International Illiquidity*

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Abstract

We build a parsimonious international asset pricing model in which deviations of government bond yields from a fitted yield curve of a country measure the tightness of investors' capital constraints. We compute these measures at daily frequency for six major markets and use them to test the model-predicted effect of funding conditions on asset prices internationally. Global illiquidity lowers the slope and increases the intercept of the international security market line. Local illiquidity helps explain the variation in alphas, Sharpe ratios, and the performance of betting-against-beta (BAB) strategies across countries.

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The recent financial crisis has dramatically illustrated how market frictions can impede orderly investment activity and have significant effects on asset prices.[1] These phenomena are even more prominent when looking at asset prices in an international context where specialized institutions, such as hedge funds and investment banks, are responsible for a large fraction of active cross-country investments. In order to understand the specific mechanisms at work and to test various friction-based asset pricing theories, researchers and policymakers alike require economically-motivated indicators of market stress.

In this paper, we study the effect of frictions, such as funding constraints or barriers that impede smooth cross-border movement of capital, on asset prices internationally. We generically refer to the tightness of these frictions as illiquidity. Our contribution to the existing literature is threefold. First, we develop a parsimonious international asset pricing model with constrained investors who trade in equity and bond markets globally. Second, we construct model-implied proxies for country-level and global illiquidity from daily bond market data of six developed countries. Third, in line with the model predictions, we find that global illiquidity affects the international risk-return trade-off by lowering the slope and increasing the intercept of the international security market line, stocks in countries with higher local illiquidity earn higher alphas and Sharpe ratios, and, as a result, accounting for the cross-country differences in illiquidity improves on the performance of traditional betting-against-beta (BAB) type strategies.

We measure illiquidity as the average squared deviation of observed government bond prices from those implied by a smooth fitted yield curve. This approach has been proposed by Hu, Pan, and Wang (2013), who argue that "noise" in the US Treasury yield curve contains a strong signal about the general scarcity of investment capital in financial markets. Government bonds are particularly well suited in this context because they are among the safest and most liquid assets, they are actively traded for investment purposes, and they are used as the main source of collateral to obtain funding. Moreover, their prices are known to be well described by a simple factor structure during normal times. In our model bond price deviations emerge in equilibrium because capital

[1] See, e.g., Brunnermeier and Pedersen (2009), Gârleanu and Pedersen (2011), and He and Krishnamurthy (2012, 2013).

constraints prevent investors from eliminating price discrepancies between bonds with similar risk; larger deviations indicate tighter capital constraints.

Frictions in financial markets can arise for a variety of reasons, such as regulatory capital requirements, restricted borrowing, margins, investment taxes, or endowment shocks, all with a similar effect of constraining investment activity.[2] In our model, we assume that investors have to fund a fraction of their position in each asset with their own capital. We focus on the differential impact of these frictions across countries. In an international context, more capital may be required to invest in some countries relative to others, and it may be costly for investors to move capital across borders. When this constraint binds for at least some investors, the equilibrium expected excess return on any security depends not only on its risk (e.g., duration for bonds or market beta for stocks), but also on an additional illiquidity component that is proportional to the capital required to maintain the position in this asset. Therefore, in the model, equilibrium bond price deviations are associated with distortions in the relation between risk and return of other securities, too.

We compute daily illiquidity measures for the US, Germany, the UK, Canada, Japan, and Switzerland. Unlike other funding liquidity and market stress proxies that suffer from short time series (e.g., implied volatility indices such as the VIX), are only available at very low frequency (e.g., broker-dealers' leverage), or are difficult to compare internationally (e.g., TED spread), our indicators have the same economic interpretation across all countries and are available daily for a history of more than 20 years. In terms of economic magnitude, bond price deviations are a multiple larger than government bond bid-ask spreads, in particular during crisis periods.

[2]See, e.g., Black (1972), Frazzini and Pedersen (2013), Stulz (1981), and Vayanos and Wang (2012) for examples of the latter four, respectively. Banking sector investors are subject to non risk-based capital requirements under the original Basel and Basel III Leverage Ratio regulations. The new Basel III accord also introduces global liquidity standards (in addition to capital requirements) aiming at improving banks' liquidity management during periods of high market distress; see, e.g., Strahan (2012). Historically, one famous example is the default of Carlyle Capital Corporation, a large hedge fund which went into bankruptcy on March 5, 2008, after failing to meet several margin calls on their USD 22bn fixed income portfolio, which quickly depleted its liquidity. Similarly, during the week of August 6, 2007, a number of quantitative long/short equity hedge funds experienced unprecedented losses triggered by a series of margin calls which led to fire sales and additional liquidity shocks across different trading strategies; see, e.g., Khandani and Lo (2007).

Interestingly, we find that while local illiquidity measures feature a strong common component, these measures also exhibit significant idiosyncratic variation that is not shared globally, with countries regularly moving in and out of the more illiquid group. For example, a major increase in the German and UK indicators following the British Pound withdrawal from the Exchange Rate Mechanism in September 1992 leaves the US measure largely unaffected. This provides us with more power to test the impact of illiquidity on asset returns, exploiting both its global effect and, importantly, its differences across countries.

Starting with the global effect, the theory predicts that higher average illiquidity across countries implies a higher intercept and a lower slope of the average international security market line (SML). This happens because capital-constrained investors value securities with higher exposure to the global market factor. We find strong support for this prediction of the model in international stock returns data. For instance, comparing the lowest to the highest illiquidity quintile, the intercept rises from 0.19% to 0.51% per month (their difference is statistically different from zero with a t-statistic of 1.77), while the slope flattens from 0.17% to 0.01% (the difference has a t-statistic of 3.71). When we control for other determinants of the SML, such as the size or book-to-market factor, we find the results to remain qualitatively the same.

The theory also predicts that cross-country differences in illiquidity imply a difference in risk-adjusted returns that compensates investors for the capital they have to commit to maintain their positions. More precisely, holding the beta of a security constant, its alpha increases in local illiquidity. We verify this pattern in the cross-section of illiquidity- and beta-sorted portfolios of international stocks. For example, for high beta stocks, the alpha increases from 0.40% to 0.52% per month from low to high illiquidity stocks, the difference which is 0.13% is statistically different from zero with a t-statistic of 1.97. Similarly, the annualized Sharpe ratio jumps from 0.28 to 0.37.

We proceed to test this implication further by looking at the performance of self-financing market-neutral portfolios that are constructed to take advantage of the illiquidity alpha.[3] First, betting-against-beta (BAB) strategies that exploit constrained

[3]These strategies would represent a genuine trading opportunity only for unconstrained investors who do not require a compensation for the shadow cost of the capital constraint.

investors' preference for high-beta assets should perform significantly better in more illiquid countries. We verify that the portfolio implementing the BAB strategy in countries that have high illiquidity in a given period outperforms the portfolio doing the same in low illiquidity countries by 0.74% per month with an associated t-statistic of 4.48. Second, a trading strategy that is long high illiquidity-to-beta-ratio stocks and short low illiquidity-to-beta-ratio stocks globally (betting-against-illiquidity, or BAIL) outperforms the global BAB strategy that does not take the difference in illiquidity across countries into account.

Since funding conditions could be correlated with market illiquidity, we control for the effect of the latter by orthogonalizing our illiquidity indicators with respect to the Amihud (2002) stock market illiquidity measure.[4] Using the orthogonalized indicators, we find our theoretical predictions still confirmed in the data.

There exists a large theoretical literature that studies how funding constraints affect asset prices. Our work is closest to Gârleanu and Pedersen (2011) and Frazzini and Pedersen (2013). Gârleanu and Pedersen (2011) show that deviations of the Law of One Price can arise between assets with the same cash flows but different margins. Frazzini and Pedersen (2013) model an economy where margin-constrained agents invest in more risky assets which causes their returns to decline and explains the performance of the betting-against-beta strategy. Relative to these two papers, we build an international asset pricing model that motivates our use of a novel country- and global measure of the tightness of funding constraints and in which the effect of these constraints on asset prices can be different across countries.

Our work also speaks to Miranda-Agrippino and Rey (2015) who argue that a global factor related to the constraints of leveraged global banks and asset managers explains the high degree of international stock return comovement. Relative to their work, we measure and study the asset pricing implications of both the global level of illiquidity and its differences across countries.

[4]For a theoretical link between funding and market liquidity, see, e.g., Brunnermeier and Pedersen (2009). For the US stock market, Chen and Lu (2015) find correlations of 17% to 24% between their measure of funding conditions and various market liquidity proxies.

Karolyi, Lee, and van Dijk (2012) study commonality in market liquidity for 40 different stock markets and ask whether the time variation in commonality is mainly driven by liquidity supply by financial intermediaries or liquidity demand by institutional investors. Similar to these authors, we do not find a very strong link between market liquidity and funding conditions. Our contribution shows that funding conditions have an important effect on international stock returns, even after controlling for market liquidity.

This paper is also related to several other papers that study liquidity in an international context. Amihud, Hameed, Kang, and Zhang (2015) measure market illiquidity premia in 45 different countries and find that a portfolio long illiquid stocks and short liquid stocks earns more than 9% per year even when controlling for different global risk factors. Goyenko and Sarkissian (2014) show that market liquidity of US Treasuries predicts global stock returns. Bekaert, Harvey, and Lundblad (2007) investigate different definitions of liquidity risk and assess their pricing ability for emerging market portfolios. Motivated by Acharya and Pedersen (2005), Bekaert, Harvey, and Lundblad (2007) and Lee (2011) study how liquidity risk is priced in the cross-section of different stock returns. Different from these papers, we focus on funding illiquidity, for which we construct new measures, and study its direct effect on expected returns, rather than its role as a risk factor.

The rest of the paper is organized as follows. Section 1 describes the model and derives its predictions. Section 2 describes the data and the construction of the illiquidity proxies. Section 3 analyzes the illiquidity measures, while Section 4 presents our empirical results. Finally, Section 5 concludes. All proofs are deferred to the Appendix. Additional results are available in an Online Appendix.

1 Model

In this section we build a parsimonous international asset pricing model to guide our empirical analysis. Throughout the rest of the paper, we index time by t, investors by i, countries by j, stocks by k, and bonds by h. To simplify the notation, we assume the information about the corresponding country is already contained in indices k and h, and hence only emphasize the country index j when it is explicitly needed.

1.1 Assumptions

Assets. Time is continuous and goes from zero to infinity. Uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$, on which is defined a standard $(N+1)$-dimensional Brownian motion $\left(B_{r,t}, B_t^\top \equiv (B_{t,1}, B_{t,2}, ..., B_{t,N})\right)^\top$, $t \in [0, \infty)$, with $B_{r,t}$ being independent of B_t.

We consider a world economy with a set of countries $\mathcal{J}$. In each country $j \in \mathcal{J}$ there exist a set of stocks $k \in \mathcal{K}_j$ and a set of zero-coupon bonds $h \in \mathcal{H}_j$ as financial assets. We denote the set of all stocks by $\mathcal{K} \equiv \cup_{j \in \mathcal{J}} \mathcal{K}_j$ and the set of all bonds by $\mathcal{H} \equiv \cup_{j \in \mathcal{J}} \mathcal{H}_j$. At date t, stock $k \in \mathcal{K}$ is in supply $\theta_t^k > 0$, pays a dividend D_t^k in the unique consumption good, which we assume to be driven by B_t, and its ex-dividend price is denoted by P_t^k.

Bonds are assumed to be in zero net supply. For each $h \in \mathcal{H}$, we denote the time t price of the zero-coupon bond h paying one dollar at maturity $t + \tau_h$ by Λ_t^h, and its yield by $y_t^h = -\frac{1}{\tau_h} \log \Lambda_t^h$. Agents also have access to a global riskless asset with instantaneous return (i.e., short rate) r_t given exogenously, only driven by $B_{r,t}$.[5] Finally, we assume that purchasing power parity holds and all prices are expressed in US dollars.

Agents. Stocks and bonds are held by two types of investors: buy-and-hold agents and optimizing financial institutions. We assume that buy-and-hold agents have exogenously given demand d_t^h for bond h at time t, and do not trade stocks. We understand them as the source of the net supply of bonds coming from the domestic and international official sector, preferred-habitat investors, or noise traders.[6]

Financial institutions are competitive and have mean-variance preferences over the instantaneous change in the value of their portfolios. We have in mind investors such

[5] The independence assumption on $B_{r,t}$ and B_t is to simplify our derivation, and can be relaxed without qualitative changes to our results. The assumption that the short rate r_t only depends on a one-dimensional Brownian motion implies that bond yields in the model have one common factor. The model could easily accommodate several (e.g., three) yield factors, without adding any additional economic message to our framework.

[6] These agents play a role similar to preferred-habitat investors in the term structure model of Vayanos and Vila (2009). Similar to that paper, we could allow buy-and-hold agents to have a downward-sloping demand curve for bonds, but this would not change qualitatively our results.

as investment banks, hedge funds, and fund managers, who trade actively in the international stock and bond markets, and act as marginal investors there in the short- to medium-run. In short, we refer to them as investors.

Investor $i \in \mathcal{I}$, born with wealth $W_{i,t} \geq 0$, can invest in all assets of the world economy. If $x_{i,t}^k$ and $z_{i,t}^h$ denote the dollar amount investor i holds in stock k and bond h at time t, respectively, her budget constraint is

$$dW_{i,t} = \left(W_{i,t} - \int\limits_{k \in \mathcal{K}} x_{i,t}^k dk - \int\limits_{h \in \mathcal{H}} z_{i,t}^h dh \right) r_t dt + \int\limits_{k \in \mathcal{K}} x_{i,t}^k \frac{D_t^k dt + dP_t^k}{P_t^k} dk + \int\limits_{h \in \mathcal{H}} z_{i,t}^h \frac{d\Lambda_t^h}{\Lambda_t^h} dh. \tag{1}$$

We further assume that investor i's portfolio holdings in risky securities, which include all stocks and bonds, have to satisfy the following constraint:

$$\int\limits_{k \in \mathcal{K}} m_{i,t}^k \left| x_{i,t}^k \right| dk + \int\limits_{h \in \mathcal{H}} m_{i,t}^h \left| z_{i,t}^h \right| dh \leq W_{i,t}. \tag{2}$$

The constraint implies that investing in or shorting securities requires investor i to commit the amount of her capital equal to the multiple $m_{i,t}^k$ (or $m_{i,t}^h$) of the position size. Thus, investor i maximizes the mean-variance objective

$$\max_{\left\{x_{i,t}^k\right\}_{k \in \mathcal{K}}, \left\{z_{i,t}^h\right\}_{h \in \mathcal{H}}} \mathrm{E}_t \left[dW_{i,t} \right] - \frac{\alpha}{2} \mathrm{Var}_t \left[dW_{i,t} \right] \tag{3}$$

subject to (2), where α is her risk-aversion coefficient.

1.2 Equilibrium

We write stock and bond price dynamics in the forms

$$dP_t^k = \left(\mu_{P,t}^k P_t^k - D_t^k \right) dt + P_t^k \sigma_{P,t}^{k\top} dB_t \tag{4}$$

and

$$d\Lambda_t^h = \Lambda_t^h \left(\mu_{\Lambda,t}^h dt - \sigma_{\Lambda,t}^h dB_{r,t} \right), \tag{5}$$

respectively, where $\mu^k_{P,t}$, $\mu^h_{\Lambda,t}$, and $\sigma^h_{\Lambda,t}$ are one-dimensional variables, and $\sigma^k_{P,t}$ is an N-dimensional vector. Substituting (4) and (5) into (1) and then into (3), and denoting the Lagrange multiplier of constraint (2) by $\psi_{i,t}$, we obtain the following result:

Lemma 1. *The first-order conditions of agent i imply*

$$\mu^k_{P,t} - r_t = \alpha\sigma^{k\top}_{P,t} \int\limits_{k\in\mathcal{K}} x^k_{i,t}\sigma^k_{P,t}dk + \psi_{i,t}m^k_{i,t}sgn\left(x^k_{i,t}\right) \tag{6}$$

and

$$\mu^h_{\Lambda,t} - r_t = \alpha\sigma^h_{\Lambda,t} \int\limits_{h\in\mathcal{H}} z^h_{i,t}\sigma^h_{\Lambda,t}dh + \psi_{i,t}m^h_{i,t}sgn\left(z^h_{i,t}\right), \tag{7}$$

where $sgn\,(x) = \pm 1$ if $x \gtrless 0$ and $sgn\,(x) \in [-1, 1]$ if $x = 0$.

Equation (6) states that the expected excess return investors require when investing in stock k, $\mu^k_{P,t} - r_t$, consists of two terms. The first term is a compensation for the risk of stock k: it is proportional to the return volatility, $\sigma^k_{P,t}$, and the proportionality term depends on the aggregate amount of B_t risk investor i takes, $\int_{k\in\mathcal{K}} x^k_{i,t}\sigma^k_{P,t}dk$. The second term depends on how constrained investor i is: it is zero if the constraint does not bind ($\psi_{i,t} = 0$); otherwise it depends on whether investor i is long or short stock k, captured by $sgn\left(x^k_{i,t}\right)$, and is larger in absolute terms for assets with higher capital requirement $m^k_{i,t}$. Intuitively, being long one dollar in a stock with higher requirement ties down more capital of the investor, who then requires a higher compensation on this asset. Similarly, (7) states that the expected excess return investors require when investing in bond h, $\mu^h_{\Lambda,t} - r_t$, increases with the volatility of bond h, $\sigma^h_{\Lambda,t}$, and the aggregate interest rate risk agent i takes, $\int_{h\in\mathcal{H}} z^h_{i,t}\sigma^h_{\Lambda,t}dh$. Moreover, as long as the capital constraint of agent i binds, she also requires a higher compensation on bonds with higher margin, $m^h_{i,t}$.

Since financial institutions have to take the other side of the trade in stock and bond markets, the market-clearing conditions for stocks are given by

$$\int\limits_{i\in\mathcal{I}} x^k_{i,t}di = \theta^k_t, \tag{8}$$

and for bonds by

$$\int_{i\in\mathcal{I}} z_{i,t}^{h} + d_{t}^{h} = 0, \tag{9}$$

for all t, k, and h.

Aggregating (6) across all investors $i \in \mathcal{I}$ and imposing market-clearing condition (8), we get

$$\mu_{P,t}^{k} - r_t = \alpha\sigma_{P,t}^{k\top} \int_{k\in\mathcal{K}} \theta_t^k \sigma_{P,t}^{k} dk + \int_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^{k} sgn\left(x_{i,t}^{k}\right) di. \tag{10}$$

Let us define a global market index, G_t, that is the dollar-supply weighted average of all stocks; the value and the dynamics of this global index, respectively, are given by

$$G_t = \int_{k\in\mathcal{K}} \theta_t^k dk \text{ and } dG_t = \int_{k\in\mathcal{K}} \theta_t^k \frac{D_t^k dt + dP_t^k}{P_t^k} dk. \tag{11}$$

Combining (10) and (11), after some algebra, we obtain the following result:

Theorem 1. *The equilibrium expected excess return of security k is*

$$\mu_{P,t}^{k} - r_t = \beta_{P,t}^{k}\lambda_t + \phi_t^k - \beta_{P,t}^{k}\phi_t^G, \tag{12}$$

with

$$\phi_t^k = \int_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^{k} sgn\left(x_{i,t}^{k}\right) di \text{ and } \phi_t^G = \int_{k\in\mathcal{K}} \phi_t^k \frac{\theta_t^k}{\int_{k\in\mathcal{K}} \theta_t^k dk} dk, \tag{13}$$

where $\beta_{P,t}^{k} = Cov_t\left[\frac{D_t^k dt + dP_t^k}{P_t^k}, \frac{dG_t}{G_t}\right] / Var_t\left[\frac{dG_t}{G_t}\right]$ is the beta of security k returns with respect to the global market portfolio return, and $\lambda_t = \mu_{G,t} - r_t$ is the expected excess return of the global market portfolio.[7]

[7]The term ϕ_t^G appears in (12) because excess returns on the global market portfolio are themselves in part driven by the compensation for the constraint.

Next we look at the equilibrium prices of bonds.[8] As in the case of stocks, aggregating (7) across all investors $i \in \mathcal{I}$ and imposing market-clearing condition (9), we obtain:

$$\mu_{\Lambda,t}^{h} - r_t = -\alpha\sigma_{\Lambda,t}^{h} \int_{h\in\mathcal{H}} d_t^h \sigma_{\Lambda,t}^{h} dh + \int_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^{h} sgn\left(z_{i,t}^{h}\right) di. \tag{14}$$

In the Appendix we characterize zero-coupon bond prices in the general case, derived from (14) without further assumptions. For the purpose of this paper, however, it is more illustrative to consider a special case in which a closed-form solution is available:

Theorem 2. *Suppose the dynamics of the short rate r_t under the physical probability measure are given by the mean-reverting process*

$$dr_t = \kappa\left(\bar{r} - r_t\right) dt + \sigma dB_{r,t},$$

where κ, $\bar{r}$, and σ are all positive constants. Moreover, suppose that $m_{i,t}^{h}$, d_t^h, $m_{i,t}^{k}$, θ_t^k, and $W_{i,t}$ are constant over time but can vary across investors and assets. Then the equilibrium yield on bond h is given by

$$y_t^h = \mathcal{A}\left(\tau_h\right) + \mathcal{B}\left(\tau_h\right) r_t + \mathcal{C}_h\left(\tau_h\right),$$

where the first two terms describe a standard affine yield curve that depends only on the maturity of a bond and the short-rate factor r_t, and

$$\mathcal{C}_h\left(\tau_h\right) = \frac{1}{\tau_h} \int_{h'<h} \int_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^{h'} sgn\left(z_{i,t}^{t+\tau_{h'}}\right) di dh' \tag{15}$$

represents a deviation from this smooth yield curve, specific to bond h.[9]

[8] Due to the assumptions, our setting allows us to discuss stock and bond pricing separately, but it could easily accommodate a non-zero correlation between the risk of stocks and bonds. In this case, the combined stock and bond global portfolio becomes the relevant market portfolio. Zero correlation allows us to follow Frazzini and Pedersen (2013), who use asset-class-specific market portfolios. In either case, qualitative predictions regarding the effect of illiquidity on excess returns are the same.

[9] In the general case, when fundamentals are not constant over time, $\mathcal{C}_h\left(\tau_h\right)$ depends on the current and expected future values of $\psi_{i,t}$, $m_{i,t}^{h}$, and $sgn\left(z_{i,t}^{t+\tau_h}\right)$; see Lemma 2 of the Appendix.

1.3 Discussion and Predictions

The model above provides us with a minimal setting that introduces a friction into investors' myopic risk-return tradeoff in both stock and bond markets. The friction takes the form of a capital constraint similar to Gârleanu and Pedersen (2011), He and Krishnamurthy (2012), and Frazzini and Pedersen (2013).[10]

Our focus is on the effect of such constraints at the country level. In the model, illiquidity conditions can differ across countries for two reasons. Capital requirements $m_{i,t}^k$ and $m_{i,t}^h$ are asset- and investor-specific. In particular, they have a country-specific component and can be higher for foreign investors. In the latter case, a constrained investor located in a given country will have a different effect on the illiquidity component of expected returns domestically and internationally, as can be seen from (10) and (14). Cross-border flows may require more capital because such investments involve a higher degree of intermediation. Also, the effect of higher cross-border capital requirements is isomorphic to that of barriers to international investments studied in Stulz (1981), Bhamra, Coeurdacier, and Guibaud (2014), and Gârleanu, Panageas, and Yu (2015).[11]

We abstract away from exchange rate risk by imposing purchasing power parity; see, e.g., Bekaert, Harvey, and Lundblad (2007) who make a similar assumption. Thus, from investors' perspective, countries differ only in the capital required to fund positions there. In practice, however, the costs of hedging in foreign exchange markets and exchange rate movements themselves can also depend on funding conditions (see, e.g., Ivashina, Scharfstein, and Stein (2015) and Mancini, Ranaldo, and Wrampelmeyer (2013), respectively), and thus could be indirectly captured in our analysis.

Based on Theorems 1 and 2, we derive five predictions that link illiquidity phenomena in international bond and stock markets. First, noise in the yield curve is informative about illiquidity conditions in a given country.

[10] A model with non-myopic agents can deliver further predictions: In addition to taking the current capital constraint into account in their optimization, investors will want to hedge against the future states where the constraint will bind; see, e.g., Kondor and Vayanos (2015) and Malamud and Vilkov (2015). Testing whether international illiquidity, in addition to its myopic effect on asset returns, is also a priced risk factor is an interesting extension that we leave for future research.

[11] At the same time, our setting is different from the market segmentation model of Errunza and Losq (1985), where foreign investors are prohibited from taking any position in a subset of domestic stocks.

Proposition 1. *In each country there exists a smooth theoretical yield curve, but bond yields can be off the curve. Everything else being equal, deviations are larger in countries where investors are more constrained, i.e., illiquidity is higher.*

Proposition 1 follows from Theorem 2. In particular, (15) shows that the deviation of bond h from a standard affine yield curve depends on the capital required to invest in this bond and the position (long or short) that investors take in it. For instance, bonds for which buy-and-hold investors have a short (long) net position will tend to be more expensive (cheap) compared to the frictionless benchmark yield curve to compensate constrained investors for the capital they have to commit to take the other side of the trade.[12] Everything else being equal, the average magnitude of such deviations across all bonds in country j is higher if it is more difficult to fund positions in this country, and if investors are more constrained. We refer to this average effect of capital constraints at the country level as local illiquidity, and proxy for them by the average yield curve deviation.

Next, we look at the effect of country-level and average global illiquidity on expected stock returns. Similarly to the bond market, equation (13) shows that the terms ϕ_t^k capturing the effect of constraints on stock returns depend on capital requirements and shadow prices of capital $\psi_{i,t}$. Because constrained investors trade in both stocks and bonds, the latter are the same in both markets. In addition, we think of capital requirements as having a country-level component that affects the funding of positions in all securities in a given country. Thus, ϕ_t^k is related to our local illiquidity indicators, derived from bond yields, while ϕ_t^G to their global average.

Proposition 2. *There is an* average *global security market line (SML) with slope decreasing in global illiquidity and intercept increasing in global illiquidity.*

Proposition 2 is a simple rearrangement of (12) in Theorem 1:

$$\mu_{P,t}^k - r_t = \underbrace{\phi_t^G}_{\text{average intercept}} + \beta_t^k \underbrace{\left(\lambda_t - \phi_t^G\right)}_{\text{slope of SML}} + \underbrace{\left(\phi_t^k - \phi_t^G\right)}_{\text{country effect}},$$

[12] For instance, as reported in the FR 2004 statistical reports by the Federal Reserve Bank of New York, primary dealers in the US Treasuries market hold long positions in some maturities and short positions in others.

where the dollar-supply weighted average of security-specific terms $\phi_t^k - \phi_t^G$ is zero by construction. Equation (12) also yields:

Proposition 3. *Holding illiquidity constant, a higher beta means lower alpha. Because securities can lie off the security market line due to the asset-specific term ϕ_t^k, holding beta constant, the alpha increases in the local illiquidity.*

Alphas with respect to the global market, $\phi_t^k - \beta_{P,t}^k \phi_t^G$, arise because constrained investors pay a premium for high beta stocks that allow them to get a higher exposure to the global market factor per unit of capital. For the same reason, investors require additional compensation for securities that are difficult to use as collateral or to bypass intermediation barriers to international investments. The combination of these two effects characterize the distribution of risk-adjusted returns across securities. We focus on the country-level component of ϕ_t^k, that we can refer to as local illiquidity and denote by ϕ_t^j, for which average bond price deviations in that country provide a good signal.

Finally, we derive two propositions regarding the self-financing market-neutral portfolios constructed to insulate and take advantage of these risk-adjusted returns. We think about the performance of these strategies as the extra gain to an investor who does not face funding constraints and is small enough not to affect prices.

Proposition 4. *Everything else being equal, the expected excess return of a self-financing market-neutral portfolio that is long in low-beta securities and short in high-beta securities of country j, with the appropriate leverage applied to the two legs (betting-against-beta or BAB), is positive and increasing in country-specific illiquidity.*

Proposition 4 states that the BAB portfolio, proposed by Frazzini and Pedersen (2013), performs better in countries where investing is more difficult to fund. To see this, consider two portfolios composed of country-j securities with respective average betas $\beta^{HB} > \beta^{LB}$. The excess return on the BAB portfolio in country j is then

$$\mu_t^{j,BAB} - r_t = \frac{1}{\beta^{LB}} \left[\beta^{LB}\lambda_t + \phi_t^j - \beta^{LB}\phi_t^G\right] - \frac{1}{\beta^{HB}} \left[\beta^{HB}\lambda_t + \phi_t^j - \beta^{HB}\phi_t^G\right] = \phi_t^j \left[\frac{1}{\beta^{LB}} - \frac{1}{\beta^{HB}}\right],$$

which is positive by assumption, and, keeping betas constant, increases in ϕ_t^j.

Proposition 5. *The expected excess return of a self-financing market-neutral portfolio that is long in high illiquidity-to-beta ratio securities and short in low illiquidity-to-beta ratio securities (betting-against-illiquidity or BAIL) is positive and higher than the expected return on a similar long-short trading strategy that ignores sorting on illiquidity.*

Proposition 5 states that taking into account the difference in country-level illiquidity could improve on the performance of the global BAB portfolio. Formally, consider two global portfolios with respective average betas β^H and β^L and illiquidities ϕ_t^H and ϕ_t^L. Creating a long-short portfolio of them, in which the leverage applied to the two legs are inversely proportional to betas, has an excess return of

$$\mu_t^{H-L} - r_t = \frac{1}{\beta^H}\left[\beta^H \lambda_t + \phi_t^H - \beta^H \phi_t^G\right] - \frac{1}{\beta^L}\left[\beta^L \lambda_t + \phi_t^L - \beta^L \phi_t^G\right] = \frac{\phi_t^H}{\beta^H} - \frac{\phi_t^L}{\beta^L}.$$

If $\phi_t^H/\beta^H > \phi_t^L/\beta^L$, the excess return on this risk-neutral BAIL portfolio is positive, and increases in the difference of the illiquidity-to-beta ratios.

In the following, we take our main model predictions to the data. To this end, we first introduce model-implied illiquidity proxies for six different countries.

2 Data

In this section we describe our data and the construction of country-level and global illiquidity measures.

2.1 Bond Data

We collect raw data on government bonds and stock return data from Datastream. The frequency is daily, running from 1 January 1990 to 31 December 2012, leaving us with 6,001 observations in the time-series.

The bond data spans six different countries: the United States, Germany, the United Kingdom, Canada, Japan, and Switzerland.[13] We obtain a daily cross-section of end-of-

[13]The country choice is driven by two main factors: data availability and credit risk considerations. For example, while there is enough data available on some Eurozone countries, these sovereign bonds feature quite a large credit risk component, especially after 2008 (see, e.g., Pelizzon, Subrahmanyam, Tomio, and Uno (2016)), an aspect absent from our model.

day bond prices for our sample period for all available maturities. We use mid prices to avoid any discrepancies between the prices of similar bonds due to the bid-ask spread.

Furthermore, we collect information on accrued interest, coupon rates and dates, and issue and redemption. Following Gürkaynak, Sack, and Wright (2007), we apply several data filters in order to obtain securities with similar liquidity and avoiding special features. The filters can vary by country, but in general they are as follows: (i) We exclude bonds with option like features such as bonds with warrants, floating rate bonds, callable and index-linked bonds. (ii) We consider only securities with a maturity of more than one year at issue (this means that, for example, for the US market we exclude Treasury bills). We also exclude securities that have a remaining maturity of less than three months to alleviate concerns that segmented markets may significantly affect the short-end of the yield curve.[14] Moreover, short-maturity bonds are not very likely to be affected by arbitrage activity, which is the objective of our paper. (iii) We exclude bonds with a remaining maturity of 15 years or more as in an international context they are often not very actively traded (see, e.g., Pegoraro, Siegel, and Tiozzo 'Pezzoli' (2013)). (iv) For the US we exclude the on-the-run and first-off-the-run issues for every maturity. These securities often trade at a premium to other Treasury securities as they are generally more liquid than more seasoned securities (see, e.g., Fontaine and Garcia (2012)). Other countries either do not have on-the-run and off-the-run bonds in the strict sense, as they for example reopen existing bonds to issue additional debt, or they do not conduct regular auctions as the US Treasury does. We therefore do not apply this filter to the international sample. (v) Additionally, we exclude bonds if the reported prices are obviously wrong. While the data quality for the US is reasonably good, there are a lot of obvious pricing errors in the international bond sample, which requires substantial manual data cleaning.

Panel A of Table 1 provides details of our international bond sample. We note that on average we have 71 bonds every day to fit the yield curve and 60 bonds to construct the illiquidity measure. Japan and the US are the most active markets, while the average number of bonds in Switzerland and the UK are lower. The cross-section

[14] Duffee (1996), for example, shows that Treasury bills exhibit a lot of idiosyncratic variation and have become increasingly disconnected from the rest of the yield curve.

varies considerable over time: During the years 2001 and 2007, the number of bonds available dropped considerably for all countries except Japan, which was a response to the banking crisis in the years 2000.

[Insert Table 1 here.]

2.2 Stock Data

To assess the asset pricing implications of our proxies of illiquidity, we collect daily stock returns, volume, and market capitalization data for the six countries from Datastream. The initial sample covers more than 10,000 stocks. We only select stocks from major exchanges, which are defined as those in which the majority of stocks for a given country are traded. We exclude preferred stocks, depository receipts, real estate investment trusts, and other financial assets with special features based on the specific Datastream type classification. To limit the effect of survivorship bias, we include dead stocks in the sample. We exclude non-trading days, defined as days on which 90% or more of the stocks that are listed on a given exchange have a return equal to zero. We also exclude a stock if the number of zero-return days is more than 80% in a given month. Excess returns are calculated versus the US Treasury bill rate and the proxy for the global market is the MSCI world index. Panel B of Table 1 reports summary statistics.

We follow Frazzini and Pedersen (2013) to construct ex-ante betas for our dataset of international stocks from rolling regressions of daily excess returns on market excess returns. The estimated beta for stock k at time t is given by:

$$\hat{\beta}^k_{t,\mathrm{TS}} = \hat{\rho}^k_t \frac{\hat{\sigma}^k_t}{\hat{\sigma}^G_t},$$

where $\hat{\sigma}^k_t$ and $\hat{\sigma}^G_t$ are the estimated volatilities for the stock and the market and $\hat{\rho}^k_t$ is their correlation. Volatilities and correlations are estimated separately. First, we use a one-year rolling standard deviation for volatilities and a five-year horizon for the correlation to account for the fact that correlations appear to move more slowly than volatilities. To account for non-synchronous trading, we use one-day log returns to estimate volatilities and three-day log returns for correlation. Finally, we shrink the time-series estimate of the beta towards the cross-sectional mean ($\beta^k_{t,\mathrm{CS}}$) following Vasicek (1973):

$$\hat{\beta}^k_t = \omega \hat{\beta}^k_{t,\mathrm{TS}} + (1-\omega)\,\hat{\beta}^k_{t,\mathrm{CS}},$$

where we set $\omega = 0.6$ for all periods and all stocks, in line with Frazzini and Pedersen (2013).

2.3 Country-Level Illiquidity Proxies

To construct country-specific illiquidity measures, we follow Hu, Pan, and Wang (2013) who employ the Svensson (1994) method to fit the term structure of interest rates.[15]

The Svensson (1994) model assumes that the instantaneous forward rate is given by

$$f_{m,b} = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right),$$

where m denotes the time to maturity and $b = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ are parameters to be estimated. By integrating the forward rate curve, we derive the zero-coupon spot curve:

$$\begin{aligned} s_{m,b} \;=\;& \beta_0 + \beta_1 \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} \\ &+\beta_2 \left(\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} - \exp\left(-\frac{m}{\tau_1}\right)\right) \\ &+\beta_3 \left(\left(1 - \exp\left(-\frac{m}{\tau_2}\right)\right)\left(\frac{m}{\tau_2}\right)^{-1} - \exp\left(-\frac{m}{\tau_2}\right)\right). \end{aligned}$$

A proper set of parameter restrictions is given by $\beta_0 > 0, \beta_0 + \beta_1 > 0$, $\tau_1 > 0$, and $\tau_2 > 0$. For long maturities, the spot and forward rates approach asymptotically β_0, hence the value has to be positive. $(\beta_0 + \beta_1)$ determines the starting value of the curve at maturity zero. (β_2, τ_1) and (β_3, τ_2) determine the humps of the forward curve. The hump's magnitude is given by the absolute size of β_2 and β_3 while its direction is given by the sign. Finally, τ_1 and τ_2 determine the position of the humps.

[15] We also use the Nelson and Siegel (1987) and a cubic spline method. All three approaches lead to qualitatively very similar results. We chose the Svensson (1994) method over the other two as it is the most widely used and also the most flexible.

To estimate the set of parameters b_t^j for each country j and day t, we minimize the weighted sum of the squared deviations between actual and model-implied prices:

$$b_t^j = \arg\min_b \sum_{h=1}^{H_t^j} \left[\left(P^h(b) - P_t^h \right) \times \frac{1}{D_t^h} \right]^2,$$

where H_t^j denotes the number of bonds available in country j on day t, $P^h(b)$ is the model-implied price for bond $h = 1, ..., H_t^j$, P_t^h is its observed bond price, and D_t^h is the corresponding Macaulay duration. We verify that our yield curve estimates are reasonable by comparing our term structures with the estimates published by central banks or the international yield curves used in Wright (2011) and Pegoraro, Siegel, and Tiozzo 'Pezzoli' (2013).[16]

The illiquidity measure for country j is then defined as the root mean square error between the model-implied yields and the market yields, i.e.,

$$\text{Illiq}_t^j = \sqrt{\frac{1}{H_t^j} \sum_{h=1}^{H_t^j} \left[y^h\left(b_t^j\right) - y_t^h \right]^2},$$

where $y^h\left(b_t^j\right)$ is the model-implied yield corresponding to bond h and y_t^h is the market yield.

While we calculate the term structure using a wide range of maturities, we calculate the measure only using bonds with maturities ranging between one and ten years. Similar to Hu, Pan, and Wang (2013), we also apply data filters to ensure that the illiquidity measures are not driven by single observations. In particular, we exclude any bond whose associated yield is more than four standard deviations away from the model yield.

To get a measure of global illiquidity, we construct a market-capitalization weighted average of each country-level illiquidity measure in line with our theoretical model proposed in the previous section:

$$\text{Illiq}_t^G = \frac{1}{\text{total market cap}} \sum_{j=1}^{6} \text{market cap}_t^j \times \text{Illiq}_t^j.$$

[16]We thank Fulvio Pegoraro and Luca Tiozzo 'Pezzoli' for sharing their codes.

We note that the unconditional correlation between the weighted average and the first principal component of our country-level illiquidity is 95%, and using either of them leads to very similar results.

2.4 Other Illiquidity Proxies

As alternative proxies for illiquidity or capital constraints, we also consider the TED spread and the volatility index VIX. The TED spread is defined as the difference between the three-month Eurodollar LIBOR rate and the three-month US Treasury bill rate. The VIX is obtained from CBOE, the LIBOR and Treasury bill rates are from Datastream.

We also compare our proxies to the Amihud (2002) market liquidity measure. We construct country-level Amihud liquidity measures using our international stock data set. In line with the literature, we add a constant to the Amihud measure and take logs to reduce the impact of outliers. The stock-level measure is defined as:

$$\text{Amihud}_t^k = \log\left(1 + \sum_{t=1}^{T} \frac{|r_t^k|}{\text{vol}_t^k}\right),$$

where $|r_t^k|$ is the absolute return of stock k on day t, vol_t^k is the trading volume in the local currency of stock k on day t, obtained by multiplying the number of shares traded by the closing price, and T corresponds to the length of the window. Similar to Karolyi, Lee, and van Dijk (2012), we calculate Amihud_t^k for each stock based on daily data over a non-overlapping three-month rolling window. We first restrict the sample to stocks from major exchanges, except for Japan, where we use data from two exchanges (Osaka and Tokyo). We require that a stock has at least 10 valid daily observations (return and volume) during the three months. We delete stock days where the trading volume is below USD 100 and remove extreme observations manually. We use data from 1990 onwards, except for Germany, where we use data after 1999, because the daily trading volume is not available for most German stocks before that date. To get a country-level measure, we take a market capitalization weighted average for each stock.

3 Illiquidity Facts

In this section we document the key time series and cross-sectional properties of our illiquidity measures, and compare them to other market stress indicators.

3.1 Properties of Illiquidity

The time-series of all country-specific illiquidity measures, in basis points, are plotted in Figure 1. In Panel A of Table 2, we report summary statistics. Overall, the average pricing errors are quite small, ranging from 2.8 basis points (bp) for the US to 6.2 bp for Switzerland. The larger pricing errors also come with an overall larger volatility that ranges from 4.5 bp (Switzerland) to 1.37 bp (US). To put the magnitude of these pricing deviations into economic perspective, we can compare them to the bid-ask spreads. In the US Treasuries market, for which we have detailed bid-ask spread studies, pricing errors are larger than the bid-ask spread on average, and in particular are several times larger during illiquidity spikes. For instance, Engle, Fleming, Ghysels, and Nguyen (2013) report spreads for five-year Treasuries that do not exceed 1 bp in normal times and 2 bp at the height of the crisis in 2009.

One might suspect that in the time-series countries which tend to be more illiquid remain illiquid in the cross-section over long periods of time. To this end, we sort each country-level illiquidity proxy at the end of each month into three different bins, depending on their illiquidity: The low (high) illiquidity bin contains the countries which are in the lowest (highest) tercile of illiquidity. Panel B reports the average fraction of how many months each country-level illiquidity measure is in the low, medium, or high bin out of the 276 months (ranging from January 1990 to December 2012). We note that except for Japan and Switzerland, the frequency of being in either of the three bins is quite equally distributed for the other four countries. Japan is in the low illiquidity bin 2/3 of the time, while Switzerland is in the high illiquidity bin 77% of all times.

[Insert Table 2 and Figures 1 and 2 here.]

The time-series variation in country-level illiquidity exhibits significant commonality. Pairwise correlations between illiquidity proxies reported in Panel C of Table 2 are all positive and range between 20% (US and Japan) and 74% (Germany and Japan). Panel D of Table 2 reports loadings from the following regression:

$$\text{Illiq}_t^j = \beta_0^j + \beta_1^j \text{Illiq}_t^G + \epsilon_t^j,$$

where Illiq_t^j is the illiquidity proxy of country j and Illiq_t^G is the global illiquidity proxy. Unsurprisingly, we find that all country-level measures co-move positively with the global illiquidity factor, and that the latter explains a significant proportion of the variation in the country-level illiquidity with R^2 ranging between 39% and 66%. We note that the high unconditional correlation between country-level illiquidities is driven by a few crisis episodes. Figure 3 plots the average conditional correlation among the different illiquidity proxies, calculated on daily data using a three-year rolling window. The average correlation peaks during periods of distress such as the dotcom bubble burst or the most recent financial crisis where the correlation reaches almost 80%, but is significantly lower otherwise. We also note an upward trend in the conditional correlation that could point towards higher market integration.

[Insert Figure 3 here.]

Next, we look at the dispersion of illiquidity across countries. As can be seen from Table 2, the levels of country-level illiquidity are relatively close on average. In other words, there are no large permanent differences in illiquidity between the countries we consider. This is perhaps not surprising given that we include only developed financial markets in our analysis. However, illiquidity can become significantly dispersed when some countries experience idiosyncratic illiquidity episodes. Figure 4 illustrates the cross-sectional standard deviation of illiquidity measures.

[Insert Figure 4 here.]

Overall, illiquidity exhibits significant country-specific variation: country- or region-specific events seem to be reflected in spikes in the respective local illiquidity measures that are not shared globally. For example, the Japanese measure is highly volatile in the early 1990s, especially around the Asian crisis of 1996–1997. It displays further spikes again around the dot-com bubble burst in 2001 and during the most recent financial crisis. The German illiquidity proxy is especially volatile after 1992 and during the most recent financial crisis. The heightened level of the illiquidity proxy after 1990 can be explained by the large uncertainty surrounding the German reunification in October 1990. German interest rates had climbed relentlessly during 1991 and 1992 and then started to fall after the outbreak of the ERM crisis in September 1992 steadily through 1994. Moreover, the autumn of 1992 has witnessed massive speculative currency attacks (see, e.g., Buiter, Corsetti, and Pesenti (1998)). The repercussions of the ERM crisis are also found in the illiquidity proxies of the UK and Switzerland, where we see large jumps during the year 1992. Interestingly, these stark movements are completely absent in the US illiquidity proxy, which displays only moderate movements until 1997 (Asian crisis), except around the first Gulf War in 1991. Further illustration can be found in Figure 5 where we plot the model-implied yields together with the data for Black Wednesday (16 September 1992) both for Germany and the US. As we can see, the observed yields are far off the fitted curve in German (upper right panel), while the observed yields in the US nicely track the model-implied ones (lower left panel). Finally, the global measure is mainly characterized by four large spikes: The ERM crisis, the Asian crisis, the dot-com bubble burst, and the Lehman default.

[Insert Figure 5 here.]

Finally, we note that the global illiquidity measure is mainly characterized by four large spikes (the ERM crisis, the Asian crisis, the dot-com bubble burst, and the Lehman default) while summarizing the properties of the different country-level proxies. For example, the high volatility before 1995 can be attributed to rather Europe-specific events such as the British Pound leaving the ERM or the German elections in 1994 that were surrounded by large uncertainty. The downgrade of GM and Ford in May 2005, on the other hand, is a US specific event, which is not reflected in the other five country-level illiquidity proxies. Another noteworthy observation is that there seems to be a downward trend in global illiquidity, which intuitively points towards the fact that over time more arbitrage capital has become available and, hence, constraints are less binding.

3.2 Comparison with Other Illiquidity Measures

In the following, we compare our illiquidity measures to other market stress indicators. To save space, we report all results in the Online Appendix. As a first measure, we look at the Amihud measure as it is one of the most widely used proxies of market liquidity. In Section OA-1 of the Online Appendix, we report that the unconditional correlation between country-level stock market illiquidity measures constructed following Amihud (2002) and our illiquidity measures is positive and ranges between 10% (Germany) and 43% (US). The correlation is largely driven by the 2008 period.

There is an intimate link between funding liquidity and market volatility, and the causality of the relationship can possibly go in either direction.[17] Brunnermeier and Pedersen (2009), among others, suggest the VIX index as a proxy for funding liquidity itself. In Section OA-2 of the Online Appendix, we compare our illiquidity proxies with country-level VIX for the longest time-series available.[18] We note that overall the correlation between the time-series is quite high ranging from 49% (Japan) to 66% (Germany and Switzerland).

In addition, we also test for Granger causality between our illiquidity measures, Amihud (2002) market illiquidity, and volatility in each country. The results show only limited evidence for causality linkages between our illiquidity and Amihud market illiquidity, which is perhaps not surprising given the relatively low correlation between them.

[17]For example, Karolyi, Lee, and van Dijk (2012) find that international illiquidity measures comove more during periods of high volatility. Hedegaard (2014) finds a large effect from margins onto volatility in the commodity market. Hardouvelis (1990) and Hardouvelis and Peristiani (1992), on the other hand, argue that more stringent margins lead to lower stock market volatility in the US and in Japan, respectively. While from a policy perspective it is interesting to study how margins affect volatility, the relationship can also go the opposite direction. For options and futures, margin requirements are set based on volatility itself. For example, the Chicago Mercantile Exchange (CME) uses the so called SPAN (Standard Portfolio Analysis of Risk) method that calculates the maximum likely loss that could be suffered by a portfolio. The method consists of 16 different scenarios which are comprised of different market prices and volatility. For more information see `http://www.cmegroup.com/clearing/files/span-methodology.pdf`. Similarly, on the London Stock Exchange, the initial margin is calculated based on the maximum loss according to volatility and investors' leverage.

[18]We could not find any data on the Canadian equivalent of VIX.

We find stronger support for volatility causing both stock market and our illiquidity, as well as a reverse causality link.

Finally, in Section OA-3 of the Online Appendix, we compare our global proxy with a range of other illiquidity measures that are not available for countries other than the US. The unconditional correlation ranges between 4% (Fontaine and Garcia (2012) measure) and 65% (Goyenko, Subrahmanyam, and Ukhov (2011) proxy).

4 Empirical Results

In this section we use the illiquidity measures to test the predictions of our theory that assumes investors being capital constrained when forming their optimal portfolio.

4.1 Global Illiquidity and the Security Market Line

Proposition 2 states that the slope of the average SML should depend negatively on the tightness of global margin constraints, while the intercept is positively related to it. As a first illustration, we follow the procedure in Cohen, Polk, and Vuolteenaho (2005) and divide our monthly data sample into quintiles according to the level of global illiquidity. We then examine the pricing of beta-sorted portfolios in these quintiles and estimate the empirical SML. Figure 6 depicts the average intercept and slope of the SML for different levels of global illiquidity ranging from low illiquidity (bin 1) to high illiquidity (bin 5).

[Insert Figure 6 here.]

We note that in line with our prediction, the slope coefficient is decreasing with global illiquidity, while the intercept is increasing. For example, for low illiquidity states the average intercept is 0.191% with a slope of 0.171%, whereas for high illiquidity, the intercept increases to 0.51% and the slope decreases to 0.008%. The difference between the low and high illiquidity bin intercept is 0.32% per month which is statistically different from zero with a t-statistic of 1.77. Similarly, the difference in slope coefficients which is 0.16% is highly statistically different from zero with a t-statistic of 3.71.

Next, we study in more detail how the intercept and the slope are affected by global illiquidity risk. To this end, we consider Fama and MacBeth (1973) regressions and regress excess returns on the basis assets on a constant and the portfolios' trailing-window post-ranking beta:

$$\mathrm{rx}_t^j = \alpha_t + \phi_t \times \beta_t^j + \epsilon_t^j,$$

where rx_t^j is the excess return of the j-th β-sorted portfolio and β_t^j is the post-ranking beta of portfolio j. This gives us the time-series of the intercept α_t and the slope ϕ_t of the SML for each quintile of global illiquidity. In the second stage, we now estimate the following two regressions:

$$\begin{aligned}
\alpha_t &= a_1 + b_1 r_t^M + c_1 r_t^S + d_1 r_t^B + e_1 \mathrm{Illiq}_{t-1}^G + u_{1,t}, \\
\phi_t &= a_2 + b_2 r_t^M + c_2 r_t^S + d_2 r_t^B + e_2 \mathrm{Illiq}_{t-1}^G + u_{2,t},
\end{aligned}$$

where r_t^G, r_t^S and r_t^B are the excess returns on the global market, size, and book-to-market portfolios, respectively. While the global size and book-to-market portfolios are not accounted for in our theory, we control for these variables as it is well known that these factors have an effect on the shape of the SML as well (see, e.g., Hong and Sraer (2016)). The estimated coefficients are presented in Table 3.

[Insert Table 3 here.]

In line with our theoretical predictions, we find that global illiquidity has a positive (negative) effect on the intercept (slope) of the SML. When we only include the global market excess returns and global illiquidity, the coefficient on the intercept regression has a value of 0.008 with an associated t-statistic of 1.83 and the illiquidity coefficient for the slope regression is -0.013 with an associated t-statistic of 1.87. Adding other factors like the global size or book-to-market variables does not alter the results: The estimated coefficient for the intercept is 0.009 with a t-statistic of 2.04 and for the slope regression with find that the coefficient is -0.009 with a t-statistic of -1.70.

4.2 *Local Illiquidity and Alpha*

We now inspect how returns vary in the cross-section of illiquidity and beta-sorted stocks. Propositions 3 states that holding local illiquidity constant, a higher beta means lower alpha; holding beta constant, the alpha increases in the local illiquidity. Table 4 reports the results using our international stock data set. We consider three beta- and two illiquidity-sorted portfolios and document their average excess returns, alphas, market betas, volatilities, and Sharpe ratios. Consistent with the findings of Frazzini and Pedersen (2013), we find that alphas decline from the low-beta to the high-beta portfolio: for low (high) illiquidity stocks, the alpha decreases from 0.527% to 0.395% (0.547% to 0.522%), and similarly, Sharpe ratios drop from 0.49 to 0.28 (0.50 to 0.37). On the other hand, keeping betas constant, we find that alphas increase from the low illiquidity stocks to high illiquidity stocks. For example, the alpha for low beta stocks increases from 0.527% per month to 0.547%, for medium beta it increases from 0.471% to 0.540%, and for high beta stock it increases from 0.395% to 0.522%.

[Insert Table 4]

Proposition 4 (building on Proposition 3) states that, everything else being equal, a BAB strategy should perform better in countries with higher local illiquidity. In order to test this proposition we construct a BAB strategy within each country, and then sort in each month the country-level BAB strategies into high and low illiquidity bins. The summary statistics of the two trading strategies are reported in Table 5.

[Insert Table 5 here.]

We find that the high-illiquidity BAB portfolio produces significantly higher excess returns than a corresponding low-illiquidity BAB portfolio: The average monthly return on the former is 0.989% (t-statistic of 5.12) whereas the latter has an average return of 0.247% (1.46). The alpha of the high illiquidity portfolio is 1.01% and the annualized Sharpe ratio is 1.08. If we would construct a high illiquidity minus low illiquidity portfolio, we would have earned a monthly alpha of 0.75% with a t-statistic of 4.09, and an annualized Sharpe ratio of 0.94. Overall we conclude that conditioning on illiquidity yields very attractive returns with highly significant alphas.

Proposition 5 provides us with an alternative way to test the importance of country-level illiquidity. It states that a portfolio that is globally long high illiquidity-to-beta-ratio stocks and short sells low illiquidity-to-beta-ratio stocks (BAIL) should on average outperform the global betting-against-beta (BAB) portfolio. In order to test Proposition 5, we start by constructing the ratio of the corresponding local illiquidity, Illiq_t^j, and the estimated beta, $\hat{\beta}_t^k$, for each stock k, and then rank them in ascending order.[19] The ranked securities are assigned into two different bins: high illiquidity-to-beta stocks and low illiquidity-to-beta stocks. We long the former and short the latter. We weight each stock in order for the portfolio to have a beta of zero. The BAIL strategy is then a self-financing zero-beta portfolio that is long a high illiquidity-to-beta portfolio and short a low illiquidity-to-beta portfolio.

The summary statistics for the BAB and BAIL portfolios are presented in Table 6. In line with our prediction, we find that on average, the BAB strategy performs worse than the BAIL strategy: the average excess return is 0.741% per month, 11% lower than that of the BAIL strategy. In terms of alpha, again the strategy performs worse then BAIL: the monthly alpha is 0.731%, or 8% lower.

[Insert Table 6 here.]

While the difference in excess returns and alpha of the BAB and BAIL portfolios over our whole sample has the sign predicted by the theory, it is not very large and results in similar Sharpe ratios. To gauge in more detail the differences of the two trading strategies over time, in Figure 7 we plot cumulative returns of the BAB and BAIL strategies for the past ten years. The two strategies move almost in lock-step until after the Lehman default late 2008, whereas BAIL performs much better than BAB after that. For the period January 2003 (1990) to December 2012, a $1 investment would have lead

[19] Note that with our illiquidity measures we are able to capture only one dimension along which the margins on stocks can differ, namely the country-level effect. We are agnostic about the other dimensions (e.g. industry) that could improve the sorting on illiquidity and thereby enhance the performance of the BAIL portfolio, and simply assume that the effect of any additional cross-sectional variation is averaged out at the country level.

to \$8 (\$5.7) for BAIL and \$6.5 (\$3.9) for BAB. Economically, the better performance after 2008 can be traced back to our theoretical predictions: In a world where liquidity risk matters and differently affects countries, it generates a higher difference in returns. Hence, a strategy that goes long high illiquidity assets and short low illiquidity assets and, thus, exploits this difference should perform particularly well after funding crises that hit certain countries more than others.

[Insert Figure 7 here.]

4.3 Comparison with Market Illiquidity

Finally, it is important to show that our results do not simply capture stock market liquidity that has been shown to be important for asset prices. To this end, for each country we regress our illiquidity measure onto the country-level stock market illiquidity computed following Amihud (2002), and take the residual to be our new illiquidity measure.[20] We then repeat the same exercise as in Section 4.2 and check whether there is any cross-sectional variation in returns when sorting on beta and the new illiquidity measure.

The results are reported in Table 7. In line with Proposition 3, we find that alphas still decline from the low-beta to the high-beta portfolio and that alphas increase from low illiquidity to high illiquidity stocks: For example, holding illiquidity constant, we find that for low (high) illiquidity stocks, the alpha decreases from 0.772% to 0.510% (1.033% to 0.874%), and similarly, Sharpe ratios drop from 0.41 to 0.34 (0.87 to 0.50). On the other hand, keeping betas constant, we find that alphas increase from the low illiquidity stocks to high illiquidity stocks. For example, the alpha for low beta stocks increases from 0.772% per month to 1.033%, for medium beta it increases from 0.731% to 0.951%, and for high beta stock it increases from 0.510% to 0.874%.[21] The last column

[20]Other possible measures include the Pástor and Stambaugh (2003) Gamma, the Zero measure by Lesmond, Ogden, and Trzcinka (1999) and the Hasbrouck (2004) Gibbs measure. Goyenko, Holden, and Trzcinka (2009) and Fong, Holden, and Trzcinka (2011) run horse races among different liquidity proxies and recommend the Amihud (2002) measure as a good proxy of market illiquidity.

[21]While these differences are even larger than in the non-orthogonalized results presented in Table 4, note that the data sample is also shorter because of the limited availability of volume data required to calculate the Amihud (2002) measure.

presents the BAIL strategy returns. The alpha is 0.783% per month and statistically significant (t-statistic of 2.22) at the same time, the annualized Sharpe ratio is 0.59.

[Insert Table 7 here.]

5 Conclusion

This paper investigates the effect of capital constraints on asset returns across different countries. We construct daily country-specific illiquidity proxies from pricing deviations on government bonds. While the overall correlation between the country-specific measures is high, the measures display distinct idiosyncratic behavior especially during country-specific political or economic events. The average level of illiquidity and the difference in illiquidity across countries have an important effect on asset prices. In line with the prediction of a parsimonious international CAPM with constraints, higher global illiquidity affects the international risk-return trade-off by lowering the slope and increasing the intercept of the average international security market line. In the same way, differences in local illiquidity are associated with significant differences in alpha: trading strategies that condition on illiquidity yield attractive returns with highly significant alpha and Sharpe ratios.

Our country-specific illiquidity proxies can be used in several related avenues. Idiosyncratic variation in the cross-section of illiquidity could be applied to test market segmentation. Further, it is possible to study whether innovations in global and local illiquidity are priced risk factors when explaining the cross-section of international stock returns. Moreover, our model is silent on countries' default risk, a paramount aspect in relation to the recent Eurozone crisis. It would be interesting to extend our dataset by countries with different credit risk to study the feedback between sovereign risk and illiquidity and its effect on asset prices. We leave these tasks for future research.

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Appendix A Proofs and derivations

Proof of Lemma 1. We substitute (4) and (5) into (1) to obtain wealth dynamics

$$\begin{aligned} dW_{i,t} &= \left[r_t W_{i,t} + \int\limits_{k\in\mathcal{K}} x_{i,t}^k \left(\mu_{P,t}^k - r_t \right) dk + \int\limits_{h\in\mathcal{H}} z_{i,t}^h \left(\mu_{\Lambda,t}^h - r_t \right) dh \right] dt \\ &\quad + \left[\int\limits_{k\in\mathcal{K}} x_{i,t}^k \sigma_{P,t}^k dk \right]^\top dB_t - \left[\int\limits_{h\in\mathcal{H}} z_{i,t}^h \sigma_{\Lambda,t}^h dh \right] dB_{r,t}. \end{aligned} \tag{A-1}$$

Combining (2), (3), and (A-1), and denoting the Lagrange multiplier of (2) by $\psi_{i,t}$, the optimization problem is equivalent to maximizing the following expression by choosing positions $x_{i,t}^k$ and $z_{i,t}^h$:

$$\begin{aligned} &\int\limits_{k\in\mathcal{K}} x_{i,t}^k \left(\mu_{P,t}^k - r_t \right) dk + \int\limits_{h\in\mathcal{H}} z_{i,t}^h \left(\mu_{\Lambda,t}^h - r_t \right) dh - \frac{\alpha}{2} \left[\int\limits_{k\in\mathcal{K}} x_{i,t}^k \sigma_{P,t}^k dk \right]^\top \left[\int\limits_{k\in\mathcal{K}} x_{i,t}^k \sigma_{P,t}^k dk \right] \\ &\quad - \frac{\alpha}{2} \left[\int\limits_{h\in\mathcal{H}} z_{i,t}^h \sigma_{\Lambda,t}^h dh \right]^2 - \psi_{i,t} \left[\int\limits_{k\in\mathcal{K}} m_{i,t}^k \left| x_{i,t}^k \right| dk + \int\limits_{h\in\mathcal{H}} m_{i,t}^h \left| z_{i,t}^h \right| dh - W_{i,t} \right]. \end{aligned}$$

Pointwise differentiation with respect to $x_{i,t}^k$ and $z_{i,t}^h$ then yields the first-order conditions

$$0 = \mu_{P,t}^k - r_t - \alpha \sigma_{P,t}^{k\top} \int\limits_{k\in\mathcal{K}} x_{i,t}^k \sigma_{P,t}^k dk - \psi_{i,t} m_{i,t}^k sgn \left(x_{i,t}^k \right)$$

and

$$0 = \mu_{\Lambda,t}^h - r_t - \alpha \sigma_{\Lambda,t}^h \int\limits_{h\in\mathcal{H}} z_{i,t}^h \sigma_{\Lambda,t}^h dh - \psi_{i,t} m_{i,t}^h sgn \left(z_{i,t}^h \right),$$

respectively, which are equivalent to (6) and (7). □

Proof of Theorem 1. From (4) and (11), we can write

$$\frac{dG_t}{G_t} = \mu_{G,t} dt + \sigma_{G,t} dB_t = \frac{\int\limits_{k\in\mathcal{K}} \theta_t^k \mu_{P,t}^k dk}{\int\limits_{k\in\mathcal{K}} \theta_t^k dk} dt + \frac{\int\limits_{k\in\mathcal{K}} \theta_t^k \sigma_{P,t}^{k\top} dk}{\int\limits_{k\in\mathcal{K}} \theta_t^k dk} dB_t,$$

where

$$\mu_{G,t} - r_t = \frac{1}{\int\limits_{k\in\mathcal{K}} \theta_t^k dk} \left(\alpha \left[\int\limits_{k\in\mathcal{K}} \theta_t^k \sigma_{P,t}^k dk \right]^\top \left[\int\limits_{k\in\mathcal{K}} \theta_t^k \sigma_{P,t}^k dk \right] + \int\limits_{k\in\mathcal{K}} \theta_t^k \left[\int\limits_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^k sgn \left(x_{i,t}^k \right) di \right] dk \right).$$

Moreover, it is easy to confirm that

$$\mathrm{Cov}_t\left[\frac{D_t^k dt + dP_t^k}{P_t^k}, \frac{dG_t}{G_t}\right] = \mathrm{Cov}_t\left[\mu_{P,t}^k dt + \sigma_{P,t}^{k\top} dB_t, \frac{1}{\int\limits_{k\in\mathcal{K}} \theta_t^k dk} \int\limits_{k\in\mathcal{K}} \theta_t^k \left(\mu_{P,t}^k dt + \sigma_{P,t}^{k\top} dB_t\right) dk\right]$$
$$= \frac{\sigma_{P,t}^{k\top} \int\limits_{k\in\mathcal{K}} \theta_t^k \sigma_{P,t}^k dk}{\int\limits_{k\in\mathcal{K}} \theta_t^k dk} dt$$

and

$$\mathrm{Var}_t\left[\frac{dG_t}{G_t}\right] = \mathrm{Var}_t\left[\frac{1}{\int\limits_{k\in\mathcal{K}} \theta_t^k dk} \int\limits_{k\in\mathcal{K}} \theta_t^k \frac{D_t^k dt + dP_t^k}{P_t^k} dk\right] = \frac{\left[\int\limits_{k\in\mathcal{K}} \theta_t^k \sigma_{P,t}^k dk\right]^\top \left[\int\limits_{k\in\mathcal{K}} \theta_t^k \sigma_{P,t}^k dk\right]}{\left(\int\limits_{k\in\mathcal{K}} \theta_t^k dk\right)^2} dt.$$

Frome here, (10) becomes

$$\mu_{P,t}^k - r_t = \alpha \sigma_{P,t}^{k\top} \int\limits_{k\in\mathcal{K}} \theta_t^k \sigma_{P,t}^k dk + \int\limits_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^k sgn\left(x_{i,t}^k\right) di$$
$$= \beta_{P,t}^k \left(\mu_{G,t} - r_t\right) + \int\limits_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^k sgn\left(x_{i,t}^k\right) di - \beta_{P,t}^k \frac{\int\limits_{k\in\mathcal{K}} \theta_t^k \left[\int\limits_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^k sgn\left(x_{i,t}^k\right) di\right] dk}{\int\limits_{k\in\mathcal{K}} \theta_t^k dk}$$
$$= \beta_{P,t}^k \left(\mu_{G,t} - r_t\right) + \phi_t^k - \beta_{P,t}^k \phi_t^G,$$

which completes the proof of Theorem 1. □

Proof of Theorem 2. Before providing the proof, we start by expressing bond yields in a general form, to illustrate which ingredients are necessary and sufficient to obtain deviations from the smooth yield curve in equilibrium. Then we provide the proof in the specific case discussed by Theorem 2.

Lemma 2. *The equilibrium yield of bond h is given by*

$$y_t^h = \frac{1}{\tau_h} E_t\left[\int\limits_t^{t+\tau_h} \int\limits_{i\in\mathcal{I}} \psi_{i,s} m_{i,s}^h sgn\left(z_{i,s}^{t+\tau_h - s}\right) dids\right]$$
$$+ \frac{1}{\tau_h} E_t\left[\int\limits_t^{t+\tau_h} \left(r_s - \alpha \sigma_{\Lambda,s}^{t+\tau_h - s} \int\limits_{h\in\mathcal{H}} d_s^{t+\tau_h - s} \sigma_{\Lambda,s}^{t+\tau_h - s} dh - \frac{1}{2}\left(\sigma_{\Lambda,s}^{t+\tau_h - s}\right)^2\right) ds\right]. \tag{A-2}$$

Proof of Lemma 2. From (5) the log price dynamics at time s of a bond that matures at time $t+\tau$ is

$$d\log \Lambda_s^{t+\tau-s} = \left[\mu_{\Lambda,s}^{t+\tau-s} - \frac{1}{2}\left(\sigma_{\Lambda,s}^{t+\tau-s}\right)^2\right] ds - \sigma_{\Lambda,s}^{t+\tau-s} dB_{r,s}.$$

Integrating up between time t and $t+\tau$ and using the fact that bond yields are the average expected returns until maturity, we obtain

$$\tau y_t^{t+\tau-t} \equiv \log \Lambda_{t+\tau}^0 - \log \Lambda_t^{t+\tau-t} = \int_t^{t+\tau} d \log \Lambda_s^{t+\tau-s}$$

$$= \int_t^{t+\tau} \left[\mu_{\Lambda,s}^{t+\tau-s} - \frac{1}{2} \left(\sigma_{\Lambda,s}^{t+\tau-s} \right)^2 \right] ds - \int_t^{t+\tau} \sigma_{\Lambda,s}^{t+\tau-s} dB_{r,s}.$$

Substituting in (14), and taking the expectation of both sides as of time t, we obtain (A-2). □

For the proof of Theorem 2, let us assume that $m_{i,t}^h$, d_t^h, $m_{i,t}^k$, θ_t^k and $W_{i,t}$ are constant over time and that the short rate r_t follows (2). We conjecture and later verify that bond prices are in the form

$$\Lambda_t^h = \exp \left\{ -A(\tau_h) - B(\tau_h) r_t \right\}, \tag{A-3}$$

where $B(0) = 0$. Applying Ito's lemma on (A-3), we obtain (5) with

$$\mu_{\Lambda,t}^h = A'(\tau_h) + B'(\tau_h) r_t - B(\tau_h) \kappa (\bar{r} - r_t) + \frac{1}{2} B^2(\tau_h) \sigma^2 \text{ and } \sigma_{\Lambda,t}^h = B(\tau_h) \sigma. \tag{A-4}$$

We conjecture that there exists an equilibrium in which each investor invests a constant proportion of her wealth into each bond and stock. Rewriting (2) in the form

$$\int_{k \in \mathcal{K}} m_{i,t}^k \left| \frac{x_{i,t}^k}{W_{i,t}} \right| dk + \int_{h \in \mathcal{H}} m_{i,t}^h \left| \frac{z_{i,t}^h}{W_{i,t}} \right| dh \leq 1,$$

our assumption means that all $\left| x_{i,t}^k / W_{i,t} \right|$ and $\left| z_{i,t}^h / W_{i,t} \right|$ ratio is constant over time, thus the LHS is also constant over time. Therefore, either the constraint never binds for investor i, with $\psi_{i,t} = 0$ for all t, or it always binds: $\psi_{i,t} = \psi_i > 0$ for all t.

Combining (14) and (A-4), making use of the assumptions of the Corollary, and realizing that these further imply that $sgn\left(z_{i,t}^h\right)$ is also constant over time for each investor and asset, we obtain

$$A'(\tau_h) + B'(\tau_h) r_t - B(\tau_h) \kappa (\bar{r} - r_t) + \frac{1}{2} B^2(\tau_h) \sigma^2 - r_t = -\alpha \sigma^2 B(\tau_h) \int_{h' \in \mathcal{H}} d^{h'} B(\tau_{h'}) dh'$$

$$+ \int_{i \in \mathcal{I}} \psi_i m_i^h sgn\left(z_i^h\right) di.$$

Collecting the r_t terms, we get $B'(\tau_h) + B(\tau_h)\kappa - 1 = 0$, i.e., B is the standard Vasicek coefficient

$$B(\tau_h) = \frac{1 - e^{-\kappa \tau_h}}{\kappa},$$

and $\mathcal{B}(\tau_h) = B(\tau_h) / \tau_h$. On the other hand, collecting constant terms and rearranging implies

$$A'(\tau_h) = \left[\kappa \bar{r} - \alpha \sigma^2 \int_{h' \in \mathcal{H}_j} d^{h'} B(\tau_{h'}) dh' \right] B(\tau_h) - \frac{1}{2} \sigma^2 B^2(\tau_h) + C'(\tau_h), \tag{A-5}$$

where $C(0) = 0$ and

$$C'(\tau_h) \equiv \int_{i \in \mathcal{I}} \psi_i m_i^h sgn\left(z_i^h\right) di. \tag{A-6}$$

To obtain $A(\tau_h)$ and hence bond prices, we need to solve the simple ODE (A-5). Notice that integrating the first two terms of the RHS of (A-5) provides a term structure coefficient similar to that of standard affine models, which only depends on maturity τ_h and the aggregate amount of interest rate risk in the economy, $\int_{h' \in \mathcal{H}} d^{h'} B(\tau_{h'}) dh'$.

Finally, if there exist some $i \in \mathcal{I}$ with $\psi_i > 0$, the last term of (A-5), denoted by $C'(\tau_h)$, is non-trivial, and depends on margins and asset positions; thus, it can vary both across countries and across bonds of a specific country, too. As $sgn(.)$ is a discontinuous function, this term oscillates around zero, and can create the non-smooth component in bond yields. In fact, integrating $C'(.)$ for all assets $h \in \mathcal{H}_j$ with maturity up to τ_h and dividing by τ_h we obtain the last component of yields, $\mathcal{C}_h(\tau_h)$, given in (15)that picks up *noise* in the term structure of country j. $\square$

Proof of Propositions 4 and 5. Suppose an investor creates a market-neutral portfolio by going long a security or a portfolio with beta β_t^H and illiquidity ϕ_t^H, and going short a security or portfolio with beta β_t^L and illiquidity ϕ_t^L, and applying leverages of $1/\beta_t^H$ and $1/\beta_t^L$ to the two legs, respectively. From (12), the expected excess return on this self-financing portfolio is:

$$\mu_t^{H-L} - r_t = \frac{1}{\beta_t^H}\left[\mu_t^H - r_t\right] - \frac{1}{\beta_{P,t}^L}\left[\mu_t^L - r_t\right] = \left[\lambda_t + \frac{\phi_t^H}{\beta_t^H} - \phi_t^G\right] - \left[\lambda_t + \frac{\phi_t^L}{\beta_t^L} - \phi_t^G\right] = \frac{\phi_t^H}{\beta_t^H} - \frac{\phi_t^L}{\beta_t^L}. \tag{A-7}$$

It is easy to see that if both legs are constructed from assets of the same country j, and the long part consists of low-beta securities while the portfolio goes short in high-beta securities, we obtain a BAB portfolio of country j with expected excess return

$$\mu_t^{j,BAB} - r_t = \phi_t^j \left[\frac{1}{\beta_t^{LB}} - \frac{1}{\beta_t^{HB}}\right] > 0.$$

Keeping the term in the parentheses constant, the return of the country-j BAB portfolio is increasing in ϕ_t^j, which confirms Proposition 4. In the meantime, a global BAB portfolio that goes long in low-beta assets and short in high-beta assets maximizes $1/\beta_t^{LB} - 1/\beta_t^{HB}$, but by ignoring illiquidity, might not maximize the right-hand side of (A-7). It can therefore be dominated by a BAIL strategy that goes long assets with the highest ϕ_t^H/β_t^H possible and goes short in assets with the lowest ϕ_t^L/β_t^L possible. This confirms Proposition 5. $\square$

Appendix B Tables

Table 1
Data Summary Statistics

This table reports summary statistics of the bonds (Panel A) and stocks (Panel B) used for six different countries: United States (US), Germany (GE), United Kingdom (UK), Canada (CA), Japan (JP), and Switzerland (SW). Panel A reports the average number of bonds used each day to calculate the term structure (ts) and the illiquidity proxy (illiq). To estimate the term structure, we use bonds of maturities ranging from 3 months to 10 years. To calculate the illiquidity measure, we eliminate bonds of maturities less than one year. Panel B shows country-level summary statistics, monthly mean and volatility, for the stocks used in our sample. The data runs from January 1990 to December 2012.

Panel A: Bonds Summary Statistics

	US		GE		UK		CA		JP		SW	
	ts	illiq	ts	illiq	ts	illiq	ts	illiq	ts	illiq	ts	illiq
1990-2000	124	99	151	130	16	13	44	35	100	92	31	27
2001-2007	77	61	52	42	12	9	20	16	155	133	15	10
2008-2013	146	122	39	32	17	13	27	21	164	138	12	9
All	115	93	105	90	17	13	37	30	127	111	23	19

Panel B: Stocks Summary Statistics

	All	US	GE	UK	CA	JP	SW
Number of Stocks Considered	10,891	2,385	1,149	2,951	945	3,105	356
Average Number of Traded Stocks	3,973	1,082	323	560	309	1,567	132
Mean Return (monthly percentage)	0.67	1.18	0.55	0.70	1.22	0.13	0.82
Return Volatility (annualized)	17.0	16.9	17.5	19.7	22.5	25.2	17.7
Mean Excess Return	0.39	0.91	0.28	0.43	0.95	-0.15	0.54
Excess Return Volatility	17.1	16.9	17.6	19.8	22.6	25.4	17.8

Table 2
Summary Statistics Illiquidity Proxies

Panel A reports summary statistics (mean, standard deviation, maximum and minimum) for six different country specific illiquidity proxies in basis points. The countries are the United States (US), Germany (GE), United Kingdom (UK), Canada (CA), Japan (JP), and Switzerland (SW). Panel B reports the fraction of being in the low, medium, or high illiquidity bin. Panel C reports the unconditional correlation between the country-specific illiquidity measures. Panel D reports the estimated coefficients with the associated t-statistic and R^2 from the following regression: $\text{Illiq}_t^j = \beta_0^j + \beta_1^j \text{Illiq}_t^G + \epsilon_t^j$, where Illiq_t^j is the illiquidity proxy of country j and Illiq_t^G is the global illiquidity proxy. t-statistics are calculated using Newey and West (1987). Data is weekly and runs from January 1990 to October 2013.

Panel A: Summary Statistics

	US	GE	UK	CA	JP	SW
Mean	2.819	4.169	5.211	4.907	3.120	6.210
StDev	1.375	2.247	3.319	3.286	2.317	4.533
Max	11.203	11.566	18.078	14.306	11.213	19.286
Min	1.028	0.756	1.051	1.103	0.719	1.225

Panel B: Illiquidity Bin

	US	GE	UK	CA	JP	SW
low illiq	52.90%	13.04%	31.16%	30.07%	67.39%	5.43%
med illiq	28.26%	42.03%	47.83%	41.30%	23.55%	17.03%
high illiq	18.84%	44.93%	21.01%	28.62%	9.06%	77.54%

Panel C: Cross Correlation

	US	GE	UK	CA	JP	SW
US	100.00%					
GE	32.38%	100.00%				
UK	49.09%	68.14%	100.00%			
CA	32.12%	57.91%	66.44%	100.00%		
JP	19.46%	74.37%	43.85%	41.92%	100.00%	
SW	38.15%	68.43%	66.53%	67.43%	61.04%	100.00%

Panel D: Loading on Global Illiquidity Proxy

	US	GE	UK	CA	JP	SW
β_0	0.625	0.091	-0.526	0.303	-0.797	-1.412
	(1.60)	(0.21)	(-1.48)	(0.41)	(-1.49)	(-1.86)
β_1	3.802	7.067	9.943	7.980	6.789	13.211
	(4.45)	(7.79)	(12.02)	(4.68)	(5.72)	(8.08)
Adj. R^2	51.46%	66.64%	60.41%	39.58%	57.75%	57.15%

Table 3
Regression Intercept and Slope of SML

This table reports OLS regression coefficient of the intercept and slope of the SML on global market, size, and book-to-market portfolio returns and global illiquidity:

$$\begin{aligned}\alpha_t &= a_1 + b_1 r_t^G + c_1 r_t^S + d_1 r_t^B + e_1 \text{Illiq}_{t-1}^G + u_{1,t},\\ \phi_t &= a_2 + b_2 r_t^G + c_2 r_t^S + d_2 r_t^B + e_2 \text{Illiq}_{t-1}^G + u_{2,t},\end{aligned}$$

where r_t^G, r_t^S and r_t^B is the excess return on the global market (mrkt), size (sml) and book-to-market (hml) portfolio. The intercept (α_t) and slope (ϕ_t) are estimated using the Fama and MacBeth (1973) methodology. t-statistics reported in parentheses are adjusted according to Newey and West (1987). Data is monthly and runs from January 1990 to December 2012.

	a	mrkt	smb	hml	illiq	Adj. R^2
Intercept	-0.004	0.208			0.008	13.72%
t-stat	(-1.34)	(5.43)			(1.83)	
Slope	0.010	0.651			-0.013	51.41%
t-stat	(2.12)	(12.89)			(-1.87)	
Intercept	-0.004	0.198	0.220	0.065	0.009	17.48%
t-stat	(-1.38)	(5.63)	(2.72)	(1.36)	(2.04)	
Slope	0.010	0.629	0.502	0.149	-0.009	59.81%
t-stat	(2.85)	(13.70)	(4.64)	(1.92)	(-1.70)	

Table 4
Illiquidity and Beta Sorted Portfolios

This table reports portfolio returns of illiquidity-to-beta sorted portfolios. At the beginning of each calendar month, we sort stocks in ascending order on the basis of their country-level illiquidity and the estimated beta at the end of the previous month. The ranked stocks are then assigned to six different bins: Low/High illiquidity, and low/mid/high beta. CAPM Alpha is the intercept in a regression of monthly excess returns onto the global market excess return. Returns and alphas are in monthly percent, t-statistics are shown below the coefficient estimates, and 5% statistical significance is indicated in bold. Beta (ex ante) is the average estimated beta at portfolio formation. Beta (realized) is the realized loading on the market portfolio. Volatilities and Sharpe ratios are annualized.

	Low Illiq			High Illiq		
	low β	mid β	high β	low β	mid β	high β
Excess Return	0.609	0.587	0.561	0.651	0.674	0.678
t-stat	(2.40)	(1.87)	(1.28)	(2.41)	(2.10)	(1.75)
CAPM Alpha	0.527	0.471	0.395	0.547	0.540	0.522
t-stat	(2.80)	(2.08)	(1.24)	(3.06)	(2.83)	(2.10)
Beta (ex ante)	0.56	1.02	1.51	0.63	1.01	1.54
Beta (realized)	0.61	0.85	1.23	0.77	0.99	1.16
Volatility (annualized)	14.82	17.80	24.24	15.64	18.51	22.25
Sharpe Ratio (annualized)	0.49	0.39	0.28	0.50	0.44	0.37

Table 5
High versus Low Illiquidity BABs

This table reports estimated excess returns and alphas of a trading strategy that each month constructs a betting-against-beta strategy in each country and then sorts according to their liquidity level into two bins (low and high). HML is the high-illiquidity minus the low-illiquidity portfolio. Alphas are in monthly percent and t-statistics are adjusted according to Newey and West (1987). Data runs from January 1990 to December 2012.

	low	high	HML
Excess return	0.247	0.989	0.742
t-stat	(1.46)	(5.12)	(4.48)
CAPM alpha	0.382	1.011	0.753
t-stat	(1.76)	(4.11)	(4.09)
Volatility (annualized)	9.58	10.98	9.37
Sharpe Ratio (annualized)	0.31	1.08	0.94

Table 6
BAIL versus BAB

This table reports estimated excess returns and alphas for the BAIL and BAB trading strategies. BAIL is a self-financing portfolio that is long the high illiquidity to beta stocks and short the low illiquidity to beta stocks. BAB is long the low-beta portfolio and short the high-beta portfolio. The alphas are calculated from regressions of monthly excess returns onto the market (CAPM). Alphas are in monthly percent and t-statistics are adjusted according to Newey and West (1987). Data runs from January 1990 to December 2012.

	BAB	BAIL
Excess Returns	0.741	0.827
t-stat	(3.51)	(3.53)
CAPM alpha	0.731	0.791
t-stat	(2.48)	(3.53)
Volatility (annualized)	12.10	13.51
Sharpe Ratio (annualized)	0.73	0.73

Table 7
Illiquidity and Beta Sorted Portfolios Orthogonalized

This table reports portfolio returns of illiquidity-to-beta sorted portfolios where illiquidity measures have been orthogonalized with respect to the Amihud (2002) market illiquidity measure. At the beginning of each calendar month, we sort stocks in ascending order on the basis of their country-level illiquidity and the estimated beta at the end of the previous month. The ranked stocks are then assigned to six different bins: Low/High illiquidity, and low/mid/high beta. CAPM Alpha is the intercept in a regression of monthly excess returns onto the global market excess return. Returns and alphas are in monthly percent, t-statistics are shown below the coefficient estimates, and 5% statistical significance is indicated in bold. Beta (ex ante) is the average estimated beta at portfolio formation. Beta (realized) is the realized loading on the market portfolio. Volatilities and Sharpe ratios are annualized. Using these orthogonalized illiquidity measures, we build a BAIL strategy that is long the high illiquidity to beta stocks and short the low illiquidity to beta stocks.

	Low Illiq			High Illiq			
	low β	mid β	high β	low β	mid β	high β	BAIL
Excess Return	0.521	0.745	0.794	0.887	0.963	1.056	0.783
t-stat	(1.53)	(1.69)	(1.27)	(3.26)	(2.51)	(1.85)	(2.21)
CAPM Alpha	0.772	0.731	0.510	1.033	0.951	0.874	0.783
t-stat	(2.29)	(2.36)	(1.80)	(3.52)	(4.32)	(5.06)	(2.22)
Beta (ex-ante)	0.53	0.87	1.41	0.56	0.88	1.39	0.00
Beta (realized)	0.51	0.85	1.44	0.57	0.86	1.33	0.14
Volatility (annualized)	15.32	19.61	28.05	12.13	17.26	25.52	15.81
Sharpe Ratio (annualized)	0.41	0.45	0.34	0.87	0.67	0.50	0.59

Appendix C Figures

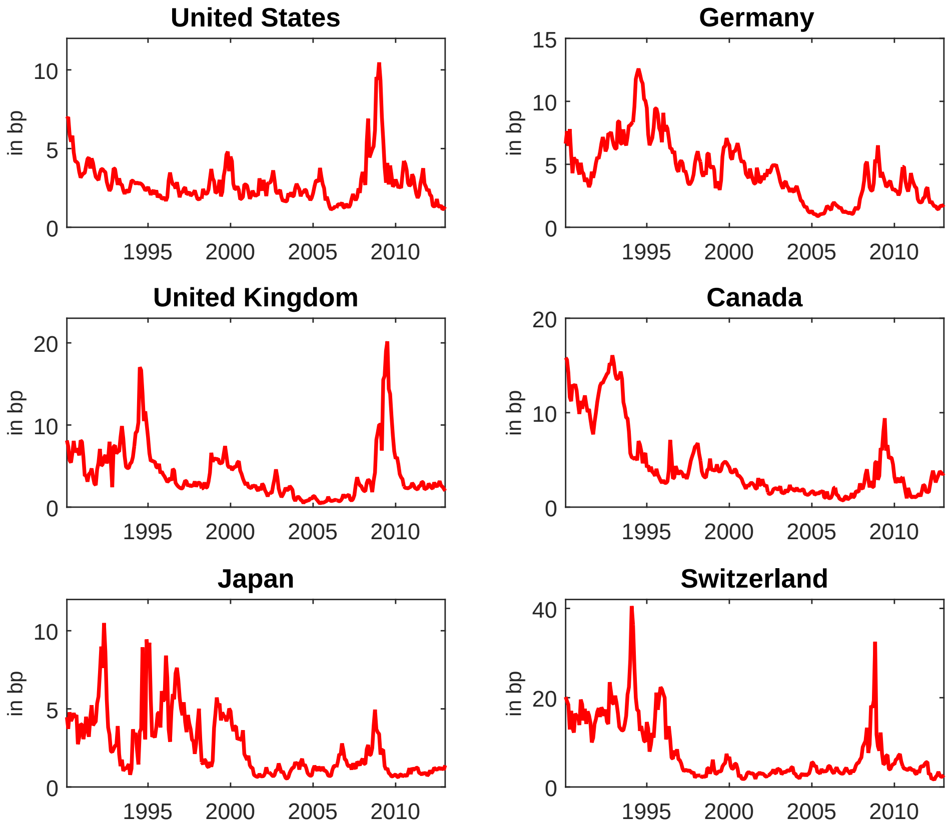

Figure 1. Illiquidity Measures All Countries

This figure plots country-level illiquidity proxies for six different countries: United States, Germany, United Kingdom, Canada, Japan, and Switzerland. Illiquidity is calculated as the average squared deviation of observed bond prices from those implied by a fitted yield curve using the method of Svensson (1994). Measures are in basis points. Data is monthly and runs from January 1990 to December 2012.

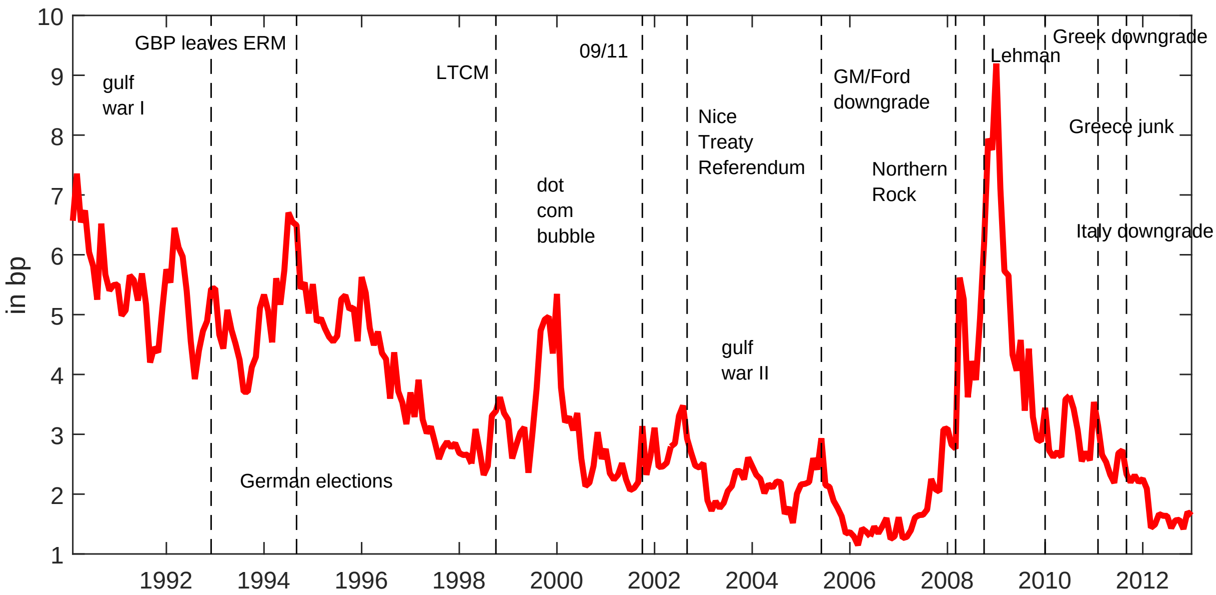

Figure 2. Global Illiquidity

This figure presents global illiquidity in basis points. Global illiquidity is calculated as the market capitalization-weighted average from the six country-specific illiquidity proxies (United States, Germany, United Kingdom, Canada, Japan, and Switzerland). Data is monthly and runs from January 1990 to December 2012.

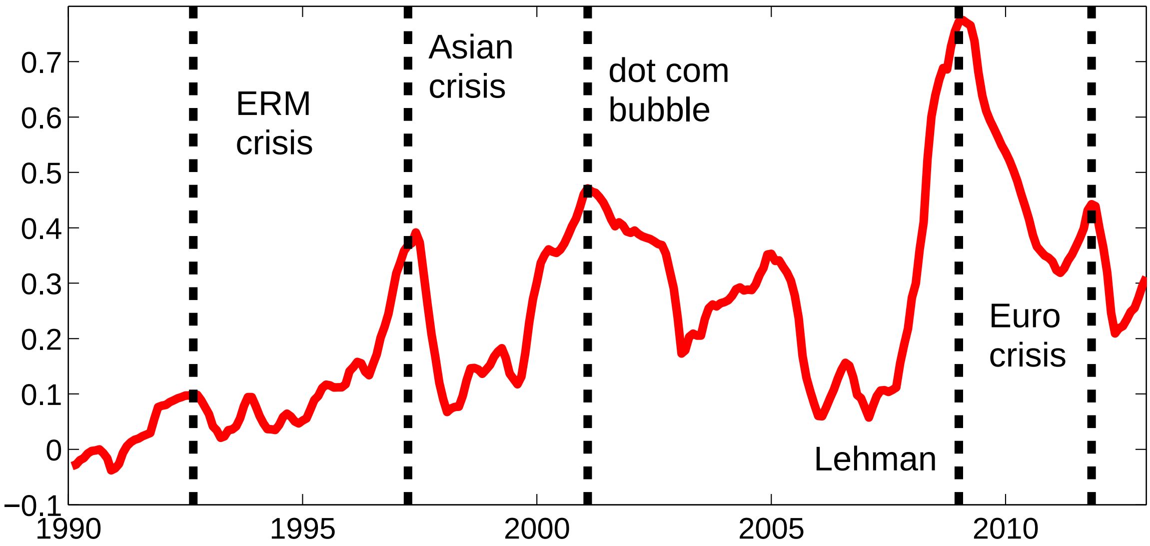

Figure 3. Average Conditional Correlation of Country-Specific Illiquidity Measures

This figure present the conditional average correlation among all six country-specific illiquidity proxies (United States, Germany, United Kingdom, Canada, Japan, and Switzerland). Conditional correlations are calculated using a rolling window of three years using daily data. Data is sampled monthly and runs from January 1990 to December 2012.

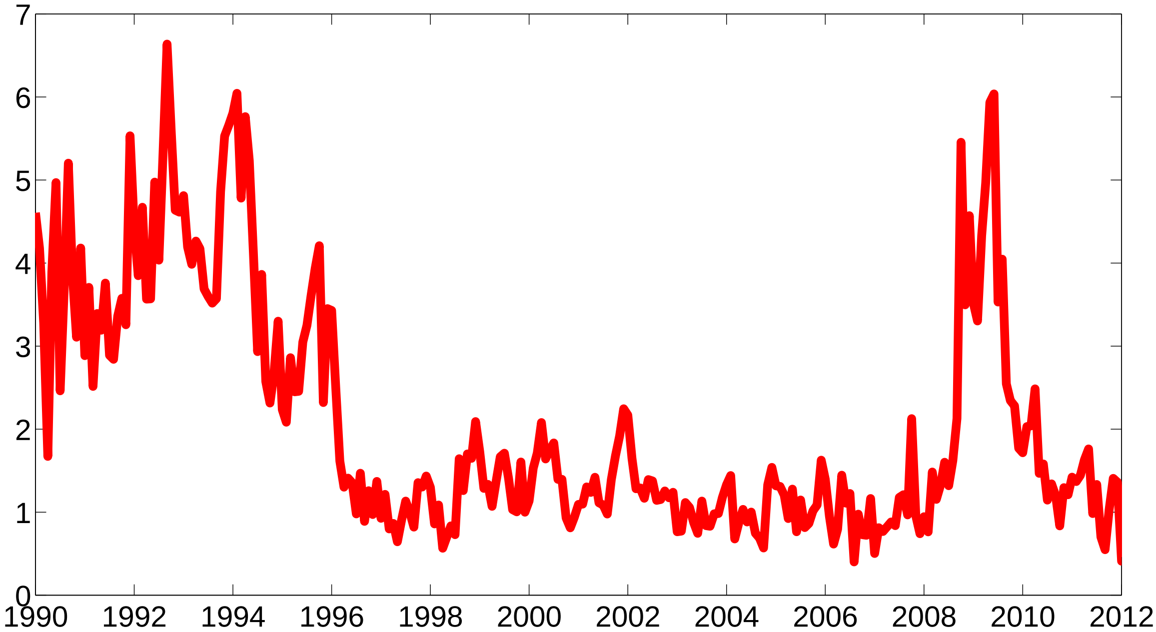

Figure 4. Cross-Sectional Standard Deviation of Country-Specific Illiquidity Measures

This figure present the cross-sectional standard deviation of country-specific illiquidity measures (United States, Germany, United Kingdom, Canada, Japan, and Switzerland). The standard deviations are calculated monthly, the data runs from January 1990 to December 2012.

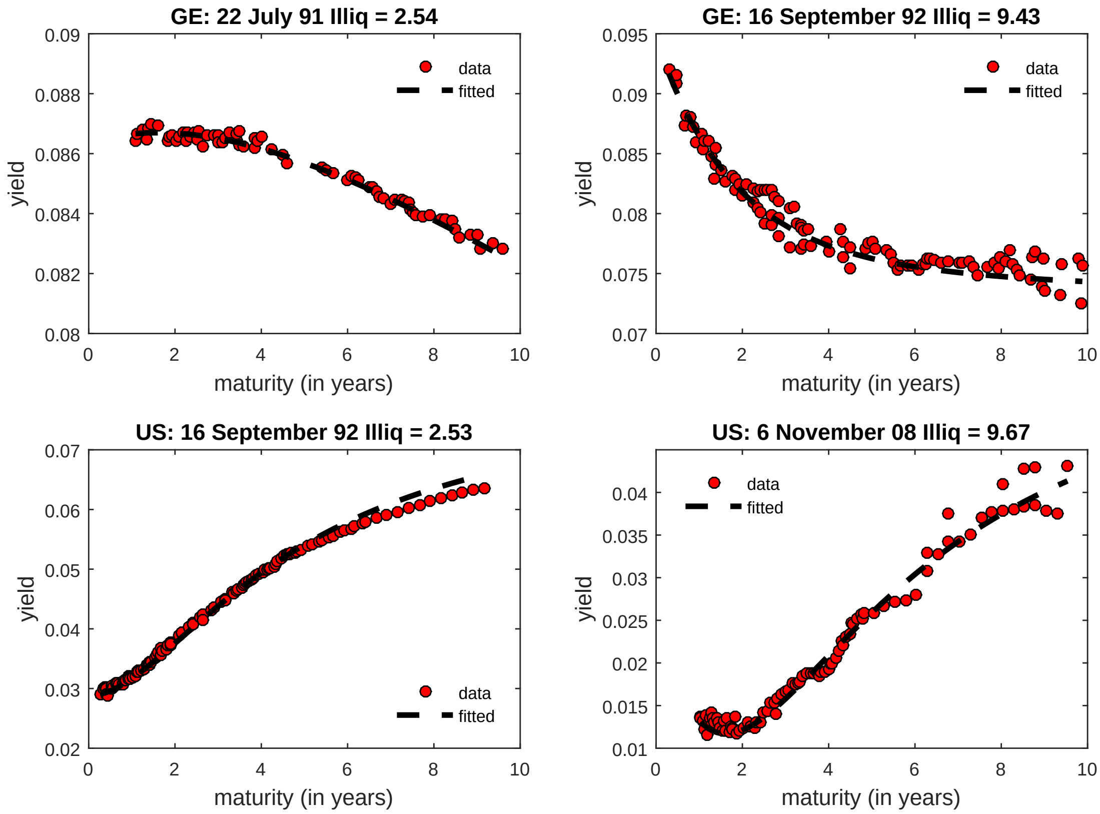

Figure 5. International Term Structures Different Days

This figure presents data and model-implied yields for Germany and the US for three specific days. The dots are observed yields (in percent) for different maturities. The dashed line is the fitted curve using the Svensson (1994) method. The upper left panel plots the German term structure on 22 July 1991. The upper right panel plots the German term structure on the day when the British Pound exited the European Exchange Rate Mechanism, 16 September 1992. The lower left panel plots the US term structure on this day and the lower right panel plots the US term structure on 6 November 2008.

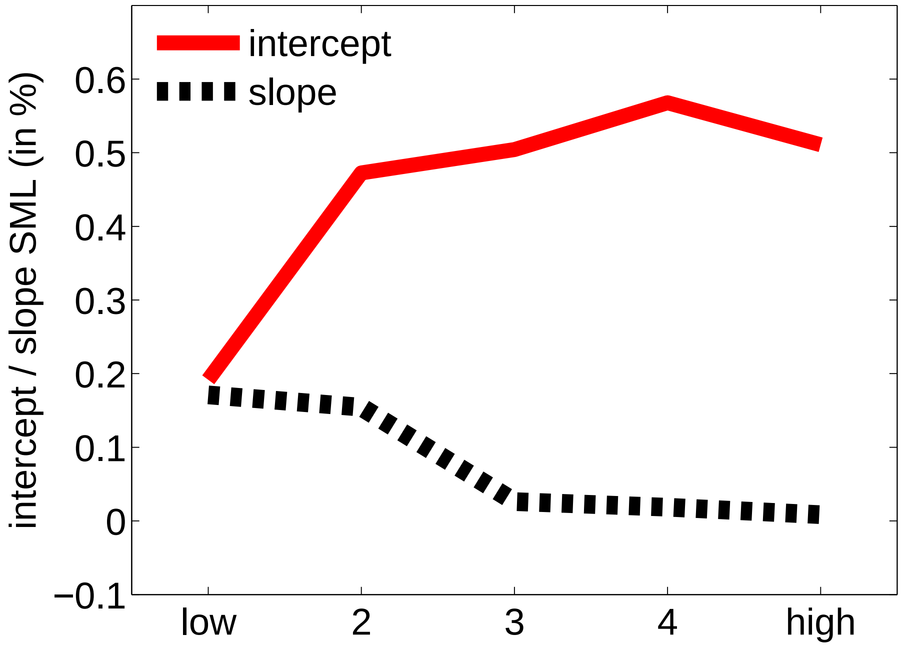

Figure 6. Intercept and Slope Security Market Line

This figure plots the average intercept and slope of the security market line for different global illiquidity quintiles. The full sample period is from January 1990 to December 2012.

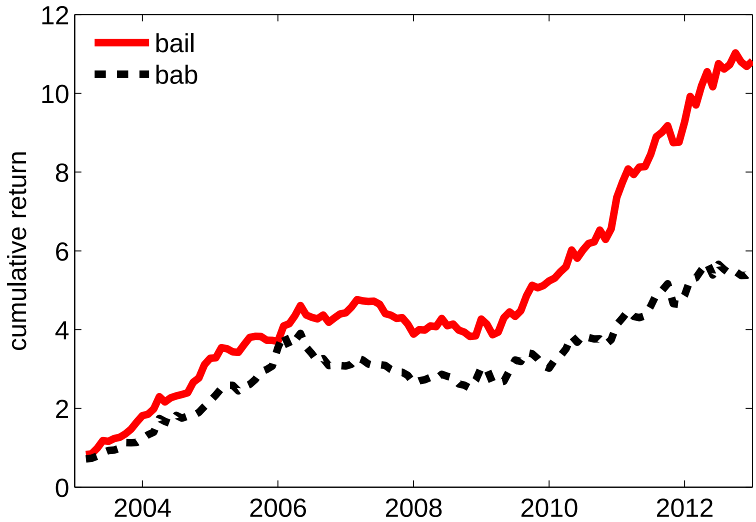

Figure 7. BAIL versus BAB cumulative returns

This figure plots the cumulative return of BAIL and BAB. Data is monthly and starts in 2003 and ends in 2012.