

# Amplification and Spillover with Financial Arbitrage, Production and Collateral Constraints

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# Highlight

- general equilibrium of collateral constrained arbitrage in a production economy
- agenda:
  - ▶ price difference between identical assets
  - ▶ collateral constraints
  - ▶ financial markets and production sector
  - ▶ spillover and amplification

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  - ▶ Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999)
- tractable equilibrium model
  - ▶ explicit theory for production and arbitrage with financial friction
  - ▶ closed-form solutions in special cases
- welfare analysis
  - ▶ spillover and amplification
  - ▶ possible relevance for policy debates
    - \* systemic risk
    - \* margin requirement regulation

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# Motivation I – Lessons from Financial Crisis

- 1998 financial crisis

- ▶ Arbitrage

- ★ hedge funds bet on convergence of prices of similar-payoff assets
    - ★ during crisis, prices diverged.
    - ★ hedge funds experienced heavy losses + distress
    - ★ force to liquidate profitable positions

- 2008 financial crisis

- ▶ shocks from the housing sector spill over into financial sectors and reinforce with each other

- asset prices and liquidity:

- ▶ prices pushed away from fundamentals
  - ▶ liquidity dried up
  - ▶ cross-sector contagion

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## Motivation II

- empirical evidence about persistent price differences
  - ▶ “Siamese-twin” stocks:
    - ★ Rosenthal and Young (1990) and Dabora and Froot (1999)
  - ▶ Before 1998, the British government bonds vs German bonds
- market segmentation
  - ▶ Before Nov 2014, A shares in mainland China and H shares in Hongkong
  - ▶ Post-Brexit without financial “passport”
- links between financial friction and macroeconomy
  - ▶ Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999)

# Main Conclusions

- with perfect foresight of market demand
  - ▶ self-recovery
- with inaccurate estimation of future market demand
  - ▶ looser collateral constraints trigger recession
    - ★ spillover and amplification
  - ▶ tighter collateral constraints stabilize the economy

# Related Literature

- Vayanos and Gromb (2002, 2015)
  - ▶ No production sector
  - ▶ The financial constraints must cover the maximum loss of the arbitrageurs.
- Brunnermeier and Sannikov (2014)
  - ▶ no financial arbitrage

# Baseline Model

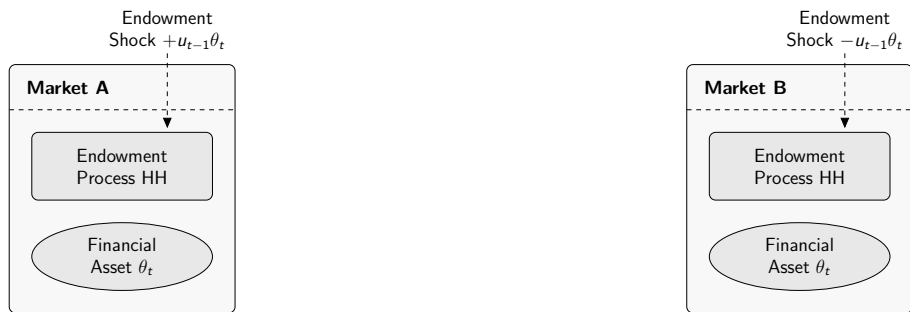


Figure: The structure of the economic system.

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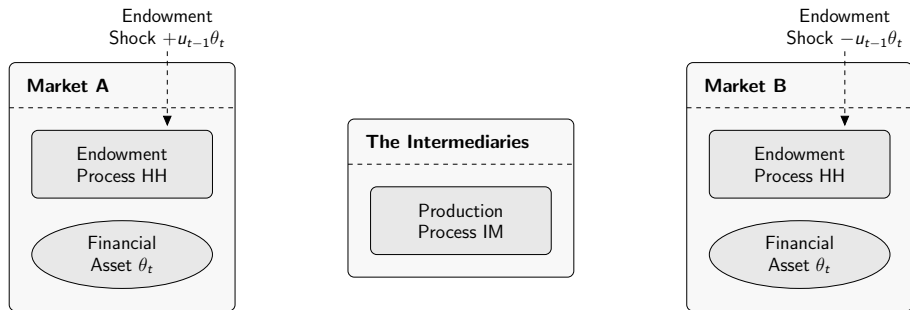


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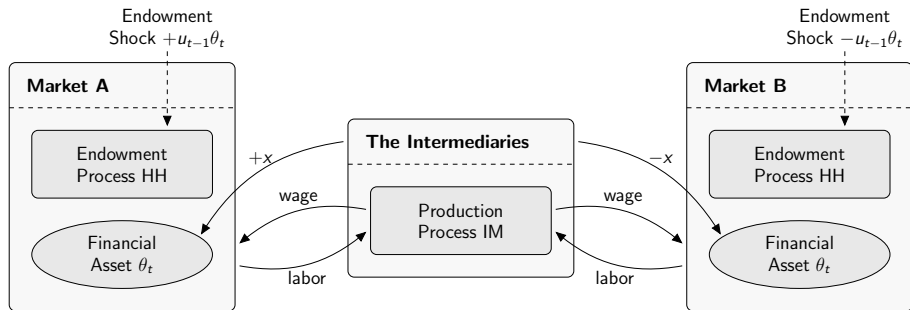


Figure: The structure of the economic system.

# Agents

- a continuum of competitive IM and HH
- only one perishable consumption goods
  - ▶ IM can convert consumption and capital
- IM are both arbitrageurs and entrepreneurs
  - ▶ IM invest capital and HH offer labor.

$$y_t = F(K_{t-1}) = aK_{t-1}^\alpha L_t^\gamma$$

- ▶ separate collateral posting with capital investment



# Exogeneous Shocks

- HH's fixed-size production / natural endowment

$$y_{i,t} = bK_H + u_{i,t-1}\theta_t, \quad i \in \{A, B\}, \quad t \in \{1, 2, \dots\}.$$

- ▶  $\theta_t$  follows a symmetric distribution around zero on  $[-\bar{\theta}, \bar{\theta}]$ .
  - ▶ the shock intensities/market demand  $u_{A,t} = -u_{B,t} =: u_t$ .
- opposite shocks, opposite hedging demand

# Financial Assets

- identical financial assets in each market
  - ▶ dividend  $\theta_t$  mimicks the shock
  - ▶ long-lived, in zero net supply
  - ▶ IM and HH's position  $x_{i,t}^{\text{IM}}$  and  $y_{i,t}^{\text{HH}}$ .
- prices differ across markets
  - ▶ opposite hedging demand A :  $-u_{A,t} = -u_t$ ; B :  $-u_{B,t} = u_t$ .
- IM exploit arbitrage profit

# Collateral Constraints

- IM take identical but opposite positions  $x_{A,t} = -x_{B,t} = x_t$ .
  - ▶ measure of market liquidity
- collateral constraints
  - ▶ separately post capital input as collateral
  - ▶ cover HH's maximum loss if IM default or walk away from their positions
  - ▶ total collateral limit: IM's capital rent  $\alpha F(K_t)$ .

# IM's Optimization Problems

$$\max_{c_s^{\text{IM}}, x_{i,s}, K_s} \mathbb{E} \left[ \sum_{s=t}^{\infty} \rho^s \log \left( c_s^{\text{IM}} \right) \right], \quad i \in \{A, B\}.$$

subject to

$$c_t^{\text{IM}} = \underbrace{\sum_{i \in \{A, B\}} x_{i,t-1}^{\text{IM}} p_{i,t}}_{\text{value of previous period's investment in financial asset } i} - \underbrace{\sum_{i \in \{A, B\}} x_{i,t}^{\text{IM}} p_{i,t}}_{\text{current cost of taking new positions or immediate arbitrage profit}} + \underbrace{a(1-\gamma)K_{t-1}^\alpha L^\gamma - K_t}_{\text{entrepreneur income: net production output minus wage and investment}}$$

$$\sum_{i \in \{A, B\}} \min \left\{ \min_{p_{i,t+1}, \theta_{t+1}} \left\{ x_{i,t}^{\text{IM}} (p_{i,t+1} + \theta_{t+1}) \right\}, 0 \right\} + \alpha F(K_t) \geq 0.$$

# HH's Optimization Problems

$$\max_{c_{i,s}^{\text{HH}}, y_{i,s}^{\text{HH}}} \mathbb{E} \left[ \sum_{s=t}^{\infty} \beta^s \log \left( c_{i,s}^{\text{HH}} \right) \right]$$

subject to

$$c_{i,t}^{\text{HH}} = \underbrace{(y_{i,t-1}^{\text{HH}}(p_{i,t} + \theta_t) - y_{i,t}^{\text{HH}} p_{i,t})}_{\text{income from trading financial asset}} + \underbrace{\frac{1}{2} a \gamma K_{t-1}^{\alpha} L^{\gamma}}_{\text{labor Income}} + \underbrace{(bK_H + u_{i,t-1} \theta_t)}_{\text{endowment}}$$

- Ideally,  $y_{i,t-1}^{\text{HH}} = -u_{i,t-1}$  so that households are fully protected from the endowment shock  $\theta_t$ .

# Competitive Equilibrium

For any initial capital endowment, an equilibrium is described by the price process  $p_{i,t}$ , IM's capital investment  $K_t$ , financial asset positions  $y_{i,t}^{\text{HH}}$  and  $x_{i,t}^{\text{IM}}$ , and consumption choices  $c_t^{\text{IM}}$  and  $c_{i,t}^{\text{HH}}$  for  $i \in \{A, B\}$  such that

- all agents solve their optimization problems given prices;
- markets clear for financial assets, that is  $y_{i,t}^{\text{HH}} + x_{i,t}^{\text{IM}} = 0$ .

# Riskless Arbitrage

When shock intensity is known to agents, there exist one steady state

- if  $\rho \geq \frac{(2 \max\{|u_t|\}\bar{\theta})^{\frac{1-\alpha}{\alpha}}}{a^{\alpha\alpha}}$ , then  $\alpha\rho F'(K^*) = 1$ ,  $C^* = (1 - \alpha\rho)F(K^*)$ ,  $x_t = u_t$ , price difference  $|\psi^*|$  is 0.
- otherwise if  $u_t$  is constant, then  $K_t$  converges over time to a unique  $K^*$ , with price discrepancy  $|\psi^*| > 0$  and market liquidity  $x^* = \frac{\alpha F(K^*)}{2\theta + |\zeta^*|} < u$ .

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# Steady State with Binding Collateral Constraints

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- net zero financial income
- without arbitrage,  $\rho F'(K_n^*) = \frac{1}{\alpha}$
- $\rho F'(K_n^*) = \frac{1}{\alpha'}$ , capital elasticity  $\alpha' = \alpha + \alpha \frac{1-\rho}{\rho} \frac{|\psi^*|}{2\bar{\theta} + |\zeta^*|} > \alpha$
- $K^* > K_n^*$
- nominal zero-interest debt, leveraged production
- reduced marginal cost of physical investment

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# Equilibrium Existence and Uniqueness

## Proposition

*When the shock intensity  $u_t$  is constant, a unique competitive equilibrium exists in which the price difference  $\psi_t$ , intermediaries' capital investment  $K_t$  and the positions of the financial assets  $x_t^i$  are deterministic.*

# Shock Reactions–Self-recovery

In case of a sudden loss in capital input or financial income,

- immediate reaction

price difference:  $|\psi_{t+1}| > |\psi^*|$ , market liquidity:  $|x_{t+1}| < x^*$ ,  
 $K_{t+1} < K^*$ .

- ▶ marginal return of capital
- ▶ arbitrage profitability

- long term

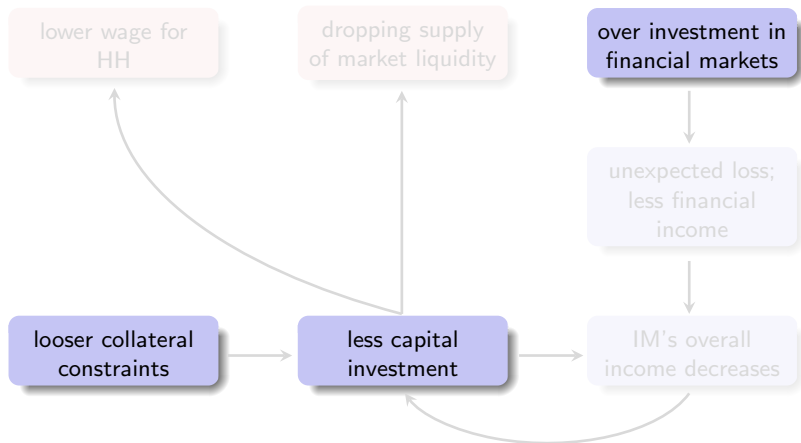
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# Risky Arbitrage – Looser Collateral Constraints

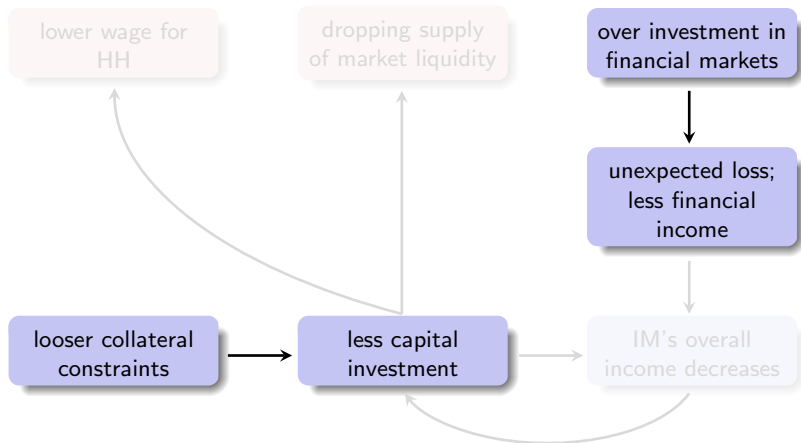
With uncertainty, agents might underestimate or overestimate the market demand.

- underestimate case
  - ▶ looser collateral constraints
  - ▶ overinvestment in financial markets
  - ▶ spillover and amplification
  - ▶ trigger recession & systemic risk

# Spillover and Amplification Effects

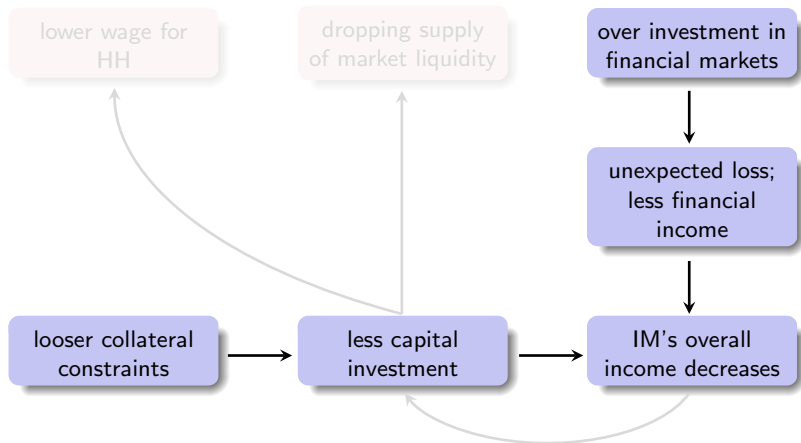


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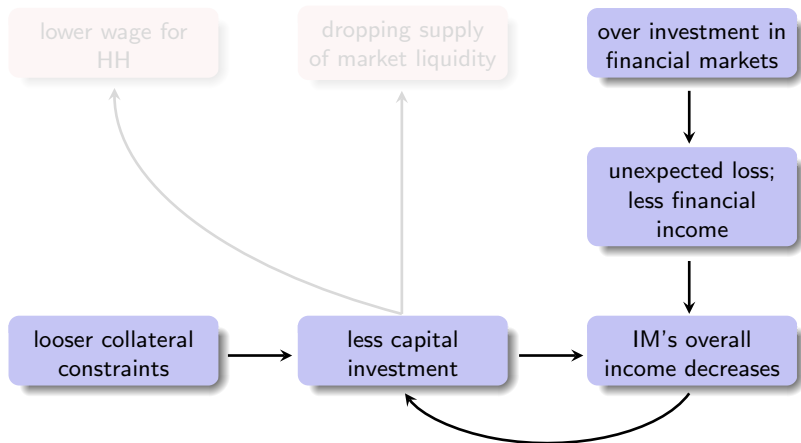




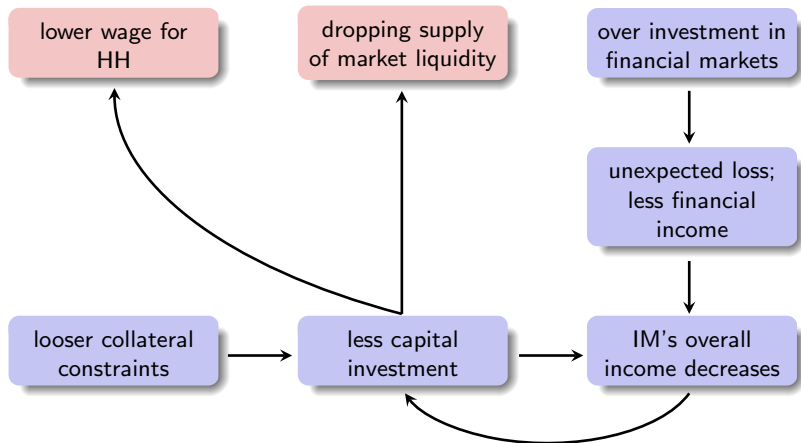
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# Risky Arbitrage – Tighter Collateral Constraints

overestimate case

- tighter collateral constraints
- underinvestment in financial markets
- higher income for both IM and HH
- stabilize the economy by boosting production at the cost of market liquidity

# Policy Implication

with overheated financial market and shrinking production sector,

- tightening collateral constraints
  - ▶ alternative exogenous collateral constraints
  - ▶ independent of estimate over future prices

# Extensions

- heterogeneous belief between IM and HH (Ally Q Zhang 2015)
  - ▶ information friction
  - ▶ HH's emotional trading
  - ▶ IM's misjudgement of HH's estimate
- segmented production (Ally Q Zhang 2016)
  - ▶ local production, local collateral
  - ▶ heterogeneous financial frictions
  - ▶ impact on foreign investment, market liquidity and local employment

# Q&A

Thank you!