Adapting Against the Curse

On solution methods for heterogeneous agents models with aggregate uncertainty and multidimensional states

Luca Mazzone

May 8, 2017

The main issue in general equilibrium models with heterogeneous agents and aggregate uncertainty lies in the intractability of the object that characterizes the cross-sectional distribution of agents. Solving for individual policy rules in presence of aggregate uncertainty requires agents to forecast aggregates using the distribution of past choices, when this distribution is an infinite-dimensional object. To overcome this limitation, three main methodologies have emerged: Projection via simulation as in Krusell and Smith (1998), Projection with parametrized distribution, as in Algan, Allais and Den Haan (2008), or Den Haan and Rendahl (2010) and Perturbation, as in Reiter (2009) or Winberry (2015). By "projection", here, I refer to the notion that aggregate variables enter explicitly into agents' individual problems, which are computed on different points of the (aggregate) state space: in the case of Krusell and Smith, the points of the state space are the outcome of a simulation.

Using an example model with incomplete markets, aggregate uncertainty, and firm entry and exit, based on Arellano, Bai, and Kehoe (2012), I show the strengths and limitations of these approaches. Establishing accurate solution methods has important policy implications, as it determines the sign, not only the magnitude, of the effects of the so-called *uncertainty shocks*, that is, second-moment shocks. In my model example, the economy receives shock to the second moment of the exogenous process for firm productivity. Since the economy is populated by infinite heterogeneous firms that can borrow using bonds (but have no access to state-contingent assets), and are forced to choose over labor with a period in advance, the measure of firms that are "active" (i.e. the incumbents, minus the defaulting firms, plus the new entering ones) varies over time. In this framework, I propose a global solution method which employs parametrized distributions with flexible functional forms and allows to deal with multidimensional states without suffering the curse of dimensionality by relying on adaptive sparse grids. Adaptive sparse grids, introduced in economics by Brumm and Scheidegger (2016), guide the approximation of the aggregate law of motion to be computed in a way such that more resources are spent to "learn" the behavior of the economy in specific areas of the state space.

The algorithm benefits from being extremely parallelizable, and achieves higher speed as well as higher accuracy, using various error measures, compared to relevant competitors. Thanks to the support of the Alphacruncher platform, I am able to run my algorithm using a combination of MPI and OpenMP architectures (relying, respectively, on distributed and shared memory protocols), which allows me to run large-scale models using extensive computational resources.
What I find is that, although Krusell and Smith is probably going to mantain its popularity due to its versatility, in problems where distributional channels are important, like the one proposed, alternative solution methods can reach superior performances. The approach proposed in this work has promising results in terms of accuracy, and being easily parallelizable it can deliver important speed gains, allowing results of more complicated models to be computable