. (2001) find a very strong and robust relationship between the ability of a country to borrow internationally in its own currency and its willingness to tolerate exchange rate volatility vis-a-vis the interest rate volatility. Calvo and Reinhart (2002) develop a model where fear of floating arises from the combination of the lack of credibility (as manifested by risk-premium shocks), high exchange rate pass-through and inflation targeting. Caballero and Krishnamurthy (2001) relate fear of floating to the inelastic supply of external funds in times of financial crisis which causes an exchange rate overshooting. We show that there can be an entirely different reason for the fear of floating for developed and developing countries alike. Namely, a switch from the fear of floating (or monetary union) to flexible exchange rate regime may adversely impact stability of closely interconnected multinational financial networks.

The rest of the paper is structured as follows. Section 1 outlines the basic model, i.e. the model without an interbank deposit market. Section 2 describes the model of interbank deposit market and contrasts complete and incomplete market structure. Section 3 is the core of the paper. It contrasts conditions for an economy-wide contagion under different assumptions about the exchange rate regime and completeness of interbank links. Section 4 concludes the paper. Proofs are relegated to the Appendix.

1 The Basic Model

1.1 Model Setup

Consider a world economy in which there are two countries, a 'large' and a 'small' one. In the large country there are three regions, A, B and C. The small country consists of a single region which we denote as region D.[13]

Each country has its own currency and a Central Bank (to be described later). Dollars are the currency in the large country, while pesos are the currency of the small country. We assume that absolute purchasing power parity holds. On the other hand, nominal peso-dollar exchange rate, and hence, the price level in the small country, may vary over time. Without loss of generality, we set the initial peso-dollar exchange rate and the price level in both countries equal to one.

Each region is populated by a continuum of ex ante identical agents. There are three dates 0, 1, 2. Agents are born in period 0 and have an endowment of one unit of a tradable consumption good. The price of consumption good in the world market is constant and equals one dollar. We assume that any agent can freely exchange dollars for goods at any moment of time. Consumption good can be invested in a long-run constant-returns-to-scale technology which yields $r < 1$ units of consumption good in period 1 and $R > 1$ units of consumption good in period 2. Alternatively, agents can invest their endowment in the world market. In this case one unit of consumption good invested in period 0 produces one unit of consumption good either in period 1 or in period 2.[14]

In each region agents have Diamond-Dybvig preferences (see Diamond and Dybvig (1983)). When born in period 0 agents do not know their type. In period 1 each agent discovers his type. With probability λ he is impatient, i.e. derives utility only from consumption in period 1, and his utility function is $u(x)$, where x is his first-period consumption. Function $u(.)$ is smooth, strictly increasing, strictly concave and satisfies Inada conditions. With probability $1 - \lambda$ agent is patient and derives utility from the real value of holdings of the currency between periods 1 and 2 and from consumption in period 2. Following Chang and Velasco (2000b), utility of patient agents is $u(\chi(m)+y)$, where y is their consumption in period 2, and m is the money holdings between periods 1 and 2 deflated by the price level of period 2. Function $\chi(.)$ that describes demand for money

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[13]The terms 'large' and 'small' for the economies imply only that one of these economies consists of three identical regions while the other one consists of only one such region. Importantly, the small economy is nonnegligible, i.e. events occuring in it may have repercussions for the large economy.

[14]Instead of investment in the world market, we can assume a storage technology with the gross rate of return equal to 1.

holdings is smooth, strictly concave and satisfies $\chi(0) = 0, \lim_{m \to 0} \chi'(m) = \infty$ and $\chi'(\bar{m}) = 0$ for some $\bar{m} > 0$. Level $\bar{m}$ can be regarded as a satiation level of money holdings.[15]

The realization of each agent's type is private information to that agent. Residents of the small country can invest but not borrow in the world market.

1.2   The Social Planner's Problem

In each region, the Social Planner maximizes the expected utility of the representative agent. Due to the symmetry of the problem, the Social Planner's Problem is identical across the four regions and reads as follows:

$$U = \lambda u(x) + (1 - \lambda)u(\chi(m) + y) \tag{1}$$

subject to:

$$k + b \leq 1 \tag{2}$$

$$\lambda x \leq b + rl \tag{3}$$

$$(1 - \lambda)y \leq R(k - l) \tag{4}$$

$$x \leq \chi(m) + y \tag{5}$$

$$x, y, m, k, l, b \geq 0$$

Optimization is done with respect to x, y, m, k, b and l. Here, x is the consumption of an impatient agent in period 1, y is the consumption of a patient agent in period 2, m is the real money balances provided to a patient agent in period 1, b is the per-capita investment in the world market, k is the per-capita investment in long-term technology and l is the first-period termination of the illiquid technology. Equation (2) is the aggregate resource constraint in period 0 in per-capita terms. It shows that the sum of investment in long-term technology, k, and investment in the world market, b, cannot exceed the initial endowment. Equations (3) and (4) are the aggregate resource constraints in periods 1 and 2, respectively. Equation (3) shows that consumption of an impatient agent comes from the return on storage technology, b, and period 1 termination of the long-term investment, l. It also takes into account that the share of impatient agents in the

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[15]The assumption of the money in the utility function is not critical in the model. The main results of the paper - Propositions 1-4 - do not depend on the functional form of the utility function. See footnote 16 for an alternative assumptions about the money demand. However, in the paper we follow assumptions of Chang and Velasco (2000b) to make our results directly comparable to theirs.

population is λ. Equation (4) shows that consumption of the patient agents comes from the return on remaining illiquid technology and takes into account that the share of patient agents is $1 - \lambda$. Finally, equation (5) is the incentive-compatibility constraint. It shows that a patient agent has no incentive to misrepresent himself in period 1 and claim that he is impatient. It is worth noting that m is not present in the left-hand side of any constraint. This is because money is costless to produce, and hence the Social planner can create money up to the satiation level, $\bar{m}$. The analysis of the problem (1)-(5) is provided in the Appendix. Note that it is never optimal to interrupt the long-term technology in period 1, and it is never optimal to leave any resources unused. Therefore, in equilibrium $l = 0$ and inequalities (2)-(4) are satisfied as equalities. Furthermore, the social planner is able to provide unlimited amount of money balances to agents. Thus, without loss of generality we may assume that $m = \bar{m}$. Finally, the assumptions about the utility function ensure that interior solution to the problem (1)-(5) does exist, and is unique.[16]

1.3 Competitive Equilibrium and Central Banks

Similar to the original Diamond-Dybvig model (and its numerous derivatives), the first-best allocation can be decentralized in a competitive equilibrium with banks. Commercial banks arise endogenously to provide liquidity and to insure agents against preference shocks. Each country has its own Central Bank. Central banks supply currency for the impatient agents in the respective economies. In addition, they may act as Lenders of Last Resort (LOLR). In this paper we are interested in conditions under which a crisis that spreads across all regions of the large economy may occur (we refer to this as global financial contagion). For this reason, we assume that the Central Bank of the large country does not act as a lender of last resort. On the other hand, Central Bank of the small country acts as a LOLR.

The central bank of the large country lends dollars interest-free to banks in period 1 and allows banks to use these dollars only for withdrawals of reportedly patient agents. In period 2 banks (if solvent) repay the loan.[17]

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[16]The identical allocation can be obtained if the utility of patient agents does not depend on real money holdings, i.e. if $U = u(y)$, but the second-period consumption is subject to the Clower constraint: $\kappa y \leq m$, where κ is the share of the second-period consumption financed with cash. The allocation will be identical if $\kappa = \bar{m}/y$, where y is the socially optimal consumption of the patient agents. Alternatively, we can replace the Clower constraint with a standard cash-in-advance constraint, $y \leq m$. In the latter case the command optimum will be the same, except for the real money balances kept by patient agents. The main results of the paper will still go through.

[17]We borrow this assumption from Chang and Velasco (2000b). It allows commercial banks to satisfy the money demand of reportedly patient agents without diverting real resources and hence to achieve the first-best allocation.

The central bank of the large country is the only agent that can borrow dollars on the international market. It can borrow interest-free, because world-market investors earn zero net return on their investment, and in equilibrium the central bank always repays the debt.[18] It is convenient to assume that the Central bank provides exactly $h = (1 - \lambda)\bar{m}$ in real per-agent terms. This limited role of the central bank rules out depreciation of dollars vis-a-vis the consumption good, i.e. inflation in the large economy.

Next consider the case of the small country. We assume that its Central Bank also lends the local currency (i.e., pesos) to the representative commercial bank to satisfy the real money demand of the patient agents. It can also serve as a LOLR i.e. in the case of a bank run it can provide liquidity support to the commercial bank in the small economy. In particular, if more than λ customers claim to be impatient, the central bank lends as many pesos as necessary to meet the demand of reportedly impatient depositors. In the latter case, the Central Bank obtains control over the long-term asset in period 1 and liquidates it, as needed, in order to sell the dollars to agents claiming impatience.[19] All transactions between the commercial bank and its depositors are done in the local currency - pesos.

1.4 Timeline

The sequence of events is as follows (exchange of dollars for consumption good is omitted for brevity):

Period 0

Agents are born with their endowments. Agents in the small country exchange their endowments for pesos at the Central Bank of the small country. All agents make deposits at the commercial bank of their region using their respective local currency. In other words, agents in the small country deposit pesos while agents in the large country deposit dollars. Commercial bank of the small country exchanges, at the Central Bank, all of its pesos for dollars, and makes investments in the long term technology and/or the world market. Similarly, commercial banks of the large country invest into long term technology and/or the world market.

Period 1

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[18]In a crisis (bank run) equilibrium, patient agents claim they are impatient, and hence the central bank does not need to borrow.

[19]It is convenient to assume that the central bank appropriates the share of the long-term investment which is equal to the share of patient agents claiming impatience.

All agents learn their types and report them to their banks (but they may not necessarily do so truthfully). After observing the share of reportedly impatient agents each commercial bank liquidates all of its world market investment. It also requests a loan from the central bank to satisfy the demand for money holdings of the reportedly patient agents. In the large country, in case the commercial bank faces a higher fraction of reportedly impatient agents (i.e. a bank run), it terminates the long-term investment to satisfy the withdrawal requests of depositors. In the small country, in case the commercial bank faces a higher fraction of reportedly impatient agents, it requests an emergency loan from the central bank. Central bank of the small country issues pesos and provides a loan to the commercial bank. Central bank of the large country borrows dollars in the world market and lends them to the commercial banks. Commercial banks make payments to their depositors. Reportedly impatient agents of the small country exchange pesos obtained from the commercial bank for dollars at the Central Bank. To satisfy these agents central bank of the small country may terminate (some of the) illiquid technology it received from the commercial bank in exchange for emergency loan. Truly impatient agents consume, while patient agents who claim impatience invest in the world market.

Period 2

Commercial banks liquidate all their long-term investment. Reportedly impatient (but, in fact, patient) agents liquidate their world-market investments. Commercial banks repay their debts to their central banks. The Central bank of the large country repays the debt to the world-market investors. Commercial bank in the small country exchanges dollars for pesos at the Central Bank of the small country. These pesos are used to pay off reportedly patient agents in the small country. Commercial banks of the large country pay off patient agents. Reportedly patient agents in the small country exchange their pesos for dollars. All patient agents consume.

1.5 Commercial Banks and Multiplicity of Equilibria

Because of the perfect competition among banks each representative bank in the large country strives to offer agents a deposit contract that maximizes the expected utility of the representative agent (1) subject to the following constraints:

$$k + b \leq 1 \tag{6}$$

$$\lambda x + (1 - \lambda)\frac{M}{P_1} \leq b + h + rl \tag{7}$$

$$(1 - \lambda)y - (1 - \lambda)\frac{M}{P_2} \leq R(k - l) - h\frac{P_1}{P_2} \tag{8}$$

$$\chi(M/P_2) + y \geq x \tag{9}$$

$$X, y, M, k, l, b, h, P_1, P_2 \geq 0,$$

where M is the nominal money balances (in local currency terms) lent to commercial banks in period 1, $P_1 = 1$ is the price level in period 1 and P_2 is the price level in period 2. Inequalities (6)-(8) are the budget constraints of the commercial bank in periods 0-2, respectively. Quantity h is the real amount of loan from the central bank (in *per capita* terms) to provide for the money demand of the patient agents. This loan is repaid in period 2. Expression $(1 - \lambda)\frac{M}{P_1}$ is the payment of the bank to the reportedly patient agents in period 1.

It is straightforward to see that the system (1), (6)-(9) yields the same values for x, y, k and b as the system (1)-(5) if $\frac{M}{P_1} = \frac{M}{P_2} = \bar{m}$, i.e. the real value of the currency does not change, and $h = (1 - \lambda)\bar{m}$.[20] This implies that the competitive equilibrium allocation coincides with the

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[20]Henceforth, we will use these variables at their optimal values in order to simplify the notation.

Social Planner optimum if there is no inflation and all agents truthfully reveal their types. It is important to note that the bank is unable to make the payment to a depositor conditional on his type, because the depositor type is private information. Instead, the bank has to offer a demand-deposit contract: any depositor can claim that he is impatient and withdraw quantity x in period 1. On the other hand, if a depositor claims he is patient, he can withdraw M dollars in period 1. It is customary in the literature to assume the sequential service constraint: the bank cannot condition the first-period payouts on the number of agents claiming impatience, and pays x to every agent claiming impatience and M to every agent claiming patience as long as it has any resources left. Like in the original Diamond-Dybvig framework, the demand-deposit arrangement yields multiplicity of equilibria. There is an 'honest' equilibrium, in which allocation coincides with the social optimum. In that equilibrium, due to incentive-compatibility constraint (9), patient agents have no incentive to misrepresent their type. However, there is another equilibrium, a bank run. Namely, if, for any reason, a sufficient number of patient agents believe that the commercial bank will be insolvent in period 2, they claim impatience and attempt to withdraw their deposits in period 1. The commercial bank does not have enough resources to pay all of its depositors if:

$$b + rk < x \tag{10}$$

Assuming that inequality (10) is satisfied, the bank will interrupt its long-term investment and exhaust its resources before it is able to repay all of the depositors. Hence, the patient depositors who wait until period 2 will get nothing in period 2. In that case, it would be optimal for all patient depositors to attempt to withdraw their deposits in period 1 (a bank run).

Another condition for the run of patient agents is

$$\bar{m} < x \tag{11}$$

If inequality (11) is violated, impatient agents have an incentive to pretend that they are patient. This would be a 'reversed' run.

The representative bank of the small country strives to offer agents a deposit contract that maximizes the expected utility of the representative agent (1) subject to the same constraints (6)-(9). The only difference is that all contracts are denominated in pesos, instead of dollars. When there is no run on the banks, the social optimum allocation can be achieved. Furthermore, assuming that in the small country the central bank acts as a LOLR, an emergency credit from the central

bank can prevent the bank run in the small country. In that case, there is a currency crisis instead: it is the central bank that has to terminate the long term investment to satisfy the demand for dollars of the reportedly impatient agents. As inequality (10) is satisfied, the central bank in the small country does not have enough resources to exchange pesos for dollars at the initial exchange rate equal to 1, and has to devalue the peso.

2   The Model of Interbank Deposit Market

Following Allen and Gale (2000a), we consider stylized financial networks with only four regions (A, B, C, and D). Large country consists of three regions (A, B, and C), while the small country consists of just one region, namely region D. As noted in the Introduction, four is the smallest number of regions for which one can distinguish between the connected network in which banks from each region have interbank deposits in all other regions (a network with a complete set of links) and the connected network in which banks from each region have interbank deposits in only one adjacent region (a network with incomplete set of links). Following Allen and Gale (2000a), we assume that a network structure is exogenous to the model. [21]

We now present interbank deposit market in our model. In each region agents have the same *ex ante* probability of being impatient, namely λ. Following Allen and Gale (2000a) we assume that the probability of agents being impatient may vary across regions so that interbank deposits provide insurance against liquidity shocks. There are three states of nature. In state S_1 the probability of being impatient is the same in each region and equals λ. In states S_2 and S_3 there are two possible values of this probability for each region, a high value and a low value, denoted by w_H and w_L, respectively, where $0 < w_L < w_H < 1$ and $\lambda = (w_H + w_L)/2$. States S_2 and S_3 are equally likely.[22] Possible realizations of liquidity shocks are given in Table 1.

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[21]Note that this assumption is not innocuous. Allen and Babus (2009) discuss endogenous network formation and argue that under certain conditions some types of interbank links may not materialize at all. They use this argument to explain why, during the recent crisis, global interbank market froze up. Freixas and Holthausen (2005) demonstrate that, even when countries operate within the monetary union regime, when the level of informational asymmetry about the quality of assets in different countries is sufficiently high, an integrated interbank market may not be an equilibrium. On the other hand, a segmented market (a separate market for each country) is always an equilibrium in their model. In contrast to Freixas and Holthausen (2005), such informational asymmetry is absent in our model.

[22]Note that if liquidity demand in a region was high in period 1, it will be low in period 2, and vice versa

Table 1

State	A	B	C	D	Probability of the State
S_1	λ	λ	λ	λ	p
S_2	w_H	w_L	w_H	w_L	$0.5(1-p)$
S_3	w_L	w_H	w_L	w_H	$0.5(1-p)$

Given that the aggregate demand for liquidity in the two countries is the same in each state, the global social planner problem is the same as in section 2. But, to implement the social optimum in a decentralized setting, banks have to make interbank deposits in other regions. Following Allen and Gale (2000a), we compare two alternative arrangements: a complete interbank market, where all regions are linked to all other regions, and an incomplete interbank deposit structure.

2.1 Complete Interbank Deposit Structure

Commercial banks are allowed to exchange deposits in period 0. The case of the network with a complete set of links is illustrated in Figure 1 (see the Appendix).

In such market structure, each representative bank exchanges demand deposits with all other representative banks. In other words, it makes deposits in all other banks, and receives deposits from all other banks. Interbank deposits are denominated in respective local currencies, i.e. deposits made in the large country are denominated in dollars, while those made in the small country are denominated in pesos. If a commercial bank from the large country wants to make a deposit in the commercial bank of the small country, it would exchange dollars for pesos at the Central Bank of the small country (at an exchange rate of one dollar per peso). This would lead to an increase of dollar reserves of the Central Bank of the small country. These reserves equal the amount that the commercial bank of the small country deposits in the three regions of the large country.

We assume that representative bank in region j holds $(w_H - \lambda)/2 \equiv z/2$ demand deposits in each region $i \neq j$. This is the smallest amount that banks have to deposit in every other region in order to fully insure against region-specific liquidity shock. Banks from other regions have the same rights as private depositors from the region, and the deposit rates are also the same. It is straightforward to show that such an arrangement allows banks to attain the first-best allocation. Every bank offers the same terms of the demand deposit contracts in period 0 and makes the same investment decision as in autarky. When a region faces a positive liquidity shock (it has a higher share of impatient deposits, w_H), it withdraws demand deposits from other regions in period 1 and satisfies the liquidity needs of the depositors without having to interrupt long-term technology. In

17

the second period, it has a lower need for funds because of the smaller number of patient depositors, and hence it can pay off banks with the lower share of impatient/higher share of patient agents. Furthermore, it is also easy to show that different currencies do not affect the feasibility of the first-best (no-run) allocation. For example, when a bank from the large country withdraws the peso-denominated deposit from the commercial bank of the small country, the latter exchanges dollars for pesos at the Central bank of the small country and pays them to the bank from the large country. That bank, in turn, goes to the Central bank of the small country and exchanges pesos back for dollars. As a result, neither the reserves of the Central Bank of the small country, nor the amount of pesos in circulation are affected.

2.2 Incomplete Interbank Deposit Structure

The second network structure that we consider is an incomplete system of interbank links. It is shown in Figure 2 (see the Appendix). Following Allen and Gale (2000a) we assume that bank in each region can hold deposits only in one neighboring region. In particular, commercial bank from region A can hold deposits only in region B, bank in region B can hold deposits only in region C and so on. We assume that the amount of each interbank deposit is $z = w_H - \lambda$. Again, this is the smallest amount of interbank deposit that allows banks to attain the command optimum allocation for their depositors. The way banks acquire and redeem deposits denominated in different currencies is quite similar to the case of the network with a complete set of links discussed above.

3 Analysis of Financial Fragility

This section contains the core results of the paper. We study how different degrees of network interconnectedness jointly with different exchange rate regime affects the financial stability of a two-currency financial network. Towards that end we assume that there is either a bank run in one region of the large country or a currency crisis in the small country.[23] Our aim is to find out for each combination of interbank structures and exchange rate regimes (to be specified later) the crisis would spread to all other regions of the large country, i.e. under which conditions in each region of the large country patient agents would attempt to withdraw in period 1.[24] We focus on

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[23]Recall that the Central Bank in the small country can act as an LOLR and, thus, prevent a banking crisis from starting in region D.

[24]The behavior of patient agents in the small country depends on the exchange rate regime. Under the flexible exchange rate regime they never misrepresent their type, and hence there is no run.

the case when the share of impatient agents in all regions is equal to λ. In focusing on this case we follow Allen and Gale (2000a). To justify such focus, one can assume that the probability of the realization of state S_1 is exogenous and sufficiently close to 1.[25] If for a certain combination of interbank deposit structure and exchange rate regime a wider range of parameter values ensures the spread of the crisis throughout the global economy than for another combination, we say that for such combination of the interbank structure and exchange rate regime the global financial system is more fragile.

3.1 Pecking Order of Asset Liquidation and Bank Buffer

In period 1, each commercial bank can be solvent, insolvent or bankrupt. A bank is *solvent* if it can satisfy the demands of every depositor who wishes to withdraw (including banks from other regions) by using only its world-market assets and deposits in other regions, i.e. without liquidating long term assets or emergency borrowing from the Central Bank (if feasible). The bank is said to be *insolvent* if it can carry out its obligations either by liquidating a portion of its long term investments (in case of the large country) or drawing on the emergency credit of the Central Bank (in case of the small country). The commercial bank in the large country is said to be *bankrupt* if it can not satisfy the demands of its depositors even if it liquidates all of its assets. The commercial bank in the small country can not be bankrupt due to the emergency credit line from the Central Bank.[26]

We impose a specific pecking order of liquidation of assets in period 1: banks initially liquidate their world-market assets, then deposits in other regions and finally their long-term investments. Such pecking order is achieved if the following inequality holds:

$$\frac{R}{r} > \frac{y}{x - \bar{m}} > 1 \tag{12}$$

The opportunity cost of period 1 consumption (in terms of future consumption) is different for different asset classes. For the world-market asset it is unity because one unit of world-market asset is worth one unit of consumption good today. If reinvested, it will be worth one unit of consumption good tomorrow. If we withdraw one unit of demand deposit in period 1 we get x units of the consumption good at period 1 and give up y units of the consumption good in period 2

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[25]Like our paper, most of the literature on financial contagion takes aggregate liquidity shocks to be exogenous. In contrast, Cao and Illing (2011) consider a model of endogenous exposure to systemic liquidity risk.

[26]Later on we relax this assumption.

as well as the cash transfer $\bar{m}$ in period 1. So, the opportunity cost of liquidating demand deposits is $y/(x - \bar{m})$. Also, if the bank liquidates one unit of the long term asset, it gives up R units of consumption good in period 2 and receives r units of consumption in period 1. So the opportunity cost of liquidating the long term asset is R/r. Such pecking order is violated in case of bankruptcy. If the bank in some region is bankrupt then all of its depositors including banks in other regions will rush to withdraw their deposits regardless of their own pecking order.

Next we introduce the notion of a buffer. A buffer is defined as the maximum amount of dollars that can be obtained by liquidating the long term asset in period 1 without causing a run by patient depositors. When a bank is insolvent, patient depositors should be given at least $x - \chi(m)$ in period 2 in order to induce them to withdraw their deposits in period 2. Otherwise it will be better for them to attempt to withdraw in period 1. A bank with a fraction λ of impatient consumers must keep at least $(1 - \lambda)(x - \chi(m))/R$ units of long term asset in order to meet the demand of the patient agents in period 2. Thus the buffer is

$$g(\lambda) = r \left[k - \frac{(1 - \lambda)(x - \chi(m))}{R} \right] \tag{13}$$

3.2 Exchange Rate Regimes

Financial fragility in the global economy, i.e. an economy that consists of the large and the small country, critically depends upon the monetary regime adopted by the central bank of the small country. What does the central bank do when patient agents report impatience and withdraw their deposits from commercial bank in period 1: does it attempt to keep the exchange rate fixed and liquidate the long-term investment to get dollars in order to intervene in the foreign-exchange market? Or, does it allow the peso to depreciates and keeps the long-term investment until period 2? We denote the share of the long-term investment that the central bank is willing to terminate as v and contrast two polar cases: the case when the central bank is willing to terminate all of the long-term investment in order to maintain the exchange rate level, $(v = 1)$, and when it does not terminate it at all, $(v = 0)$. The former case is a proxy to the 'fear of floating' regime, when the central bank intervenes in the foreign exchange market, and is willing to use the foreign exchange reserves in order to prevent excessive fluctuations of the exchange rate. The latter case is a proxy to the flexible exchange rate regime, when the central bank does not intervene to prevent exchange rate fluctuations. For completeness, we discuss, also, the situation when there is a monetary union

(a single currency), which coincides with the analysis of Allen and Gale (2000a).

3.3 Liquidation Values

Liquidation value of a commercial bank deposit is period 1 is the value of bank assets per unit of deposit in dollar terms when the bank is bankrupt (liquidated) in the large country, or when the currency is devalued in the small country. Consider, for example, incomplete interbank deposit market and bank run in region A that does not spread to region B. If all depositors in region A decide to withdraw their deposits the bank's dollar liabilities will be $(1 + z)$ because the bank from region D holds z units of deposit and patient and impatient agents in region A together hold one unit of deposit. Its dollar assets consist of b dollars obtained from liquidation of the world-market investment, rk dollars obtained from early liquidation of long term investment and zx dollars obtained from the liquidation its deposit in the bank in region B. The equilibrium value of q^A, therefore, reads:

$$q^A = \frac{b + rk + zx}{1 + z} \tag{14}$$

In the similar fashion one can analyze other cases.

3.4 Financial Fragility of an Incomplete Network of Interbank Deposits

3.4.1 Incomplete Network of Interbank Deposits and the Fear of Floating Exchange Rate Regime

In this subsection we analyze financial fragility when the market structure of the global financial network is incomplete (see Figure 2). We begin by studying the situation when the Central Bank of the small country pursues 'fear of floating' policy and stands ready to terminate the long-term technology in period 1 in order to get dollars for allegedly impatient agents. We commence by finding conditions under which crisis that originated in region D (the small country) would spread to all other regions of the global economy, i.e. conditions for global contagion. We, then, demonstrate that the conditions for global contagion are exactly the same in cases when crisis originates in any of the three regions of the large country (A, B, or C).

In case of the financial crisis in country D, Central Bank provides emergency loans to the bank of the region preventing it from collapsing and, in return, takes its long term investment as collateral. Since the Central Bank pursues fear of floating, it liquidates these long term assets in order to

try to maintain the exchange rate level. If all depositors in the small country (including the bank domiciled in region C of the large country) withdraw their deposits, they receive $(1 + z)x$ pesos, which is equal to the demand for dollars from the Central Bank of the small country at the initial exchange rate of 1 peso per dollar. However, the dollar reserves at the Central Bank would be, at most, equal to $b + rk + zx$. If $b + rk < x$, the Central Bank has to devalue the peso, and the new exchange rate would be equal to:

$$E_{1I}^D = \frac{(1 + z)x}{b + rk + zx} > 1 \tag{15}$$

In notation for the exchange rate, the superscript stands for the region in which the crisis originates. The first subscript denotes the exchange rate regime in the small country (1 stands the fear of floating regime while 2 stands for flexible exchange rate regime). The second subscript denotes the system of interbank links (I denotes incomplete market structure while C stands for complete financial link structure). We notice that despite the desire of the Central Bank of the small country to maintain the exchange rate level, crisis in region D would lead to devaluation of its currency. It is this devaluation that could serve as a contagion trigger. Namely, after devaluation of peso, the bank in region C would suffer a loss because it would receive only zx/E_{1I}^D dollars while it should pay zx dollars to the bank in region B. Its loss would be, therefore, equal to $zx(1 - 1/E_{1I}^D)$. The bank in region C will become bankrupt if its loss exceeds the buffer:

$$zx \left(1 - \frac{1}{E_{1I}^D}\right) > g(\lambda) \tag{16}$$

It turns out that condition (16) is a necessary and sufficient condition for the global run originating in region D.

Lemma 1. In case of a network with incomplete set of links between the regions, when the central bank of the small country pursues a fear of floating regime, a crisis in region D ensures runs in regions A, B, and C if and only if conditions (10) and (16) are satisfied.

Conditions (10) and (16) ensure runs in regions B and A, because of the spillover effect. The more regions are already bankrupt, the more losses would have accumulated due to early liquidation of the long term asset in these regions. As a result, liquidation value of the interbank deposit in each subsequent bank becomes smaller and smaller.

What would happen if all depositors in region A decide to withdraw their deposits? The bank in region A would be bankrupt and maximum liquidation value of its assets, provided that the

bank in region B is not bankrupt, is given by the following expression:

$$q^A = \frac{b + rk + zx}{1 + z} \qquad (17)$$

Bankruptcy of the bank in region A will lead to the loss in the bank of the small country (region D). If the loss exceeds the buffer

$$z(x - q^A) > g(\lambda) \qquad (18)$$

then all depositors of the small country will rush to withdraw their deposits. As a result, the Central Bank of the small country would be forced to devalue the peso. In that case, the lower bound of the new exchange rate [27] would be:

$$E_{1I}^A = \frac{(1 + z)x}{b + rk + zq^A} > 1 \qquad (19)$$

It is easy to show that $E_{1I}^A > E_{1I}^D$ so that the small country would in that case experience a higher devaluation than in case when the liquidity shock starts in the small country itself. This happens because of the loss on deposits held in region A by the banks of region D.

Inspection of condition (18) verifies that it is equivalent to condition (16). From Lemma 2 follows that conditions (10) together with (16) are both necessary and sufficient for the run originating in region A to cause crisis in regions D, B and C.

Lemma 2. In case of a network with incomplete set of links between the regions, when the central bank of the small country pursues a fear of floating regime a run in region A ensures runs in regions B, C and D, if and only if conditions (10) and (16) are satisfied.

Furthermore, it turns out that conditions for the global run originating in regions B and C are exactly the same. In other words, when the small country's Central Bank pursues a fear-of-floating regime all regions are in some sense completely symmetrical. Since the Central Bank of a small country would liquidate all of its long-term assets in case of a run, then liquidation value of small country's deposits in dollar terms will be the same as the liquidation value of deposits of any large country's bank (in case of a run on that particular bank). This is due to the devaluation rule adopted by the Central Bank of the small country.

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[27]This is the lower bound on the exchange rate in case of crisis in region A. It holds if there is no run in regions B and C. If those regions succumb to the crisis as well, the exchange rate in region D would be higher.

3.4.2 Incomplete Network of Interbank Deposits and the Monetary Union Regime

It is important to contrast the case of 'fear-of-floating' exchange rate regime with the benchmark single-currency world of Allen and Gale (2000a) (for the network configuration with incomplete set of links, Figure 2). The single-currency world effectively implies that the small country becomes the fourth region of the large country. The condition for the global run is independent of the region where the crisis starts. Without loss of generality, let us assume that the run hits the region A. Then the conditions for the global run will be conditions (10) and (18). This is equivalent to condition (16). In words, the fragility condition in case of the monetary union is the same as the fragility conditions of the two-currency global economy, with the small country adopting the fear-of-floating regime. The intuition for important result is simple. Namely, if a crisis unravels under the fear-of-floating regime, the central bank in the small country does exactly what a commercial bank in the large country does. It attempts to honor the nominal commitment and liquidates long-run technology investment to satisfy the demand for dollars of allegedly impatient depositors. The banking crisis is replaced with the currency run, but contagion conditions are the same, as the liquidation of long-term investment implies the same loss of value.

3.4.3 Incomplete Network of Interbank Deposits and the Floating Exchange Rate Regime

In this subsection we still consider a network with incomplete set of links (Figure 2) but assume that the central bank of the small country pursues the floating exchange rate regime. In this case, it does not terminate the illiquid technology in order to minimize the exchange rate depreciation. In this case, the currency crisis cannot originate in region D even if the implicit short-term peso-denominated obligations are higher than the dollar denominated assets of the whole country under the exchange rate of one peso per dollar. Namely, the commitment of the Central Bank not to liquidate long term investments in period 1 can be considered as an insurance for the patient agents that they will receive promised amount of pesos and will be able to exchange them for dollars at a fixed exchange rate of one peso per dollar. So, this devaluation rule eliminates currency crisis originating in the small country. This is, essentially, insight from Chang and Velasco (2000b) study of a small open economy.

Importantly, since the region D is embedded into the network (Figure 2) (in contrast to Chang

and Velasco (2000b)), if a run occurs somewhere in the large country it can lead to devaluation of the small country's currency even in the floating exchange rate regime case. In order to see why, let us consider a case when all depositors in region A decide to withdraw their deposits. In that case, the bank in region A would be bankrupt and maximum liquidation value of its assets, given that the bank in region B is solvent, would be determined by the following equation (20):

$$q^A = \frac{b + rk + zx}{1 + z} \tag{20}$$

Here we take into account that all 4 banks withdraw their interbank deposits. Bank A suffers from the run, and hence it withdraws from B. Hence bank B has to withdraw from C even if it is not bankrupt, etc. As a result, the dollar denominated assets of the bank in region D whose maximum value is equal to $b + zq^A$ will be lower than its peso-denominated liabilities $\lambda x + zx$ under the exchange rate equal to unity. The bank in region D will ask for emergency credit from the Central Bank to cover the difference and pass the control over long term assets to the Central Bank. Since the early liquidation of the long term assets is not allowed (due to the assumed floating exchange rate regime), the Central Bank will devalue the currency without liquidation of the long term asset. The lower bound of the new exchange rate is equal to:

$$E_{2I}^A = \frac{(\lambda + z)x}{b + zq^A} \tag{21}$$

Here we take into account that all 4 banks withdraw their interbank deposits. Bank A suffers from the run, and hence it withdraws from B. Hence bank B has to withdraw from C even if it is not bankrupt, etc.

As a result of this devaluation, the bank in region C suffers a loss due to currency devaluation. It would become bankrupt if and only if its minimum possible loss exceeds the buffer:

$$zx\left(1 - \frac{1}{E_{2I}^A}\right) > g(\lambda) \tag{22}$$

Lemma 3 demonstrates that this is the main condition for the global contagion if the crisis originates region A.

Lemma 3 Under the assumptions of incomplete market structure and flexible exchange rate regime, a run in region A ensures runs in regions B and C, if and only if conditions (10) and (22) are satisfied.

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As Lemma 4 below asserts, the main condition for the global contagion originating in region B is as follows:

$$ zx \left[1 - \frac{b + \frac{z}{1+z}\left(b + rk + zq^B \right)}{(\lambda + z)x)} \right] > g(\lambda), \tag{23} $$

where

$$ q^B = \frac{b + rk + zx}{1 + z} \tag{24} $$

Lemma 4 Under the assumptions of incomplete market structure and flexible exchange rate regime a run in region B ensures runs in regions A and C, if and only if conditions (10) and (23) are satisfied.

The last equation shows the liquidation value of the deposits in the bank of region B after it suffers from a run. Inequality (23) shows the condition under which the bank in region C becomes bankrupt. The proof of Lemma 4 (in the Appendix) demonstrates that inequality (23) ensures that bank in region A becomes bankrupt as well.

Finally, Lemma 5 specifies the conditions under which the run in region C spreads to all other regions of the large economy. It turns out that the conditions are the same as in the case of the fear-of-floating exchange rate regime.

Lemma 5 Under the assumptions of incomplete market structure and flexible exchange rate regime a run in region C ensures runs in regions A and B, if and only if conditions (10) and (16) are satisfied.

The result is not unexpected given that the contagion spreads from region A to B and then to C, i.e. the regions of the large economy, and hence the exchange rate regime in the foreign economy does not affect the contagion in that case.

The Proposition 1 below contrasts conditions for global run under flexible and fear-of-floating regimes in case of incomplete market structure.

Proposition 1 *Under incomplete market structure, conditions for a global run are at least as stringent under flexible exchange rate regime as under fear of floating regime.*

Proposition 1 states that the global economy is more fragile under the fear of floating regime than under the flexible exchange rate regime when the market structure is incomplete. The proof compares the condition for the global run originating in each of the four regions. If the crisis starts in region C, the conditions are identical. If the crisis starts in regions A or B, the crisis can spread to all other regions of the large economy, but the conditions are more stringent (i.e., the global run

is less likely) under the flexible exchange rate regime. Finally, a run can originate in region D only under the fear-of-floating regime.

Proposition 1 generalizes the findings of Chang and Velasco (2000b) in case when a small open economy is embedded into a network with incomplete set of links. In both models, the flexible exchange rate arrangement allows the monetary authority to refrain from termination of the long-term technology. This, in turn, ensures that the patient agents have no incentive to run and mitigates (in this model) or completely prevents (in the model of Chang and Velasco) the financial crisis. Thus, as long as financial links between the regions are not particularly strong, it may be optimal from the point of view of global financial stability, for the small economy to maintain a floating exchange rate regime. We shall see that this is no longer true when interbank set of links is complete.

3.5 Financial Fragility of a Network with Complete Set of Links

3.5.1 Complete Network of Interbank Deposits and the Fear of Floating Regime

Under complete network structure (see Figure 1) the bank in each region holds $z/2 = (w_H - \lambda)/2$ deposits in each other regions. The claim on any region is smaller than in case of incomplete network structure, but the total amount of claims is larger. Consider what happens if all depositors in the small country decide to withdraw their deposits. The argument is the same as in the case of incomplete network structure discussed above. The value of the bank's dollar denominated assets is in that case lower than its peso-denominated liabilities, as long as the exchange rate stays fixed. This is so because:

$$(1 + 3z/2)x > b + rk + 3zx/2$$

In that case, the Central Bank of the small country will devalue the peso. The lower bound of the new exchange rate in that case is:

$$E_{1C}^D = \frac{(1 + 3z/2)x}{b + rk + 3zx/2} \tag{25}$$

It is easy to show that $E_{1I}^D > E_{1C}^D$, i.e. under the complete market structure the Central Bank devalues peso by less than under the incomplete market structure, *ceteris paribus*. Since devaluation and interbank claims are lower than under the incomplete network structure, the loss for each large country bank will be also lower than in case of incomplete market structure. Banks in regions A,

B and C will become bankrupt if the following inequality holds for every bank:

$$\frac{zx}{2}\left(1 - \frac{1}{E_{1C}^D}\right) > g(\lambda) \tag{26}$$

Because the complete market structure is symmetric, the same condition holds for each region of the large economy.

Lemma 6 When the market structure is complete, under the fear-of-floating exchange rate regime a run in region D ensures runs in regions A, B and C, if and only if conditions (10) and (26) are satisfied.

If a liquidity shock occurs in some region i of the large country (the place of origin does not matter now since the market is completely symmetric) then banks in all other regions of the large country will become bankrupt if the following inequality holds for each bank:

$$\frac{z}{2}(x - q^i) > g(\lambda) \tag{27}$$

where

$$q^i = \frac{b + rk + 3zx/2}{1 + 3z/2} \tag{28}$$

is the liquidation value of the unit of deposit in region i provided there is no run in other regions. It is straightforward to show that conditions (26) and (27) are equivalent.

Lemma 7 Under the assumptions of complete market structure and fear-of-floating exchange rate regime, a run in any region of the large economy ensures runs in all other regions, if and only if conditions (10) and (26) are satisfied.

Similar to the case of incomplete market structure, conditions for the global run do not depend on the region where the crisis originates.

We are now in a position to compare conditions for the global contagion under complete and incomplete interbank deposit network structure, provided that the small country has a fear-of-floating exchange rate regime.

Proposition 2 *Under the fear-of-floating exchange rate regime, conditions for a global run are more stringent under the complete market structure than under incomplete market structure.*

The Proposition 2 states that the global contagion is more likely when the structure of interbank links is incomplete than when it is complete. This generalizes the original Allen and Gale (2000a)

result to the case of financial networks with multiple currency and fear of floating exchange rate regimes. We have already seen that under the fear-of-floating regime the difference between the two countries becomes immaterial.

3.5.2 Complete Network of Interbank Deposits and the Monetary Union Regime

In analogy with the case of incomplete system of interbank links, it is straightforward to show that the conditions for the global run under the monetary union are identical to conditions under fear of floating regime. In the monetary union, inequalities (10) and (27) determine conditions under which a run in one of the regions spreads to all other regions. But (27) is equivalent to (26).

Thus, under either monetary union or the fear of floating regimes, higher network interconnectedness leads to more stability due to the better mutual insurance that banks in different regions are able to provide each other. As we shall see below, this result may not always hold when the exchange rate regime is flexible.

3.5.3 Complete Network of Interbank Deposits and the Floating Exchange Rate Regime

If the Central Bank in the small country commits not to liquidate the long term asset in period 1, the financial crisis cannot originate there, as patient agents will never have an incentive to request dollars in period 1. Consider what will happen if a run occurs in some region i of the large country. The run will make the liquidation value of the deposit in region i fall below x. That would cause an immediate devaluation in region D. Both effects will reinforce each other because banks i and D hold deposits in each other. However, two other banks in the large country will not be bankrupt as long as the buffer in each of them exceeds (or equals) the total loss of value of interbank deposits in regions D and i.

Let q^i denote the liquidation value of the deposit in region i (provided that the banks in the two other regions are not bankrupt) and E_{2C}^A denote the nominal exchange rate in region D after devaluation. Their values are determined implicitly by equations (29) and (30):

$$q^i = \frac{b + rk + zx + 0.5zx/E_{2C}^A}{1 + 3z/2} \tag{29}$$

and

$$E_{2C}^A = \frac{(\lambda + 3z/2)x}{b + zx + zq^i/2} \tag{30}$$

The main condition for the overall run will be:

$$\frac{z}{2}\left(2x - q^i - \frac{x}{E_{2C}^A}\right) > g(\lambda) \tag{31}$$

The left-hand side of the last inequality is the sum of two terms, $z(x - x/E_{2C}^A)/2$, and $z(x - q^i)/2$. The first term is the loss of value of deposit in region D due to devaluation. The second term is the loss of value of deposit in region i because of the run. If the total loss exceeds the buffer, the global run becomes inevitable.

Lemma 8 When the market structure is complete, under flexible exchange rate regime a run in any region of the large country ensures runs in all other regions of the large country, if and only if conditions (10) and (31) are satisfied.

The following proposition states, at a first glance, a counterintuitive result: if network of interbank deposits is complete (Figure 1), when global financial crisis originates in any of the regions of the large country, peso would devalue more under fear of floating exchange rate regime than under flexible exchange rate regime.

Proposition 3 *If a global financial crisis originates in a region of the large country, under the complete set of interbank links the exchange rate depreciation in the small country is higher under fear of floating than under flexible exchange rate regime.*

The intuition behind Proposition 3 is as follows: when the small country has floating exchange rate, the truly patient depositors have no incentive to join the run. Therefore, the demand for dollars is smaller than under the fear-of-floating regime, and the demand effect dominates the supply effect.

The following proposition states the main result of this subsection:

Proposition 4 *When network of interbank deposits is complete, conditions under which a global run originates in a region of the large country are more stringent under fear of floating than under flexible exchange rate regime. The result does not depend on the choice of utility function of representative agent.*

Previously (see Proposition 1), we have established that in case of incomplete network of inter-bank deposits (Figure 2), Chang and Velasco (2000b) single small open economy result generalizes so that, *ceteris paribus*, flexible exchange rate regime in the small country reduces a chance of global contagion with respect to fear of floating or monetary union regimes. In turns out that the incomplete network structure is crucial for such a result to hold. Namely, Proposition 4 shows that the result is reversed if the network of interbank deposits is complete (Figure 1). In that case, switching from monetary union or fear of floating to flexible exchange rate regime actually increases financial fragility and increases a change of a global contagion. Moreover, this result is quite strong in that it does not depend on the shape of the utility function of representative agent.

This result is striking, because it is obtained in a framework 'most favorable' to the floating exchange rate regime. The setup rules out a financial crisis originating in the economy with the floating exchange rate regime, and there are no other welfare-reducing effects of the exchange-rate instability present in the real world. Furthermore, the run-avoidance under the floating exchange rate regime yields a *lower* exchange-rate depreciation than under the fear-of-floating regime when there is a global run (Proposition 3). We show, however, that even under such extreme conditions, a switch from the fear-of-floating regime to the floating regime increases the financial fragility when the system of interbank links is complete. The result is even more striking as it does not depend on the functional form of the utility function of the representative agent.

Why does an existence of a separate currency with flexible exchange rate regime in one of the regions and a complete system of interbank links jointly contribute to the global financial fragility? When each region is financially linked to all other regions, the region with flexible exchange rate regime immediately 're-exports' negative shocks (including the shock of a bank run in one of the regions of the large country) to all other regions of the large economy via the exchange rate depreciation (instead of absorbing them). These regions are under dual pressure, as each of them loses a part of the value of the deposit in the region affected by the bank run, and a part of the value of the deposit in the small country. However, these regions cannot follow the small country and devalue, and so they are more likely to suffer from the financial meltdown.

3.6 Complete versus Incomplete Network of Interbank Deposits Under Flexible Exchange Rate Regime: A Comparison

Previously we have shown that under fear of floating and monetary union exchange rate regimes, Allen and Gale (2000a) results hold so that more interconnections between region leads to more stable network due to mutual liquidity insurance that regions provide to each other. Importantly, in that case there are no substantial differences between the regions. In this subsection we check whether this intuition preserves when the small country pursues flexible exchange rate policy. It turns out that it depends on the region of origin of the crisis as well as, in general, on the parameters of the model. When crisis originates in region C, the result of Allen and Gale (2000a) holds and the incomplete network of interbank deposits is more fragile than complete network.

Lemma 9 If the small country has a flexible exchange rate regime and the bank run originates in region C, then the conditions for the global contagion are more stringent under the complete market structure.

However, if the initial bank run originates in regions A or B instead, then the comparison of contagion conditions depends on the functional form of the utility function and the parameters of the model. Let us assume, as is often done, that the utility function exhibits constant relative risk aversion:

$$U(x) = \frac{x^{1-\theta} - 1}{1 - \theta}$$

and that the utility-from-holding-money function is given by the following expression

$$\chi(m) = \sqrt{\bar{m}^2 - (m - \bar{m})^2}$$

for $m \leq \bar{m}$.[28] Then, there exists a set of parameter values for which the complete market structure is more fragile, i.e. the left-hand side of inequality (31) is greater than the left-hand sides of inequalities (22) and (23). An example of such a set is $R = 1.5, r = 0.8, \bar{m} = 0.2, \lambda = 0.5, \theta = 2, z = 0.1$.

In other words, if the small country pursues flexible exchange rate regime, increasing links between various regions in a multi-currency network may lead to increased fragility of the network (in contrast to the well known results of Allen and Gale (2000a)). This reconfirms the intuition that a combination of flexible exchange rates in one part of the network together with increased interconnectedness between regions may be dangerous from the global financial stability point of view.

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[28]This function satisfies all the assumptions outlined in section 2.1.

4 Concluding Remarks

Our paper demonstrates that the effects of different levels of financial interconnectedness and different exchange rate regimes on stability of multinational financial networks should not be considered in isolation from one another. To show this, we analyze network fragility in the context of a two-country four-region model a la Allen and Gale (2000a) with open-economy monetary features of Chang and Velasco (2000b). We assume that one of the regions in the Allen and Gale framework is a separate country with its own currency and a central bank. In this framework, the major results of Allen and Gale and Chang and Velasco are obtained as special cases. In particular, under incomplete structure of interbank links, a switch from the fear-of-floating or monetary union to the flexible exchange rate regime reduces financial fragility, which is consistent with the Chang and Velasco findings. Also, under the fear-of-floating regime or monetary union regime, a switch from incomplete to complete system of interbank links, which can be interpreted as increased financial globalization, also reduces financial fragility. This is essentially the original Allen and Gale (2000a) result, as the fear-of-floating regime makes the difference between the currencies immaterial for financial contagion.

However, the combination of a complete set of interbank links and a flexible exchange rate regime yields two novel results. First, in contrast to Allen and Gale (2000a), we find that a switch from incomplete to complete set of links may, under certain conditions, *increase* rather than decrease financial fragility if the smaller country maintains a floating exchange rate regime. Also, in contrast to Chang and Velasco (2000b), when the set of links is complete, a switch from monetary union or fear-of-floating to a floating exchange rate regime in the small country unambiguously decreases financial stability of the global network. Note that this finding is quite robust and is reached in a setup that assumes away several 'real-world' problems inherent in flexible exchange rate regimes. Thus, financial links between countries with different exchange rate regimes may be a source of financial fragility, and the importance of this source of risk rises with increased level of financial interconnections between countries.

Our analysis, while simple, has potentially important policy implications. It demonstrates that the stability of multinational financial networks depends on *combination* of the degree of network interconnectedness and the exchange rate regimes that countries participating in the network pursue. Thus, decisions to increase or decrease financial links between countries (i.e. further integration or

ring fencing measures) or decisions to change exchange rate regimes (entering or exiting a monetary union, for example) cannot be made in isolation from one another. Our results rationalize an increase in financial links among European countries, and not only since 1999 (when the Euro zone was established), but in the earlier years when European countries tried to limit exchange rate fluctuations within the European Monetary System. Namely, as we have seen, under the conditions of a monetary union, or a fear of floating exchange rate regime, increased links improve stability of the network. Importantly, the model implies, also, that once close interregional ties are established, a country's exit from the monetary union or fear of floating regime may be detrimental to the stability of the network. This provides a simple additional argument against potential secession of Greece (and other countries with weak fundamentals) from the euro zone. As euro zone countries are financially highly intertwined, network-wide financial contagion would be more likely if one or more regions switch to an independent currency with a floating exchange rate regime. Because of the high degree of interconnectedness with Euro zone economies, a flexible exchange rate regime in countries adjacent to the Euro zone (Switzerland, UK, Poland, Hungary, etc.) may additionally contribute to the financial fragility of European financial networks.

There are several venues for potential future research. One issue to consider is how our results will be affected if more than one region in the network has independent currency. While this remains a venue for future research, analysis in the paper allows us to conjecture that, first, if another region becomes a separate country with a fear-of-floating exchange rate regime, then its monetary independence is immaterial. If this country has a floating exchange rate regime, then the conditions for global contagion will depend on the region where the crisis starts. The crisis cannot start any longer in the regions with flexible rate regimes. On the other hand, if the crisis originates in one of the remaining regions of the large country, contagion becomes more likely under a complete system of links. This happens because such a region will come under triple pressure: its interbank deposits will lose value in all three other regions (in the two devaluing regions, and in the region where the crisis originated). Thus, qualitative results of our model would likely preserve in that case.

In our analysis we assume away, for simplicity, the presence of a moral hazard problem. Brusco and Castiglionesi (2007) incorporate moral hazard in an extension of the single-currency model by Allen and Gale (2000a). They show that higher level of interconnectedness in that case may increase rather than decrease financial fragility of the network. It would be interesting to check whether

presence of a moral hazard would substantially change conclusions of our model, especially since discussions of moral hazard problems are at the forefront of both regulatory and political attention in the Euro zone and beyond.

As we have seen, a combination of different exchange rate regimes and financial network structures can qualitatively change conclusions of the benchmark theoretical models of financial contagion. It may be useful to revisit other standard models of financial contagion that are formulated in a single-currency setup and study their multi-currency analogues. For example, introducing different currencies in the money-center model of Freixas et al (2000), one could study stability of international payment networks (the original formulation uses single currency). Another important issue is how central banks should optimally react to systemic crises in a multi-country, multy-currency world.

Last but not least, it is important to empirically test predictions of the model. Existing empirical studies on relationship between the structure of a banking network and its stability focus, typically, on an individual country (see Degryse et al (2009), Chapter 7). Moreover, the results of the existing studies are ambiguous. While Degryse et al (2007) finds that Belgian banking system became less prone to contagion once its structure changed from complete interconnectedness to multiple money centers structure of Freixas et al (2000), Mistrulli (2007) states that the likelihood of contagion for Italian banks *increased* as a result of a similar change in network structure. Because they study financial stability of a single country, these studies are not directly comparable to our model as they ignore issues of multiple currencies in a network.[29] In the future, it would be of considerable interest to design a careful empirical (simulation) study of a joint impact of different network structures and exchange rate regimes on stability of multinational financial networks.

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[29]Elsinger et al (2006) analyze stability of interbank networks in Austria and simulate the joint impact of interest rate, exchange rate, and stock market shocks.

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Appendix

Analysis of the Social Planner's problem (1)-(5)

The Lagrangian for the maximization problem is as follows:

$$L = \lambda u(x) + (1-\lambda)u(\chi(m)+y) + \mu_1(1-k-b) + \mu_2(b+rl-\lambda x) + \mu_3(R(k-l)-(1-\lambda)y) + \mu_4(\chi(m)+y-x)$$

First-order conditions are as follows:

$$\frac{\partial L}{\partial x} = \lambda u'(x) - \lambda\mu_2 - \mu_4 \leq 0 \tag{A1}$$

$$\frac{\partial L}{\partial x}x = [\lambda u'(x) - \lambda\mu_2 - \mu_4]x = 0 \tag{A2}$$

$$\frac{\partial L}{\partial y} = (1-\lambda)u'(\chi(m)+y) - (1-\lambda)\mu_3 + \mu_4 \leq 0 \tag{A3}$$

$$\frac{\partial L}{\partial y}y = [(1-\lambda)u'(\chi(m)+y) - (1-\lambda)\mu_3 + \mu_4]y = 0 \tag{A4}$$

$$\frac{\partial L}{\partial m} = (1-\lambda)u'(\chi(m)+y)\chi'(m) + \chi'(m)\mu_4 \leq 0 \tag{A5}$$

$$\frac{\partial L}{\partial m}m = [(1-\lambda)u'(\chi(m)+y)\chi'(m) + \chi'(m)\mu_4]m = 0 \tag{A6}$$

$$\frac{\partial L}{\partial k} = -\mu_1 + R\mu_3 \leq 0 \tag{A7}$$

$$\frac{\partial L}{\partial k}k = [-\mu_1 + R\mu_3]k = 0 \tag{A8}$$

$$\frac{\partial L}{\partial b} = -\mu_1 + \mu_2 \leq 0 \tag{A9}$$

$$\frac{\partial L}{\partial b}b = [-\mu_1 + \mu_2]b = 0 \tag{A10}$$

$$\frac{\partial L}{\partial l} = (r\mu_2 - R\mu_3) \leq 0 \tag{A11}$$

$$\frac{\partial L}{\partial l}l = [r\mu_2 - R\mu_3]l = 0 \tag{A12}$$

$$\frac{\partial L}{\partial \mu_1} = 1 - k - b \geq 0 \tag{A13}$$

$$\frac{\partial L}{\partial \mu_1}\mu_1 = [1 - k - b]\mu_1 = 0 \tag{A14}$$

$$\frac{\partial L}{\partial \mu_2} = b + rl - \lambda x \geq 0 \tag{A15}$$

$$\frac{\partial L}{\partial \mu_2}\mu_2 = [b + rl - \lambda x]\mu_2 = 0 \tag{A16}$$

$$\frac{\partial L}{\partial \mu_3} = R(k - l) - (1 - \lambda)y \geq 0 \tag{A17}$$

$$\frac{\partial L}{\partial \mu_3}\mu_3 = [R(k - l) - (1 - \lambda)y]\mu_3 = 0 \tag{A18}$$

$$\frac{\partial L}{\partial \mu_4} = \chi(m) + y - x \geq 0 \tag{A19}$$

$$\frac{\partial L}{\partial \mu_4}\mu_4 = [\chi(m) + y - x]\mu_4 = 0 \tag{A20}$$

Assuming the incentive compatability constraint does not bind, from (A20) we get that $\mu_4 = 0$. (We analyze the case of binding incentive compatibility constraint later on). Consider the (only relevant) case when constraints (2)-(4) from the Social Planner's problem (1)-(5) hold as equalities. Also assume that the investment in the long-term technology, k, and the world-market investment, b, are both positive. We will consider the case $b = 0$ later on.[30] Therefore, from (A8) and (A10) we get $\mu_1 = \mu_2 = R\mu_3$.

Substituting the last results into equations (A11)-(A12):

$$\frac{\partial L}{\partial l} = r\mu_3 R - \mu_3 R \leq 0 \tag{A11*}$$

$$\frac{\partial L}{\partial l}l = [r\mu_3 R - \mu_3 R]l = 0 \tag{A12*}$$

Taking into account that $r < 1$ we conclude that $l = 0$, i.e. it is never optimal to terminate the long-term illiquid technology in period 1. This makes perfect sense. The one-period gross return on interrupted illiquid technology is smaller than the one-period gross return on world-market investment. Therefore, the Central Planner should use only world-market investment to provide for consumption of impatient agents, and only long-term illiquid technology to provide for consumption of patient agents.

Taking into account the preferences, we focus on the (only relevant) case when the consumption of an impatient agent, x, consumption of the patient agent, y, and the real money balances, m are all strictly positive. Thus, condition (A5) transforms to:

$$\frac{\partial L}{\partial m} = (1 - \lambda)u'(\chi(m) + y)\chi'(m) = 0 \tag{A5*}$$

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[30]The case $k = 0$ is not relevant, because it implies that either $C_2 = 0$ (which cannot be optimal given the assumptions on preferences), or $C_2 > 0$, but the social planner uses only world-market investment for two consecutive periods to save for period 2. However, it is clearly suboptimal, becuse the gross return on illiquid technology over two periods, R is greater than unity, the gross return on world-market investment over 2 periods.

Frpm (A5*) we get that $\chi'(m) = 0$. Taking into account $l = 0$, from the binding constraints (2)-(4), we get

$$\lambda x = b$$

$$\lambda x = 1 - k$$

and

$$kR = (1 - \lambda)y$$

Now consider the case when the incentive compatibility constraint binds, i.e. $\chi(m) + y = x$ and $\mu_4 \geq 0$. We get the following system of equations:

$$\frac{\partial L}{\partial x} = \lambda u'(x) - \lambda u_2 - \mu_4 = 0 \qquad\qquad (A1^{**})$$

$$\frac{\partial L}{\partial y} = (1 - \lambda)u'(\chi(m) + y) - (1 - \lambda)\mu_3 + \mu_4 = 0 \qquad\qquad (A3^{**})$$

$$\frac{\partial L}{\partial m} = (1 - \lambda)u'(\chi(m) + y)\chi'(m) + \chi'(m)\mu_4 = 0 \qquad\qquad (A5^{**})$$

$$\frac{\partial L}{\partial k} = -\mu_1 + R\mu_3 = 0 \qquad\qquad (A7^{**})$$

$$\frac{\partial L}{\partial b} = -\mu_1 + \mu_2 = 0 \qquad\qquad (A9^{**})$$

From (A5**) we get that

$$[(1 - \lambda)u'(\chi(m) + y) + \mu_4]\chi'(m) = 0$$

Case 1

$$(1 - \lambda)u'(x) + \mu_4 = 0 \qquad\qquad (A10^{**})$$

Substituting into (A3**), we get:

$$\frac{\partial L}{\partial y} = -(1 - \lambda)\mu_3 = 0$$

From the last equation, (A7**) and (A9**), $\mu_1 = \mu_2 = \mu_3 = 0$. Hence, from (A1**) , $u'(x) = 0$, which contradicts the assumptions about preferences for finite x.

41

Case 2

$$\chi'(m) = 0$$

From (A7**) and (A9**) we know that $\mu_1 = \mu_2 = R\mu_3$, therefore (A1**) becomes

$$\mu_4 = \lambda(u'(x) - R\mu_3)$$

At the same time (A3**) becomes:

$$\mu_4 = (1 - \lambda)(\mu_3 - u'(x))$$

For $\mu_4 > 0$ we need the following inequalities to hold simultaneously:

$$u'(x) > R\mu_3 \qquad \text{and} \qquad \mu_3 > u'(x)$$

However, since $R > 1$ and $u'(x) > 0$, these two inequalities are incompatible. Therefore, in equilibrium the incentive compatibility constraint does not bind, and $\chi(m) + y > x$.

Consider the case $b = 0$. First-order conditions (A7), (A9) and (A11) will be as follows:

$$\frac{\partial L}{\partial k} = -\mu_1 + R\mu_3 = 0 \tag{A7***}$$

$$\frac{\partial L}{\partial b} = -\mu_1 + \mu_2 \leq 0 \tag{A9***}$$

$$\frac{\partial L}{\partial l} = r\mu_2 - \mu_3 R \leq 0 \tag{A11***}$$

From (A7***) and (A9***) we get that $\mu_2 \leq \mu_1 = R\mu_3$. Since $r < 1$ then $r\mu_2 < \mu_2 \leq \mu_3 R$. Therefore, (A11***) holds as a strict inequality and $l = 0$. If both b and l are equal to zero, then $x = 0$, which contradicts our assumption.

Proofs

Proof of Lemma 1

Equations (10) and (16) are necessary and sufficient conditions for bankrupcy of the banks in region C and D. Therefore, the liquidation value of assets of the region C bank is

$$q^C = \frac{b + rk + \frac{zx}{E_{1I}^D}}{1 + z}$$

42

The bank in region B will be bankrupt if and only if its loss is greater than its buffer. The following condition should hold:

$$z(x - q^C) > g(\lambda)$$

The last condition will be satisfied if (16) holds and $q^C < x/E_{1I}^D$, which can be proven as follows. By using the definition of E_{1I}^D we have that:

$$\frac{x}{E_{1I}^D} = \frac{b + rk + zx}{1 + z}$$

As $E_{1I}^D > 1$, we have

$$q^C = \frac{b + rk + zx/E_{1I}^D}{1 + z} < \frac{x}{E_{1I}^D} = \frac{b + rk + zx}{1 + z}$$

We have shown that if conditions (10) and (16) are satisfied the bank in region B is bankrupt. The liquidation value of its assets is

$$q^B = \frac{b + rk + zq^C}{1 + z}$$

The bank in region A will be bankrupt if its loss is greater than its buffer. The following condition should hold:

$$z(x - q^B) > g(\lambda)$$

Again, we need to prove that $q^B < q^C$. We have already shown that $q^C < x/E_{1I}^D$. Thus,

$$q^B = \frac{b + rk + zq^C}{1 + z} < q^C = \frac{b + rk + zx/E_{1I}^D}{1 + z}$$

Thus, if conditions (10) and (16) are satisfied, then a run in region D will spread to all other regions. If at least one of these conditions is not satisfied, the bank in region C will not be bankrupt. Q.E.D.

Proof of Lemma 2 is completely analogous to the proof of Lemma 1 and is omitted for brevity.

Proof of Lemma 3

First, we show that $E_{2I}^A < E_{1I}^D$. This inequality is equivalent to

$$\frac{(1 + z)x}{b + rk + zx} > \frac{(\lambda + z)x}{b + zq^A}$$

Substituting $q^A = \frac{b+rk+zx}{1+z}$, cross-multuplying and collecting terms, the last inequality collapses to:

$$b + bz > \lambda b + \lambda rk + \lambda zx$$

43

From the solution to the Social Planner's problem we have that $\lambda x = b$, and therefore the analyzed inequality becomes (after some simplification),

$$x > b + rk$$

which coincides with (10).

Condition (22) ensures that the bank in region C is bankrupt and its buffer is lower than its loss:

$$zx\left[1 - 1/E_{2I}^A\right] > g(\lambda) \qquad\qquad \text{(A22***)}$$

where $E_{2I}^A = \frac{(\lambda+z)x}{b+zq^A}$. We need to show that the bank in region B is bankrupt as well. The condition for its bankruptcy is

$$z(x - q^C) > g(\lambda)$$

where

$$q^C = \frac{b + rk + zx/E_{2I}^A}{1 + Z}$$

Inequality $z(x - q^C) > g(\lambda)$ follows from (22) if $q^C < x/E_{2I}^A$. The proof of the last inequality iis as follows. We already know that $E_{2I}^A < E_{1I}^D$. Using simple algebra it is straghtforward to verify that

$$E_{1I}^D = \frac{(1+z)x}{b + rk + zx} < \frac{x}{b + rk}$$

Thus, $E_{2I}^A < E_{1I}^D < \frac{x}{b+rk}$. On the other hand, simple algebra verifies that $x/E_{1I}^D - q^C > 0$, and, therefore,

$$\frac{x}{E_{2I}^A} > \frac{x}{E_{1I}^D} > q^C$$

Hence $q^C < \frac{x}{E_{2I}^A}$. Q.E.D.

Proof of lemma 4

If condition (10) is satisfied, a run on bank in region B ensures its bankruptcy. Bank in region A will be bankrupt, if $z(x - q^B) > g(\lambda)$, where $q^B = \frac{b+rk+zx}{1+z}$.

If bank in region A is bankrupt, then the bank in region D is forced to devalue the peso and the new exchange rate will be

$$\frac{(\lambda + z)x}{b + zq^A}$$

44

where

$$q^A = \frac{b + rk + zq^B}{1 + z}$$

Bank in region C will be bankrupt, if

$$zx\left[1 - \frac{b + zq^A}{x(\lambda + z)}\right] > g(\lambda) \tag{A23***}$$

Below we prove that

$$z(x - q^B) > zx\left[1 - \frac{b + zq^A}{x(\lambda + z)}\right]$$

Therefore, condition (23) ensures run in region A, and, therefore, a global run.

Condition $z(x - q^B) > zx\left[1 - \frac{b + zq^A}{x(\lambda + z)}\right]$ is equivalent to

$$q^B < \frac{b + zq^A}{\lambda + z}$$

Taking into account that $\lambda x = b$ and expressions for q^A and q^B, and after some tedious algebraic manipulations, the last inequality can be shown to be identical to

$$(z^2 - \lambda - \lambda z)(x - b - rk) < 0$$

Taking into account that $\lambda = w_H - z$, and $z = 0.5(w_H - w_L)$, it is identical to

$$[w_L(w_L - w_H) - (w_H + w_L)](x - b - rk) < 0$$

The last inequality holds, because $b + rk < x$ and $w_L < w_H$. Q.E.D.

Proofs of Lemmas 5-8 are straightforward and are omitted for brevity.

In the following proofs we will use the notation q_1 (with an appropriate superscript) for the liquidation value under fear-of-floating exchange rate regime, and q_2 for the liquidation value under flexible exchange rate regime.

Proof of Proposition 1.

If a run starts in region A, then it is easy to prove that contagion is more likely for $v = 1$. To prove this we have to show that LHS(22)< LHS(18).

$$LHS(22) = zx\left[1 - 1/E_{2I}^A\right] \vee z(x - q_1^A) = LHS(18)$$

where $q_1^A = \frac{b+rk+zx}{1+z}$ and $E_{2I}^A = \frac{(\lambda+z)x}{b+zq^A}$.

$$zx\left[1 - 1/E_{2I}^A\right] \vee z(x - q_1^A) \Leftrightarrow q^A \vee x/E_{2I}^A \Leftrightarrow E_{2I}^A \vee E_{1I}^D$$

We have already proved that $E_{1I}^D > E_{2I}^A$.

If a run starts in region B, then we will show that (23) ensures (18), in other words, the global run is more likely for $v = 1$. Assume (23) is satisfied. Then, as we showed in the proof of Lemma 4,

$$z(x - q_2^B) > zx\left[1 - \frac{b + zq_2^A}{x(\lambda + z)}\right]$$

Given that $q_1^A = q_2^B = \frac{b+rk+zx}{1+z}$, we get

$$zx\left[1 - \frac{b + zq_2^A}{x(\lambda + z)}\right] < z(x - q_1^A)$$

The left-hand side of the last inequality is the left-hand-side of (23). The right-hand-side is the left-hand side of 18. Hence, if (23) is satisfied, then (18) is satisfied as well.

If a run starts in region C then conditions for a global run are the same under $v = 0$ and $v = 1$. Q.E.D.

Proof of Proposition 2

In the following proof we will adopt the following notations: we will use q_I (with an appropriate superscript) for the liquidation value under the incomplete market structure, and q_C for the liquidation value under complete structure.

To prove this proposition we need to show that LHS (18) is larger than LHS (27).

$$z(x - q_I^A) \vee \frac{z}{2}(x - q_C^i)$$

where

$$q_I^A = \frac{b + rk + zx}{1 + z}$$

and

$$q_C^i = \frac{b + rk + 3zx/2}{1 + 3z/2}$$

46

$z(x - q_I^A) \vee \frac{z}{2}(x - q_C^i)$ is equivalent to

$$x/2 + q_C^i \vee q_I^A$$

Below we prove that $q_C^i > q_I^A$, i.e.

$$\frac{b + rk + 3zx/2}{1 + 3z/2} > \frac{b + rk + zx}{1 + z}$$

After cross-multiplying and simplification, the last inequality is equivalent to $b + rk < x$, which is true by assumption. Hence, $x/2 + q_C^i > q_I^A$, and LHS(18) > LHS (27). Q.E.D.

Proof of Proposition 3.

If there is a global run under complete structure and fear-of-floating regime, each bank will have to liquidate all its long-term investment. For any i, $q^i = b + rk$. Hence, the exchange rate in the small country will be equal to:

$$E_{1C}^{run} = \frac{x + 3zx/2}{b + rk + 3z/2(b + rk)} = \frac{x}{b + rk}$$

Next compute the exchange rate under the complete structure and floating exchange rate regime. Liquidation values in the regions of the large country will be:

$$q^A = \frac{b + rk + q^B z/2 + q^C z/2 + z/2 * (x/E_{2C}^{run})}{1 + 3z/2}$$

$$q^B = \frac{b + rk + q^A z/2 + q^C z/2 + z/2 * (x/E_{2C}^{run})}{1 + 3z/2}$$

$$q^C = \frac{b + rk + q^A z/2 + q^B z/2 + z/2 * (x/E_{2C}^{run})}{1 + 3z/2}$$

where E_{2C}^{run} is the exchange rate in the small country when the run is global.

Analogously for the small country we have

$$x/E_{2C}^{run} = \frac{b + zq^A/2 + zq^B/2 + zq^C/2}{\lambda + 3z/2}$$

Regions A, B and C are identical, therefore $q^A = q^B = q^C = q$.

Solving for q the liquidation value equation yields:

$$q = \frac{b + rk + z/2 * (x/E_{2C}^{run})}{1 + z/2}$$

47

and

$$x/E_{2C}^{run} = \frac{b + 3zq/2}{\lambda + 3z/2}$$

The last two equations suffice to solve for the equilibrium value of E_{2C}^{run}. Straightforward algebraic manipulations yield:

$$E_{2C}^{run} = \frac{x(2\lambda + 3z + z\lambda)}{2b + 4zb + 3zrk}$$

$$E_{1C}^{run} = \frac{x}{b + rk} \vee E_{2C}^{run} = \frac{x(2\lambda + 3z + z\lambda)}{2b + 4zb + 3zrk}$$

Cross-multiplying, collecting terms, dropping positive constants and taking into account that $x = \lambda b$, the last comparison is identical to:

$$x \vee b + rk$$

By inequality (10), $x > b + rk$, therefore, $E_{1C}^{run} > E_{2C}^{run}$.

Q.E.D.

Proof of Proposition 4.

To prove that under complete market structure a global run is more likely under $v = 0$ than under $v = 1$, we need to show that LHS (31)> LHS (27).

After solving the system (29)-(30), we get the following values for q_0^i and E_{2C}^A:

$$q_0^i = \frac{(b + zx)(\lambda + 2z) + rk(\lambda + 3z/2)}{\lambda + 3z/2 + 3z\lambda/2 + 2z^2}$$

$$E_{2C}^A = \frac{x(\lambda + 3z/2 + 3z\lambda/2 + 2z^2)}{(b + zx)(1 + 2z) + rkz/2}$$

We need to show that

$$x - q_0^i + x - \frac{x}{E_{2C}^A} > x - \frac{b + rk + 3zx/2}{1 + 3z/2}$$

We will do it in 2 steps. First, we show that $x > \frac{x}{E_{2C}^A}$. Second we show that $\frac{b + rk + 3zx/2}{1 + 3z/2} > q^i$.

Step 1.

Taking into account the definition of E_{2C}^A and cross-mutiplying, we get

$$\lambda x + 3zx/2 + 3zx\lambda/2 + 2z^2 x > b + 2zb + zx + 2z^2 x + zrk/2$$

Collecting terms and taking into account that $b = \lambda * x$, the last inequaity is identical to inequality (10), i.e.

$$x > b + rk$$

Step 2.

Taking into account the definition of q^i, cross-multiplying $\frac{b+rk+3zx/2}{1+3z/2} > q^i$ and collecting terms, we get:

$$zx\lambda/2 + z^2x/4 + 3z^2x\lambda/4 > zb/2 + z^2b + z^2rk/4$$

Taking into account that $b = \lambda * x$ and collecting terms, we get:

$$x > b + rk$$

Q.E.D.

Proof of Lemma 9.

To prove the lemma, we need to show that LHS (16) > LHS(27). However, we have shown that inequality (16) is equivalent to (18). In the proof of Proposition 2 we have shown that LHS (18) > LHS (27). Q.E.D.
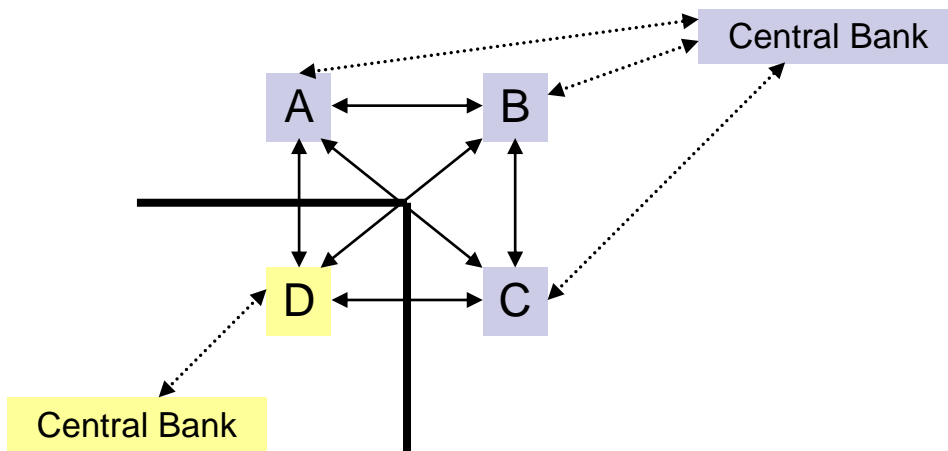
Figure 1. Complete Market Structure

Figure 2. Incomplete Market Structure