In search of economic reality under the veil of financial markets*

Second Draft

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Abstract

This paper presents a general equilibrium model with technological uncertainty, financial intermediation and imperfect information. The intermediate inputs of the financial sector consist in (state-contingent) intertemporal trading opportunities – financial products – and their pricing. They are used in the real sector for allocating the savings of households and financing investments of firms. Households have no direct information about the productivity of risky technologies in future environments. They rely on the information conveyed by the set of financial products offered in the financial markets, the pay-off promises of these products and their prices.

Because of uncertainty, prices are based on beliefs which can be more or less accurate. Therefore, things may go wrong ex post even if markets are in equilibrium. The presented framework provides a clear criterion for distinguishing "man-made" crises, for which some agents are responsible, from strokes of fate. Finally, the paper sketches a way how one might find rules for sound financial development which tie the growth of the financial sector to the dynamics of the real sector.

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1 Introduction

Most economists would share the view that financial development is an important source of prosperity of the real economy. At the same time, many people think that the financial sector is responsible for severe crises and misallocations of resources. The presented paper contributes to the understanding of this tension by considering the nature of financial markets in a general equilibrium model with technological uncertainty, financial intermediation and imperfect information. The central research questions to be answered in this framework are: What are the inputs from the financial sector required by the agents of the real economy – households and firms? How are these inputs produced and how does their quality affect the productivity of the real sector? For answering these questions we have to account carefully for the nature of economic reality. Economic objects are "things" with a price tag showing their valuation by the market. Financial markets attach to things today their value for the future. The future however results from savings and investments today, when the future is a set of ideas rather than realized things. What is then reality and what is the role of financial markets in shaping this reality? According to this paper, the basic function of the financial system is to provide intermediary inputs for transforming ideas about future possibilities into realized possibilities.

In a nutshell, the situation to be modeled can be illustrated by the following example. There is a mass of consumer households, spread over a set of different islands, and a set of islands with different firms ("entrepreneurial engineers"). The two sets of islands may overlap; the important thing is that each island hosts only one firm (or a small variety of firms) and households cannot communicate with firms across islands. Each firm knows how to construct a "machine" for producing a given good in a particular environment (engineerial capacity) and to produce output if it gets financing for employing the resources to run the "machine" (entrepreneurial capacity). Households are endowed with a resource which can be consumed or used for producing future consumption possibilities. The latter, however, requires that the resources saved by consumers are channeled to the firms and firm output is channeled back to consumers. The actual shipping of resources from households to firms and of output back to households can be done by the firms and causes no problems. The problem is that the resources have to be given away today; the goods in return are coming back only tomorrow. Moreover, since nobody knows in advance, which particular environment is realized in the future, the output from the resources invested into a particular firm is uncertain. Thus, households wish to diversify their savings across the different firm islands. The firms are willing to employ the savings they get. But how do the different islands know from each other, and in which form can firms fix promises of future output to consumers in return for resources given today? This communication problem gives rise to the role of the financial sector with agents specialized in inspecting the islands, exchanging their views among them, and communicating the result to households and firms – together with a description how to fix promises. The means of communication, the "language" used by the financial agents, consists of a set of financial products with state-contingent performance characteristics and their prices. They express – more or less reliably – the prospects of firms in the different environments, and the willingness of households to save under such prospects. The variety of financial products as well as the reliability of their characteristics and prices determine the size and the quality of the intermediary input delivered by the financial sector to the real sector.

In the further sections of the introduction, first the modeling principles and primitives of the analysis are described, then, the specific contributions of the analysis and their relation to the literature are exposed.

1.1 Modeling principles and primitives

The following traits of life in a modern economy are taken as facts. *Fact 1*: The future is partly "men-made" and partly depends from exogenous forces ("nature"). *Fact 2*: The future is uncertain and there is limited knowledge about the possible realizations of future events. *Fact 3*: There are two sources of information on which individuals can base their actions – direct inspection of real fundamentals and inference from market data. The inferred information depends on the amount of information collected by direct inspection and the quality of the transmission of collected information into market data. *Fact 4*: In a complex economic system, individuals act in specialized roles. In particular, there is specialization in the collection and transmission of information.

From these facts follow three basic requirements for the transformation of current resources to future possibilities and its modeling in an economy with a real and a financial sector. *Requirement 1 ("Measurability")*: State-contingent resource transformation from present to the future is only possible for measurable states. Therefore, the modeling of the state space and its measure is one crucial element in the analysis of this paper. *Requirement 2 ("Real investment")*: Present-day household income can only be saved for future consumption if firms invest the savings in future production possibilities. The second important element in the presented model is therefore the set of possible technologies and their state-contingent performance. *Requirement 3 ("Financial intermediation")*: The investment of household savings into production possibilities of firms requires financial instruments. The primary function of agents in the financial sector is therefore to design appropriate financial instruments ("financial products") and to trade them with households and firms at market prices. Financial products are characterized by the state-contingent pay-offs they promise or require. In order to fulfill the primary function to households and firms, a secondary function of agents in the financial sector is the pricing of the financial products. This requires direct inspection of real fundamentals and arbitrage trading in order to aggregate the information collected by direct inspection and to coordinate beliefs about the unknown part of reality.

In sum, the technically feasible possibilities to transform resources from present to an uncertain future depend on three objects: the measurable state space, the set of technologies into which real resources can be invested and their state-contingent productivity. The means of communication about these objects, in particular the means coordinating household actions and firm actions, are the set of financial products, their state-contingent pay-offs and their market prices. Therefore, the economically feasible possibilities to transform real resources of today into real opportunities tomorrow depend on the information-processing quality of the financial sector. This quality is high if the variety of financial products together with their pay-off characteristics and prices inform accurately about the measurable future state space and the future production possibilities. If the information is inaccurate, the quality of the financial sector is low. Inaccurate information is revealed ex post when pay-off promises are deceived.

1.2 Specific contributions and relation to literature

A first contribution concerns uncertainty. Standard economic reasoning thinks about the future as an event space, Ω, with a probability measure, π, assigning to each event in Ω the probability of its realization. However, as Keynes and Knight most prominently pointed out, uncertainty cannot be fully reduced to measurable risk. More recent contributions to the foundations of economic decision making under uncertainty have picked up this issue by modeling Knightian uncertainty as uncertainty about the probability distribution, where the uncertainty about the latter is captured by characterizing the set of possible distributions (see for instance Gilboa and Schmeidler [1989] or Bewley [2002]).[1] This paper follows the more radical view of "true uncertainty", which Keynes [1937, p. 214] summarized by the phrase: "We simply do not know." I account for the principle limits of our knowledge by splitting the future event space into two sets: a subset S of measurable risks, which in principle can be insured by state-contingent securities, and a subset $\bar{S}$ which is not measurable. In addition, I also consider Knightian uncertainty by allowing the

[1]See also the notion of model misspecification and robustness (Hansen and Sargent [2007]) or the literature on ambiguity following Ellsberg [1961] (see, for instance, Cerreia-Vioglio et al. [2013]).

measure on S to be more or less precise.

A second contribution concerns the relationship between specialization, diversification and robustness. The insurance possibilities provided by financial markets depend on the state-contingent performance of the technologies in which savings are invested. Like in Diamond [1967], markets are incomplete to the extent that these technologies do not cover all states of the world. Thus, the degree of completeness may change if more specialized technologies become feasible by innovation. As Diamond [1967] pointed out, there is an important informational aspect which limits these possibilities of market completion, namely "an inability to distinguish finely among the states of nature in the economy's trading" (p. 760). The limit is reflected in the presented model in two ways: First, the unmeasurability of $\bar{S}$ expresses in a formal way that events within $\bar{S}$ are not distinguishable.[2] Second, the measure π on S (the measurable part of Ω) may be imprecise. On top of the principle constraints on the measurability of events (or the precision of the measure), knowledge about the performance of a technology in a specific environment may be imperfect, too. Ultimately, there exist two possibilities to insure against future uncertainty. Either the savings are invested into a robust technology which works smoothly under any exogenous conditions; or the savings are diversified across different independent technologies, each of which works well in certain circumstances but not so well under other conditions. The latter is clearly brought out by the paper of Acemoglu and Zilibotti (1997) on risk, diversification and growth.[3] In their model diversification is limited by minimum size requirements for innovation. In contrast, in my analysis markets are incomplete because of limited knowledge about the future. As a consequence, financial innovation – allowing households and firms to save in more diversified portfolios and to invest into more specialized technologies, respectively – is not necessarily a good thing. On the one side, as long as markets are missing for

[2] Formally, $\bar{S}$ is thus a single state so that the state space is given by $S \cup \{\bar{S}\}$.

[3] At the aggregate level, the model presented here essentially coincides with their framework if perfect measuring of risks is assumed, precise knowledge about the technological uncertainty is available, and transaction costs for financial intermediation are set to zero. But neither economic growth and long-run development are considered here.

insurable risks in the measurable part of the future, financial innovations that cover additional risks increase diversification possibilities. On the other side, if the new financial products and their prices are based on imprecise information, they generate new uncertainty. This and the uncertainty in the non-measurable part of the future cannot be insured by providing more state-contingent financial instruments. In sum, any innovation based on erroneous measurement of the future generates uninsured uncertainty in the diversified part of savings and investments. As a consequence, the allocation of savings is distorted away from robust technologies to non-performing specialized technologies.

The third contribution concerns the consequences of specialization in the acquisition and dissemination of information. There are two sources of information: direct inspection of reality and inference from market data.[4] In contrast to most of the literature on the information processing role of markets,[5] in the framework presented here the window of direct inspection is not open equally to all agents. In particular, households cannot directly inspect the state-contingent performance of the technologies in which their savings are invested. Moreover, unlimited arbitrage is only possible for financial agents.[6] These assumptions model the fact that in-

[4]This relates the analysis to the literature on crises and belief volatility which has recently emerged (see Angeletos and Werning [2006] for an important example) – based on the information structure considered in Morris and Shin [1998].

[5]There, information is heterogeneous insofar as different agents receive different private signals. Access to private signals, however, is symmetric in the sense that all agents receive a signal drawn independently from the same distribution (see previous footnote for examples). Symmetric dispersion of information was suggested by Hayek‘s [1945] view that knowledge “never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals process”(p. 519). At the same time, however, he acknowledged that “different kinds of knowledge ”are “at the disposal of particular individuals” or “in the possession ... of experts” (p. 521).

[6]There are clearly exemptions. For instance, if a household invests in a house to live in. In general, however, the typical household is not owner or expert of the projects which are financed by her or his savings. Moreover, also households and firms may participate in the financial market for speculative motives. But this is not their core function; if it is, they switched roles.

dividuals act in specialized roles.[7] The agents in the real sector – households and firms, respectively – save in portfolios and invest in specific technologies; the set of financial instruments they can use as well as the pay-off promises and prices they face are given by the financial sector, that is, by the aggregate outcome of the direct inspection and arbitrage activities of the financial agents. The activities of the financial agents require resources as input. The costs are financed by pay-off differentials for borrowers and lenders. Their aggregate value represents the size of the financial sector in terms of GDP.[8] For the sake of brevity, agents representing the financial sector are also addressed as "banks" in this paper. This is of course a much broader meaning than in the theory of banking – comprehensively exposed in the book by Freixas and Rochet [1998]. Apart from brevity, it reflects the view that financial intermediation activities are not bound to deposit-taking and loan-making; they also play a substantial information processing role in designing and trading financial products for the securities market. Quite generally, the aggregate income of the financial sector – wherever earned in the financial industry – can only be generated by intermediary services which are perceived as valuable inputs to the real sector.[9] (This holds regardless of whether the incomes of the financial sector include rents or correspond to minimal costs.) According to the approach of this paper, a

[7]To think in a model of roles or functions was common in Classical Political Economy (see Schumpeter [1954, 1959, pp.554-561] on actors and agents – or 'functional classes' – in the classic modeling.) Traditional macroeconomics also separates the roles of consumer (or workers) and investors. Moreover, it takes a stand on who plays an active or a passive role in determining aggregate outcomes, as emphasized most clearly by Kalecki (see Laski [1987] for a concise introduction into his work). By contrast, in current economic modeling it has become usual that each individual acts in all roles. This misses the fact that roles emerge in a system to exploit the advantages of specialization.

[8]The transaction costs are exogenous in this paper. Thus, the presented analysis is silent about whether the financial sector exerts its intermediation function in a cost-efficient way. For a more elaborated analysis of the banking sector in a model with technological uncertainty see Studer [2013]; there, the cost function of banks and competition between banks are explicitly modeled, and financial innovation as well as the size of the banking sector are endogenously determined by the interplay of bank competition and diversification-seeking consumers.

[9]Unless one would include in the financial sector the gambling industry, which produces excitement – a consumer good – by offering bets.

specialized role for these intermediary services has emerged – to have expertise for collecting information about fundamentals, coordinating beliefs by arbitrage, and communicating the results in the financial instruments and their prices. In other words, since there are no "invisible hands" or virtual auctioneers who could provide intertemporal financing possibilities at correct prices, "embodied" financial agents act in this function.

A fourth contribution concerns the causes and responsibilities for distortions and crises. Distortions in the intertemporal allocation of resources are possible for two reasons. Households may base their portfolio choice on wrong beliefs about future risks; or the financial products they use do not reflect the true risk and productivity performance of underlying technologies. In the first case, the distorted portfolio choice leads to a distorted investment structure, but no pay-off promises are deceived; realized consumption coincides with planned consumption. In the second case, realized pay-offs do not match the promised pay-offs, and realized consumption levels deviate from consumption plans. Thus, if financial agents are careless in the assessment of fundamentals, they exert a negative externality on households.[10] A "crisis" in the sense that consumers get for their savings little back in the future can be caused by "nature" or by the financial sector. The presented framework gives us a clear criterion for distinguishing the latter from the former. "Nature" causes the bad future outcome if an uninsured event is realized in which only robust technologies work. Such an event hurts the consumers but it is not unexpected. In contrast, the financial sector is responsible for a crisis if financial products and their prices convey unfounded beliefs about risks and performance of technologies, or if financial products are provided for uninsurable events. This failure is revealed ex post by deceived pay-off promises and unfulfilled consumption plans. Thus, the

[10] In this respect the presented model is related to Magill, Quinzii and Rochet [2011] who consider a complete market model in which all possible future outcomes are insurable by securities but the risk distribution can be influenced by some agents. Although in the model presented here neither true probabilities nor true productivities can be influenced, future production and output will be distorted if the beliefs about probabilities and productivities conveyed by the price and pay-off structure of the traded financial products are inaccurate.

experience of a bad outcome is complemented by the experience of deception and a loss of confidence in the financial system. In this sense a "men-made" crisis causes more damage than "bad luck" drawn by nature. In principle, realized consumption can deviate from planned consumption in both directions depending on whether pay-off promises have been too optimistic or too pessimistic. Moreover, if states are in principle measurable but the information basis for the measurement is poor, we face a trade-off. On the one hand, financial innovations insuring such states allow more diversified portfolios, invested in more specialized technologies. On the other hand, the pay-off promises – based on poor information – may be deceived and confidence in the financial system be hurt. This leads to the question what is an optimal degree of financial innovation. In section 7 the paper also contributes to this normative question.

By focusing on the most basic traits of an economy with financial markets the paper disregards many relevant facets of these traits and ignores a lot of important issues. In particular, the paper does not address any monetary or financing aspects (like liquidity or leverage) nor dynamical issues (like herding or financial cycles), which build the core of the current discussion about financial crises and policies to deal with such crises.[11] Moreover, it does not explain the aggregation of information and coordination of beliefs within the financial sector. This function is captured by assuming that prices and pay-offs of financial products satisfy a no-arbitrage condition. That means, financial markets are successful in coordinating financial agents, but they do not necessarily provide efficient collection of information about fundamentals. The consequences for the real sector are the principle focus of this paper,

[11] As Geanakoplos [2010] puts it, an important lesson from the current crisis is "that the macro economy is strongly influenced by financial variables beyond prices" (p.6) – leverage in his case. Or in Shin's [2007] words: "...the common thread [of the literature on liquidity, JF] is the relationship between funding conditions and the resulting market prices of assets" (p. 317). In a similar vein, Brunnermeier and Sannikov [forthcoming] put financial frictions and the risk created by endogenous amplification of such frictions in the center of their macroeconomic model with a financial sector. See also Borio [2012] on the need to integrate monetary and financing constraints in macroeconomic models – on top of financial intermediation in the allocation of resources between savers and investors – and to consider medium-term financial cycles in addition to business cycles.

which is complementary to the finance and monetary macroeconomics literature. The goal of the presented analysis is to contribute to the "broad-exploration mode" (Caballero [2010]) of our discipline's thinking and to point out fundamental causes of distortions beyond liquidity issues or financial cycles.

The paper is organized as follows: In the next section, uncertainty structure, technology and boundaries of knowledge are discussed and formally outlined. Section 3 deals with the role of banks and firms. The behavior of households, in particular their savings decisions, are analyzed in section 4. In section 5, distortions and crises are considered by comparing planned and realized consumption levels to each other. Section 6 characterizes the size of the financial sector relative to GDP. Moreover, it addresses possible measures for the quality of the financial sector and discusses some aspects of higher-order functions of the financial sector. Section 7 applies the analysis to a model in which markets can in principle be completed by financial innovations but the probability assessment is more or less accurate. In this way, Knightian uncertainty can be discussed as a variant of the model. The modification allows to characterize the meaning of unfounded or careless financial innovations in a rigorous way. Moreover, as an illustration, a rule for the appropriate variety of financial products in relation to the size of the GDP is derived. Section 8 concludes.

2 Uncertainty structure, technology and boundaries of knowledge

There are two periods of time: Presence, $t = 0$, and future, $t = 1$. The future is uncertain. Let Ω be the future event space. Each $\omega \in \Omega$ describes an environment which is possibly realized in $t = 1$. Some of the future events are insurable in principle - that is, they consist of measurable risk, other events are not. Formally, space Ω is partitioned into a subspace S^* and its complement $\bar{S}^*$. S^* can be measured by a probability measure π^*_ω, $\sum_{\omega \in S^*} \pi^*_\omega = 1$. No such measure exists on $\bar{S}^*$. This reflects the fact that uncertainty is different from risk. The weights of S^* and $\bar{S}^*$ in Ω are μ^* and $1-\mu^*$, respectively, but there is no further knowledge about the structure of $\bar{S}^*$.[12] In other words, $\bar{S}^*$ and $\omega \in S^*$ are the "distinguishable" environments – the states – for which specialized technologies and state-contingent financial products can be designed in principle.

All agents share this logical structure and organize perception and reasoning according to this structure. In general, however, they have imperfect knowledge about S^*, $\pi^* \equiv (\pi^*_\omega)_{\omega \in S^*}$ and μ^*. They base their behavior on beliefs S, π and μ (about the respective objects with star). Since both π and π^* are probability measures on S, $\sum_{\omega \in S} \pi_\omega = \sum_{\omega \in S^*} \pi^*_\omega = 1$. For $S \subset S^*$, we can represent the deviation of beliefs from true probabilities by a process ϵ_ω:

$$\pi_\omega = \pi^*_\omega(1 + \epsilon_\omega) \tag{1}$$

where $\sum_{\omega \in S} \pi^*_\omega \epsilon_\omega = 1 - \pi^*_S$, $\pi^*_S \equiv \sum_{\omega \in S^*} \pi^*_\omega$. In principle, the beliefs may be heterogeneous across agents. An important thing to notice, however, is that an individual agent in the real sector, in particular a household, cannot influence financial market conditions – neither the set of financial products nor their pay-off characteristics

[12] To avoid any misunderstanding, π and μ are different measures. While set S^* is measurable (with probability measure π^*), $\bar{S}^*$ and thus Ω are not. μ^* is not a measure on Ω but on the set $\{S^*, \bar{S}^*\}$. Note also that unmeasurability of $\bar{S}^*$ does not imply that agents are unaware of $\bar{S}^*$ in the sense of Modica and Rusticchini [1999].

or prices. The trading possibilities available to a real agent are determined by the information and the beliefs collected and aggregated by the financial sector, which may reflect household beliefs more or less correctly. In any case, in the presented framework, individuals are able to infer the probability beliefs π aggregated by the financial sector from the trading possibilities provided by the financial market. How the inference works will be shown in the next section of the paper.

The output of the economy consists of one final good which can be used for consumption and investment. The price of this good is set equal to one. Production uses capital as input. I assume that technologies are linear in capital. Moreover, to focus on the role of the financial sector in helping to transform income of today into income tomorrow, it is assumed that capital fully depreciates after production so that only the capital invested in $t = 0$ matters as input for production in $t = 1$. There are two types of technologies for producing final output: a robust technology and risky technologies. All technologies produce the same final good. The robust technology works in any environment. Formally, each unit of physical capital invested into the robust technology generates, for any realization $\omega \in \Omega$, the output $a^* \geq 1$. In contrast, risky technologies are highly productive, but sensitive to the realized environment. More specifically, each technology works only in a particular environment. In $\bar{S}^*$ none of them is productive. Formally, for each $s \in S^*$, there exists a risky technology s with stochastic productivity

$$\tilde{A}_s^*(\omega) = \begin{cases} A_\omega^* & \text{if } s = \omega \text{ and } \omega \in S^*, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Thus, each unit of capital invested into technology s generates A_ω^* units of output, if $\omega = s \in S^*$ is realized and zero otherwise. Possible interpretations of this productivity generating process are, for instance, specialized seeds or research activities producing know-how for clearly specified conditions. No such specialization is possible for the indistinguishable environments in $\bar{S}^*$. There is a trade-off between specialization and robustness. On the one side, a technology is more productive if it is targeted to a more specific environment – provided that the relevant environment is realized. On the other hand, specific environments are less likely. The trade-off is modeled in the following way: There is a stock of knowledge A^* from which specialized technologies can be generated; the productivity of a technology specialized to an environment is inversely related to the probability of the environment. The following condition captures this property of the productivity generating process formally: For all $\omega \in S$,

$$\pi^*_\omega A^*_\omega = A^*, \quad A^* > a^*. \tag{3}$$

Conditional on S, the expected productivity level of all environment-specific technologies is equal to A^*, the economy's stock of knowledge for highly productive risky projects.[13] Moreover, unless $\bar{S}^*$ is realized, specialized technologies are more productive than the robust technology. It is worth noting that the unconditional expectation of an environment-specific productivity is $\mu^* A^*$ since $\omega \in S$ is realized with probability μ^*. Only share μ^* of A^* is exploitable for environment specific projects.[14]

In sum, if I_s units of capital are invested into the s-technology, the resulting output process is

$$\tilde{X}_s(\omega) = \begin{cases} \frac{A^*_\omega}{\pi^*_\omega} I_s & \text{if } s = \omega \text{ and } \omega \in S^*, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

If L units of capital are invested into the robust technology, the output is

$$X(\omega) = a^* L \tag{5}$$

whatever is ω.

Knowledge about the productivity of risky technologies may be imperfect, too.

[13]Alternatively, we could read (2) and (3) as follows: There exists risky technologies $s \in S$ which perform with productivity A_ω in a good state and do not deliver output in a bad state. The probability that for a particular technology a good state emerges is inversely related to its productivity level. The essential point is that the performance of specialized technologies in different environments is an object of uncertainty.

[14]This paper provides no comparative static analysis with respect to μ^*. One further property of specification (3) is worth noting. Rewriting (3) as $\pi^*_\omega = A^*/A^*_\omega$ and summing over $\omega \in S^*$, we obtain $A^* = [\sum_{\omega \in S^*} 1/A^*_\omega]^{-1}$ and $\pi^*_\omega = [A^*_\omega \sum_{\omega \in S^*} 1/A^*_\omega]^{-1}$. Hence, if A^*_ω is known for all $\omega \in S^*$, then one can infer A^* and π^*_ω.

Moreover, technological knowledge is distributed asymmetrically across the different types of agents. Households have no direct access to technological information. They can infer the quality of projects only through the more or less accurate lens of the pay-off structure of financial products. We will see in the next section how this can be done in the presented framework. This boundary of knowledge reflects the fact that in reality a typical household is neither an owner of the project into which her or his savings are invested nor has (s)he the instruments and the expertise for direct examination of real projects. After all, scale effects in information acquisition about investment projects are an important reason for why financial intermediation exists as a specialized role in the first place. In contrast to the households, financial agents like firms can acquire direct information. Nonetheless, the information may be imperfect so that again their actions are based on more or less accurate beliefs A_ω and A about productivity characteristics A^*_ω and A^*, respectively.[15] This limitation of knowledge is of a less fundamental nature than the one about uncertain future events. The physical performance of a technology under specified circumstances can be learned by direct inspection and experiments without market transactions. Hence, financial agents and firms can substantiate their beliefs about the performance of projects by direct inspection.

Besides production of the final good there are services from the financial sector. The services are produced in $t = 0$ by a linear technology with the final output of the production sector as input. As described later, the services are financed by fees proportional to the pay-offs of financial products. Let T be the aggregate volume of resources absorbed by the financial sector. The resources can be employed in two types of activities: inspection of fundamentals, T_I, and arbitrage, T_A. T_I determines the amount of information collected from the real sector, in particular about the performance of technologies in specific circumstances – for instance, by examining technical reports or business plans. T_A is used for aggregating the collected

[15] In principle, also beliefs about the robust technology can be wrong. It would mean that there is no robust possibility of preparing for $t = 1$, even not by accumulating physical inventories, i.e. if $a^* = 1$. Such drasitc situations are beyond the scope of this paper. Therefore, I assume that a^* is publicly known.

information and coordinating beliefs within the financial sector. For a given set S of differentiated environments, the information-processing quality of the financial sector depends on both the sector's total resources T and their allocation on the different types of activities. This paper does not provide any microfoundation of how financial agents collect and aggregate information. Instead, I assume that analogous to real production functions there exists an optimal allocation $\xi^* \equiv T_A/T_I$ of total resources T on inspection and arbitrage activities.[16] Therefore, the assessment of risks and project performance by the financial sector can be inaccurate for two reasons. First, accurate assessment may be unfeasible even under optimal allocation of T. For instance, if the measure of differentiated environments is large, the cost of acquiring and aggregating information may be high.[17] Second, the allocation of T deviates from the optimal combination ξ^* of inspection and arbitrage. In general, the further analysis does not separate the two sources explicitly. Section 7, however, focuses on the first source, assuming that the information communicated by financial markets becomes less reliable if the differentiation of environments by financial innovations is out of line with experience.

[16]For example, if the relationship between quality of the financial sector, Ξ, and allocation T_I, T_A of $T = T_I + T_A$ can be described by a homogenous function. then

$$\Xi = \Phi T f(\xi), \quad \xi = T_A/T_I$$

where f is maximal at some optimal combination ξ^*. Φ is a productivity parameter. For instance, Φ may capture the following consideration. For any T_I, T_A, the information conveyed by the pay-off promises and prices of the financial products provided by the financial sector will be less reliable in an economy with a highly differentiated state space (S) and little real experience in this space compared to an economy with less differentiated and more familiar states.

[17]One could then relate the error process ϵ_ω of deviations of A_ω from A^*_ω to these costs by arguing along the lines of Grossmann and Stiglitz [1980] that the informational efficiency of financial markets declines when the cost of information rises.

3 Banks and firms

Financial agents ("banks") offer to households and firms intermediation services for two types of financial products: a bank account, in which households can deposit income and firms can borrow at a fixed interest rate; and, for $\omega \in S$, an Arrow-security. In reality we have of course more complex financial products. From the abstract point of view of the presented model, however, the essential property of products different from deposits and loans is their capacity to provide state-contingent savings and financing opportunities. This property can be captured most simply by Arrow-securities. As a consequence, financial innovation – that is, the provision of new financial products – boils down to extending the set S of states for which a security is available. S may be smaller or larger than S^*. By designing state-contingent financial products for $\omega \in S$ the financial sector informs the market about what is considered as insurable risk. The financial agents' belief that $\omega \in S$ is an insurable risk must be shared by households and firms. Otherwise a security designed for state ω would find no market.

Firms use the financial products for financing their capital investments. Households use the products for saving. The conditions for households and firms differ due to the transaction costs (τ) of banks for the provision of the products, in particular the acquisition and aggregation of information conveyed by the characteristics of the products and their prices. A firm borrowing in $t = 0$ one unit on credit has to pay r units in $t = 1$. In contrast, a household depositing one unit of income in $t = 0$ earns

$$r_h = r(1 - \tau^-)$$

units in $t = 1$, where $0 \leq \tau^- < 1$ and $r_h > 0$. As argued in more detail below, this form of financing is used for robust technologies. Thus, in equilibrium

$$r = a^*. \tag{6}$$

The prices of Arrow-securities are denoted by $(q_\omega)_{\omega \in S}$. The gross pay-offs they promise are $(R_\omega)_{\omega \in S}$. That means, if a firm sells one unit of an ω-security, it gets

q_ω units of capital in $t = 0$ and has to pay back, in $t = 1$, R_ω units of output if ω is realized. Clearly, this product can only be used for financing a technology which is productive in ω. The corresponding pay-offs for the saving household are

$$R^h_\omega = R_\omega(1 - \tau^+), \quad \omega \in S$$

with $0 \leq \tau^+ < 1$. It seems reasonable to assume that transaction costs for risky assets are higher than those for bank accounts, that is: $\tau^+ > \tau^-$.

As outlined in the previous section, financial agents collect and aggregate information by direct inspection of risky projects in different environments and by arbitrage trading. As a result, coordinated beliefs $\pi = (\pi_\omega)_{\omega \in S}$ and $(A_\omega)_{\omega \in S}$ about the probability distribution of risks and the productivities of risky technologies, respectively, emerge within the financial sector. Since agents perceive the world in the framework of the presented model, the productivity beliefs are consistent with condition (3) so that $A_\omega = A/\pi_\omega$ where A is the belief about the conditional average productivity of risky projects. Investments in a risky technology $\tilde{A}_s(\omega)$ only perform if $\omega = s$ is realized in the future. Thus, such investments can only be financed by state-contingent securities. Since R_ω/q_ω has to be paid back per unit of invested capital, technology $\tilde{A}_s(\omega)$ will get financing if and only if financial market beliefs satisfy $A_\omega = R_\omega/q_\omega$. In sum, a financial market equilibrium satisfies the following no-arbitrage condition.

Property 1. *(No-arbitrage condition). For all $\omega \in S$,*

$$\frac{\pi_\omega R_\omega}{q_\omega} = A, \quad A_\omega = \frac{A}{\pi_\omega}. \tag{7}$$

Firms can invest into an s-contingent technology with productivity $\tilde{A}_s$ or in a robust technology with productivity a^*. All investment is assumed to be externally financed and free entry establishes equality between costs and returns. There is no asymmetry between banks and firms with respect to the information which type of technology is financed with a given instrument. Since a^* is public knowledge by assumption, the financing of robust projects by credits with fixed interest rate is straight-forward. Condition (6) guarantees zero profits for firms running the robust technology. For risky technologies the situation is less obvious. Let S^f be the firms' belief about S^*. For each $s \in S^f$ there is a mass of potential firms with business plans for highly productive projects contingent on $\omega = s$. As outlined before, firms have access to direct inspection of the performance of their project in a specified environment. Nonetheless, they may be careless and hold inaccurate beliefs A^f_ω in a range $[A^{min}_\omega, A^{max}_\omega]$ around true productivity A^*_ω. A^f_ω refers to firm beliefs which in principle could deviate from bank beliefs A_ω. In equilibrium, however, bank and firm beliefs must coincide as the following argument shows. Facing S, $(q_\omega)_{\omega \in S}$, $(R_\omega)_{\omega \in S}$, the firms' demand for investments into s-technologies is fully elastic if $s \in S^f \cap S$ and beliefs A^f_ω about A^*_ω are consistent with condition (7). Under this condition the equilibrium level and structure of investments is determined by the savings supplied by the households (see next section). If the condition is violated, investment demand is either zero or infinite, which cannot be an equilibrium if household savings are positive. Hence, the beliefs implied by the financial products and firm beliefs about the productivity of risky projects coincide in equilibrium. Either financial agents and firms assess the productivity by joint examination or one side is relying on the assessment of the other side. Superscript f in S^f and A^f_ω is therefore dropped in the further analysis. In sum, financial agents and firms are jointly responsible for wrong beliefs about S^* or $(A^*_\omega)_{\omega \in S^*}$. Finally, arbitrage in choosing the way of financing a technology is excluded if the inequality

$$A > a^* \tag{8}$$

holds. First, a firm investing into an s-technology does not get a fixed interest rate loan, because $r = a^*$ could only be paid back in state $\omega = s$. Second, financing a robust technology with ω-securities would mean that the firm has to pay back $\frac{R_\omega}{q_\omega} = \frac{A}{\pi_\omega} > A$, rather than a^*, if $\omega \in S$ is realized; the actual return though is $a^* < A$.

Equilibrium condition (6) and (7) show the terms of trades at which households can use financial products for their saving needs. Thus, their choices depend on the information-processing "quality" of the financial sector, that is on how accurate $(\pi_\omega)_{\omega \in S}$ and A assess true risk $(\pi^*_\omega)_{\omega \in S^*}$ and productivity A^*. The beliefs about productivities and risks, conveyed by the financial market, can be inferred from the returns R_ω/q_ω as follows: Rewriting (7) in the form $\pi_\omega = (q_\omega/R_\omega)A$ and summing over $\omega \in S$, we have

$$\frac{1}{A} = \sum_{\omega\in S} \frac{q_\omega}{R_\omega} \equiv Q \text{ and } \pi_\omega = \frac{q_\omega/R_\omega}{Q}, \tag{9}$$

which gives for the households A, $(\pi_\omega)_{\omega\in S}$ and $(A_\omega)_{\omega\in S}$ as functions of the prices and pay-offs observed in the market. ($Q = \sum_{\omega\in S} \frac{q_\omega}{R_\omega}$ is the present value of one unit of income available in each $\omega \in S$ of period $t = 1$, if the income is raised by purchasing Arrow securities in $t = 0$). Another important thing to notice is that financial markets convey no information on the true measure of the size of insurable environments, μ^*, compared to the size of uninsurable uncertainty, $1-\mu^*$. Problems arise if the productivity assessment by financial agents and firms deviate from true productivities. $A_\omega = A/\pi_\omega$ may deviate from $A^*_\omega = A^*/\pi^*_\omega$ for two reasons, namely: $\pi_\omega \neq \pi^*_\omega$ or $A \neq A^*$.[18] In any case, the wrong beliefs lead to false prices and pay-off promises in the financial market. They induce households to choose a distorted portfolio – compared to the optimal structure under accurate beliefs. As a result, the allocation of saved resources is distorted, and pay-off promises are deceived. The consequences of these distortions will be examined in detail in Section 5.

[18]As stated already, specification (3) implies that $(\pi_\omega)_{\omega\in S}$ and A_ω can be inferred if $(A_\omega)_{\omega\in S}$ is known. Likewise $(A_\omega)_{\omega\in S}$ can be inferred from $(\pi_\omega)_{\omega\in S}$ and A. Incorrect beliefs about $(\pi^*_\omega)_{\omega\in S}$, $(A^*_\omega)_{\omega\in S}$ and A^* arise if the following two conditions hold: i) A^*_ω is improperly assessed for some ω; ii) π^*_ω is improperly assessed, for some ω, or A^*.

4 Households and savings

In $t = 0$ the economy is endowed with a resource Y_0 which is distributed over a mass N of households. Let y_0^i be the endowment of household i. Part of this endowment, c_0^i, is consumed, the other part, denoted by b_i, is saved. The saving is done partly by purchasing a portfolio of Arrow securities, $(z_\omega^i)_{\omega \in S}$, and partly by depositing income in a bank account, l_i.The income resulting in $t = 1$ from the savings is fully spent on consumption c_ω^i. For any $\omega \in S$, the effective sum available for spending may be disturbed by an idiosyncratic process e_ω^i with $\sum_{\omega \in S^*} \pi_\omega^* e_\omega^i = 0$. In sum, conditional on the financial instruments offered by the financial sector, household i faces the following budget constraints:

$$\begin{aligned} c_0^i &= y_0^i - b_i, \\ b^i &= \sum_{\omega \in S} q_\omega z_\omega^i + l_i, \\ c_\omega^i &= \begin{cases} R_\omega^h z_\omega^i + r_h l_i + e_\omega^i, \text{ if } \omega \in S, \\ r_h l_i \text{ otherwise.} \end{cases} \end{aligned} \tag{10}$$

Households have logarithmic intertemporal preferences with a discount δ on the future. The perception of and the reasoning about the future follows the structure described in Section 2. Let $\pi_\omega^i = (\pi_\omega^i)_{\omega \in S}$ and $0 < \mu_i < 1$ be the beliefs of household i about π^* and the measure of insured states, respectively.[19] If i has correct beliefs, $\mu_i = \mu^* \sum_{\omega \in S \cap S^*} \pi_\omega^* \leq \mu^*$. If i assesses the measure μ^* correctly and otherwise follows the beliefs conveyed by the financial market, then $\mu_i = \mu^*$, as $\sum_{\omega \in S} \pi_\omega = 1$. In general, μ_i can adopt any value between zero and one. For a mixed portfolio with both savings in bank deposits and savings in risky assets we have to impose

[19] In principle, the beliefs of a household about the set of insurable states may differ from the beliefs of banks and firms. In this case we have for the single household $z_\omega^i = 0$ if $\omega \in S - S_i$, where S_i denotes the household's belief about the set of insured risks. But, since securities are in the market only for states which financial agents, firms and a positive mass of households consider as insurable, it is assumed in the following that S_i coincides with the set S for which securities are provided in the financial market.

the condition

$$\frac{r_h}{R_h} < \mu_i < 1 \tag{11}$$

where $R_h \equiv \frac{1-\tau^+}{Q}$.

The household's savings and portfolio choice is given by the program

$$\underset{b_i,\ (z^i_\omega)_{\omega\in S},\ l_i}{Max} \quad EU = \log(c^i_0) + \delta\Big[\mu_i \sum_{\omega\in S} \pi^i_\omega \log(c^i_\omega) + (1-\mu_i)\log(r_h l_i)\Big]$$

subject to (10). As shown in the Appendix, this program leads to the following savings decisions and consumption plans.

Proposition 1. *For a household with beliefs* $(\pi^i_\omega)_{\omega\in S}$, μ_i,

a) the optimal savings portfolio is given by:

$$\begin{aligned} b_i &= \frac{\delta y^i_0 - \tilde{E}e^i}{1+\delta} \\ l_i &= (1-\mu_i)\frac{b_i + \tilde{E}e^i}{1 - r_h/R_h} \\ \sum_{\omega\in S} q_\omega z^i_\omega &= b_i - l_i \end{aligned} \tag{12}$$

with $R_h \equiv \frac{1-\tau^+}{Q} = (1-\tau^+)A$ *and* $\tilde{E}e^i \equiv \frac{1}{1-\tau^+}\sum_{\omega\in S}\frac{q_\omega}{R^h_\omega}e^i_\omega$.

b) The planned individual consumption levels are:

$$c^i_\omega = \mu_i \frac{\pi^i_\omega}{\pi_\omega} R_h\Big(b_i + \tilde{E}e^i\Big),\ \textit{if}\ \omega \in S \tag{13a}$$

$$c^i_\omega = r_h l_i\ \textit{otherwise.} \tag{13b}$$

Thus, financial markets promise to fully insure any potential individual income risk e^i_ω, $\omega \in S$. This is a consequence of the fact that a complete set of securities is offered for S. In contrast, the financial market provides no such insurance for the variance of consumption between S and $\bar{S}$. More important, a consumer fully bears the risk resulting from deviations of his or her beliefs π^i_ω from π_ω, the beliefs conveyed by the financial product characteristics and their prices.[20] As equation

[20]This is consistent with the no-arbitrage condition (7), because by assumption households do not participate in pure arbitrage trade.

(13a) shows us, c^i_ω is fully insured on S if and only if $\pi^i_\omega = \pi_\omega$. According to (9), the consumer can infer π_ω from $(q_\omega)_{\omega\in S}$ and $(R_\omega)_{\omega\in S}$ and eliminate this risk by adjusting π^i_ω to π_ω. If consumer i is insecure about π^i_ω he or she will do so. If however, i is convinced that π^i_ω is closer to the true value than π_ω, then the returns $R^h_\omega/q_\omega = R_h/\pi_\omega$ are considered as distorted and he or she plans to consume less than the financial markets suggest, if $\pi^i_\omega < \pi_\omega$, and more, if $\pi^i_\omega > \pi_\omega$. Clearly, the households also pay the costs of unreliable risk and productivity assessment by financial agents. If $A \neq A^*$ or $\pi_\omega \neq \pi^*_\omega$, actual pay-off, A^*_ω, will deviate from pay-offs R^h_ω and consumer plans will be deceived. Finally, it is worth noticing that e^i_ω enters the savings decisions by its financial market valuations, $\tilde{E}e^i$. Thus, in general, not only the structure of the portfolio $(l_i, b_i - l_i)$ but also the level of savings, b_i, depend on the financial sector's beliefs about the future. For instance, if $S = S^*$ we have $\tilde{E}e^i = \frac{Cov^*(\epsilon,e^i)}{R_h}$, where $Cov^*(\epsilon,e^i) \equiv \sum_{\omega\in S^*} \pi^*_\omega \epsilon_\omega e^i_\omega$ is the covariance of belief distortion and exogenous income shocks.[21] If markets believe that positive income shocks dominate, then the present value of exogenous future income is positive. As a consequence, the savings level declines and the portfolio structure shifts in favor of deposits, ceteris paribus. This effect is moderated, if due to euphoric beliefs about the productivity of risky technologies the promised pay-offs of risky assets are high; and enhanced if the beliefs are pessimistic. For a household with $Cov^*(\epsilon,e^i) < 0$, we have the opposite effects on the savings behavior. In the further analysis the channel $\tilde{E}e^i$ will vanish due to the aggregation assumption I impose for simplicity reasons. But it is worth keeping in mind that in general financial market valuations also affect accumulation through the value assigned to exogenous future opportunities, which are not generated by accumulation.

Before we turn to aggregation, I want to summarize the conditions for an equilibrium.

Definition 1. *(Equilibrium). An economy with a financial sector – characterized by* S, $(R_\omega)_{\omega\in S}$, $(q_\omega)_{\omega\in S}$, r *and* τ^-, τ^+ *– and a real sector – with firm beliefs* S^f, $(A^f_\omega)_{\omega\in S^f}$

[21] According to (9), $\tilde{E}e^i = \frac{1}{R_h}\sum_{\omega\in S} \pi_\omega e^i_\omega$, where $R_h = \frac{1-\tau^+}{Q} = (1-\tau^+)A$ has been used. Now, $\sum_{\omega\in S} \pi_\omega e^i_\omega = \sum_{\omega\in S} \pi^*_\omega e^i_\omega + \sum_{\omega\in S} \pi^*_\omega \epsilon_\omega e^i_\omega$, where, for $S = S^*$, $\sum_{\omega\in S} \pi^*_\omega e^i_\omega = 0$ by assumption.

and household beliefs π^i and μ^i – is in equilibrium in $t = 0$, if the following conditions hold: a) $r = a^*$, where a^* is the known productivity of the robust technology. b) S coincides with S^f and $(R_\omega)_{\omega \in S}$, $(q_\omega)_{\omega \in S}$ satisfy no-arbitrage conditions (7) *and* (8) *for beliefs $(\pi_\omega)_{\omega \in S}$ and A so that $(A/\pi_\omega)_{\omega \in S}$ coincides with $(A^f_\omega)_{\omega \in S}$. c) For all households i, savings decisions b^i, l^i, $(z_\omega)_{\omega \in S}$ and consumption plans $(c^i_\omega)_{\omega \in S}$ satisfy optimality conditions* (12) *and* (13).

Since under conditions *a)* and *b)* investment demand by firms is fully elastic, equalization of aggregate savings and investment is automatically guaranteed by *a)-c)*. In $t = 1$, no actions can be taken and households are exposed to the consequences of the equilibrium realized in $t = 0$. The future output of the economy is determined by the aggregate investment levels.

Provided that $b_i > 0$ and $l_i > 0$ for all i, the equations in (12) and (13) can be easily aggregated if the following assumption is made.

Assumption 1. *(a) There exists a household h so that for almost all i: $\mu_i = \mu_h$ and $\pi^i_\omega = \pi^h_\omega$. (b) For all $\omega \in S$, $\int_N e^i_\omega di = 0$.*

Under this assumption we obtain from (12) for total savings, B, savings in bank accounts, L, and savings in state-contingent securities:[22]

$$\begin{aligned} B &= \frac{\delta}{1-\delta} Y_0, \\ L &= \frac{1-\mu_h}{1-\rho_h} B, \\ B - L &= \frac{\mu_h - \rho_h}{1-\rho_h} B, \end{aligned} \tag{14}$$

where $\rho_h \equiv \frac{r_h}{R_h}$, which can also be expressed in the form

$$\rho_h = rQ/\tau, \, \tau \equiv \frac{1-\tau^+}{1-\tau^-}. \tag{15}$$

For owning one unit of income in each state $\omega \in S$, one would need to buy $z_\omega = \frac{1}{R_\omega}$ units of each security. This requires $Q = \sum_{\omega \in S} \frac{q_\omega}{R_\omega}$ units of cash in $t = 0$. Putting

[22]Note that $\int_N \tilde{E} e^i di = \tilde{E} \int_N e^i_\omega di = 0$ as $\int_N e^i_\omega di = 0$ by assumption.

this into the bank account would give rQ in each $\omega \in \Omega$. Thus, rQ describes the terms of trade between having one unit of income if the future lies in S or one unit of income for sure. Representation $\rho_h = rQ/\tau$ gives us the look at the world when we see it through the lens of the valuation by the financial markets. We can represent ρ_h also in terms of the productivity beliefs on which banks and firms base their decisions. By using (6) and (9), we obtain

$$\rho_h = \frac{a^*}{\tau A}. \tag{16}$$

This represents the look at the economy from the real perspective.

The third equation in (14) shows aggregate savings in form of securities. The allocation of $B - L$ on the single securities determines the amount available to firms as capital investment into a particular technology. Let I_ω denote the amount invested into a technology that works if $\omega \in S$ is realized. It is given by $I_\omega = \int_N q_\omega z^i_\omega di$. Solving the household's budget constraint, given by (10), for z^i_ω and aggregating, we get

$$Z_\omega \equiv \int_N z^i_\omega di = \frac{1}{R^h_\omega}(C_\omega - r_h L), \tag{17}$$

where, according to (13a) and Assumption 1,

$$C_\omega \equiv \int_N c^i_\omega di = \frac{\mu_h \pi^h_\omega}{\pi_\omega} R_h B. \tag{18}$$

Substituting (18) for C_ω and (14) for L into (17), and multiplying by q_ω, we have

$$I_\omega = \pi_\omega \frac{\frac{\pi^h_\omega}{\pi_\omega}\mu_h - \rho_h + \mu_h \rho_h (1 - \frac{\pi^h_\omega}{\pi_\omega})}{1 - \rho_h} B \tag{19}$$

where $\frac{q_\omega}{R^h_\omega} = \pi_\omega \frac{1}{(1-\tau^+)A} = \frac{\pi_\omega}{R_h}$ has been used from (9). Thus, I_ω is increasing in $\pi^h_\omega / \pi_\omega$. This shows, although households play a passive role in the financial markets, their beliefs can in principle have an influence on the structure of realized projects. A higher share of savings goes to technologies that perform well in states which households belief to be more likely than banks and firms do. Nonetheless, this only happens if almost all households unanimously share the same belief about the probabilities of risky events. More realistically, if there is no collective coordination by market independent institutions, the average household will follow the beliefs which can be inferred from the financial markets, which according to (9) are given by $\pi_\omega = \frac{q_\omega/R_\omega}{Q}$.

Assumption 2. *For all* $\omega \in S$, $\pi^h_\omega = \pi_\omega$.

With this assumption, (19) reduces to

$$I_\omega = \pi_\omega(B - L) \quad \text{for } \omega \in S. \tag{20}$$

In an economy with no uninsurable uncertainty, aggregate accumulation in the presented framework coincides with the model of Acemoglu and Zilibotti (1997) if agents have correct beliefs and transactions costs for financial intermediation are absent.[23] In their framework a technology delivering A exists for all $\omega \in \Omega$ and the measure of states insured by state-contingent technologies is only limited by fixed costs of innovations. An increase of μ with the size of the market is therefore always beneficial. In contrast, in the framework presented here not all future states are insurable and even under correctly priced securities financial innovations are only productive as long as S is a strict subset of S^*.

[23]Specify $S^* = \Omega = [0,1]$ with true risk measured by the uniform distribution. Moreover, set $S = [0,\mu]$, $\pi_\omega = 1/\mu$, $\mu_h = \mu$, $\tau = 1$ and define $R \equiv \mu A$. Then, according to (6) and (16), $\rho_h = \frac{r}{R}\mu$ so that (14) and (20) reduce to $L = \frac{R(1-\mu)}{R-r\mu}$, $B - L = \frac{\mu(R-r)}{R-r\mu}$, $I_\omega = \frac{R-r}{R-r\mu}$

5 Financial beliefs meet reality: Planned vs. realized consumption

In $t = 0$, household savings, B, is invested in risky technologies, I_ω, $\omega \in S$, and robust technologies, L, respectively. This allocation is based on the valuation of future events, as reflected in the prices and pay-off promises of assets offered in the financial market and on the households' belief μ_h. The instruments provided by the financial market as well as the information these instruments convey are summarized in the following conditions:

$$\begin{aligned} r_h &= \bar{a}^*, \\ R_h &= \frac{1-\tau^+}{Q} = \bar{A}, \\ \pi_\omega &= \frac{q_\omega}{R_\omega Q}, \quad \omega \in S \end{aligned} \tag{21}$$

where $\bar{a}^* = (1-\tau^-)a^*$, $\bar{A} = (1-\tau^+)A$ and $Q = \sum_{\omega \in S} \frac{q_\omega}{R_\omega} = 1/A$.

Thus, there are essentially two potential sources of misguided savings and investment decisions: wrong household beliefs with regard to the measure of insurable future events ($\mu_h \neq \mu^*$); and a distorted return structure R_ω/q_ω, conveying wrong productivity and risk expectations ($\pi_\omega \neq \pi^*_\omega$, $\bar{A} \neq \bar{A}^*$ and $\bar{A}_\omega = \bar{A}/\pi_\omega \neq \bar{A}^*/\pi^*_\omega$). As a consequence of misguided savings and investment decisions, the households' consumption plans may not be sustained by reality. This section compares, at the aggregate level, for the different sources of distortions the optimal plans with the actually realized outcome.

5.1 Planned consumption levels

According to (13b), (14) and (18), under Assumption 1 and 2, the consumption levels planned for $t = 1$ aggregate to

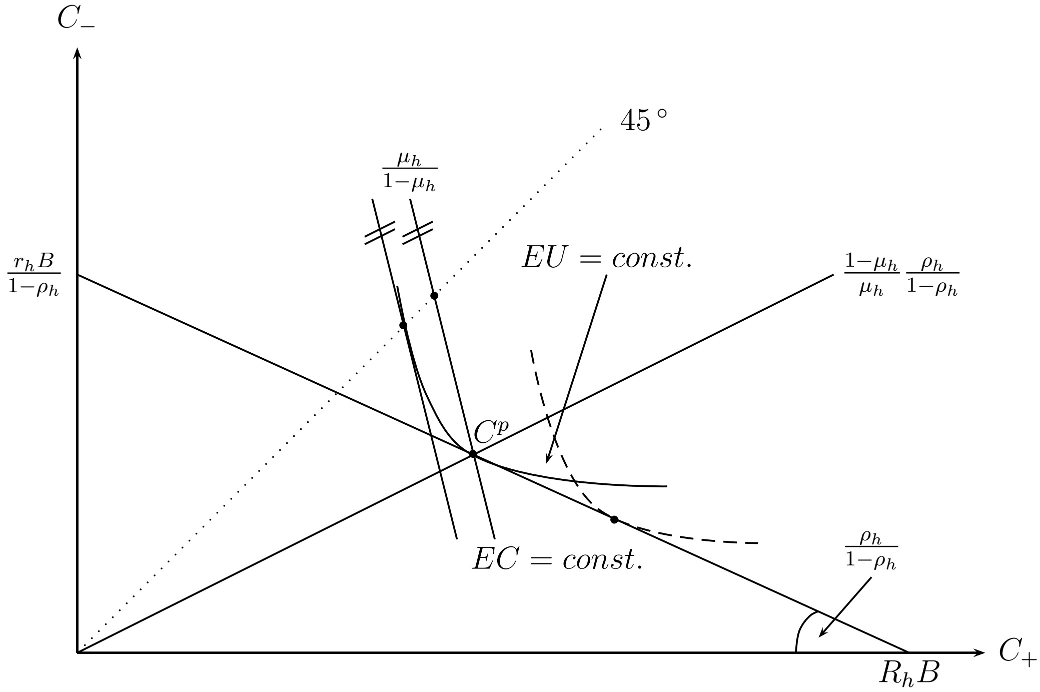

Figure 1: Consumption plan under μ_h, ρ_h

$$C^p_\omega = \begin{cases} \mu_h R_h B \equiv C^p_+, \text{ if } \omega \in S, \\ r_h L = \frac{1-\mu_h}{1-\rho_h} r_h B \equiv C^p_-, \text{ if } \omega \in \bar{S}. \end{cases} \tag{22}$$

This gives us for the ratio of consumption planned for the future with unmeasurable uncertainty, $\bar{S}$, relative to the future with measurable risk, S:

$$\frac{C^p_-}{C^p_+} = \frac{1-\mu_h}{\mu_h}\frac{\rho_h}{1-\rho_h}. \tag{23}$$

For a graphical illustration it is useful to express also the households' intertemporal budget constraint in terms of C_+, C_-. Using (17), we can rewrite this constraint in the form:[24]

$$\rho_h C^p_+ + (1-\rho_h) C^p_- = r_h B. \tag{24}$$

Figure 1 shows expansion path (23) and budget constraint (24) in the (C_+, C_-)-space. Intersection point C^p represents the consumption plan resulting under μ_h and ρ_h. In addition, the the figure shows the locus of (C_+, C_-)-combinations with $EC^p = \mu_h C^p_+ + (1-\mu_h) C^p_-$. Finally, we define an aggregate utility index $EU(C_+, C_-) \equiv$

[24]Note first: $B = \sum_{\omega \in S} q_\omega Z_\omega + L = \sum_{\omega \in S} \frac{q_\omega}{R^h_\omega} R^h_\omega Z_\omega + L$. Applying (17) we get from this: $B = \sum_{\omega \in S} \frac{q_\omega}{R^h_\omega}(C^p_\omega - C^p_-) + L = \frac{Q}{1-\tau^+}(C^p_+ - C^p_-) + C^p_-/r_h$, which finally gives us (23). Note that $\rho_h = rQ/\tau = r_h/R_h$.

$\mu_h \log C_+^p + (1-\mu_h) \log C_-^p$ and calculate

$$\frac{dC_-^p}{dC_+^p}\Big|_{EU=const.} = -\frac{\mu_h}{1-\mu_h}\frac{C_-^p}{C_+^p}. \tag{25}$$

On the certainty line ($C_- = C_+$) the slope of the iso-EU curve coincides with the slope of the iso-EC curve. To the right of the 45°-line, the slope diminishes – becoming tangential to the budget constraint at C^p. Therefore, aggregated consumption plans can be represented as optimum of a representative household which maximizes $EU(C_+, C_-)$ subject to (24).

An immediate implication of (25) is that the representative indifference curve through a given point C^p rotates clockwise if μ_h increases. *Ceteris paribus*, this shifts the optimal plan south-east; the expansion path rotates clockwise, too.

5.2 Realized consumption levels

In $t = 1$, households get the output of the production capacities created by their savings invested in $t = 0$. But, part of their savings is absorbed by the transaction costs of financial intermediaries. Total transaction costs for the savings in bank accounts are given by $(r - r_h)L = r\tau^- L$, the value of which is at $t = 0$

$$T^- = \tau^- L.$$

Total transaction costs for the savings in security $\omega \in S$ amount to $(R_\omega - R_\omega^h)Z_\omega = \tau^+ R_\omega Z_\omega$ in $t = 1$. Discounting this by the return rate for this security, which is R_ω / q_ω, we get for the cost in $t = 0$ the equivalent $\tau^+ q_\omega Z_\omega$. Summing over $\omega \in S$, we have

$$T^+ = \tau^+(B - L).$$

Thus, the aggregate net savings of households are: $L^n = (1 - \tau^-)L$ and $I_\omega^n = (1 - \tau^+)I_\omega$, $\omega \in S$. They are invested by firms into technology (4) and (5) respectively. If $S^* \subset S$, then $X_s = 0$ for $s = \omega \in S - S^*$ and all savings in securities $\omega \in S - S^*$ is lost. In the following $S \subset S^*$ is assumed. Then the savings accumulated in period

$t = 0$ allow to produce in $t = 1$ the following state-contingent output levels:

$$Y_\omega = \begin{cases} \frac{\bar{A}^*}{\pi^*_\omega} I_\omega + \bar{a}^* L, \text{ if } & \omega \in S, \\ \bar{a}^* L, \text{ if } & \omega \in \bar{S}. \end{cases}$$

Using $C^r_\omega = Y_\omega$ and substituting (20) for I_ω we obtain the realized consumption levels:

$$C^r_\omega = \begin{cases} (\frac{\pi_\omega}{\pi^*_\omega}) \bar{A}^* (B - L) + \bar{a}^* L, & \text{if } \omega \in S, \\ \bar{a}^* L \equiv C^r_-, & \text{if } \omega \in \bar{S}. \end{cases} \tag{26}$$

Comparison with (22) shows us immediately that $C^r_- = C^p_- \equiv C_-$. For $\omega \in S$, however, the situation is different. By using $\pi_\omega = \pi^*_\omega(1+\epsilon_\omega)$, C^r_ω can be decomposed in the following way:[25]

$$\begin{aligned} C^r_\omega &= C^r_+ + \epsilon_s \bar{A}^* (B - L), \\ C^r_+ &\equiv C^p_+ + \Delta, \end{aligned} \tag{27}$$

where C^p_+ is the planned consumption level given by (22) and Δ is defined by

$$\Delta \equiv \Big(\frac{A^*}{A} - 1\Big) R_h (B - L). \tag{28}$$

We see that realized consumption can deviate from planned consumption for three reasons. First, the average consumption level in S deviates from the planned level if financial markets reflect wrong productivity expectations. Second, households may hold wrong beliefs about the measure of the set of insured states, compared to uninsured events. These two effects are summarized by Δ. We have

$$\begin{aligned} &\Delta < (= ,\ >)\ 0 \text{ if } A > (= ,\ <) A^* \\ &\text{and, if } \Delta \neq 0, \\ &\frac{\partial |\Delta|}{\partial \mu_h} > 0. \end{aligned} \tag{29}$$

Δ reflects the deception (surprise) if promised pay-offs are not matched by the real outcome. Deception or surprise is higher if more savings are invested in risky assets because of an optimistic view about the "relative size" of the insured future (μ_h).

[25] Note first that $C^r_+ = C^r_\omega - \epsilon_\omega \bar{A}^*(B - L) = C_- + \bar{A}^*(B - L)$, according to (26). Substituting (14) for L and $B - L$, we get from this $C^r_+ - C^p_+ = r_h B \frac{1-\mu_h + (\bar{A}^*/r_h)(\mu_h - \rho_h) - (\mu_h/\rho_h)(1-\rho_h)}{1-\rho_h}$, which reduces to $\Delta = r_h B \frac{1-\mu_h/\rho_h + (A^*/A)(\mu_h/\rho_h - 1)}{1-\rho_h} = \Big(\frac{A^*}{A} - 1\Big) \frac{\mu_h - \rho_h}{1-\rho_h} \frac{r_h B}{\rho_h}$ and gives us (28).

A third source of distortions is erroneous risk assessment, $\pi_\omega = \frac{q_\omega/R_\omega}{Q}$, implied by the return structure of risky assets. They induce volatility of future consumption within S, even though the events in S are meant to be fully insured by the available securities. This volatility is given by the (conditional) variance of C^r_ω in S, which according to (27) is

$$VAR(C^r_\omega) = \left[\bar{A}^*(B-L)\right]^2 VAR(\epsilon). \qquad (30)$$

In the worst case, if beliefs are very far from reality, C^r_ω approaches C_- as can be seen by setting $\epsilon_\omega = -1$. In this case, the "deception" $C^p_\omega - C^r_\omega = \frac{\mu_h}{\rho_h} r_h B - C_-$ is equal to $\frac{\mu_h - \rho_h}{1-\rho_h}\frac{r_h B}{\rho_h} = R_h(B-L)$. All savings in risky assets are lost. Section 6 and Section 7 will deepen the discussion of effects resulting from careless risk assessment.

Figure 2 illustrates "average deception" Δ between planned and realized consumption levels for $R_h > \bar{A}^*$, that is, for euphoric financial markets.[26] If the euphoric prospects, promised by the risky assets, would be moderated by cautious household beliefs about the extent of insurable future events ($\mu'_h < \mu_h$), then the planned consumption would move closer to the certainty line and average deception would diminish. In contrast to distorted financial prices and pay-offs, wrong household beliefs alone do not lead to deception or surprises. They remain undetected in our two-period framework. According to (28), $\Delta = 0$ if $A = A^*$ regardless of the value of μ_h. Nevertheless, wrong household beliefs affect the intertemporal allocation of resources. For an example, consider the case of overconfidence ($\mu_h > \mu^*$). Compared to the first-best solution C^*, households are willing to accept too high a sacrifice of consumption possibilities in uninsured future environments ($\bar{S}$) in order to obtain high consumption levels in insurable situations (S). Yet, if the productivity and risk valuation by financial markets is correct, no unhonored financial promises will reveal that households overestimate the extent of the insurable future and underestimate the extent of uninsurable future events.

[26]Note that, according to (26), for $\epsilon_\omega = 0$, the actual resource constraint of the economy is given by $C^r_+ = C_-(1 - \bar{A}^*/\bar{a}^*) + \bar{A}^* B$, which is equivalent to (24) for $\rho_h = \rho^*_h \equiv \bar{a}^*/\bar{A}^*$.

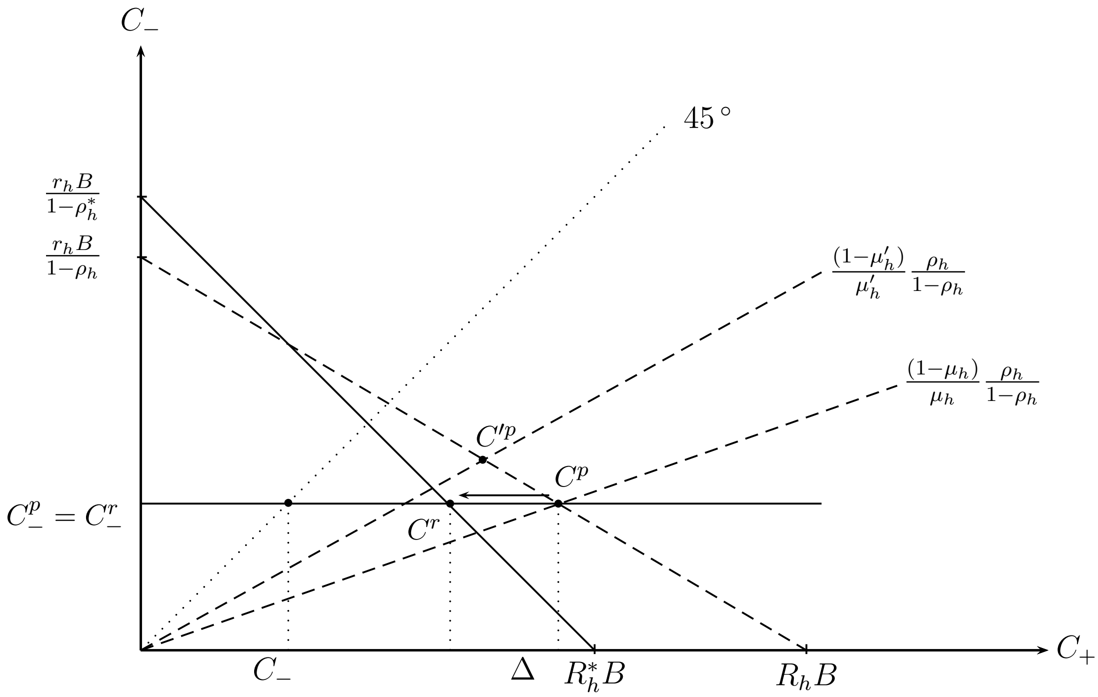

Figure 2: Deception of households under euphoric financial markets ($R_h > R_h^* = \bar{A}^*$, $\rho_h < \rho_h^* = r_h/R_h^*$)

6 Size and quality of the financial sector

The financial services are provided in $t = 0$, when savings and investment decisions are made. For determining the share of the financial sector in Y_0, we have to calculate the present value of total transaction costs charged to the real sector. The aggregate transaction costs charged to the real sector correspond to the income earned in the financial sector. They represent the resources absorbed by the financial sector collecting and aggregating information about future possibilities, and to communicate the information by providing a set of financial products characterized by their pay-off promises and prices. The costs are paid by the savers. This means, the household sector spends this part of GDP on financial services. The quality of the financial sector depends on the assessment of risks and the valuation of financed projects. According to the approach of this paper, the efforts spent by financial agents on direct inspection of reality are decisive for an accurate assessment; trading within the financial sector serves the purpose to aggregate information and coordinate beliefs among financial agents.

6.1 The size of the financial sector

Since fee parameter τ^- and τ^+ are exogenous in the presented model, total transaction costs depend on the volume and structure of savings. As discussed in the previous section, aggregate transaction costs for the savings in bank accounts and securities amount to $T^- = \tau^- L$ and $T^+ = \tau^+(B - L)$, respectively. Total transaction costs are thus $T = \tau^+ B - (\tau^+ - \tau^-)L$. Substituting (14) for L and B, we obtain for the relative size of the financial sector in GDP, T/Y_0:

$$\frac{\delta}{1-\delta}\left[\tau^+ - (\tau^+ - \tau^-)\frac{1-\mu_h}{1-\rho_h}\right] \equiv \chi. \tag{31}$$

Obviously, the banking sector grows with the saving rate, $\frac{\delta}{1-\delta}$, and with the charged fees. The more interesting question is, how distortions affect the size of the financial sector relative to the real sector. A first observation is, they only matter if there is a differential in the costs of different forms of savings. Under the reasonable assumption that the costs of services for risky investments are higher than the cost of services for bank accounts, the effects of distortions have the following sign:

$$\frac{\partial \chi}{\partial \mu_h} > 0, \quad \frac{\partial \chi}{\partial \rho_h} < 0.$$

For any given ρ_h, euphoric household beliefs – about the extent of insurable future events – blow up the financial sector, pessimistic beliefs let it shrink. Euphoric (pessimistic) assessment of risky projects, implying a decline (rise, resp.) of ρ_h, has similar effects.

Finally, it is worth noticing that a selective rise in τ^+ relative to τ^-, (for instance, if more complex financial products are offered) affects the size of the financial sector in two ways. On the one side, there is a direct positive effect. On the other side, there is a negative indirect effect through ρ_h. Differentiation establishes that the direct effect dominates.[27]

6.2 The quality of the financial sector

The economic functions of financial markets are to support the transformation of current resources into future output and to insure against risks. A possible indicator for the quality of the transformation function is the correct expectation of future consumption, given the aggregate volume of saved resources (B) and their structure ($L, B - L$). In an analogous way, we can take the correct variance of future consumption as indicator for the insurance function. For the correct expectation and variance of future output we have to weigh C^r_ω, $\omega \in \Omega$, with the true measures π^*_ω for $\omega \in S^*$ and μ^* for S^*. Let $E^*[C^r] = \mu^* \sum_{\omega \in S^*} \pi^*_\omega C^r_\omega + (1 - \mu^*) C^r_-$ denote the correct expectation of realized consumption and let $\text{VAR}^*[C^r] = \mu^* \sum_{\omega \in S^*} \pi^*_\omega (C^r_\omega - E^*[C^r])^2 + (1 - \mu^*)(C_- - E^*[C^r])^2$ be the correct variance. $E^*[C^r]$ and $\text{VAR}^*[C^r]$

[27] $1 > \partial \Big[\frac{(\tau^+ - \tau^-)(1 - \mu_h)}{1 - \rho_h} \Big] / \partial \tau^+$ is equivalent to $\frac{1 - \rho_h}{1 - \mu_h} > 1 - \frac{\rho_h}{1 - \rho_h} \frac{\tau^+ - \tau^-}{1 - \tau^+}$, where $\frac{\partial \rho_h}{\partial \tau^+} = \frac{\rho}{1 - \tau^+}$ has been used. Since $\mu_h < \rho_h$, the left side of this inequality is larger than one. This proves $\partial \chi / \partial \tau^+ > 0$.

are hypothetical values, which in general deviate from the values that households have in mind when making their savings decisions.[28] Using (26), we obtain:[29]

$$E^*[C^r] = \mu^* \bar{A}^*(B - L) + \bar{a}^* L. \tag{32}$$

The correct variance $\mathrm{VAR}^*[C^r]$ can be written in the form[30]

$$\mathrm{VAR}^*[C^r] = \mu^* D[\bar{A}(B - L)]^2 \tag{33}$$

with

$$D \equiv \sum_{\omega \in S} \pi^*_\omega \epsilon^2_\omega + \sum_{\omega \in S^* - S} \pi^*_\omega + 1 - \mu^*.$$

The first-best outcome would result if savings portfolios $(L, B - L)$ were formed on the basis of correct beliefs and correctly priced assets, that is, if $S = S^*$, $\mu_h = \mu^*$, $\rho_h = \rho^*_h$ and $\epsilon_\omega = 0$ for all $\omega \in S^*$. To isolate the quality of the financial sector for which financial agents are responsible, we can assume that households form correct beliefs. In this case we have $\mu_h = \mu^* \sum_{\omega \in S} \pi^*_\omega \leq \mu^*$. Extending S towards S^* by financial innovations shifts the savings portfolio towards risky assets by increasing μ_h. As a consequence, $E^*[C^r]$ increases, provided that risky asset holding is an optimal first-best outcome ($\mu^* > \bar{a}^*/\bar{A}^*$) to begin with. This is obvious from (32) and the fact that L is declining in μ_h. At the same time, the variance of future consumption increases, as (33) shows. If financial innovations are based on correct performance and risk assessment, this is no problem as prices inform households correctly and they consciously choose the higher variance in exchange for the increase of expected future consumption. A problem arises if the innovations are based on an erroneous assessment of fundamental values. The consequences of an erroneous

[28] The values are hypothetical because households have no possibility to observe true possibilities. Nonetheless, they can ask themselves: What outcome – in terms of average and deviation from average – are to be expected if financial market conditions and savings decisions are based on a wrong model? Such thought experiments may seem useless; a household cannot change the terms at which financial products are traded. Reflection about the consequences of model uncertainty serve a different purpose. They are useful for understanding the causes and consequences of imperfect information-processing in financial markets. Potentially, the understanding then leads to insights on how to improve the robustness of the financial system.

[29] Note $\sum_{\omega \in S} \pi_\omega = \sum_{\omega \in S} \pi^*_\omega (1 + \epsilon_\omega) = 1$

[30] The proof is in the appendix.

performance assessment have been examined in the discussion of Δ in (29). The consequences of errors in the risk assessment are shown by term D in (33). The term shows a tension: On the one side, more states can be insured by financial innovations, which reduces $\sum_{\omega \in S^* - S} \pi^*_\omega$. On the other side, if financial innovations are based on unreliable information they weaken the precision of risk assessment, which increases the variance of ϵ. From the point of view of prudence, it is the latter effect that has to be kept under control. Less reliable risk assessment makes surprises more likely or severe – positive and negative ones – and may undermine the confidence in the financial sector. In particular, negative surprises imply deceived promises, which, if severe, can have far-reaching consequences like a fundamental crisis of the system. Section 7 will elaborate this idea in more detail. Before that I want to address some issues which are beyond the scope this paper.

6.3 Some remarks on higher-order finance

This paper considers the role of the financial sector from an outsider point of view – an outsider sitting in the real economy rather than knowing the finance business from the inside. Under this perspective, the input of the financial sector to the real economy is given, after financial markets have settled at the no-arbitrage condition. The no-arbitrage condition is established by arbitrage trades within the financial sector. This involves instruments the complexity of which goes far beyond the financial products considered in this paper. I call them higher-order financial instruments, to distinguish them from the basic financial products which span the insured set of events in the real system. From the point of view taken in this paper the decisive criterion for the value of higher-order financial business is its impact on the reliability of the information communicated through the prices and pay-offs of the basic financial products.

One argument is that, loosely formulated, an increase in the volume of financial transactions by higher-order instruments like derivatives or high-frequency trading generate information even if they are not tied to the real economy. In the presented framework the objects about which one would like to have accurate information are the productivity performance ($\tilde{A}_s^*$) of risky technologies, the measure of insurable environments (μ^*) and the probability distribution (π^*) of the risks which are insurable. The only reliable sources for generating information about these objects are direct inspection or learning from realizations of events ω. The latter is excluded in the two-period framework of this paper. More generally, it is limited by the number of "real" realizations, which cannot be substituted by increasing the number of financial transactions not tied to such realizations. Nonetheless, higher-order trades can add value to the information-processing quality of financial markets by aggregating information and coordinating beliefs among financial agents. It would be desirable to complement the framework presented here by a theory which explains how the size of the financial sector measured by the volume of transaction – as opposed to the size of the financial sector in terms of its share in GDP – is related to the quality of information conveyed by financial services to the real sector. As reality and scientific studies show, the immense growth of this volume does not necessarily improve the assessment of fundamentals by financial markets. Without proof of the contrary, it is wise to be cautious with respect to the information-improving role of financial products with no clear relation to reality.

A second argument is that the insurance function of the financial sector is not constrained to risks emerging from the real economy but also has to cover genuine risks in the financial market itself. This raises the question where such risks come from. Ultimately, since neither systemic risks nor true uncertainty are insurable, the two possible sources of insurable financial risks are idiosyncratic exposure of agents to real risks or idiosyncratic beliefs about the real fundamentals. The first source is dealt with in the analysis of this paper. The second source arises if information is spread randomly across agents. So we are back in the discussion about the higher-order information-processing function of financial markets. Moreover, since the potential for belief heterogeneity increases with the frequency of trades and the complexity of used instruments, one may find here a source for self-feeding financial trading.

A third argument is that financial markets have an important function in providing liquidity. In the presented framework this function is not modeled any further. The need of liquidity for real transactions, or transactions in the financial market related to the real economy, is determined by the volume of transactions in the real economy and the frequency of real events. Therefore, the provision of liquidity should be in line with the growth dynamics of the real economy, as we know from the discussion about monetary policy rules. If more liquidity is provided, an inflation risk emerges which makes people feel insecure and undermines the information-processing function of market prices. So what about the liquidity needs of the financial market as such? In the presented model an unforeseen liquidity need arises to fulfill the promised pay-offs if promises were based on euphoric beliefs. This is a systemic need which can only be satisfied by the central bank – at the risk of inflation. In addition, liquidity is needed for the higher-order financial business, which leads us back to the discussion about the value of this business for the real sector. Apart from the question how much reliable information about fundamentals can be generated by financial events which are not related to realizations of real events, the following problem arises: On the one hand, monetary economics tells us that provision of liquidity which is out of line with the real economy destroys the information-processing quality of markets. How can, on the other hand, the provision of liquidity in the financial market – by financial innovations and transactions not aligned to the dynamics of the real system – support this quality? Are there no rules for determining a sound rate of growth of financial markets – analogous to the rules for money supply? In the next section, I sketch a possible strategy how one might reasonably approach these questions in the framework presented in this paper.

7 Knightian uncertainty and the cost of unreliable financial innovations

Instead of modeling "true" uncertainty as unmeasurable set of future environments, one can consider future events as uncertain if they are drawn from a measurable set with unknown measure. As discussed briefly in the introduction, this view corresponds to the approach pursued by studies on Knightian uncertainty. The view can be integrated in the presented framework by splitting $S^* \subset \Omega$ into subsets and varying the precision of the risk distribution across subsets. For simplicity, let us set $S^* = \Omega$ and assume that the true probability measure can be accurately assessed for $\omega \in S_0$, but is unknown for $\omega \in \bar{S}_0$. That means, financial markets can provide full insurance for S_0 by providing state-contingent securities for $\omega \in S_0$ if they carefully assess the risk distribution. In contrast, if they provide state-contingent securities for $S_1 \subset \bar{S}_0$, the provided insurance possibilities are not reliable. They are based on imperfect information by assumption. More formally, there is a lower bound $\underline{\epsilon}$ on the possible errors ϵ_ω of beliefs about the probability of $\omega \in S_1$. Whether and how far the beliefs conveyed by the characteristics and prices of the financial products for S_1 deviate from these lower bounds depends on the quality of financial markets. In the further analysis I assume that they accurately assess risks wherever this is possible in principle.

Agents perceive and reflect economic fundamentals in the framework used so far. In particular, they have in mind that robust technologies are working in any environment at low productivity, whereas specialized technologies are highly productive but vulnerable to shocks as specified in (2) and (3). Let $(\hat{\pi}_\omega)_{\omega \in \Omega}$ be the true probability measure on Ω and $S \subset \Omega$ be the set of states for which state-contingent securities are offered in the market. Then, $\mu(S) = \sum_{\omega \in S} \hat{\pi}_\omega$ is the true measure of S; π^*_ω and π_ω, which refer in the presented framework to conditional probabilities on S, are given by:

$$\pi^*_\omega = \frac{\hat{\pi}_\omega}{\mu(S)} \text{ and } \pi_\omega = \pi^*_\omega (1 + \epsilon_\omega).$$

By assumption, for $\omega \in S_0$ the deviations of beliefs from true probabilities can be fully eliminated by careful risk assessment. In $S - S_0$ such deviations cannot be avoided. Like before we exclude systematic errors. That is, $\sum_{\omega \in S} \pi^*_\omega \epsilon_\omega = 0$ and $\sum_{\omega \in S} \pi^*_\omega = \sum_{\omega \in S} \pi_\omega$. For $\omega \in S_0$, the productivity assessment is assumed to be accurate. We have $A^*_\omega = A^*/\pi^*_\omega$. This implies $\rho_h = \rho^*_h = \bar{a}^*/\bar{A}^*$. For $\omega \in \bar{S}_0$, neither $\hat{\pi}_\omega$ nor A^*_ω are known. The financial sector can be prudent and accept that there is no reliable informational basis for trading securities contingent on $\omega \in \bar{S}_0$. In this case, we have $S = S_0$ with $\pi_\omega = \pi^*_\omega = \hat{\pi}_\omega/\mu_h$ and $\mu_h = \mu_0$, $\mu_0 \equiv \sum_{\omega \in S_0} \hat{\pi}_\omega$ in the previous analysis. We can contrast the case with a scenario in which financial agents also provide state-contingent securities for a subset S_1 of $\bar{S}_0$, guessing $\pi_\omega = \pi^*_\omega(1+\epsilon_\omega)$ and thus $A_\omega = A^*/\pi_\omega$. In this scenario, $S = S_0 \cup S_1$ and $\pi^*_\omega = \hat{\pi}_\omega/\mu_h$ with $\mu_h = \mu_0 + \mu_1$, $\mu_1 \equiv \sum_{\omega \in S_1} \pi_\omega = \sum_{\omega \in S_1} \hat{\pi}_\omega$. Under the assumption that risky assets involve higher transaction costs, the size of the banking sector is larger under the second scenario compared to the first one. Moreover, as the previous analysis showed, a larger share of savings is allocated to risky projects, yielding on average a higher future output at the cost of uncertainty. The uncertainty hits the economy in $t = 1$ in the form of a deviation of realized from planned consumption, and, if luck is bad, non-honored pay-off promises.

For a rigorous comparison we consider first the case of a prudent financial sector. I refer to this scenario by labeling savings and output variables with superscript zero. In this case, we have by (14) and (20):

$$\begin{aligned} L^0 &= \frac{1-\mu_0}{1-\rho^*_h}, \\ B - L^0 &= \frac{\mu_0 - \rho^*_h}{1-\rho^*_h}, \\ I^0_\omega &= \pi^*_\omega (B - L^0), \end{aligned} \tag{34}$$

In contrast, if financial agents design financial products and act as intermediary for $\omega \in S_1$, they guess π_ω and $A_\omega = A^*/\pi_\omega$ and trade ω-contingent securities with $q_\omega/R_\omega = \pi_\omega/R_h$, $R_h = \bar{A}^*$ for $\omega \in S_1$, too. If households trust in the financial markets, their savings decisions lead to the following investment structure:

$$\begin{aligned} L^1 &= \frac{1-(\mu_0+\mu_1)}{1-\rho_h^*}, \\ B - L^1 &= \frac{(\mu_0+\mu_1)-\rho_h^*}{1-\rho_h^*}, \\ I_\omega^1 &= \begin{cases} \pi_\omega^*(B-L^1) & \text{if } \omega \in S_0, \\ \pi_\omega(B-L^1) & \text{if } \omega \in S_1, \end{cases} \end{aligned} \tag{35}$$

where superscript one is used to refer to the scenario with financial innovations based on unreliable information.

Looking now at the realization of events in $t = 1$, we see that unreliable financial innovation makes deception of households more likely. The chance that $\omega \in S_1$ is realized in $t = 1$ is given by $\mu_1^* \equiv \sum_{s\in S_1} \pi_s^*$. Thus with probability μ_1^* we end up in an environment in which realized consumption levels deviate from planned consumption levels. As shown in Section 5, for $\omega \in \bar{S}$, consumption plans C_-^p are in line with realized consumption C_-^r. They are given by

$$C_- = \bar{a}^* L^1. \tag{36}$$

Furthermore, plans and reality coincide if $\omega \in S_0$ is realized in $t = 1$. Since $\rho_h = \rho_h^* = \bar{a}^*/\bar{A}^*$ and $\mu_h = \mu_0 + \mu_1$ in the present context, consumption levels in $\omega \in S_0$, where $\pi_\omega = \pi_\omega^*$, are given by[31]

$$\begin{aligned} C_+ &= [\bar{A}^*(B-L^1) + \bar{a}^* L^1] \\ &= (\mu_0+\mu_1)\bar{A}^* B, \end{aligned} \tag{37}$$

according to (26) and (35). For $\omega \in S_1$, however, the realized consumption level deviates from the planned level C_+^p by the term

$$\begin{aligned} \Delta C_t &= \epsilon_\omega \bar{A}^*(B-L^1) \\ &= \epsilon_\omega \bar{A}^* \frac{(\mu_0+\mu_1)-\rho_h^*}{1-\rho_h^*} B, \end{aligned} \tag{38}$$

according to (27) (where $\Delta = 0$ since $A = A^*$ in the present context). If $\epsilon_\omega < 0$, firms cannot deliver the promised pay-off $0R_\omega$. If $\epsilon_s > 0$, there are unexpected gains.

[31]Using (35) in (26), we have, for $\omega \in S_0$, $C_\omega^r = [\bar{A}^*(\mu_h - \rho_h^*) + \bar{a}^*(1-\mu_h)]/(1-\rho_h^*)$, which can be rewritten as $\bar{A}^*[\mu_h - \rho_h^* + \rho_h^*(1-\mu_h)]/(1-\rho^*)$ and reduces to (37)

Figure 3 illustrates the possible disturbances created by unfounded financial innovations. In the absence of such innovations C_0 would result in $t = 1$. If securities are traded for $\omega \in S_1$, too, then household portfolios are adjusted so that C_1 results as planned consumption point in the (C_+, C_-)-space. Actually, however, some point in the bracketed interval around C_1 is realized. The size of the bracketed interval depends on the size of the error, $\epsilon_\omega \in [-\underline{\epsilon}, \bar{\epsilon}]$. One could argue that the error is relatively small as long as S_1 is a small extension beyond S_0; and increases, if a larger extension S_1' with $\mu_1' > \mu_1$ is covered by financial innovations without reliable basis of information about true risks. This is illustrated by the expansion paths for different values of μ_h. (With μ_h increasing the expansion path rotates downwards in the (C_+, C_-)-space.). Whatever is the distribution of good luck and bad luck within the bracket around C_1 of C_2, there is a fundamental asymmetry. A realization of C_ω^r which falls short of the planned level is not just a loss of consumption opportunities. It also means a loss of confidence in the system, because pay-off promises are broken. The two-period framework presented here is silent about what happens after the period in which the unreliability of financial products has been revealed. Therefore, the costs of such a a loss are outside the model. But they have negative external effects beyond the foregone consumption in $t = 1$. Then, the valuation of the gains of financial innovations beyond S_0 in terms of expected consumption against their unreliability is more than an individual consumer's trade-off under risk-aversion. It is a judgment about the reliability of the economic system. Looking from this perspective on the results of the presented model, one can try to find a rule which relates the tolerable rate of financial innovations to real economic development. The following example illustrates how this could be done. Let, for $\mathbf{\Gamma} = [0, \Gamma]$, $0 < \Gamma < \infty$, $\{S_\gamma\}_{\gamma \in \mathbf{\Gamma}}$ be a series of monotonously increasing subsets of $S^* = \Omega$. More specifically, assume that Ω and S_γ are Borel sets with Lebesgue measure $\lambda(S_\gamma) = \lambda(S_0)(1 + \gamma)$. Assume further that the "size" of the event space is related to GDP, so that $\lambda(\Omega) = \Lambda Y$ for some $\Lambda > 0$ and $\lambda(S_0)/\lambda(\Omega) = \lambda_0$ for a constant $\lambda_0 < \frac{1}{1+\Gamma}$. Then $\mu(\gamma) \equiv \int_{S_\gamma} \hat{\pi}_\omega d\omega$ is a monotonously increasing function of γ. Let $\pi_\omega^* = \hat{\pi}_\omega / \mu(\gamma)$ be the true conditional probability of $\omega \in S_\gamma$. ($\hat{\pi}$ denotes the true risk distribution over Ω.) Assume that with increasing γ the informational basis for assessing true risks becomes less and

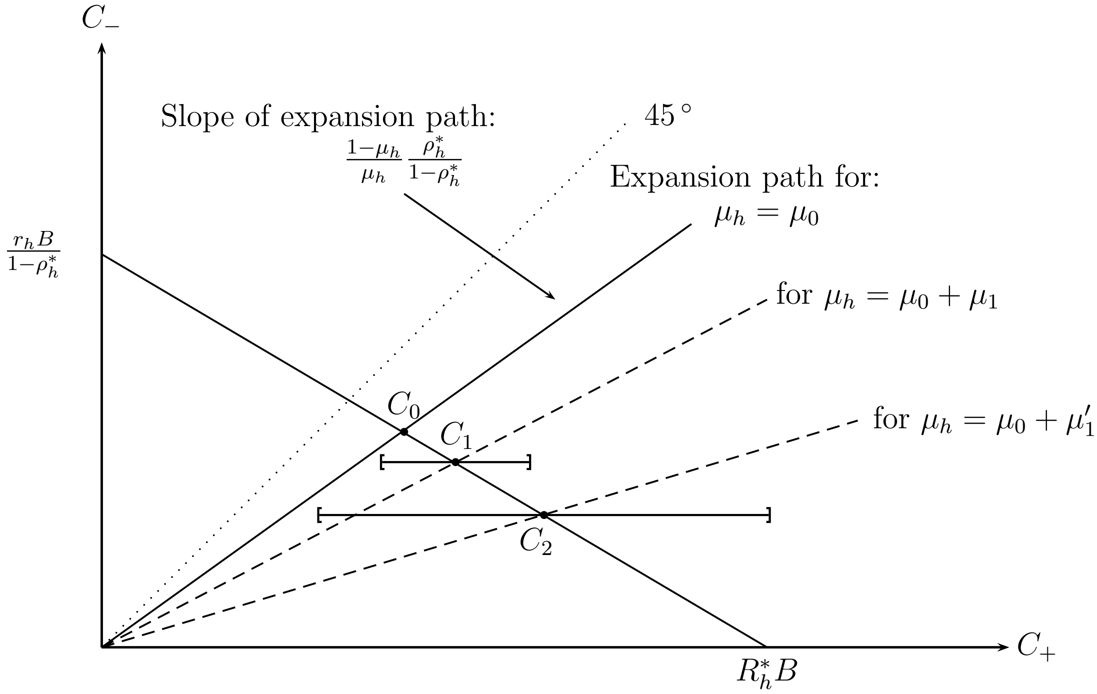

Figure 3: Insecurity generated by unreliable financial innovations ($\mu_1' > \mu_1 > 0$)

less reliable. Formally, let $\pi_\omega = \pi_\omega^*(1 + \epsilon_\omega(\gamma))$ and $\epsilon_\omega(\gamma)$ be an error process with $\int_{S_\gamma} \pi_\omega^* \epsilon_\omega(\gamma) d\omega = 0$. Define $\sigma_\epsilon(\gamma) \equiv \int_{S_\gamma} \pi_\omega^* [\epsilon_\omega(\gamma)]^2 d\omega$.

Assumption 3. *$\sigma_\epsilon(\gamma)$ is increasing in γ, starting at $\sigma_\epsilon(0) = 0$.*

Assumption 3 means there is a terrain S_0 in which risks can be assessed accurately. The further we move away from this terrain, the less reliable is the informational basis for risk assessment.

Suppose now that the frontier of financial innovations stands at some $\gamma \geq 0$, that is, financial products are traded for all $\omega \in S_\gamma$. Denote by $\sigma_C(\gamma)$ the variance of consumption in S_γ. According to (30), this variance is proportional to $\sigma_\epsilon(\gamma)$. We have

$$\sigma_C(\gamma) = \left[\bar{A}^* \frac{\mu(\gamma) - \rho_h^*}{1 - \rho_h^*} B\right]^2 \sigma_\epsilon(\gamma). \tag{39}$$

The planned consumption in S_γ also increases with γ. According to (22), we have

$$C_+^p = \mu(\gamma) \bar{A}^* B. \tag{40}$$

Suppose finally that a society agrees that deception of promises should be kept in some tolerable relationship to the plans based on the promises. Formally, $\sigma_C \leq \beta C_+^p$

is required for some parameter β, which may be called crisis tolerance.[32] Then, according to (39) and (40), the frontier of financial innovations is bounded by the condition

$$\sigma_\epsilon(\gamma) \leq \beta \frac{\mu(\gamma)}{(\mu(\gamma) - \rho_h^*)^2} \frac{(1-\rho_h^*)^2}{\bar{A}^* B}. \tag{41}$$

The left side of the equation is an increasing function of γ starting at $\sigma_\epsilon(0) = 0$. The term at the right-hand side is positive for $\gamma = 0$ and declining in γ, as $\mu(\gamma)$ is increasing in γ. Hence, condition (41) imposes an upper limit $\bar{\gamma}(\beta)$ on financial innovations with $\bar{\gamma}(0) = 0$ and $\bar{\gamma}'(\beta) > 0$. More specifically, the measure $\lambda(S_\gamma)$ of states to be covered by financial products is limited by

$$\lambda(S_\gamma) \leq \lambda(S_{\bar{\gamma}}) = \lambda(S_0)\left[1 + \bar{\gamma}(\beta)\right], \tag{42}$$

where $\lambda(S_0) = \lambda_0 \Lambda Y$. Hence, ultimately financial innovations should be tied to the development of the GDP to keep "men-made" crises within a certain tolerance bound.

Policies for implementing such ties to financial innovations to real economic development would complement regulations of volumes and frequencies of financial trading on a given set of financial products. In the approach presented here, the latter regulations affect the allocation (ξ) of the financial sector's resources, T, on direct inspection and arbitrage trade. The should reduce potential inefficiencies within the financial sector and bring the errors contained in the prices and pay-offs, the real economy is facing, closer to the bounds $\epsilon(\gamma)$ imposed by principal limits of knowledge about the future. This goal is realized in the above exercise by assumption. A policy rule based on condition (42) would limit the unavoidable errors, within $\epsilon(\gamma)$, arising from overconfidence in the possibilities of financial markets to generate knowledge about the future.

[32] I use this label to avoid confusion with the notion "uncertainty aversion" in Gilboa and Schmeidler (1989) which is related to Bewley's (2002) "inertia assumption". For $\beta = 0$ only $S_\gamma = S_0$ would be tolerable.

8 Conclusion

What are the real fundamentals in a world in which objects are composed of (in the broad sense of the word) physical properties and the valuation of these properties by financial markets? This paper tried to disentangle the mixture of “men-made” and exogenous fundamentals – at a logical level and in practical terms. The endeavor was driven by the firm conviction that, if rationality should have any meaning, economic analysis needs to reflect the principle limits of knowledge and perception, in particular with regard to the future. Basic principals of the proposed framework were the distinction between risk and uncertainty and a careful tracking of what agents know, what they do not know, what they know from market information and what they might know more or less by direct inspection of reality. The answer was different for different types of agents, depending on their roles, namely on whether they act as firm, as household or as financial agent (“bank”). The analysis clearly distinguished savings and investment decisions in the real sector from financial intermediation. Regarding the latter an important modeling element was that financial intermediation is not done by a Walrasian auctioneer but by a resource absorbing financial sector which processes and communicates information about future opportunities by providing and trading financial products.

The main message following from the analysis was that things can go wrong because of bad luck as well as sloppy evaluation of reality and careless dealing with uncertainty. In the first case, the economy experiences a fatal shock, which hurts but is no surprise in a world of incomplete knowledge. In the second case, the distortion is men-made and a crisis of confidence can emerge. According to the presented model, in a crisis households experience non-honored pay-off promises and deceived consumption plans, which reveal that the information conveyed by the financial market was based on unreliable assessment of future productivities and risks. Under a fatal shock, which by definition is uninsurable, no contingent contract on the shock exists. Essentially, two sources of financial market failures were made responsible for household deception: First, the allocation of financial sector resources on direct inspection of reality, on the one hand, and arbitrage , on the other hand, may be biased. Second, financial innovations provide state-contingent securities for uncertain environments without reliable information about the measure of events or the real performance of projects in these environments. To account for the second problem, a policy rule was outlined in the last section, which attempts to bring the reliability of financial innovations in line with the crisis tolerance of a society by tying the number of financial products to GDP development.

A series of problems remained open. In particular, the inside of the financial sector was taken as a black box in this paper, which is silent on finance issues in a more narrow sense of the word. Some issues were addressed in section 6 which discussed how a more complex view of the financial sector – with higher-order financial instruments for arbitrage trading and liquidity provision – fit into the analysis. The bottom line was that these topics are complementary to the presented analysis. They concern the trading within the financial sector, which aggregates the information collected by the financial agents and coordinates their beliefs in the model. As mentioned, if financial sector resources employed in arbitrage trade are not aligned to the resources employed for collecting information by direct inspection of reality, the information communicated by financial product characteristics and prices to the real sector is unreliable. This explains why regulations concerning the frequency and volume of financial transactions, which were not analyzed here, can make sense from the point of view of this paper, too.

Finally, learning from real market data was excluded in the presented two-period framework. By extending the model to more than two periods, one could possibly substantiate a hypothesis about the frequency of crises. According to the uncertainty and knowledge structure defined in this paper, the learning about π and A depends on the number of observed realizations of events in S^* generated by a stationary stochastic process π^*, $\tilde{A}^*$. If there is a structural break after a long stationary period, or if financial innovations outgrow the speed of learning, then there is a danger of a crisis in the sense of this paper. In the first case, the learning has to start again. In the second case, we have to tame an excessive financial business. In contrast, the learning about μ_h depends on the frequency of observations of $\omega \in S^*$ as opposed to $\omega \in \Omega - S^*$. The latter means we have to experience bad luck with only robust technologies working properly. That is, we have to experience fatal shocks, in which the economy works at a low productivity but which are no surprise and not accompanied by deceived promises.

A Appendix

A.1 Optimal progam of household *i*

The first-oder conditions for *max EU* s.t. (10) are for b_i, z^i_ω and l_i, respectively:

$$\frac{1}{y^i_0 - b_i} = \lambda_i, \tag{A.1a}$$

$$\frac{\delta\mu_i\pi^i_\omega}{c^i_\omega}R^h_\omega = \lambda_i q_\omega, \quad \omega \in S, \tag{A.1b}$$

$$\delta\mu_i \sum_{\omega\in S}\frac{\pi^i_\omega}{c^i_\omega}r_h + \delta\frac{r_h(1-\mu_i)}{r_h l_i} = \lambda_i, \tag{A.1c}$$

where λ_i is the Langrange multiplier for constraint $c^i_0 \leq y^0_i - b_i$. Using (A.1b) we can rewrite (A.1c) in the form

$$\lambda_i r_h \frac{Q}{1-\tau^+} + \frac{\delta(1-\mu_i)}{l_i} = \lambda_i, \tag{A.2}$$

with $Q \equiv \sum_{\omega\in S}\frac{q_\omega}{R_\omega}$.

With $R_h \equiv \frac{1-\tau^+}{Q}$ equation (A.2) can be rewritten in the form

$$l_i = \frac{\delta}{\lambda_i}\frac{1-\mu_i}{1-r_h/R_h} \tag{A.3}$$

and (A.1b) together with budget constraint (10) for c^i_ω, $\omega \in S$, gives us:

$$z^i_\omega = \frac{\delta}{\lambda^i}\frac{\mu_i\pi^i_\omega}{q_\omega} - \frac{r_h l_i + e^i_\omega}{R^h_\omega} \tag{A.4a}$$

$$q_\omega z^i_\omega = \frac{\delta}{\lambda_i}\mu_i\pi^i_\omega - (r_h l_i + e^i_\omega)\frac{q_\omega}{R^h_\omega} \tag{A.4b}$$

$$\sum_{\omega\in S} q_\omega z^i_\omega = \frac{\delta}{\lambda_i}\mu_i - \frac{r_h}{R_h}l_i - \tilde{E}e^i \tag{A.4c}$$

with $\tilde{E}e^i \equiv \sum_{\omega\in S}\frac{q_\omega}{R^h_\omega}e^i_\omega$.

Substituting (A.4c) into the budget constraint $b_i = \sum_{s\in S} q^h_s z^i_s + l_i$, we get $b_i = l_i(1-\frac{r_h}{R_h}) + \frac{\delta}{\lambda_i}\mu_i - \tilde{E}e^i$. Substituting (A.3) for l_i, we have

$$\frac{\delta_i}{\lambda_i} = b_i + \tilde{E}e^i \tag{A.5}$$

This, (A.1a) and (A.3) give us

$$\begin{aligned} b_i &= \frac{\delta y_0^i - \tilde{E}e^i}{1+\delta}, \\ l_i &= (1-\mu_i)\frac{b_i + \tilde{E}e^i}{1 - r_h/R_h}. \end{aligned} \tag{A.6}$$

Using then (A.5) in (A.1b) and $\frac{R_\omega^h}{q_\omega} = \frac{1-\tau^+}{\pi_\omega Q}$ from (9) and accounting for $R_h \equiv \frac{1-\tau^+}{Q}$, we obtain

$$c_\omega^i = (b_i + \tilde{E}e^i)\mu_i \frac{\pi_\omega^i}{\pi_\omega} R_h. \tag{A.7}$$

Finally, according to the budget constraint for c_ω^i, $\omega \notin S$,

$$c_\omega^i = r_h l_i. \tag{A.8}$$

A.2 Derivation of the VAR$^*(C^r)$

Use $\pi_S^* \equiv \sum_{\omega \in S} \pi_\omega^*$, $\sum_{\omega \in S} \pi_\omega^* \epsilon_\omega = 1 - \pi_S^*$ and $X_+ \equiv \bar{A}(B-L)$ to calculate $(C_\omega^r - E^* C^r)^2 = (1 + \epsilon_\omega - \mu^*)^2 X_+^2$ for $\omega \in S$, and $(C_\omega^r - E^* C^r)^2 = (\mu^* X_+)^2$ for $\omega \notin S$. Thus,

$$\begin{aligned} \text{VAR}^*[C^r] &= \mu^* \Big[\sum_{\omega \in S} \pi_\omega^* (1 + \epsilon_\omega - \mu^*)^2 X_+^2 + (1 - \pi_S^*)(\mu^* X_+)^2\Big] + (1-\mu^*)(\mu^* X_+)^2 \\ &= \mu^* X_+^2 \Big\{\sum_{\omega \in S} \pi_\omega^* (1 + \epsilon_\omega - \mu^*)^2 + [\mu^*(1-\pi_S^*) + (1-\mu^*)]\mu^*\Big\} \\ &= \mu^* X_+^2 \Big[\sum_{\omega \in S} \pi_\omega^* (1 + \epsilon_\omega - \mu^*)^2 + (1 - \mu^* \pi_S^*)\mu^*\Big]. \end{aligned}$$

The square-bracketed term in the last equation reduces to D, as the following calculation shows:

$$\begin{aligned} &\sum_{\omega \in S} \pi_\omega^* \epsilon_\omega^2 + 2(1-\mu^*)(1-\pi_S^*) + \pi_S^*(1-\mu^*)^2 + (1-\mu^*\pi_S^*)\mu^* \\ &\quad = \sum_{\omega \in S} \pi_\omega^* \epsilon_\omega^2 + (1-\mu^*)(1 - \pi_S^* + 1 - \mu^* \pi_S^*) + \mu^*(1 - \mu^* \pi_S^*) \\ &\quad = \sum_{\omega \in S} \pi_\omega^* \epsilon_\omega^* + 1 - \pi_S^* + 1 - \mu^* = D. \end{aligned}$$

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