Servicing Securitisation through Excessive Foreclosure*

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Abstract

How does securitisation distort the foreclosure decision of non-performing mortgages? In a model in which informed securitisers raise liquidity by *jointly* designing the mortgage-backed security and the foreclosure policy, we find that securitisers optimally adopt an *excessive* foreclosure policy while retaining the junior tranche to signal positive information to investors in the senior tranche. In order to commit to the optimal foreclosure policy, securitisers can either outsource the foreclosure decisions to mortgage servicers who are intrinsically "tough" or offer the servicers "biased" servicing contracts. Policies that aim to restore ex post efficient foreclosures may inadvertently *reduce* mortgage originators' screening effort. (*JEL* D8, G21, G23, G24)

Keywords. Security design, mortgage-backed securities, mortgage foreclosure, mortgage servicers, asymmetric information, commitment

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1 Introduction

In the aftermath of the subprime mortgage crisis, the United States has seen more than 14 million properties with foreclosure filings between 2008 and 2014.[1] This wave of mortgage foreclosures, often referred to as the "foreclosure crisis", has raised concerns from the general public and was extensively covered by media outlets. In response, policy makers have taken legal actions against large financial institutions and launched large-scale incentive schemes to reduce mortgage foreclosures.[2,3]

Several recent empirical studies about the subprime mortgage crisis have suggested that the securitisation of mortgages has hindered the modification of non-performing (delinquent) mortgages and caused excessive foreclosures.[4] These studies conjecture that some features in the securitisation process might have impeded the modification of securitised loans: the prinicpal-agent problem between the investors and the mortgage servicer, who is granted the discretion to foreclose or modify delinquent loans; the servicer's biased compensation contracts towards foreclosure; and the complex liability structure of the mortgage pools. It is imperative to recognise, however, that these features are *endogenously designed* by the securitising banks (henceforth "securitisers") and the investors in the first place. Therefore, the exact mechanism driving excessive foreclosure is yet to be fully understood.[5]

To study the economic role played by foreclosure policy in securitsation, we develop a model of asset-backed securitisation with *endogenous* foreclosure. As in DeMarzo (2005), due to liquidity needs, an informed securitiser in our model would

[1]According to a recent report by RealtyTrac (2015).

[2]For example, in a historical settlement known as the National Mortgage Settlement (NMS), the five largest mortgage servicers were required by the federal government to provide $26 billion in relief to distressed homeowners and in direct payments to the states and federal government.

[3]For example, the HOPE for Homeowners act was signed into law in 2008 to help homeowners refinance their mortgages into affordable fixed-rate ones. The Home Affordable Modification Program (HAMP) introduced in 2009 provides monetary incentives to mortgage servicers to encourage loan modification instead of foreclosure. For a detailed description and an empirical evaluation of HAMP, see Agarwal et al. (2012).

[4]Piskorski et al. (2010), Agarwal et al. (2011), Zhang (2013),and Kruger (2016) have shown that, conditional on being delinquent, mortgages in a securitised pool are more likely to be foreclosed than mortgages of similar quality in banks' portfolios. Using an earlier sample from 2005 to 2007 and a different methodology, Adelino et al. (2013) do not find statistically different probability of modification between securitised and portfolio loans. See Agarwal et al. (2011) and Kruger (2016) for a discussion on the different findings of Adelino et al. (2013).

[5]Another argument for excessive foreclosure is to discourage strategic default by borrowers. While this argument might apply to all securitised and portfolio loans, it is not clear why borrowers would choose to strategically default *more* when their mortgages are securitised.

like to design and sell a security backed by the cash flow from her mortgage pool to uninformed investors.[6] In addition, in the baseline model, the securitiser can publicly announce and commit to a policy to foreclose or to modify any fraction of delinquent mortgages in the future.[7] In order to avoid any lemon discount, a securitiser with a high-quality mortgage pool (or "a high-type securitiser") optimally signals her private information to investors in equilibrium by designing *both* the mortgage-backed security (MBS) and the foreclosure policy.

Excessive foreclosure policy arises endogenously in our model in order to mitigate informational frictions in securitisation. In a separating equilibrium, the high-type securitiser commits to an ex post excessive foreclosure policy and issues a risky debt (the senior tranche) to signal the quality of the mortgage pool to the uninformed investors. While the sale of the senior debt tranche and hence the retention of the junior equity tranche as a costly signal has been established in DeMarzo (2005) and Chemla and Hennessy (2014), to the best of our knowledge, the signalling role of the foreclosure policy is novel.

Two key properties of mortgage foreclosure drive our result. First, a foreclosure policy matters more for the securitiser with a low-quality, more prone-to-default mortgage pool. Second, foreclosure, as opposed to modification, lowers the exposure of the mortgage pool's cash flow to the borrower re-default risk, which is likely to be driven by aggregate uncertainties in future unemployment and property prices. Specifically, foreclosing a delinquent mortgage and selling the underlying property provides a safe cash flow, whereas modification delivers a higher (lower) cash flow when the once-defaulted borrower recovers (re-defaults). Put differently, foreclosure is a (potentially costly) way to transfer cash flow from the recovery state to the re-default state. We emphasise that we assume neither frictions in implementing a foreclosure policy (e.g. incomplete contract) nor that foreclosure is inherently inefficient. In fact, from an expected cash flow perspective, there is an efficient, interior level of foreclosure and the securitiser can freely commit to it.

To see how these properties of mortgage foreclosure give rise to the signalling role of an *excessive* foreclosure policy, consider the problem faced by the high-type

[6] In the remainder of the paper we will refer to a securitiser as "she" and a servicer as "he".

[7] For simplicity, we do not distinguish modification from forbearance, i.e. simply continuing the mortgage contract with the defaulted borrowers. As it will soon be clear, there is no qualitative difference in the interpretation of our model.

securitiser: she wants to maximise a weighted sum of the expected value of the mortgage pool and the proceeds from the sale of the MBS to the investors, but is limited by the low-type securitiser's mimicking behaviour. By designing the same foreclosure policy and MBS as the high type's, the low-type securitiser can sell her MBS at a premium, as if it is backed by a high-quality mortgage pool. Anticipating the low type's strategic behaviour, the high-type securitiser designs *jointly* the foreclosure policy and the MBS to reduce this mimicking premium, in order to realise more gains from securitisation. On the security design, the optimal MBS is a risky debt, whose payoffs are less sensitive to changes in the underlying mortgage pool's cash flow in states with higher cash flows. Because of this decreasing cash flow sensitivity, foreclosing mortgages *increases* the expected payoff of the debt, as foreclosure essentially transfers cash flows from relatively good states to worse states. Crucially, foreclosure increases the expected value of a debt backed by a low-quality pool *more* than the same debt backed by a high-quality pool, because the low-quality pool is more likely to return low cash flows. Foreclosure thus reduces the mimicking premium. In equilibrium, the hype-type securitiser optimally commits to an excessive foreclosure policy that trades off the benefit of achieving more gains from securitisation against the cost of inefficient foreclosure.

The above reasoning uncovers a broader theoretical contribution of this paper, namely the importance of the *joint* determination of security design and the foreclosure policy. The high-type securitiser's optimal foreclosure policy is excessive precisely because the optimal MBS is a risky debt. The opposite distortion towards an overly lenient foreclosure policy is never optimal given that the MBS is a risky debt. This insight on the joint determination of multiple signalling devices is general and should apply to many economic setting in which an informed owner of an asset or a firm tries to raise funds by selling claims backed by the asset while he can commit to take certain actions that affect the asset/firm's future cash flow.

The mechanism behind excessive foreclosure as a costly signal relies on the securitiser's commitment power, i.e., the ability to implement the ex ante optimal foreclosure policy ex post when defaults occur. Without commitment power, the securitiser could ex post profitably deviate to an overly lenient foreclosure policy in a manner akin to risk shifting, as her retained junior tranche benefits only from the upside, when the delinquent mortgage recover. While junior retention is necessary to signal quality ex ante, it distorts the securitiser's incentive to foreclose delinquent mortgages ex post.

In light of the securitiser's limited ability to commit, our result suggests an economic rationale to hire an external mortgage servicer as a commitment device. In practice, mortgage servicers feature prominently in the mortgage industry and they are granted substantial discretion over the foreclosure decision of delinquent mortgages.[8] By contracting with a mortgage servicer, the securitiser can effectively commit to an excessive foreclosure policy by either choosing a servicer known to be "tough", or providing the servicer with an incentive contract that is biased towards excessive foreclosure. The first interpretation corroborates with the economically significant servicer fixed effects in predicting foreclosure probability as emphasised by Agarwal et al. (2011). The second interpretation can explain why servicers' compensation contracts are endogenously biased towards foreclosure. Such biases have been documented by Levitin and Goodman (2009), Thompson (2009) and Kruger (2016). Finally, it is plausible that investors of an MBS infer information from the identity of the servicer and/or the mortgage servicing contract, as such information are provided in the Pooling and Servicing Agreement (PSA) alongside the prospectus of the MBS issue.

Last we extend the model to endogenise the securitiser's ex ante screening effort choice at origination and derive policy implications of foreclosure policy regulations. Our message is a cautionary one in the Lucas' critique fashion: policies aiming to restore ex post efficient foreclosure such as HAMP would inadvertently *reduce* the securitiser's incentive to screen mortgages diligently, leading to lower average quality of the mortgage pools and overall welfare in the economy. When a securitiser with a high-quality mortgage pool can no longer signal her quality effectively with an excessive foreclosure policy, her initial incentive to exert screening effort in order to form a high-quality mortgage pool is weakened. Although information asymmetry in securitisation always results in under-provision of the securitisers' screening effort, ex post excessive foreclosure in our model is a remedy, instead of a symptom, of the problem. Finally, we conclude with several novel, testable empirical implications of our model regarding the relationship between mortgage pool quality and foreclosure

[8]The servicers perform duties including collecting the payments, forwarding the interest and principal to the lenders, and negotiating new terms if the debt is not being paid back (loan modification), or supervising the foreclosure process.

policies as well as servicer-specific characteristics.

Our model starts from a discrete cash flow version of models of liquidity-based security design, such as DeMarzo and Duffie (1999), DeMarzo (2005) and Biais and Mariotti (2005). We depart from the literature by allowing the securitisers to take actions that affect the distribution of the underlying asset's cash flow. In the context of mortgage securitisation, foreclosure policy of delinquent mortgages is one of such actions. Endogenising the securitiser's foreclosure policy enables us to provide a theoretical explanation for the empirically documented causal relationship between mortgage-backed securitisation and excessive foreclosure (e.g., Piskorski et al., 2010; Agarwal et al., 2011; Zhang, 2013; Kruger, 2016). While excessive foreclosure emerges as an additional dimension of the signal in equilibrium, the optimal security in our model is debt, consistent with the classical literature on security design with a privately informed issuer, started with Myers and Majluf (1984) and Nachman and Noe (1994).

Several papers have highlighted the incentive problems associated with securitisation. In a setting of securitisation under adverse selection similar to ours, Chemla and Hennessy (2014) and Vanasco (2016) analyse how liquidity in the MBS market affects ex ante loan originators' screening effort. Hartman–Glaser et al. (2012) and Malamud et al. (2013) also study the optimal design of the originator's compensation contracts to incentivise screening effort in a dynamic setting. These papers do not study foreclosure policies of the delinquent mortgages. In contrast, we first characterise the optimal foreclosure policy and then assess its effect on the originator's screening incentives.

Our results contribute to the understanding of the role of servicers and their incentive contracts. Mooradian and Pichler (2014) study the asset composition (pooling) of the mortgage pool and show that a non-diversified pool alleviates the servicer's moral hazard problem. Our paper instead focuses on the securitisation (tranching) problem under asymmetric information for a mortgage pool of given quality and shows that even in the absence of principle-agent frictions, it is *ex ante* optimal to have an *ex post* inefficient foreclosure policy.

Our paper also relates to but differs from the literature on optimal loan modification and foreclosure policy. Wang et al. (2002) and Riddiough and Wyatt (1994) argue that borrowers' strategic default incentives lead lenders to adopt a tough foreclosure policy in order to deter non-distressed borrowers from opportunistic behaviour. However, these papers do not analyse the securitisation of the loans and their models based on borrower strategy default do not readily explain the difference in the foreclosure rates between securitised and portfolio loans. Gertner and Scharfstein (1991) focuses on the free-riding problem among multiple creditors. Our paper highlights that information asymmetry in securitisation can be another important factor that determines foreclosure decisions.

The rest of the paper is organised as follows. Section 2 describes the model setup. Section 3 carries out the main analysis of the equilibrium with endogenous foreclosure policy. Section 4 highlights the importance of commitment over the foreclosure policy and discusses the role of mortgage servicers in enabling such commitment power. Section 5 extends the model to consider ex ante screening incentives of the securitiser in relation to the subsequent foreclosure policy. Section 6 lists the model's empirical implications, and Section 7 concludes.

2 Model setup

This section sets up the model and comments on the assumptions which are central to the model.

There are four dates: 0, 1, 2 and 3. The model's participants consist of a bank and a continuum of outside investors. The main analysis of this paper (Section 3.1–4) concerns only $t = 1, 2, 3$. We extend the model to an ex ante stage $t = 0$ only in Section 5.

All agents are risk neutral. The outside investors are deep-pocketed and competitive. The banks are impatient and have a discount factor $\delta < 1$ between $t = 1$ and $t = 3$. This follows the assumption of DeMarzo and Duffie (1999) and can be interpreted as the bank's liquidity needs.The outside investors have no such discount. Hence, there are gains from trade between the bank and the investors.[9]

[9]Modelling gains from trade as a discount factor $\delta < 1$ is standard in the literature to capture liquidity needs stemming from, e.g., capital constraints, new investment opportunities, risk-sharing, etc. (see Holmström and Tirole (2011)).

Mortgage pool and foreclosures

The underlying asset in our model is a pool of assets containing a continuum of ex ante identical mortgages that pay off at $t = 3$. For the main analysis, we focus on the foreclosure of the mortgages when they become delinquent, as detailed below. In Section 5 we extend the model to consider the ex ante screening effort choice, which also has an effect on the cash flow of the mortgage pool.

We model the mortgage pool as a well-diversified portfolio of mortgages. The mortgage pool is thus only exposed to aggregate risks, which affect the ability for all borrowers to repay.[10] Specifically, with probability π, the mortgage pool is in a good state (G) and no borrowers default. In state G, the mortgage pool returns a riskless cash flow $Z_G > 0$. With probability $1 - \pi$, the mortgage pool is in a bad state (B) and each borrower defaults with some i.i.d. probability. Thanks to the diversification benefit, the proportion of the mortgages that become delinquent at $t = 2$ is fixed. We normalise the measure of the delinquent mortgages in the pool to 1. The remaining performing mortgages continue to return a riskless cash flow $Z_B < Z_G$ at $t = 3$. Since mortgage delinquency only occurs in the bad state, we will focus primarily on the sequence of events after a realisation of the bad state to study mortgage foreclosures.

When a mortgage becomes delinquent at $t = 2$, it can be foreclosed or modified.[11] In the case of foreclosure, the mortgage contract is terminated. The collateral property is repossessed and sold to outside investors. Alternatively, if the delinquent mortgage is modified, the modified mortgage pays off a riskless cash flow $X > 0$ with probability θ at $t = 3$ (recovery) or zero otherwise (re-default). For simplicity, we assume that the recovery (and re-default) of delinquent mortgages in a give pool are perfectly correlated. This is also in line with the assumption of a well-diversified mortgage pool so that only aggregate risks affect the repayment of the borrower. Finally, we further assume that $Z_G \geq Z_B + X$, so that the value of a mortgage in the good state is at least as high as in a bad state, even if all delinquent mortgages are modified and subsequently resume payments in the bad state. Intuitively,

[10] Such aggregate risks can be aggregate properties prices and employment opportunities for the borrower.

[11] Throughout the paper, we use "mortgage modification", "mortgage renegotiation", and "mortgage forbearance" interchangeably. Because we abstract from the renegotiation process between the mortgage lender and the borrower, one can interpret the cash flows to the mortgage pool following a decision of no foreclosure as the un-modelled optimal renegotiation outcome.

the difference accounts for the value of the temporary missing payments and the modified lower principal and interest repayment.

The mortgage pool's exposure to the aggregate risks is characterised by the probability of entering state G. This probability $\pi \in \{\pi_H, \pi_L\}$, where $\pi_H > \pi_L$, is mortgage-pool specific and is the source of information asymmetry between the bank and outside investors, as detailed in the next section. We interpret π_i as the "quality" of the mortgage pool (subscript "H" stands for "High" and "L" for "Low"). A high-quality pool is less exposed or more resilient to the aggregate risks and hence is more likely to have no delinquent mortgages (be in the good state G). At $t = 1$ all model participants have the prior belief that $\pi = \pi_H$ with probability γ.[12] We make the following assumption to ensure that the bad state (B) arises with sufficient probability to guarantee its relevance, which ensures the concavity of the objective function in the analysis.

Assumption 1. *$\pi_L < \pi_H < 1 - (1-\delta)\frac{\theta}{1-\theta}$, where $1 - \delta < \frac{1-\theta}{\theta}$.*

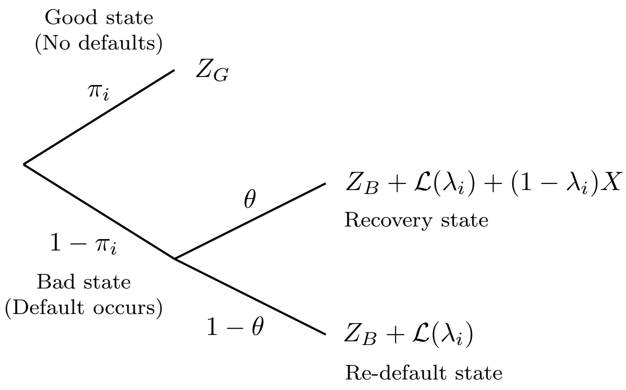
The focus of the paper is to study what proportion of the delinquent mortgages is chosen to be foreclosed in equilibrium and how securitisation affects this decision. The foreclosure policy can be summarised by $\lambda_i \in (0, 1)$, the fraction of delinquent mortgages foreclosed in a mortgage pool of quality i (i.e. $(1 - \lambda)$ fraction of delinquency mortgages modified). Denote by $\mathcal{L}(\lambda_i)$ the total liquidation proceeds from repossessed properties. For a given foreclosure policy, the overall cash flow from a type i mortgage pool at $t = 3$ is then Z_G with probability π_i (the "Good" state), $Z_B + \mathcal{L}(\lambda_i) + (1-\lambda_i)X$ with probability $(1-\pi_i)\theta$ (the "Recovery" state), and $Z_B + \mathcal{L}(\lambda_i)$ with probability $(1 - \pi_i)(1 - \theta)$ (the "Re-default" state), as illustrated in Fig 1.

The exact functional form of the liquidation proceeds $\mathcal{L}(\lambda)$ depends on the characterisation of the market for distressed properties as well as the direct and indirect costs associated with foreclosures. We abstract from these considerations to keep the analysis general and make the following intuitive assumption on the foreclosure technology.

Assumption 2. *For $\lambda \in (0, 1)$, (i) $\mathcal{L}(\lambda)$ is strictly increasing and concave; (ii) $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} \in (0, X)$; and (iii) $\lim_{\lambda \to 0^-} \frac{\partial \mathcal{L}}{\partial \lambda} > \theta X > \lim_{\lambda \to 1^+} \frac{\partial \mathcal{L}}{\partial \lambda}$.*

[12] In Section 5 we endogenise this probability γ in an ex ante stage $t = 0$ through the bank's screening effort choice.

Figure 1: Mortgage pool cash flow



Assumption 2 states that, first, $\mathcal{L}(\lambda)$ is strictly increasing and concave in λ. The decreasing marginal liquidation value of the foreclosed loans could be due to un-modelled heterogeneity in the market value of the underlying properties at $t = 2$. All else equal, a delinquent mortgage backed by a property of higher market value would be foreclosed first. Alternatively, the decreasing marginal liquidation value of the foreclosed loans could also be due to either scarce capital or scarce expertise in making the renovation needed to realise the value of the properties. Secondly, the marginal liquidation value of the mortgage is below the full repayment value of the mortgage $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} \in (0, X)$ for any $\lambda \in (0, 1)$. Intuitively, there are costs associated with liquidating a mortgage, due to, for example, renovation and repair costs associated with investing in distressed property, as well as other outstanding liabilities such as unpaid fees and taxes. The last part of this assumption is a technical assumption to ensure an interior optimal foreclosure policy in the first-best case.

Securitisation

Because of the liquidity discount δ, at $t = 1$, the bank who owns the mortgage pool would like to design and sell a security backed by the cash flow of the mortgage pool at $t = 3$ to outside investors. We will henceforth refer to the bank as the "securitiser" and the security as the mortgage-backed securities (MBS). The securitiser thus receives the cash proceed from selling the MBS at $t = 1$, and retains any residual cash flow from the mortgage pool after paying off the investors at $t = 3$.

We mentioned earlier that there is asymmetric information between the securitiser and the investors. This creates friction in the securitisation process akin to the classical lemon's problem in Akerlof (1970). Specifically, at the beginning of $t = 1$, the securitiser receives private information regarding the quality of the mortage pool $\pi_i \in \{\pi_H, \pi_L\}$. The source of private information could come from new information produced during the process of structuring the individual mortgages into a pool for securitisation, as in DeMarzo and Duffie (1999).[13]

The focus of this paper is the foreclosure decision, which interacts with the securitisation process because the foreclosure policy affects the cash flows of the mortgage pool. We therefore model the securitisation process as follows. The securitiser with information π_i offers the outside investors a security $\mathcal{F}_i$ and promises a foreclosure policy λ_i. The security $\mathcal{F}_i$ is contracted upon the cash flows at $t = 3$, specifying payments to the MBS investors for each realisation of the cash flow. We restrict our attention to monotone securities.[14]

We assume that the securitiser is able to commit to the foreclosure policy promised at $t = 1$ and then implement it as $t = 2$, when mortgage defaults occur. The securitiser's ability to commit is crucial to our main results. We show how the results change if the securitiser did not have commitment power and discuss how the securitiser can establish commitment power over her foreclosure policy by contracting with a mortgage servicer in Section 4.

After observing the offer $(\mathcal{F}_i, \lambda_i)$, the competitive investors form a posterior belief $\hat{\pi}$ regarding the private information of the securitiser, and bid the price of the security p to its fair value. At $t = 3$, after paying investors according to $\mathcal{F}_i$ from

[13]DeMarzo and Duffie (1999) solves the ex ante security design problem, whereas we solve for the ex post security design problem after the banks learn about their private information. As shown by DeMarzo (2005) and DeMarzo et al. (2015), similar intuition carries through in the ex post problem, although the problem becomes more complicated as the design itself becomes a signal.

[14]That is, a higher realisation of the mortgage pool cash flow should leave both the outside investors and the securitiser a (weakly) higher payoff. Although this implies some loss of generality, it is not uncommon in the security design literature, e.g. Innes (1990) and Nachman and Noe (1994). One potential justification provided by DeMarzo and Duffie (1999) is that, the issuer has the incentive to contribute additional funds to the assets if the security payoff is not increasing in the cash flow. Similarly, the issuers has the incentive to abscond from the mortgage pool if the security leaves the issuer a payoff that is not increasing in the cash flow. If such actions cannot be observed, the monotonicity assumption is without loss of generality.

the mortgage pool cash flow, the securitiser consumes any residual cash flow.

Time line and the equilibrium concept

The timeline of the model is summarised in Table 1. The main analysis carried out in Section 3.1–4 concerns only $t = 1, 2, 3$. We extend the model to an ex ante stage $t = 0$ in Section 5.

Table 1: Model timeline

$t = 0$	Securitiser exerts screening effort (Section 5 only)
$t = 1$	Securitiser observes π_i and offers $(\mathcal{F}_i, \lambda_i)$
$t = 2$	Mortgage defaults in state B Securitiser implements foreclosure policy λ_i
$t = 3$	Final payoffs realise

The equilibrium concept in this model is the perfect Bayesian equilibrium (PBE). Formally, a PBE consists of a security $\mathcal{F}_i$ issued by the securitiser of each type $i \in \{H, L\}$, the foreclosure policy λ_i of the securitiser of each type, and a system of beliefs such that i) the securitiser chooses the security and the foreclosure policy at $t = 1$ to maximise her expected payoff, given the equilibrium choices of the other agents and the equilibrium beliefs, and ii) the beliefs are rational given the equilibrium choices of the agents and are formed using Bayes' rule (whenever applicable). As there can be multiple equilibria in games of asymmetric information, we invoke the Intuitive Criterion of Cho and Kreps (1987) to eliminate equilibria with unreasonable out-of-equilibrium beliefs. This allows us to restrict attention to only the least cost separating equilibrium.

3 Securitisation with Endogenous Foreclosure

In this section we analyse our model of securitisation with endogenous foreclosure. In order to contrast with the distortion in the foreclosure policy created by information asymmetry, we first present the benchmark case under full information, before we proceed to solve the model under asymmetric information.

3.1 First-best and the full-information benchmark

In this section we first characterise the first-best foreclosure policy. We then analyse the benchmark equilibrium under full information, and show that the first best is achieved in the full-information equilibrium.

The first-best foreclosure policy maximises the value of the mortgage pool $V_i(\lambda)$.

$$\lambda_i^{FB} = \arg\max_{\lambda} \; V_i(\lambda) \tag{1}$$

$$\text{where} \quad V_i(\lambda) \equiv \pi_i Z_G + (1-\pi_i)[Z_B + \mathcal{L}(\lambda) + (1-\lambda)\theta X] \tag{2}$$

The solution is characterised by the first order condition, $\frac{\partial \mathcal{L}(\lambda_i^{FB})}{\partial \lambda} = \theta X$. That is, since the marginal value obtained from foreclosure is decreasing with the fraction of foreclosed loans, the first-best level of foreclosure is determined such that the the margin value from foreclosure is equal to the expected recovery value given modification, conditional on the bad state (B). Furthermore, as the H and L type mortgage pools only differ in the probability of entering state (B), the first-best level of foreclosure is identical across types. Denote the solution to this first order condition by $\lambda^{FB} \in (0,1)$.

We now characterise the equilibrium under full information. Firstly consider the optimal security issued in the securitisation process at $t = 1$. Since any retention of the cash flows by the securitiser incurs a liquidity discount, it is optimal for the security issued to be a full equity pass-through security to the investors, when all securities are fairly priced given full information. Secondly, given that the entire cash flow is securitised, the securitiser optimally commits to the first-best foreclosure policy λ^{FB} to maximise the value of the mortgage pool and hence her payoff.

The following proposition thus summarises the full-information benchmark results. All proofs are in the Appendix.

Lemma 1. *In the full-information benchmark, the securitiser of both types issues a pass-through equity security backed by the cash flows, and chooses the first-best foreclosure policy* λ^{FB}.

Denote henceforth the expected payoff to a type i securitiser in the full-information benchmark as $U_i^{FB} \equiv V_i(\lambda^{FB})$. We would like to conclude this section by stressing the fact that the first-best foreclosure policy is achieved in the full-information benchmark equilibrium. Therefore, any inefficiency in the equilibrium foreclosure policy in this paper is driven by information asymmetry between the securitiser and the outside investors. We shall turn to this asymmetric information problem in the next section.

3.2 Excessive foreclosure policy as a costly signal

In this section, in order to highlight the main result of excessive foreclosure policy, we solve the model under the simplifying assumption that the equilibrium securities are debt securities with endogenous face values.

At $t = 1$, the securitiser with private information π_i issues an MBS $\mathcal{F}_i$ backed by the cash flows of the mortgage pool, and promises a foreclosure policy λ_i. Observing the offer from the securitiser $(\mathcal{F}_i, \lambda_i)$, the investors form a belief about the quality $\hat{\pi}$ of the mortgage pool. We focus our attention to the least cost separating equilibrium, which is the unique equilibrium that satisfies the Intuitive Criterion in our model. We state this result in the following lemma and formally prove it in the Appendix.

Lemma 2. *The unique equilibrium that satisfies the Intuitive Criterion is the least cost separating equilibrium.*

Let's now start the analysis with the securitiser who owns a low quality mortgage pool (high default probability). In a separating equilibrium, the low-type securitiser always receives the fair price on the security issued. Therefore the securitiser maximises her expected payoff by selling the entire cash flow from the mortgage to outside investors, and promising the first-best level of foreclosure policy. There is no distortion in the form of either inefficient retention or inefficient foreclosure for the low type. Denote by U_i^* the expected payoff to a type i securitiser in equilibrium. The payoff to the low-type securitiser in a separating equilibrium is thus equal to the first-best level, $U_L^* = U_L^{FB}$, while her foreclosure policy in equilibrium is $\lambda_L^* = \lambda^{FB}$. We denote henceforth with superscript $*$ all equilibrium quantities.

The high-type securitiser, on the other hand, has to issue a security and promise a foreclosure policy such that in equilibrium it is not profitable for the low type to deviate and mimic. Since we restrict the security to debt, we denote by F_H the face value of the security issued by the high-type securitiser. We focus on the case in which $F_H \in (Z_B + \mathcal{L}(\lambda_H), Z_G)$, i.e. a risky debt, for any λ_H that the securitiser promises. We show in Section 3.3 that the optimal security is indeed a risky debt (Proposition 3).

Denote by $p_i(F, \lambda)$ the value of the MBS given a face value of F and a committed foreclosure policy λ, backed by a mortgage pool of quality i. That is,

$$p_i(F, \lambda) = \pi_i F + (1 - \pi_i)[\theta \min\{Z_B + \mathcal{L}(\lambda) + (1 - \lambda)X, F\} + (1 - \theta)(Z_B + \mathcal{L}(\lambda_H))] \tag{3}$$

In a separating equilibrium, after observing (F_H, λ_H), the investors believe that the issuer of the security with face value F_H who promises a foreclosure policy of λ_H is of the high type. The market price of the MBS is therefore equal to $p_H(F_H, \lambda_H)$.

In the least cost separating equilibrium, the high-type securitiser maximises the value of the proceeds from securitisation plus the value of the residual cash flow by choosing the face value of her debt and her promised foreclosure policy λ_H. Her equilibrium payoff is given by

$$\begin{aligned} U_H^* = max_{(F_H, \lambda_H)} \quad & p_H(F_H, \lambda_H) + \delta\left[V_H(\lambda_H) - p_H(F_H, \lambda_H)\right] \\ s.t. \quad (IC) \quad & U_L^{FB} \geq p_H(F_H, \lambda_H) + \delta\left[V_L(\lambda_H) - p_L(F_H, \lambda_H)\right] \end{aligned} \tag{4}$$

where (IC) is the incentive compatibility constraint for the low type not to mimic the offer (F_H, λ_H) of the high type. Denote by (F_H^*, λ_H^*) the unique solution to the above optimisation programme.

The following proposition highlights a key property of the equilibrium foreclosure policy, which is the main result of the paper.

Proposition 1. *In the least cost separating equilibrium, the high-type securitiser adopts a (weakly) excessive foreclosure policy in equilibrium, whereas the low-type securitiser adopts the first-best foreclosure policy. That is,*

$$\lambda_H^* \geq \lambda^{FB} = \lambda_L^* \tag{5}$$

The weak inequality is strict if and only if

$$G(\lambda^{FB}) \equiv \frac{(\pi_H - \pi_L)}{\pi_L(1-\delta)}(1-\theta)(1-\lambda^{FB})X - Z_G + Z_B + \mathcal{L}(\lambda^{FB}) + (1-\lambda^{FB})X > 0 \quad (6)$$

As shown in the Appendix, the condition $G(\lambda^{FB}) > 0$ given by Eq. 6 implies that the equilibrium face value of the debt satisfies $F_H^* < Z_B + \mathcal{L}(\lambda_H^*) + (1-\lambda_H^*)X$. That is, the optimal debt security in equilibrium is protected by high retention and relatively safe, so that the MBS does not default in the recovery state, but only in the re-default state. Proposition 1 states that, in an equilibrium in which the MBS issued by the high type is relatively safe, the equilibrium foreclosure policy of the high type deviates from the first-best policy and is qualitatively excessive. $G(\lambda^{FB}) > 0$ is more likely to hold when i) the discount factor δ is high, and/or ii) the extent of asymmetric information as measured by $\frac{\pi_H - \pi_L}{\pi_L}$ is high, so that a large fraction of the cash flows must be retained by the high type in order to signal her quality.

To clarify the trade-off faced by the high-type securitiser when choosing her foreclosure policy in equilibrium, we can rewrite the expected payoff to the high type issuer as follows, which consists of two components.

$$\delta V_H(\lambda_H) + (1-\delta)p_H(F_H, \lambda_H) \quad (7)$$

The first term represents the discounted value of the mortgage pool enjoyed by the securitiser without securitisation, and the second term represents the gains from securitising given the security choice and the foreclosure poplicy.

The equilibrium foreclosure policy $\lambda_H^* > \lambda^{FB}$ trades off the efficiency loss associated with excessive foreclosure $\frac{\partial V_H(\lambda_H^*)}{\partial \lambda_H} < 0$ against increased gains from securitisation. The latter force is stated by the following corollary. Denote by $\hat{p}(\lambda_H)$ the highest securitisation proceeds the high-type securitiser can obtain by optimising the face value of the debt issued in a separating equilibrium in which her foreclosure policy is λ_H. That is, $\hat{p}(\lambda_H) \equiv \max_{F_H} \; p_H(\lambda_H, F_H)$ *s.t.* (IC).

Corollary 1. *In a separating equilibrium, excessive foreclosure allows the high-type securitiser to receive higher securitisation proceeds. That is,* $\left.\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda}\right|_{\lambda_H = \lambda^{FB}} \geq 0$, *where the inequality is strict if and only if Eq. 6 holds.*

The high-type securitiser commits to excessive foreclosure in equilibrium, because an excessive foreclosure policy reduces the signalling cost she must incur at the securitisation stage. We shall illustrate the intuition behind such signalling cost reduction effect as follows: a marginally more excessive foreclosure policy relaxes the low type's no-mimicking, incentive compatibility constraint, which can be re-written as

$$\delta V_L(\lambda_H) + \underbrace{p_H(F_H, \lambda_H) - \delta p_L(F_H, \lambda_H)}_{\text{premium from securitising with } (F_H, \lambda_H)} \leq U_L^{FB} \tag{8}$$

Relaxing the incentive constraint is equivalent to lowering the low type's mimicking payoff, i.e. the left-hand-side of (Eq. 8), as U_L^{FB} is not affected by the high type's action (F_H, λ_H). The mimicking payoff comprises of two parts – i) the discounted value of the low type's portfolio $\delta V_L(\lambda_H)$ following deviation in her foreclosure policy from the first-best to that of the high type, λ_H, and ii) the premium (in utility units) from securitising under the terms offered by the high type, where the low-type securitiser receives $p_H(F_H, \lambda_H)$ from the investors whilst only giving up a security that is worth $\delta p_L(F_H, \lambda_H)$ to her. We exclusively focus on the premium $p_H(\cdot) - \delta p_L(\cdot)$ in the following discussion of the intuition because the expected value of the pool V_L is not affected by a marginal deviation from λ^{FB} as $\frac{\partial V_L(\lambda^{FB})}{\partial \lambda} = 0$.

The key driving forces behind the result that excessive foreclosure reduces the premium $p_H(\cdot) - \delta p_L(\cdot)$ come from the sensitivity differential of $p_H(\cdot)$ and $p_L(\cdot)$ with respect to changes in foreclosure rate λ_H and face value F_H. More precisely, the low type's valuation of the MBS is more sensitive to an increase in λ but less sensitive to an increase in F_H than the high type's, as shown in

$$(1-\pi_L)(1-\theta)\,\mathcal{L}'(\lambda) = \frac{\partial p_L(F,\lambda)}{\partial \lambda} > \frac{\partial p_H(F,\lambda)}{\partial \lambda} = (1-\pi_H)(1-\theta)\,\mathcal{L}'(\lambda) \tag{9}$$

$$1-(1-\pi_L)(1-\theta) = \frac{\partial p_L(F,\lambda)}{\partial F} < \frac{\partial p_H(F,\lambda)}{\partial F} = 1-(1-\pi_H)(1-\theta) \tag{10}$$

Equipped with these properties of the MBS valuation, we can show that the high type does strictly better under an excessive foreclosure policy than under the first-best one. Starting from the separating offer (λ^{FB}, F_H^{FB}) that binds (IC), we construct an alternative offer by with a marginal increase in λ_H and a simultaneous marginal decrease in F_H such that $p_H(\cdot)$ remains unchanged. This alternative offer overall increases $p_L(\cdot)$ as it increases more with the increase in λ_H and decreases less with the decrease in F_H than $p_H(\cdot)$, thereby reducing the premium $p_H(\cdot) - \delta p_L(\cdot)$ and relaxing the (IC).[15] Finally, as proven in the Appendix (as part of the proof of Proposition 3), a slack (IC) implies that the high type can issue an MBS with lower retention that gives her strictly higher ex ante utility, given her (marginally) excessive foreclosure policy. Since the low-type securitiser receives the first-best payoff in the separating equilibrium, excessive foreclosure by the high-type not only improves the high type's equilibrium payoff, but also achieves an equilibrium allocation that *Parato-dominates* any allocation with first-best foreclosure.

The final missing piece in understanding the intuition behind excessive foreclosures reducing signalling cost is what drives the sensitivity differential properties, in particular Eq (9). Several important features of the model give rise to this property. First, owing to the information asymmetry between the securitisers and the investors, the optimal MBS issued by the high type is a risky debt, whose payoffs are more sensitive to a change of the mortgage pool's cash flow in the worse states. Particularly, when Eq. 6 holds, the MBS has a face value that is strictly lower than the cash flow in the recovery state. This MBS's payoff is only sensitive to a change in the mortgage pool's cash flow in the re-default state. Second, an increase in the foreclosure rate essentially transfers cash flow from the recovery state to the re-default state. This in turn *increases* the expected payoffs of the risky debt backed by either type of mortgage pool, as a consequence of the aforementioned difference in the sensitivity of the MBS in different states. Finally, as the low-quality pool has a higher probability of delinquency, foreclosure raises the value of the MBS backed by a low-quality pool

[15] A more formal argument of this alternative offer: increase λ_H by a small positive amount ϵ and decrease F_H by $\frac{\partial p/\partial \lambda_H}{\partial p/\partial F_H}\epsilon$. By construction, the price of the MBS $p(F_H, \lambda_H)$ is the same because by total differentiation and first-order approximation

$$dp_H(F_H, \lambda_H) \approx \frac{\partial p_H}{\partial F_H} dF_H + \frac{\partial p_H}{\partial \lambda_H} d\lambda_H = \frac{\partial p_H}{\partial F_H}\left(-\frac{\partial p_H/\partial \lambda_H}{\partial p/\partial F_H}\epsilon\right) + \frac{\partial p_H}{\partial \lambda_H}\epsilon = 0 \tag{11}$$

Meanwhile, this deviation increases the low type's valuation of the MBS $p_L(F_H, \lambda_H)$ as

$$dp_L(F_H, \lambda_H) \approx \frac{\partial p_L}{\partial F_H} dF_H + \frac{\partial p_L}{\partial \lambda_H} d\lambda_H = \underbrace{\left[\frac{\partial p_L}{\partial F_H}\left(-\frac{\partial p/\partial \lambda_H}{\partial p/\partial F_H}\right) + \frac{\partial p_L}{\partial \lambda_H}\right]}_{>0 \text{ by Eq. (9) and (10)}}\epsilon > 0 \tag{12}$$

This offer thus relaxes (IC) while leaving the high type indifferent.

more than the same MBS backed by a high-quality pool.

To summarise, the high-type securitiser commits to excessive foreclosure $\lambda_H^* \geq \lambda^{FB}$ in equilibrium, in order to generate greater gains from securitisation. This is because excessive foreclosure reduces the exposure of the mortgage pools to the risk of re-default of the delinquent mortgages, which discourages mimicking by the low type, reducing the signalling cost the high type must incur at the time of securitisation.

This effect is further illustrated by the following comparative statics: Firstly, an increase in the quality of the high type, π_H, exacerbates the information asymmetry because it creates greater mimicking incentives. As a result, there is more distortion towards excessive foreclosure in equilibrium. Secondly, an increase in the discount factor of the securitiser, δ, reduces the distortion in the high-type securitiser's foreclosure policy. Since the high-type chooses an excessive foreclosure policy in order to reduce the cost of retention, an increase in δ decreases such cost, mitigating the need for excessive foreclosure.

Proposition 2. *The equilibrium foreclosure policy of the high-type securitiser, and hence the distortion in the foreclosure policy, is increasing in her quality, and decreasing in the discount factor δ. That is, $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$ and $\frac{\partial \lambda_H^*}{\partial \delta} < 0$ when Eq. 6 holds.*

3.3 The optimality of debt

In this section we endogenise the security design problem of the securitisers, and show that indeed a risky debt is an optimal security in this model.

As analysed in Section 3.2, the low-type securitiser optimally issues a full pass-through security to outside investors, because she always receives the fair price on her security in any separating equilibrium.

We therefore focus our analysis on the high-type securitiser. In conjunction with promising a foreclosure policy λ_H, the high type designs a security $\mathcal{F}_H = (f_1, f_2(c_2), f_3(c_3))$, which maps the realisation of the mortgage pool cash flows to a set of payoffs to the outside investors, as summarised in Table 2, for any given foreclosure policy λ. We suppress the dependency of c_2 and c_3 on λ whenever it does not create confusion.

Table 2: Payoffs of a generic security backed by the mortgage pool cash flows

Realisation of cash flow	Security payoff $\mathcal{F}$
$c_1 \equiv Z_G$	f_1
$c_2(\lambda) \equiv Z_B + \mathcal{L}(\lambda) + (1-\lambda)X$	$f_2(c_2)$
$c_3(\lambda) \equiv Z_B + \mathcal{L}(\lambda)$	$f_3(c_3)$

Notice that $f_2(c_2)$ and $f_3(c_3)$ also depends indirectly on the foreclosure policy λ through c_2 and c_3. For brevity we also suppress the dependency of $f_j(c_j)$ on c_j, $j \in \{2,3\}$, whenever it is clear. The value of the MBS $\mathcal{F}$ backed by a mortgage pool of quality i, given a committed foreclosure policy λ, is thus given by

$$p_i(\mathcal{F},\lambda) = \pi_i f_1 + (1-\pi_i)[\theta f_2 + (1-\theta) f_3] \tag{13}$$

In the least cost separating equilibrium, the high-type securitiser maximises the proceeds from securitisation plus the residual cash flow by choosing the security $\mathcal{F}_H$ to offer and her promised foreclosure policy λ_H, while ensuring that it is not profitable for the low type to deviate and mimic.

$$\begin{aligned}
& max_{(\mathcal{F}_H,\lambda_H)} && p_H(\mathcal{F}_H,\lambda_H) + \delta\left[V_H(\lambda_H) - p_H(\mathcal{F}_H,\lambda_H)\right] \\
s.t. \quad & (IC) && U_L^{FB} \geq p_H(\mathcal{F},\lambda_H) + \delta\left[V_L(\lambda_H) - p_L(\mathcal{F}_H,\lambda_H)\right] \\
& (MNO) && f_1 \geq f_2 \geq f_3 \geq 0 \;\forall\lambda \in (0,1) \text{ and } \frac{\partial f_j(c_j)}{\partial c_j} \geq 0 \;\forall j \in \{2,3\} \\
& (MNI) && c_1 - f_1 \geq c_2 - f_2 \geq c_3 - f_3 \geq 0 \;\forall\lambda \in (0,1) \text{ and} \\
& && \qquad \frac{\partial}{\partial c_j}(c_j - f_j(c_j)) \geq 0 \;\forall j \in \{2,3\}
\end{aligned} \tag{14}$$

The optimisation programme given by Eq. 14 takes into account the security design problem. Therefore it has two additional monotonicity constraints regulating the security design when compared to Eq. 4: (MNO), the outside investors' monotonicity constraint, and (MNI), the insider residual claim's monotonicity constraint. These constraints state that, respectively, the payoff of the security and the residual payoff to the securitiser are weakly increasing in the realisation of the cash flows.[16]

[16]We specify the two monotonicity constraints for any generic security $\mathcal{F}$ and for all possible realisation of the cash flows. Since the foreclosure policy λ_H is pre-committed, only three cash flows occur in equilibrium, namely c_1, $c_2(\lambda_H)$ and $c_3(\lambda_H)$. The equilibrium security is uniquely defined for these cash flows that occur in equilibrium. Although the payoff of the optimal security

The following proposition establishes the optimality of debt, which was taken as given in the discussion in Section 3.2.

Proposition 3. *For any pre-committed foreclosure policy* λ_H*, a risky debt backed by the mortgage pool with face value* $F_H \in (c_3(\lambda_H), c_1)$ *is an optimal security for the high-type securitiser. Formally, the MBS* $\mathcal{F}_H = \{f_1, f_2, f_3\}$ *is given by* $f_j = \min\{c_j, F_H\}\ \forall\lambda \in (0,1),\ \forall j \in \{1,2,3\}$.

This result is consistent with the classic literature on the pecking order of outside financing, e.g. Myers (1984) under asymmetric information. In order to discourage the low type from mimicking, the optimal security issued by the high-type is a debt security ($F_H < c_1$). Moreover, the high-type exhausts her capacity of issuing risk-free debt ($F_H > c_3(\lambda_H)$), since risk-free securities are free from the information asymmetry problem.[17]

The high-type securitiser's retention of residual claims of future cash flow could be seen as a necessary signalling cost in order to separate from the low type, a well-established result in the security design literature such as Leland and Pyle (1977) and DeMarzo and Duffie (1999). As discussed in the previous section, the main result of this paper is to show that excessive foreclosure helps to mitigate such signalling cost for the high-type securitiser. The extent of excessive foreclosure in equilibrium thus trades off the direct cost of inefficient foreclosure against the greater gains from securitisation.

4 Mortgage servicers as commitment devices

Thus far we have made the important assumption that the securitiser is able to commit to a foreclosure policy at $t = 1$ and then implement it at $t = 2$ when delinquencies occur. In this section, we first show how the results change if the securitiser could not commit. We then introduce mortgage servicers into the model and argue that they enable the securitisers to effectively commit to a foreclosure policy.

may not be uniquely pinned down for the cash flows associated with off-equilibrium foreclosure policies, this is inconsequential for solving the optimal foreclosure policy.

[17]Technically, the cash flow distribution in our model satisfies the (HRO) property, which is weaker than the Monotone Likelihood Ratio Property (MLRP) commonly assumed in signalling environments. DeMarzo et al. (2015) show that the (HRO) is a sufficient condition to ensure the optimality of debt security in a signalling framework with liquidity needs.

4.1 The effect of commitment

In order to formally study the importance of the securitiser's ability to commit to a foreclosure policy, we modify the sequence of events in the model, as shown in Table 3.

Table 3: Model timeline without commitment

$t = 1$	Securitiser observes π_i and offers $\mathcal{F}_i$
$t = 2$	Mortgage defaults in state B Securitiser chooses foreclosure policy λ_i
$t = 3$	Final payoffs realise

The crucial change here is that the securitiser chooses the foreclosure policy *after* she has sold the MBS to the investors. This implies that promised foreclosure policies at $t = 1$ are no longer credible and thus cannot signal any information because the investors anticipate that the securitiser would always choose the foreclosure policy that maximises the value of her residual claim when mortgage defaults occur at $t = 2$. We defer the characterisation of the equilibrium without commitment to the Appendix, and discuss the main intuition below.

To demonstrate the incentive problem associated with the lack of commitment, consider the high-type securitiser's incentive to foreclose the delinquent mortgage at $t = 2$ in state B, for a given MBS issued at $t = 1$. Recall that when Eq. 6 holds, an optimal MBS with commitment is a debt security with face value $F_H^* \in (Z_B + \mathcal{L}(\lambda_H^*), Z_B + \mathcal{L}(\lambda_H^*) + (1 - \lambda_H^*)X)$ and the securitiser would like to commit to an excessive foreclosure policy $\lambda_H^* > \lambda^{FB}$ (as described in Proposition 1).

Without commitment, however, this foreclosure policy is not incentive compatible at $t = 2$. Given the MBS with face value F_H^* issued at $t = 1$, the securitiser holds the residual levered-equity claim at $t = 2$, which only pays off in the recovery state. Therefore, instead of implementing the promised foreclosure policy λ_H^*, the securitiser would be better off deviating to a more lenient foreclosure policy. By allowing more loan modification, the securitiser increases the cash flow of the mortgage pool in the recovery state, and thus her expected payoff.[18]

[18] The illustrated profitable deviation to a more lenient foreclosure policy always exists. There might exist other profitable deviations, as the optimal security under commitment $\mathcal{F}_H^*$ is not

The above reasoning highlights the stark contrast between the securitiser's incentive to foreclose delinquent loans at $t = 1$ and at $t = 2$. The high-type securitiser optimally issues a debt security at $t = 1$ and retains the residual equity stake to signal the quality of her mortgage pool. In order to reduce the retention cost, she would like to commit to an excessive foreclosure policy. Yet precisely because of the retained equity stake necessary as a signal at $t = 1$, the securitiser at $t = 2$ faces distorted incentives, which prevent her from implementing the ex ante desired excessive foreclosure policy.

The lack of commitment thus limits the foreclosure policy the securitiser can credibly choose in equilibrium. This in turn reduces the efficiency of securitisation in equilibrium, as demonstrated by the following proposition. Denote by U_i^{NC} the expected payoff to a securitiser with a type i mortgage pool in the least cost separating equilibrium without commitment, where NC stands for "No Commitment". Proposition 4 establishes Eq. 6 as a sufficient condition for the constraint imposed by the lack of commitment to bind and lead to efficiency loss.

Proposition 4. *If Eq. 6 holds, compared to the scenario without commitment, the low-type securitiser in the least cost separating equilibrium with commitment is equally well off whereas the high-type securitiser is strictly better off. That is,*

$$U_L^{FB} = U_L^* = U_L^{NC} \quad \text{and} \quad U_H^{FB} > U_H^* > U_H^{NC} \tag{15}$$

4.2 Mortgage servicers enable commitment

The high-type securitiser has a commitment problem because her incentives at $t = 1$ to signal her quality through excessive foreclosure conflicts with her incentives at $t = 2$ to maximise the value of her residual claim. Outsourcing the foreclosure decision at to a third-party agent could thus avoid such conflict and hence effectively restore the securitiser's commitment power. Indeed, this argument provides a raison d'être for hiring mortgage servicers, a common practice in the mortgage securitisation industry. As we show below, this interpretation corroborates with some empirical findings regarding the servicers' impact on foreclosures and their compensation contracts.

uniquely defined.

We propose two potential ways in which the securitiser can commit to a foreclosure policy through contracting with a mortgage servicer. First, if servicers have different mortgage foreclosure capacities or modification capacities and such heterogeneity is public information, the high-type securitiser can signal and effectively commit to a "tough" foreclosure policy by hiring a servicer with high foreclosure capacity (or low modification capacity). This mechanism rests on the assumption that it is costly for servicers to alter their capacity significantly. We believe this assumption is realistic. For example, Thompson (2009), among others, argue that foreclosing and modifying a delinquent mortgage require substantial human capital and expertise hence it is very difficult and costly for a servicer expand the capacity quickly. This interpretation of heterogeneity among mortgage servicers corroborates with the economically and statistically significant servicer fixed effect in predicting foreclosure probability, as emphasised by Agarwal et al. (2011).

Alternatively, the securitiser can hire a servicer and provide him a compensation contract that encourages foreclosure. This can explain why mortgage servicing contracts with biased incentives are offered in practice. Thompson (2009) and Kruger (2016) argue that the biased incentives of the servicers are a key friction causing excessive foreclosures. Contracting with servicers can implement ex post inefficient foreclosures as long as there are frictions preventing the contract from being renegotiated. Empirically Kruger (2016) has shown that indeed mortgage servicing contracts are rarely renegotiated, mainly because by law, it requires the consent of the securitiser, the servicer, and the dispersed MBS investors.

We formalise the above ideas in the two extensions below respectively.

4.2.1 Mortgage servicers with heterogeneous foreclosure capacity

In this section, we extend the model to allow the securitiser to choose a mortgage servicer of known foreclosure capacity at $t = 1$. More specifically, there exists a continuum of servicers with different foreclosure capacity $\tau \in (0, 1)$, which is public information. The foreclosure capacity affects the cost of foreclosure. At $t = 2$, if the servicer with capacity τ forecloses a fraction λ of the delinquent mortgages, he incurs a quadratic private cost of $\frac{\kappa}{2}(\lambda - \tau)^2$ for some $\kappa > 0$. The servicer chooses the foreclosure policy at $t = 2$ to minimise the cost. It follows that the servicer chooses to foreclose a fraction $\lambda = \tau$ of the delinquent mortgages. We henceforth refer to τ as the intrinsic "toughness" of the servicer.

At $t = 1$, after learning the quality of her mortgage pool π_i, the securitiser can choose to hire a servicer with capacity τ_i. Each servicer has a reservation utility of 0, and is therefore willing to work for a securitiser if chosen. The securitiser then announces the identity of the servicer alongside the security $\mathcal{F}$ that she offers to the investors. The overall timeline of this extension of the model is summarised in Table 4.

Table 4: Model timeline with heterogeneous servicers

$t = 1$	Securitiser observes π_i Securitiser chooses servicer t_i and offers $\mathcal{F}_i$
$t = 2$	Mortgage defaults in state B Servicer's foreclosure policy $\lambda_i = \tau_i$
$t = 3$	Final payoffs realise

It is important to notice that the intrinsic "toughness" of the servicers directly affects the subsequent foreclosure policy in equilibrium, because the foreclosure decision is now made by the servicer. Therefore the low-type securitiser would like to choose a "neutral" sevicer with $\tau_L = \lambda^{FB}$ to implement the first-best foreclosure policy, whereas the problem faced by the high-type securitiser at $t = 1$ can be re-written as

$$\begin{aligned} &\max_{F_H, \tau_H} \quad p_H(F_H, \lambda_H) + \delta[V_H(\lambda_H) - p_H(F_H, \lambda_H)] \\ s.t \quad &\lambda_H = \tau_H \\ &\text{and } (IC) \text{ as given by Eq. 4} \end{aligned} \tag{16}$$

As discussed in Section 3, at $t = 1$, the high-type securitiser benefits from committing to an excessive foreclosure policy. In this extensions, this can be exactly achieved by hiring a servicer who is known to be "tough", as stated below.

Proposition 5. *In the least cost separating equilibrium, the high-type securitiser hires an excessively "tough" servicer, whereas the low-type securitiser hires a "neutral" servicer. That is, $\tau_H^* = \lambda_H^* \geq \lambda^{FB} = \tau_L^*$. The equilibrium foreclosure policy is $(\lambda_H^*, \lambda_L^*)$.*

Proposition 5 highlights that, not only do the servicers hired in equilibrium implement the ex ante optimal foreclosure policy, they also serve as a signalling device at the securitisation stage. By hiring an excessively "tough" servicer, the high-type securitiser signals to the investors the quality of her mortgage pool, and reduces the costly retention in equilibrium.

4.2.2 Mortgage servicing contract with biased incentives

In this section, we extend the model to allow the securitiser to explicitly contract with the servicer at $t = 1$. Unlike in the previous subsection, in this extension we assume that the servicer is free to choose any foreclosure policy without incurring any cost, consistent with the setup in the baseline model in which the foreclosure decision is made by the securitiser. This then creates a role for incentive contracts, in order for the securitiser to induce the servicer to make the desired foreclosure policy at $t = 2$.

We assume that at $t = 1$, after learning the quality of her mortgage pool π_i, the securitiser offers a servicing contract to the servicer. A contract (α_i, β_i) specifies a percentage fee to the servicer based on the repayments from the performing (or recovered) mortgages $\alpha_i > 0$, and a percentage fee based on the foreclosure proceeds $\alpha_i \beta_i \geq 0$.[19] The parameter β_i measures the relative pay from foreclosure compared to modification. The expected payoff to the servicer, given a foreclosure policy of λ, is given by

$$\hat{\pi}\alpha_i \left[\beta_i \mathcal{L}(\lambda) + \theta(1-\lambda)X\right] \tag{17}$$

where $\hat{\pi}$ is the servicer's expectation about the securitiser's type π_i, and plays no role in characterising the equilibrium foreclosure policy. The servicer has a reservation utility of 0, and is therefore willing to accept any servicing contract of non-negative value.

Once accepted by the servicer, the servicing contract (α_i, β_i) is public information at $t = 1$. The securitiser then offers a security $\mathcal{F}_i$ to the investors. The timeline of

[19] This specification of the contract resembles a servicing agreement in practice, and is without loss of generality. Any non-decreasing contract based on the cash flows of the mortgage pool in the bad state can be written as a contract (α, β). We also abstract from payments for servicing the non-defaulting loans, as they play no role in affecting how the servicing contracts affect the foreclosure of delinquent mortgages.

this version of the model is summarised in Table 5.

Table 5: Model timeline with servicing contracts

$t = 1$	Securitiser observes π_i Securitiser offers the servicer a servicing contract (α_i, β_i) Securitiser offers investors a security $\mathcal{F}_i$ and discloses (α_i, β_i)
$t = 2$	Mortgage defaults in state B Servicer chooses foreclosure policy λ_i
$t = 3$	Final payoffs realise

We now solve for the servicing contracts and foreclosure policy in equilibrium backwards. At $t = 2$, when the mortgages default in the bad state, the servicer chooses the foreclosure policy that maximises his expected payoff as specified by the servicing contract. The resulting foreclosure policy λ_s, where the subscript stands for servicer, is characterised by the first order condition $\beta_i \frac{\partial \mathcal{L}(\lambda_s)}{\partial \lambda} - \theta X = 0$. It is useful to notice that the incentive for the servicer to foreclosure the delinquent mortgages is determined by the parameter β_i of the contract, as stated in the following lemma.

Lemma 3. *For a given servicing contract (α_i, β_i), the foreclosure decision of the servicer at $t = 2$ is strictly increasing in β_i, and independent of α_i. The servicer chooses the first-best foreclosure decision λ^{FB} if and only if $\beta_i = 1$.*

We will henceforth refer to a contract with $\beta_i = 1$ as an "unbiased" contract, and a contract with $\beta_i > 1$ as a "biased" contract towards foreclosure. Anticipating the incentive effects of the servicing contract, the securitiser chooses the contract as well as the security to offer at $t = 1$, in order to maximise her payoff. We defer the full characterisation of the equilibrium to the Appendix, and discuss the main intuition below.

Proposition 6. *In the least cost separating equilibrium, the high-type securitiser provides the servicer with biased incentives towards foreclosure $\beta^*_H \geq 1$, whereas the low-type securitiser provides the servicer with unbiased incentives $\beta^*_L = 1$. The equilibrium foreclosure policy is $(\lambda^*_H, \lambda^*_L)$.*

The low-type securitiser offers an *unbiased* servicing contract to the servicer in equilibrium, in order to implement the first-best foreclosure policy. The high-type securitiser, on the other hand, would prefer an excessive foreclosure policy, because excessive foreclosure reduces the signalling cost she must incur in order to separate from the low type. Therefore, the high-type securitiser optimally chooses to offer a *biased* servicing contract towards foreclosure in equilibrium.

5 Ex ante screening effort and welfare

So far we have treated the ex ante probability γ of the mortgage pool being high quality as exogenous. In this section, we extend the model to incorporate an ex ante stage $t = 0$, at which time the securitiser can endogenously exert non-verifiable costly effort to increase the probability of receiving a high-quality mortgage pool at $t = 1$. The main finding is that while information asymmetry leads to underinvestment in screen effort, committing to an ex post excessive foreclosure policy mitigates this underinvestment problem and the associated inefficiency.

At $t = 0$, the securitiser is endowed with \$1 and can invest in a mortgage pool. When investing, the securitiser can exerts non-contractible effort to affect $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, the probability that the mortgage pool is of high quality at $t = 1$. Such effort can be interpreted as, for example, time and resources spent to screen out borrowers with suspicious income or to form mortgage pool with better diversification property. The effort incurs a quadratic cost of $\frac{1}{2}k(\gamma - \underline{\gamma})^2$. We assume $k \geq \frac{U_H^{FB} - U_L^{FB}}{\bar{\gamma} - \underline{\gamma}}$ to guarantee an interior optimal level of effort, and $U_L^{FB} \geq 1$ so that investing in the mortgage pool is always efficient.

5.1 Optimal screening effort

In this section we solve for the optimal screening effort of the securitiser in equilibrium. The securitiser is willing to exert costly effort because the expected payoff of being a high type U_H is higher than that of being a low type U_L . Since U_H and U_L will be potentially affected by the information environment, the security design, and the foreclosure policy in the subsequent stages of the model, the optimal screening effort chosen by the securitiser will be indirectly affected.

Notice that since the subsequent securitisation stage is in the least cost separating equilibrium, the equilibrium outcome does not depends on γ, the prior probability that the mortgage pool is of high quality. We can therefore consider any generic pair of $\{U_H, U_L\}$ that represents the expected payoffs to the securitiser in the separating equilibrium at the securitsation stage $t = 1$. At $t = 0$, the securitiser chooses the optimal level of effort to maximise her ex ante expected payoff

$$\max_{\gamma} \quad \gamma U_H + (1-\gamma)U_L - \frac{1}{2}k(\gamma - \underline{\gamma})^2 \tag{18}$$

The optimal effort is thus

$$\gamma^*(U_H, U_L) = \underline{\gamma} + \frac{U_H - U_L}{k} \tag{19}$$

The optimal effort chosen by the securitiser is increasing in the difference in the expected payoff $(U_H - U_L)$ between a high-quality and a low-quality pool. We will look at how this difference changes under symmetric and asymmetric information, and under different foreclosure policy.

First note that the low-type securitiser can always attain the highest possible payoff given her type, i.e. $U_L = U_L^{FB}$, because she suffers no information friction and hence optimally chooses the efficient foreclosure policy λ^{FB} and sells a full pass-through security. On the other hand, the high type is strictly worse off under asymmetric information because of the signalling cost (Proposition 4). As a result, the securitiser exerts *strictly less* effort.

Proposition 7. *Comparing to the symmetric information case, the securitiser expends less screening effort under asymmetric information. The ability to commit to a foreclosure policy at $t = 1$ enhances screening effort at $t = 0$. That is,*

$$\gamma^*(U_H^{FB}, U_L^{FB}) > \gamma^*(U_H^*, U_L^*) \geq \gamma^*(U_H^{NC}, U_L^{NC}) \tag{20}$$

where the inequality is strict when Eq. 6 holds.

As commitment power over foreclosure policy not only improves efficiency (Proposition 4) but also enhances the ex ante screening effort as stated in the above proposition, this result further points to an additional benefit of mortgage servicers as commitment devices.

5.2 Foreclosure policy and screening effort

Next we turn to the question of how regulatory interventions of foreclosure policies can affect the screening effort. As shown by our main result, committing to an excessive foreclosure policy allows the high-type securitiser to reduce the signalling costs incurred in equilibrium. Our extension reveals that this also creates a stronger incentives for the securitiser to screen to create a high-quality mortgage pool, further increasing higher welfare. The following proposition summarises the effect of a regulatory intervention in the foreclosure policy on ex ante screening effort and on welfare.

Proposition 8. *If the government imposes a foreclosure policy λ_H different from the equilibrium policy λ_H^*, including the ex post efficient policy λ^{FB}, the securitiser exerts less screening effort at $t = 0$, hence reducing the total welfare.*

Proposition 8 highlights a novel unintended consequence of government regulation of the foreclosure decision in the mortgage securitisation market. Due to information asymmetry, imposing any foreclosure policy different from λ_H^* on the securitiser *reduces* her payoff in the case of receiving a high-quality mortgage pool. This in turn lowers her incentive to exert screening effort to obtain a high-quality pool. This under-provision of value-enhancing screening effort decreases social welfare.

6 Empirical implications

This section summarises the novel empirical implications of our model related to the role of foreclosure policies in the mortgage-backed securitisation industry.

1. *Securitised mortgage pools on average have a higher foreclosure rate conditional on delinquency than comparable bank-held mortgages. Conditioning on the quality, the high-quality mortgages drive the difference.* This is the main result of the model (Propsition 1) that the high-type securitiser distorts the foreclosure policy to mitigate the information friction in the process of securitisation, while there is no distortion for the low-quality pool and the bank-held mortgages. This provides an economic explanation of the existing empirical findings of Piskorski et al. (2010), Agarwal et al. (2011) and Kruger (2016). In particular,

Piskorski et al. (2010) show that while securitised loans on average have a 3% to 7% higher foreclosure rate in absolute terms than bank-held loans, the effects are larger among borrowers with better credit quality.

2. *Foreclosing the marginal delinquent mortgage in a securitised pool returns on average less than the mortgage's expected recovery value, or the marginal delinquent mortgage in a bank-held portfolio.* As foreclosures in a securitised pool on average are excessive, the foreclosure proceeds of the marginal delinquent mortgage are lower than the loan's expected recovery value. The excessive foreclosure in securitised pools is driven by informational frictions in the securitisation process. Since bank-held mortgages are free of this friction, they are foreclosed at the ex post efficient rate, where the value of the marginal foreclosure is equated to the loan's expected recovery value.

3. *Servicers of securitised mortgages on average have compensation contracts that are biased towards foreclosure. Conditioning on the quality of the mortgage pool, the servicers of high-quality mortgage pools drive the difference.* Our model suggests that securitisers would offer optimal incentive contracts to servicers in order to implement the ex ante optimal foreclosure policy. Since the optimal policy appears excessive comparing to the ex post efficient benchmark, the servicers' incentives have to be biased accordingly towards foreclosure. Thompson (2009) and Kruger (2016) document biases in the servicers' incentives consistent with our prediction.

4. *Intrinsic servicer-specific biases towards foreclosure are positively related to the quality of the mortgage pools.* Our model allows also the interpretation that securitisers of high-quality mortgage pools seek intrinsically "tough" servicers as commitment to excessive foreclosure. While works by Agarwal et al. (2014) and Agarwal et al. (2012) uncover the importance of servicer-specific factors relating to their pre-existing organisational capabilities towards foreclosure, our model predicts a testable relationship between such servicer-specific characteristics and the quality of the mortgage pools.

7 Conclusion

This paper studies the relationship between the foreclosure decision of delinquent mortgages and the securitisation of mortgages. We propose a novel mechanism in which excessive foreclosure policies, in addition to the retention of junior securities, serves as costly signals to reduce informational frictions inherent in the securitisation process. We list the empirical predictions coming from our model, some of which explain several important observed patterns and empirical findings in the mortgage securitisation industry.

Our paper also suggests that mortgage servicers could have the important role as commitment devices, allowing the securitiser to optimally commit to ex post excessive foreclosure policies. As a result, the mortgage servicing contracts appear to have biased incentives towards foreclosure, and the servicer-specific capacity related to foreclosure can be informative about the quality of the underlying mortgage pool. These results are broadly consistent with empirical findings and yield new predictions for future empirical work.

For a normative perspective, our results caution that policies attempting to restore ex post foreclosure efficiency can have the unintended consequence of reducing the securitisers' ex ante screening effort, thereby worsening the average quality of the mortgage pools and reducing social welfare.

We conclude with some conjecture of directions for future work and extensions. First, this framework can be extended to a setting with multiple securitisers to study the spillover effects of foreclosure. For instance, it would be interesting to study the interaction between the excessive foreclosure policies due to securitisation and the fire-sale externality in the distressed property market. It could also be fruitful to analyse, in a general equilibrium, the potential impact of securitisation on the quantity, quality, and the prices of mortgages originated. Finally, a dynamic framework could shed lights on how the excessive foreclosure due to securitisation interacts with property prices across business cycles.

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Appendices

A Proofs

A.1 Proof of Lemma 1

This result follows immediately from the discussion.

A.2 Proof of Lemma 2

We will first show no pooling PBE satisfy the Intuitive Criterion. And then we show the same for any separating PBE other than the least-cost-separating PBE.

The logic of the proof is as follows: for any candidate pooling PBE (U_H^P, U_L^P) with an offer $\{\mathcal{F}^P, \lambda^P\}$, we construct an off-equilibrium pooling offer $\{\mathcal{F}', \lambda^P\}$ that prunes the candidate PBE with Intuitive Criterion. Since we do not involve changing λ^P in the following analysis, for the ease of notation we will simply denote an offer with $\mathcal{F}$ whenever it does not create confusion.

We begin by applying the Intuitive Criterion to our two-type model as follows: a PBE fails to satisfy the Intuitive Criterion if there exists an unsent offer $\mathcal{F}'$, such that the type H is strictly better off than at the posited PBE by proposing $\mathcal{F}'$ for all best responses with beliefs focused on H, and the type L is strictly better at the posited PBE than at $\mathcal{F}'$ for all best responses for all beliefs in response to $\mathcal{F}'$.

Define $J_H(\mathcal{F}')$ and $J_L(\mathcal{F}')$ as the payoff of the H and L type when they deviate to the off-equilibrium offer $\mathcal{F}'$ under a belief focused on H

$$\begin{aligned} J_H(\mathcal{F}') &\equiv p_H(\mathcal{F}') + \delta[V_H - p_H(\mathcal{F}')] \\ J_L(\mathcal{F}') &\equiv p_H(\mathcal{F}') + \delta[V_L - p_L(\mathcal{F}')] \end{aligned} \tag{21}$$

Therefore a pooling PBE (U_H^P, U_L^P) does not satisfy the intuitive criterion if there exists an $\mathcal{F}'$ such that $J_H(\mathcal{F}') > U_H^P$ and $J_L(\mathcal{F}') < U_L^P$.

We begin the proof with establishing some useful properties of any pooling PBE

(U_H^P, U_L^P). First, the payoffs can be computed as follows:

$$U_H^P \equiv \bar{p}(\mathcal{F}^P) + \delta[V_H - p_H(\mathcal{F}^P)]$$
$$U_L^P \equiv \bar{p}(\mathcal{F}^P) + \delta[V_L - p_L(\mathcal{F}^P)] \tag{22}$$

where $\bar{p}(\mathcal{F}) = \bar{\pi} f_1 + (1-\bar{\pi})[\theta f_2 + (1-\theta) f_3)]$ and $\bar{\pi} \equiv \gamma\pi_H + (1-\gamma)\pi_L$.

Second, in any pooling PBE that satisfies Intuitive Criterion, both types must attain weakly higher payoffs than the least-cost-separating (LCS) payoffs (U_H^*, U_L^*). The following claim establishes this property formally.

Claim 1. *For any pooling PBE (U_H^P, U_L^P) that satisfies the Intuitive Criterion, $U_H^P \geq U_H^*$ and $U_L^P \geq U_L^*$.*

Proof. This claim is proved by contradiction. First of all, $U_L^P < U_L^*$ cannot be a PBE because the low type can always attain at least the LCS payoffs U_L^* by deviating to the first-best offer of the low type.

Suppose now $U_H^P < U_H^*$ and $U_L^P \geq U_L^*$. To invoke the Intuitive Criterion, consider a set of beliefs that all deviations are done by the high type. Then by deviating to (F_H^*, λ_H^*), the high type achieves her LCS payoff $U_H^* > U_H^P$ whereas the low type's payoff $p_H(F_H^*, \lambda_H^*) + \delta[V_L(\lambda_H^*) - p_L(F_H^*, \lambda_H^*)]$, is also equal to her LCS payoff U_L^* because (F_H^*, λ_H^*) is the solution of the LCS problem in Eq. 4 and the (IC) therein is binding at the solution. Now consider another offer $\{F', \lambda_H^*\}$ with $F' = F_H^* - \epsilon$ for some arbitrarily small and positive ϵ such that the high type's payoff with this off-equilibrium offer is $U'_H \in (U_H^P, U_H^*)$. Such an F' exists because $U_H^P < U_H^*$ and $F_H^* > c_3$ (Proposition 3). Finally the low type's payoff with the offer $\{F', \lambda_H^*\}$ is $U'_L < U_L^* \leq U_L^P$. □

The third property is shown in the following claim

Claim 2. *In any pooling PBE with offer $\{\mathcal{F}^P, \lambda^P\}$, $f_1^p > c_3(\lambda^P)$.*

Proof. Suppose instead $f_1^P \leq c_3(\lambda^P)$. Because of (MNO), $c_3 \geq f_1^P \geq f_2^P \geq f_3^P$

$$U_L^P \leq \delta V_L(\lambda^P) + (1-\delta)c_3(\lambda^P) < V_L(\lambda^P) \leq V_L(\lambda^{FB}) \equiv U_L^*$$

which contradicts the fact that $U_L^P \geq U_L^*$. □

We are now equipped to construct the PBE pruning offer $\mathcal{F}'$ for any pooling PBE with offer $\mathcal{F}^P$. First, we parametrise a series of offers with y such that

$$\mathcal{F}(y) = \{f_1^P - y, f_2^P - \max\{y - (f_1^P - f_2^P), 0\}, f_3^P\} \tag{23}$$

for $y \in [0, f_1^P - f_3^P]$. Note that $\mathcal{F}(0) = \mathcal{F}^P$ and the domain of y is non-empty thanks to Claim 2 and $f_3^P \leq c_3$ due to limited liability. The rest of the proof involves two claims with the parametrised offer $\mathcal{F}(y)$.

Claim 3. *There exists a unique* $\tilde{y} \in (0, f_1^P - f_3^P)$ *that satisfies* $J_L(\mathcal{F}(\tilde{y})) = U_L^P$

Proof. The proof is based on the Intermediate Value Theorem. First, $J_L(\mathcal{F}(\epsilon)) > U_L^P$ with $\epsilon \to 0$ because

$$\begin{aligned} J_L(\mathcal{F}(\epsilon)) - U_L^P &= p_H(\mathcal{F}(\epsilon)) - \bar{p}(\mathcal{F}^P) - \delta[p_L(\mathcal{F}(\epsilon)) - p_L(\mathcal{F}^P)] \\ &= p_H(\mathcal{F}^P) - \bar{p}(\mathcal{F}^P) > 0 \qquad \text{as } \epsilon \to 0 \end{aligned}$$

Second, $J_L(\mathcal{F}(f_1^P - f_3^P)) < U_L^P$ as $\mathcal{F}(f_1^P - f_3^P) = \{f_3^P, f_3^P, f_3^P\}$, $f_3^P \leq c_3$ due to (LL), and following the same argument as in Claim 2,

$$J_L(\mathcal{F}(f_1^P - f_3^P)) \leq \delta V_L + (1-\delta)c_3 < V_L(\lambda^P) \leq V_L(\lambda^{FB}) = U_L^* \leq U_L^P$$

Finally, $J_L(\mathcal{F}(y))$ is strictly decreasing and continuous in y

$$\frac{\partial J_L(\mathcal{F}(y))}{\partial y} = \begin{cases} -\pi_H + \delta\pi_L < 0 & \text{for } y \in [0, f_1^P - f_2^P) \\ (1-\theta)(\delta\pi_L - \pi_H) - \theta(1-\delta) < 0 & \text{for } y \in [f_1^P - f_2^P, f_1^P - f_3^P) \end{cases} \tag{24}$$

Therefore, the Intermediate Value Theorem applies. □

Claim 4. $J_H(\mathcal{F}(\tilde{y})) > U_H^P$

Proof. This result relies on two properties:

(i) $J_H(\mathcal{F}(\epsilon)) - U_H^P = J_L(\mathcal{F}(\epsilon)) - U_L^P = p_H(\mathcal{F}^P) - \bar{p}(\mathcal{F}^P) > 0$ as $\epsilon \to 0$;

(ii) $0 > \dfrac{\partial J_H(\mathcal{F}(y))}{\partial y} > \dfrac{\partial J_L(\mathcal{F}(y))}{\partial y}$ for $y \in [0, f_1^P - f_3^P]$

(i) is immediate from the definition of J_H while (ii) from the direct comparison between Eq. 24 and

$$\frac{\partial J_H(\mathcal{F}(y))}{\partial y} = \begin{cases} -\pi_H + \delta\pi_H < 0 & \text{for } y \in [0, f_1^P - f_2^P) \\ (1-\theta)(\delta\pi_H - \pi_H) - \theta(1-\delta) < 0 & \text{for } y \in [f_1^P - f_2^P, f_1^P - f_3^P) \end{cases} \tag{25}$$

These two properties imply that the wedges $J_H - U_H^P$ and $J_L - U_L^P$ are the same when y is arbitrarily close to zero. As y increases, J_L decreases strictly faster then J_H . Therefore, at $\tilde{y}$, the wedge of $J_L - U_L^P$ is zero while the wedge $J_H - U_H^P$ is strictly positive. □

The last step of constructing the PBE pruning $\mathcal{F}'$ is to set $\mathcal{F}' = \mathcal{F}(\tilde{y} + \epsilon_y)$ with an arbitrarily small but positive ϵ_y such that $J_H(\mathcal{F}') > U_H^P$. This ϵ_y exists because $J_H(\mathcal{F}(\tilde{y})) > U_H^P$ as in Lemma 4. And by the properties of $\tilde{y}$ in Lemma 3 and J_L, $J_L(\mathcal{F}') < J_L(\mathcal{F}(\tilde{y})) = U_L^P$. As a result, the posited pooling PBE (U_H^P, U_L^P) cannot satisfy the Intuitive Criterion.

The proof for showing that no separating PBE other than the LCS PBE can satisfy Intuitive Criterion is very similar to Claim 1. Consider a separating PBE (U_H, U_L), by definition of LCS, $U_H \leq U_H^*$ and $U_L \leq U_L^*$ with at least one strict inequality. First U_L cannot be strictly less than U_L^* because the low type can always achieve at least U_L^* by giving the first-best offer. The relevant class of separating PBE is thus with $U_H < U_H^*$ and $U_L = U^*L$. The remaining argument of the proof follows exactly the same as the one in Claim 1 and therefore is omitted.

A.3 Proof of Proposition 1

The least cost separating equilibrium is characterised by Eq. 4. We prove this proposition by solving the optimisation programme and then highlighting the properties of the equilibrium foreclosure policy.

Firstly, we establish that any optimiser of the programme must bind the (IC). We prove this by contradiction. Suppose there exists (F_H, λ_H) that is an optimiser of the programme such that the (IC) is slack. Then there exists $F_H' > F_H$ such that the (IC) is still satisfied at (F_H', λ_H). However, the objective function is strictly greater at (F'_H, λ_H) than at (F_H, λ_H). This contradicts with the supposition that (F_H, λ_H) is an optimiser of the programme. Therefore any optimiser of the programme must bind the (IC).

We then solve this optimisation given a binding (IC) in two separate cases.

(i) $F_H \in [Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X, Z_G)$

(ii) $F_H \in (Z_B + \mathcal{L}(\lambda_H), Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X)$

Case (i): $F_H \in [Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X, Z_G)$

The solution in this case can be expressed as follows

$$\begin{aligned} & max_{(F_H,\lambda_H)} \quad p_H(F_H, \lambda_H) + \delta\pi_H(Z_G - F_H) \\ s.t. \quad & (IC) \quad U_L^* = p_H(F_H, \lambda_H) + \delta\pi_L(Z_G - F_H) \end{aligned} \tag{26}$$

where the market value of the high type's security is

$$\begin{aligned} p_H(F_H, \lambda_H) = \pi_H F_H + (1-\pi_H)\theta[Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X] \\ + (1-\pi_H)(1-\theta)[Z_B + \mathcal{L}(\lambda_H)] \end{aligned} \tag{27}$$

The binding (IC) implies that the implicit derivative of F_H w.r.t. λ_H is

$$\frac{\partial F_H}{\partial \lambda_H} = -\frac{\frac{\partial p_H(F_H,\lambda_H)}{\partial \lambda_H}}{\frac{\partial p_H(F_H,\lambda_H)}{\partial F_H} - \delta\pi_L} \tag{28}$$

We substitute the (IC) into the objective function to eliminate F_H. The solution λ_H^* is then characterised by the first order condition obtained by total differentiation:

$$FOC_{(i)} = \frac{\partial p_H(F_H, \lambda_H)}{\partial \lambda_H} + \left(\frac{\partial p_H(F_H, \lambda_H)}{\partial F_H} - \delta\pi_H\right)\frac{\partial F_H}{\partial \lambda_H} \tag{29}$$

The above first order condition is equal to zero at λ^{FB} because $\frac{\partial p(F_H,\lambda_H)}{\partial \lambda_H}\Big|_{\lambda_H=\lambda^{FB}} = 0$. Therefore for case (i), $\lambda^* = \lambda^{FB}$, and F_H^* is given by the binding (IC) at λ^{FB}:

$$F_H^* = Z_G - \frac{V_H(\lambda^{FB}) - V_L(\lambda^{FB})}{\pi_H - \delta\pi_L} \tag{30}$$

Next, we confirm that the second order condition is satisfied, so that $FOC_{(i)}$ indeed characterises the optimal λ_H^*. After differentiating some algebraic manipulation:

$$SOC_{(i)} = \frac{\delta(1-\pi_H)(\pi_H - \pi_L)}{\pi_H - \delta\pi_L}\mathcal{L}''(\lambda_H) < 0 \tag{31}$$

Finally we provide the condition for this case to exist. That is, $F_H^* \geq Z_B + \mathcal{L}(\lambda^{FB}) + (1-\lambda^{FB})X$. Expanding the expression and collecting terms yields the following, which is the complimentary case of Eq. 6.

$$\frac{(\pi_H - \pi_L)(1-\theta)(1-\lambda^{FB})}{\pi_L(1-\delta)}X \leq Z_G - \left[Z_B + \mathcal{L}(\lambda^{FB}) + (1-\lambda^{FB})X\right] \tag{32}$$

Case (ii): $F_H \in (Z_B + \mathcal{L}(\lambda_H), Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X)$

The solution in this case can be expressed as follows

$$\begin{aligned} U_H^* = max_{(F_H,\lambda_H)} \quad & (1-\delta)p_H(F_H,\lambda_H) + \delta V_H(\lambda_H) \\ s.t. \quad (IC) \quad U_L^* = & \; p_H(F_H,\lambda_H) + \delta\pi_L\left[(Z_G - F_H)\right. \\ & \left. +(1-\pi_L)\theta\left(Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X - F_H\right)\right] \end{aligned} \tag{33}$$

where the market value of the high type's security is

$$p_H(F_H,\lambda_H) = [\pi_H + (1-\pi_H)\theta]F_H + (1-\pi_H)(1-\theta)[Z_B + \mathcal{L}(\lambda_H)] \tag{34}$$

The binding (IC) implies that the implicit derivative of F_H w.r.t λ_H is

$$\frac{\partial F_H}{\partial \lambda_H} = -\frac{\frac{\partial p_H(F_H,\lambda_H)}{\partial \lambda_H} + (1-\pi_L)\theta[\mathcal{L}'(\lambda_H) - X])}{\frac{\partial p_H(F_H,\lambda_H)}{\partial F_H} - \delta[\pi_L + (1-\pi_L)\theta]} \tag{35}$$

We substitute the (IC) into the objective function to eliminate F_H. The solution λ_H^* is then characterised by the first order condition obtained by total differentiation:

$$\begin{aligned} FOC_{(ii)} = & \frac{\partial p_H(F_H,\lambda_H)}{\lambda_H} + (1-\pi_H)\theta[\mathcal{L}'(\lambda_H) - X] \\ & + \left(\frac{\partial p_H(F_H,\lambda_H)}{\partial F_H} - \delta[\pi_H + (1-\pi_H)\theta]\right)\frac{\partial F_H}{\partial \lambda_H} \end{aligned} \tag{36}$$

At λ^{FB}, $\mathcal{L}'(\lambda^{FB}) = X$ and the above first order condition is strictly greater than zero:

$$FOC_{(ii)}\big|_{\lambda_H=\lambda^{FB}} = -\theta(1-\theta)X \times \underbrace{\left[(1-\pi_H) - \frac{(1-\delta)[\pi_H+(1-\pi_H)\theta]}{[\pi_H+(1-\pi_H)\theta]-\delta[\pi_L+(1-\pi_L)\theta]}(1-\pi_L)\right]}_{<0} > 0 \tag{37}$$

This implies that the solution for case (ii) is $\lambda_H^* > \lambda^{FB}$. The face value of the debt issued in the case F_H^* is given by the binding (IC) at λ_H^*:

$$F_H^* = \frac{U_L^* - \delta\pi_L Z_G - \delta(1-\pi)\theta[Z_B+\mathcal{L}(\lambda_H^*)] - (1-\pi_H)(1-\theta)[Z_B+\mathcal{L}(\lambda_H^*)+(1-\lambda_H^*)X]}{[\pi_H+(1-\pi_H)\theta]-\delta[\pi_L+(1-\pi_L)\theta]} \tag{38}$$

Next, we confirm that the second order condition is satisfied, given Assumption 1. After some algebraic manipulation:

$$SOC_{(ii)} = \overbrace{[(1-\pi_H)-\theta(1-\delta+1-\pi_H)]}^{>0 \text{ by Assumption 1}} \times \frac{\delta(\pi_H-\pi_L)}{(1-\theta)(\pi_H-\delta\pi_L)+\theta(1-\delta)}\mathcal{L}''(\lambda_H) < 0 \tag{39}$$

We now provide the condition for this case to exist. That is, $F_H^* < Z_B + \mathcal{L}(\lambda_H^*) + (1-\lambda_H^*)X$, which is equivalent to $G(\lambda_H^*) > 0$ where $G(\cdot)$ is defined in Eq. 6. As $G(\lambda_H^*)$ is strictly increasing in λ_H^* for all $\lambda_H^* \geq \lambda^{FB}$, the condition Eq. 6 implies that Case (ii) exists as $G(\lambda_H^*) \geq G(\lambda^{FB}) > 0$.

Finally, while a solution in Case (ii) may still exist when Eq. 6 does not hold, the existence of a solution in Case (i) implies that the solution to the overall optimisation programme given by Eq. 4 is the solution in Case (i), which provides the high-type securitiser with a strictly higher payoff than any solution in Case (ii), given that the Case (i) solution has both an efficient foreclosure policy $\lambda_H^* = \lambda^{FB}$ and a higher face value of the debt issued.

To summarise, the equilibrium foreclosure policy by the high-type securitiser is such that $\lambda_H^* \geq \lambda^{FB}$, where the inequality is strict if and only Eq. 6 holds.

A.4 Proof of Corollary 1

We establish this corollary considering the two separate cases discussed in Appendix A.3. Notice that $\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda_H}$ can be expressed as $\frac{\partial p(F_H,\lambda_H)}{\partial F_H}\frac{\partial F_H}{\partial \lambda} + \frac{\partial p(F_H,\lambda_H)}{\partial \lambda}$, where $\frac{\partial F_H}{\partial \lambda}$ is the implicit derivative of F_H w.r.t. λ_H given by a binding (IC).

In Case (i), Eq. 26–28 imply that $\left.\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda_H}\right|_{\lambda_H=\lambda^{FB}} = 0$.

In Case (ii), Eq. 33–35 imply that

$$\left.\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda_H}\right|_{\lambda_H=\lambda^{FB}} = \frac{\delta\theta(1-\theta)(\pi_H-\pi_L)}{(1-\theta)(\pi_H-\delta\pi_L)+\theta(1-\delta)}X > 0 \tag{40}$$

The face value of the debt issued in the least cost separating equilibrium binds the (IC). The equilibrium is given by Case (ii) if and only if Eq. 6 holds. We therefore have $\left.\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda_H}\right|_{\lambda_H=\lambda^{FB}} \geq 0$, with the inequality strict if and only if Eq. 6 holds.

A.5 Proof of Proposition 2

When Eq. 6 holds, the equilibrium foreclosure policy is implicitly defined by the first order condition (Eq. 36). After some algebraic manipulation, the equilibrium foreclosure policy can be implicitly defined by

$$\mathcal{L}'(\lambda_H^*) = \frac{\delta - [\theta(1-\pi_H)+\pi_H]}{(1-\delta)\theta+(1-\pi_H)(1-\theta)}\theta X \tag{41}$$

Implicitly differentiating the above equation yields that $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$, because the RHS of the above equation is strictly decreasing in π_H.

Similarly, implicitly differentiating the above equation yields that $\frac{\partial \lambda_H^*}{\partial \delta} < 0$, because the RHS of the above equation is strictly increasing in δ.

Therefore, the equilibrium foreclosure policy is such that $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$ and $\frac{\partial \lambda_H^*}{\partial \delta} < 0$ when Eq. 6 holds.

A.6 Proof of Proposition 3

The proof is constructed by establishing several claims in succession. For any given λ_H, an optimal security is maximises the high-type securitiser's expected payoff

$$\delta V_H(\lambda_H) + (1-\delta)p_H(\mathcal{F}_H, \lambda_H)$$

subject to the constrains (IC), (MNO) and (MNI). Since $V_H(\lambda_H)$ is not affected by the security design, the security maximises the selling proceeds $p_H(\mathcal{F}_H, \lambda_H) = \pi_H f_1 + (1-\pi_H)[\theta f_2 + (1-\theta)f_3]$. Since λ_H plays no role in this proof, we suppress the selling proceeds to $p_H(\mathcal{F}_H)$ for the ease of notation.

As explained in Section 3.3, given a committed foreclosure policy λ_H, there can only be three cash flow realisations c_1, $c_2(\lambda_H)$, and $c_3(\lambda_H)$ in equilibrium. Denote by f_1^*, f_2^*, and f_3^* the payoffs of the optimal security for these equilibrium cash flow realisations respectively. Claim 5–8 below aim to establish the properties that the equilibrium payoffs of the optimal security must satisfy. We finally characterise the properties of the full security and show that a risky debt as described in Proposition 3 is indeed an optimal security.

Claim 5. *For an optimal security $\mathcal{F}_H^*$, $f_1^* < c_1$.*

Proof. If $f_1^* = c_1$, by (MCI), $f_2^* = c_2(\lambda_H)$ and $f_3^* = c_3(\lambda_H)$. This security (full equity) violates (IC). □

Claim 6. *For any optimal security $\mathcal{F}_H^*$, the (IC) must bind.*

Proof. Suppose instead the (IC) is slack for some optimal security with payoffs $\{f_1^*, f_2^*, f_3^*\}$. By Claim 5, $f_1^* < c_1$. Unless $c_1 - f_1^* = c_2(\lambda_H) - f_2^*$, there exists a security $\hat{\mathcal{F}}$ with payoffs $\{\hat{f}_1, f_2^*, f_3^*\}$ with $\hat{f}_1 > f_1^*$ that satisfies the (IC). As $p_H(\cdot)$ strictly increases with f_1, $p_H(\hat{\mathcal{F}}) > p_H(\mathcal{F}_H^*)$, contradicting the supposition that the security is optimal.

If $f_1^* < c_1$ and $c_1 - f_1^* = c_2(\lambda_H) - f_2^*$, one can increase the objective function $p_H(\cdot)$ by increasing both f_1^* and f_2^* by some $\epsilon > 0$ without violating the (IC), unless $f_2^* = c_2(\lambda_H)$ or $c_2(\lambda_H) - f_2^* = c_3(\lambda_H) - f_3^*$. Note that $f_2^* = c_2(\lambda_H)$ implies $f_1^* = c_1$ hence violates Claim 5.

Suppose now $f_1^* < c_1$ and $c_1 - f_1^* = c_2(\lambda_H) - f_2^* = c_3(\lambda_H) - f_3^*$, similarly one can increase all f_1^*, f_2^*, f_3^* without violating the (IC) to strictly increase $p_H(\cdot)$, unless $f_3^* = c_3(\lambda_H)$. And $f_3^* = c_3(\lambda_H)$ implies $f_1^* = c_1$ hence violates Claim 5.

Since we have shown that any security with a slack (IC) can be improved upon, the (IC) must be binding at any optimal security. □

Claim 7. *For any optimal security $\mathcal{F}_H^*$, $f_1^* > c_3(\lambda_H)$.*

Proof. Suppose instead that $f_1^* \leq c_3(\lambda_H)$. By (MNO), $c_3(\lambda_H) \geq f_1^* \geq f_2^* \geq f_3^*$. This implies that the (IC) is slack because the mimicking payoff

$$\delta V_L(\lambda_H)+p_H(\mathcal{F}_H^*)-\delta p_L(\mathcal{F}_H^*) \leq \delta V_L(\lambda_H)+(1-\delta)c_3(\lambda_H) < V_L(\lambda_H) \leq V_L(\lambda^{FB}) = U_L^*$$

By Claim 6, a slack (IC) contradicts the optimality of $\mathcal{F}_H^*$. □

Claim 8. *Any optimal security $\mathcal{F}_H^*$ has either*

1. $f_1^* = f_2^* > f_3^* = c_3(\lambda_H)$ *or*

2. $f_1^* > f_2^* = c_2(\lambda_H) > f_3^* = c_3(\lambda_H)$

Proof. Consider a security that pays off $\hat{f}_1$, $\hat{f}_2$, and $\hat{f}_3$ for cash flows c_1, $c_2(\lambda_H)$ and $c_3(\lambda_H)$ respectively, such that with the (IC) binds. Using the (IC), write $\hat{f}_1$ as a function of $\hat{f}_2$ and $\hat{f}_3$

$$\hat{f}_1(\hat{f}_2, \hat{f}_3) = \frac{(1-\delta)U_L^* - [(1-\pi_H) - \delta(1-\pi_L)](\theta\hat{f}_2 + (1-\theta)\hat{f}_3)}{\pi_H - \delta\pi_L} \tag{42}$$

Substitute this $\hat{f}_1$ into the objective function. After some algebraic manipulation, the objective function becomes

$$\delta V_H + (1-\delta)\left[\frac{\pi_H}{\pi_H - \delta\pi_L}(1-\delta)U_L^* + \delta\frac{\pi_H - \pi_L}{\pi_H - \delta\pi_L}(\theta\hat{f}_2 + (1-\theta)\hat{f}_3)\right] \tag{43}$$

which is strictly increasing in $\hat{f}_2$ and $\hat{f}_3$. Since $\hat{f}_2$ is bounded above by either $c_2(\lambda_H)$ or $\hat{f}_1$, and $\hat{f}_3$ only by $c_3(\lambda_H)$, any optimal security $\mathcal{F}_H^*$ must have $f_3^* = c_3(\lambda_H)$ and $f_2^* = \min\{f_1^*, c_2(\lambda_H)\}$. Finally, by Claim 7, $f_1^* > c_3(\lambda_H)$ and hence $f_2^* > c_3(\lambda_H)$. □

Having now analysed the properties of an optimal security's equilibrium payoffs $\{f_1^*, f_2^*, f_3^*\}$, we now consider the security's payoffs associated with the off-equilibrium cash flow realisations, i.e. $f_2(c_2)$ and $f_3(c_3)$ $\forall\lambda \in (0,1)$.

Claim 9. *For any optimal security* $\mathcal{F}_H^*$, $f_3(c_3) = c_3(\lambda)\ \forall\lambda \le \lambda_H$, *and either*

1. $f_1^* = f_2^* = f_2(c_2(\lambda))\ \forall\lambda \le \lambda_H$, *or*

2. $f_1^* > f_2(c_2) = c_2(\lambda)$ *and* $f_3(c_3) = c_3(\lambda)\ \forall\lambda \ge \lambda_H$

Proof. Notice that these payoffs do not affect either the objective function or the (IC). Therefore they are only restricted by the (MNO) and the (MNI). By Claim 8, $f_3^* = c_3(\lambda_H)$. The (MNI) thus implies that $f_3(c_3) = c_3(\lambda)\ \forall\lambda \le \lambda_H$, because $c_3(\lambda)$ is increasing in λ.

By Claim 8, there are two cases. In the first case, $f_1^* = f_2^*$. The (MNO) then implies that $f_1^* = f_2^* = f_2(c_2(\lambda))\ \forall\lambda \le \lambda_H$, because $c_2(\lambda)$ is decreasing in λ. In the second case, $f_2^* = c_2(\lambda_H) > f_3^* = c_3(\lambda_H)$. The (MNI) then implies that $f_2(c_2) = c_2(\lambda)$ and $f_3(c_3) = c_3(\lambda)\ \forall\lambda \ge \lambda_H$. □

Finally we can now verify that a risky debt with face value $F_H \in (c_3(\lambda_H), c_1)$, as defined in Proposition 3, indeed is an optimal security as it satisfies Claim 5–9.

A.7 Proof of Proposition 4

In order to establish this result, we first characterise fully the least cost separating equilibrium without commitment. As in the main text, we denote all equilibrium quantities with NC for the case with no commitment.

We again start the analysis with the low-type securitiser. Since the low type issues a full pass-through equity security in a separating equilibrium, she retains no cash flow. Maintaining the assumption that in this case she makes the first-best foreclosure decision to maximise the value of the mortgage pool, $\lambda_L^{NC} = \lambda_L^{FB}$, the payoff to a low-type securitiser is therefore equal to $U_L^{NC} = U_L^{FB}$.

The high-type securitiser chooses an optimal security $\mathcal{F}_H$ at $t = 1$ to maximise her expected payoff, taking into account the subsequent foreclosure policy λ_H^{NC} chosen at $t = 2$ given the security issued. In the least cost separating equilibrium, the high type's problem without commitment is

$$
\begin{aligned}
& \max_{(\mathcal{F}_H, \lambda_H^{NC})} p_H(\mathcal{F}_H, \lambda_H^{NC}) + \delta[V_H(\lambda_H^{NC}) - p_H(\mathcal{F}_H, \lambda_H^{NC})] \\
s.t. \quad & (IC) \quad U_L^{NC} \geq p_H(\mathcal{F}_H, \lambda_H^{NC}) + \delta[V_L(\lambda_H^{NC}) - p_L(\mathcal{F}_H, \lambda_H^{NC})] \\
& (IC^s) \quad \lambda_H^{NC} = \arg\max_{\lambda} \quad \theta\,[c_2(\lambda) - f_2] + (1-\theta)\,[c_3(\lambda) - f_3] \\
& (MNO) \text{ and } (MNI) \text{ as given by Eq. 14}
\end{aligned} \tag{44}
$$

Except for the constraint (IC^s), the high-type's problem without commitment has the same objective function and constraints as the problem with commitment in Eq. 14. This additional (IC^s) constraint captures the fact that, the securitiser is not able to pre-commit to a foreclosure policy. Instead, the foreclosure policy λ_H^{NC} is chosen at $t = 2$ to maximise the expected value of her residual claim, given the security $\mathcal{F}_H$ issued. Similarly, should the low-type securitiser mimic the security issued by the high type, she also subsequently chooses the same incentive compatible foreclosure policy λ_H^{NC}. This is reflected in the low type's no-mimicking constraint (IC).

We can now prove the remaining parts of this proposition. $U_H^* \geq U_H^{NC}$ follows immediately from the above observation that the opitmisation problem without commitment is the problem with commitment in Eq. 14 with the additional constraint (IC^s). As such, the solution to the problem without commitment is a feasible offer for the problem with commitment. Therefore, the high type securitiser can achieve at least U_H^{NC} when she can commit to a foreclosure policy.

We finally show that $U_H^* > U_H^{NC}$ if Eq. 6 holds. To do this, we make use of some property of the equilibrium optimal MBS $\mathcal{F}_H^*$ issued by the high-type securitiser established in the proof of Proposition 1. If Eq. 6 holds, $f_1^* = f_2^* = F_H^* < c_2(\lambda_H^*)$ and $\lambda_H^* > \lambda^{FB}$. Claim 9 then implies that $f_2(c_2) = F_H^*\ \forall \lambda \leq \lambda_H$. In this case, however, the offer $(\mathcal{F}_H^*, \lambda_H^*)$ does not satisfies the (IC^s) in the problem without commitment, as there a marginal decrease in λ from λ_H^* would increase the securitiser's expected payoff at $t = 2$, given by $\theta[c_2(\lambda_H) - F_H^*]$. As a result, any solution $(\mathcal{F}_H^*, \lambda_H^*)$ of the problem with commitment is not admissible in the more constrained problem without commitment if Eq. 6 holds, $U_H^{NC} < U_H^*$.

A.8 Proof of Proposition 5

This proposition follows immediately from the discussion.

A.9 Proof of Lemma 3

This lemma follows immediately from examining the property of the first order condition $\beta_i \frac{\partial \mathcal{L}(\lambda_s)}{\partial \lambda} - \theta X = 0$. By implicitly differentiating λ_s w.r.t. β_i we have

$$\frac{\partial \lambda_s}{\partial \beta_i} = -\frac{\frac{\partial \mathcal{L}(\lambda_s)}{\partial \lambda}}{\beta_i \frac{\partial^2 \mathcal{L}(\lambda_s)}{\partial \lambda^2}} > 0 \tag{45}$$

Further, since the first-best foreclosure policy is characterised by $\frac{\partial \mathcal{L}(\lambda^{FB})}{\partial \lambda} = \theta X$, $\lambda_s = \lambda^{FB}$ if and only if $\beta_i = 1$.

A.10 Proof of Proposition 6

We fully characterise the least cost separating equilibrium of this extension and establish the results.

We start the analysis with the low-type securitiser. Given any servicing contract, the low type issues a full pass-through equity security in a separating equilibrium. Therefore she chooses a servicing contract to maximise her payoff

$$\begin{aligned} &\max_{(\alpha_L, \beta_L)} \quad V_L(\lambda_s) - \pi_L \alpha_L \left[\beta_L \mathcal{L}(\lambda_s) + \theta(1-\lambda_s)X\right] \\ &s.t. \quad \hat{\pi}\alpha_L \left[\beta_L \mathcal{L}(\lambda_s) + \theta(1-\lambda_s)X\right] \geq 0 \end{aligned} \tag{46}$$

where λ_s is given by the first order condition $\beta_L \frac{\partial \mathcal{L}(\lambda_s)}{\partial \lambda} - \theta X = 0$. The solution to the above problem is $\beta_L = 1$ and $\alpha_L \to 0$. As a result, the equilibrium foreclosure policy of a low-type mortgage pool is λ^{FB}, and the low-type securitiser receives a payoff is equal to U_L^{FB}.

The high-type securitiser, given a servicing contract, chooses an optimal security $\mathcal{F}_H$ at $t = 1$ to maximise her expected payoff, taking into account the subsequent foreclosure policy induced by the servicing contract. We again restrict attention to a risky debt security with face value $F_H \in (Z_B + \mathcal{L}(\lambda_s), Z_G)$. As shown in Proposition 3, risky debt is indeed the optimal security. The high-type securitiser chooses to offer a servicing contract and the security to maximise the proceeds from securitisation plus the residual cash flow less the fees paid to the servicers.

$$\begin{aligned} & max_{F_H,(\alpha_H,\beta_H)} \quad p(F_H,\lambda_H)+\delta\left[\pi_H(Z_G-F_H)\right. \\ & \qquad\qquad\qquad +(1-\pi_H)\theta\max\{Z_B+\mathcal{L}(\lambda_s)+(1-\lambda_s)X-F_H,0\}] \\ & \qquad\qquad\qquad -\pi_H\alpha_H[\beta\,\mathcal{L}(\lambda_s)+(1-\lambda_s)X] \\ s.t. \quad & \hat{\pi}\alpha_L\left[\beta_L\,\mathcal{L}(\lambda_s)+\theta(1-\lambda_s)X\right]\geq 0 \\ & (MC) \text{ and } (IC) \text{ are given by Eq. 4} \end{aligned} \tag{47}$$

where λ_s is given by the first order condition $\beta_H\frac{\partial\mathcal{L}(\lambda_s)}{\partial\lambda}-\theta X=0$. The solution to the above problem is $\beta_H\geq 1$ such that $\lambda_s=\lambda_H^*\geq\lambda^{FB}$, and $\alpha_H\to 0$. As a result, the equilibrium foreclosure policy of a high-type mortgage pool is λ_H^*, and the high-type securitiser receives a payoff equal to U_H^*.

A.11 Proof of Proposition 7

This proposition follows immediately from Eq. 19 and Proposition 4.

A.12 Proof of Proposition 8

Denote with $U_i(\lambda)$ the expected payoff obtained by the high-type securitiser in the least cost separating equilibrium, for a given foreclosure policy. In this equilibrium, the high-type securitiser chooses a security to offer at $t=1$ to maximise her expected payoff, while preventing mimicking from the low type. Formally, $U_i(\lambda)$ is equal to the value of the optimisation programme Eq. 3, given $\lambda_H=\lambda$.

By definition of λ_H^* as the optimiser of Eq. 3 , $U_H(\lambda_H^*)>U_H(\lambda_H)$ for any $\lambda_H\neq\lambda_H^*$. Thus the screening effort γ^* decreases as

$$\gamma^*(U_H(\lambda_H),U_L^{FB})<\gamma^*(U_H(\lambda_H^*),U_L^{FB})\quad\forall\lambda_H\neq\lambda_H^*$$

For efficiency, we only need to look at the securitiser's expected payoff as the investors are always indifferent. The expected payoff is lower when λ_H^* is replaced with λ_H, i.e.

$$\begin{aligned}
&\gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H) + [1-\gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H), U_L^{FB}) \\
&< \gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H^*) + [1-\gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H), U_L^{FB}) \\
&\leq \gamma^*(U_H(\lambda_H^*), U_L^{FB})U_H(\lambda_H^*) + [1-\gamma^*(U_H(\lambda_H^*), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H^*), U_L^{FB})
\end{aligned}$$

The first inequality comes from $U_H(\lambda_H) < U_H(\lambda_H^*)$ and the second weak inequality follows from the definition of optimal γ^*. Finally, λ_H^{FB} is one of the possible $\lambda_H \neq \lambda_H^*$ if and only if Eq. 6 holds.