Leverage and Risk-Taking*

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Abstract

Contrary to the prediction of static models, risk-taking is non-monotonic in leverage in dynamic models. If lenders rationally anticipate risk-shifting of high-leverage firms, then equity-holders bear the cost of risk-shifting via higher debt interest rates. The higher cost of risk-shifting makes equity holders *avert* risk at medium levels of leverage. Averting risk today preserves the option to issue safe, i.e., cheap, debt tomorrow. The same friction responsible for risk-taking of high-leverage firms leads medium-leverage firms to avert risk. Our model is able to reconcile contradictory empirical results on the relation between risk and leverage, predicts that firms with medium leverage are subject to investment distortions, and helps explain the low-leverage puzzle.

Keywords: leverage, risk-taking incentives, low-leverage puzzle

JEL: G3, G31, G32, G33

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1 Introduction

The relationship between leverage and risk-taking is a fundamental question in corporate finance. The prevalent view in the literature – shaped by Jensen and Meckling (1976) – suggests that levered firms have an incentive to engage in risk-shifting. In this paper, we aim to challenge this prevalent view. In particular we argue that risk-taking incentives are strikingly different to the Jensen and Meckling (1976) results in a multi-period setting. We show that risk-shifting incentives in a multi-period model follow a U-shaped pattern: while highly levered firms have an incentive to risk-shift, firms with medium leverage avert risk.

Our model consists of two periods and a firm that needs debt financing to invest. In both periods, the firm has access to two projects; a safe project and a risky project that simply adds a mean-preserving spread to the safe project. The project choice is unobservable to outsiders who need to provide debt financing. To make the model interesting, we assume bankruptcy costs so that risk-shifting induces deadweight costs. A one-period version of our model yields the standard result: if firm leverage is high, then risk-shifting occurs. As usual, lenders rationally anticipate risk-shifting so that equity holders bear the deadweight costs of risk-shifting via higher debt interest rates.

The fact that deadweight costs of risk-shifting are borne by equity holders in a one-period model has powerful implications in a two-period setting: it makes equity holders avert risk in the first period whenever there is the possibility to end up in a situation where risk-shifting incentives become costly for equity holders in the subsequent period. In the initial period, firms therefore avert risk for medium levels of leverage. The key insight of our model is that firms with medium level of leverage reduce risk *precisely because* of risk-shifting incentives of highly levered firms.

Our model is not the first model that reevaluates the risk-shifting predictions from Jensen and Meckling (1976). Existing models can be broadly classified into two categories. First, early models abstract from conflicts of interest between equity and debt holders and assume that equity holders simply maximize overall firm value. One of the implications is that, in the presence of bankruptcy costs, a highly levered firm can maximize firm value by choosing low-risk projects.[1] Second, several models introduce additional frictions such as managerial risk aversion and career concerns[2], the (exogenous) existence of future positive NPV projects or charter values that provide an incentive for the firm to survive the initial period[3], or exogenously given covenants that induce equity holders of highly levered firms to avert risk in order to avoid a loss of control.[4]

The key distinction of our model vis-à-vis the prior literature is that risk-shifting incentives for highly levered firms directly induce risk-reduction incentives for medium levered firms, without a need for any additional friction. Precisely the same friction that induces risk-shifting for highly levered firms leads medium-leveraged firms to avoid risk. The prospect of having to refinance debt at unfavorable terms in the interim period due to anticipated risk-shifting problems is sufficient to induce aversion of risk in the initial period of our model.

This disciplining effect works as long as a firms' debt needs to be refinanced in the interim period. The interim financing arises endogenously in our model. This is because a long-term debt contract entails larger deadweight costs due to risk-shifting, and thus a larger need of internal financing compared to a series of short-term debt contracts. Even though our model relies on refinancing in the interim period and endogenously generates short-term debt, it is crucially different from models that use short-term debt as a disciplining device (Barnea, Haugen, and Senbet (1980), Calomiris and Kahn (1991), and Diamond and Rajan (2001)). In these models, short-term debt reduces the potential for risk-shifting to affect asset values. In our model, short-term debt induces firms to actively avert risk to avoid the deadweight cost of future risk-shifting.

Our model also relates to the large body of literature on dynamic capital structure models. Early models solve for the optimal capital structure by trading off bankruptcy costs and taxes (Leland (1994), Leland and Toft (1996)).[5] while more subsequent models

[1]See Mayers and Smith (1982), Smith and Stulz (1985), and Mayers and Smith (1987).

[2]See Amihud and Lev (1981), Mayers and Smith (1982), Smith and Stulz (1985), Mayers and Smith (1987), Holmstrom (1999), Eckbo and Thorburn (2003), and Bertrand and Mullainathan (2003).

[3]See Froot, Scharfstein, and Stein (1993), Almeida, Campello, and Weisbach (2011) for general models as well as Marcus (1984) and Keeley (1990) for charter value models in the banking.

[4]See in particular Smith and Warner (1979), Amihud and Lev (1981), Leland (1994), and Acharya, Amihud, and Litov (2011).

[5]Section VII in Leland (1994) provides comparative statics for the value of equity as a function of asset risk. He concludes that equity holders have an incentive to decrease risk if covenants are strict enough,

have added agency costs and debt issuance costs (Dangl and Zechner (2018) and Geelen (2019)). A recent strand of the literature analyzes leverage dynamics of firms who cannot commit to a debt issuance policy ex ante (Brunnermeier and Oehmke (2013), Admati, DeMarzo, Hellwig, and Pfleiderer (2018), He and DeMarzo (2016), He and Milbradt (2016)). In these models, a firm's debt issuance policy is endogenous, while asset risk is treated as an exogenous variable. We complement this literature by treating asset risk as an enogenous variable and analyzing asset risk choices in a multi-period setting. While these papers typically find that leveraged firms resist actions that decrease decrease bankruptcy risk, we find that optimal risk choices can work in the opposite way.

Our model has three key empirical implications. First, we show that risk-shifting incentives are a U-shaped function of firm leverage. This result can help to reconcile a set of contradictory results in the empirical finance literature. Surveying the empirical literature, we find that approximately 1/3 of the empirical papers find a positive relation between risk and leverage[6], 1/3 finds a negative relation[7], while one third finds a non-monotonic or no relation between leverage and risk-taking.[8] Table 1 provides more details.[9] Our results suggest that risk-taking incentives are a U-shaped function of leverage, and thus the relation between leverage and risk-taking will highly depend on the distribution of firm leverage in the sample at hand.

Second, our model predicts that firms' investment decisions are distorted for both highly levered firms (who may prefer high-risk projects to low-risk projects even if the latter have a higher net present value) and for medium levered firms (who may prefer low-risk projects to high-risk projects even if the latter have a higher net present value). Only firms with a low leverage follow a simple net present value rule. Thus, our model contributes to the low leverage puzzle, i.e., the empirical observation that firm leverage

and equity holders might be better off ex ante signing debt with restrictive covenants. However, he does not solve for an optimal level of risk-taking as a function of leverage.

[6]See in particular Eisdorfer (2008), Pichler, Stomper, and Zulehner (2008), Becker and Stromberg (2012), and Cocco and Volpin (2007).

[7]See in particular Haushalter (2000), Gilje (2016), Asquith, Gertner, and Scharfstein (1994), and Rauh (2009).

[8]See in particular Purnanandam (2008), Aretz, Banerjee, and Pryshchepa (2018), Andrade and Kaplan (1998), Pedersen (2019) and Graham and Harvey (2001).

[9]Additional studies in the banking industry include Esty (1997), Landier, Sraer, and Thesmar (2015), and Rampini, Viswanathan, and Vuillemey (2019). However, the banking industry is clear special due to explicit and implicit government guarantees and regulations on leverage. It is therefore unclear whether our model predictions also apply to the banking industry.

seems to be too low to be explained by standard frictions (Miller (1977), Graham (2000), Korteweg (2010)).

Third, our model provides an explanation for the way covenants are being written in long-term debt contracts. In particular, our model predicts that covenants bite "early", i.e. covenants bind when there still exists significant positive net worth. These early binding covenants are in line with empirical observation (Roberts and Sufi (2009)). The effect of such covenants is not only to avoid risk-shifting, but to induce the aversion of risk to preserve future debt capacity.

2 Illustrative example

2.1 One period

The following example illustrates the well-known incentive of equity-holders with limited liability to risk-shift (Jensen and Meckling (1976)). A firm has two projects, a safe one and a risky one (Figure 1). Both require an investment of 80. The safe project pays off 80 and the risky project adds a mean-preserving spread of ± 20 to the safe project (the zero net present value of projects is not important). The (inside) equity is 10 so the equity-holder raises debt worth 70 from lenders. The equity-holder's project choice is non-contractible, she has limited liability, and bankruptcy incurs a fixed cost of 10.

The equity holder has an incentive to choose the risky project. To see this, suppose she goes for the safe project. Lenders require debt with face value of 70, which generates a payoff of 10 to the equity-holder. However, with a face value of debt of 70, the equity-holder prefers the risky project because it generates an expected payoff of 15 ($0.5 \cdot (100 - 70) + 0.5 \cdot 0$ – there is default when the payoff is 60).

Choosing the risky project hurts the equity-holder because she has to bear the expected bankruptcy costs from risk-shifting. Lenders anticipate the equity-holder's incentive to risk-shift and require a face value of debt of 90 (because $0.5 \cdot 90 + 0.5 \cdot (60 - 10) = 70$). The expected payoff to the equity-holder drops to 5 ($0.5 \cdot (100 - 90) + 0.5 \cdot 0$). The difference between this payoff and initial equity of 10 with a zero NPV investment is the expected bankruptcy cost ($0.5 \cdot 10 = 5$).

2.2 Two periods

Our two-period example overturns the one-period intuition. In each period, there is the choice between the safe and the risky investment project. As in the one-period example, the payoff of the safe project is equal to the investment, and the risky project adds a mean-preserving spread of ± 20. In the first period, the investment need is 100. Like in any standard binominal tree, the second period starts where the first period ends (with 80, 100 or 120 depending on the outcome of the first period). Panel (a) of Figure 2 shows the entire payoff tree, while Panel (b) shows all possible paths in the first period and only the equilibrium paths in the second period.

The following example depends on two key assumptions: first, we assume one-period debt. In our formal model the debt maturity is endogenous, but simply assuming one-period debt simplifies exposition. Second, the equity-holder cannot simply pay back the first-period debt by liquidating (part of) the assets, but needs to refinance the first-period debt. This assumption is important: if liquidation were costless, the equity-holder could always avoid agency costs by liquidating the entire firm. We discuss liquidation costs in more detail in Section 3.

We assume that the initial value of equity is 30, so the equity-holder raises debt worth 70 in the first period. First-period debt is safe because a face value of 70 always allows the equity-holder to refinance debt in the interim period, even with the worst possible first-period payoff (80).

If the equity-holder chooses the risky project in the first period and the payoff is 120 then she can easily refinance the 70 first-period debt. Second-period debt is safe, she is indifferent between choosing the safe project and the risky project in the second period, and the payoff to the equity-holder after the second period is 50.[10] If, on the other hand, the payoff of the risky project in the first period is 80, then the second period is exactly as in the previous one-period example. The equity-holder needs to obtain new debt worth 70 $(80 - (80 - 70))$ and has an incentive to choose the risky project in the second period. Lenders anticipate this, require a face value of 90 in the second period and the equity-holder's expected payoff for the second period is 5. The overall expected

[10] The expected value of equity is $120 - 70 = 50$, or $0.5 \cdot (100 - 70) + 0.5 \cdot (140 - 70) = 50$.

payoff to the equity-holder from choosing the risky project in the first period therefore is 27.5 $(0.5 \cdot 50 + 0.5 \cdot 5)$.

The equity-holder averts risk in the first period because this preserves the option to issue safe debt in the second period. When she chooses the safe project in the first period, the payoff is 100, which makes it possible to issue safe debt to finance the second-period investment of 70 $(100 - (100 - 70))$. Second-period debt is safe because a face value of 70 is above the lowest payoff at the end of the second period (80). The overall expected pay-off to the equity-holder from choosing the safe project in the first period is therefore 30 $(1 \cdot 30)$. In the first period the equity-holder prefers the safe project (payoff 30) to the risky project (payoff 27.5) because otherwise she may have an incentive to risk-shift in the second period. Possibly not being able to issue safe debt in second period, the equity-holder bears an expected costs of bankruptcy of 2.5 $(0.5 \cdot 0.5 \cdot 10)$. The equity holder prefers the safe project in the first period precisely because she does not want to end up in the risk-shifting region in the second period.

3 Model

3.1 One-period model

3.1.1 Set-up

There is one period, starting at $t = 1$ and ending at $t = 2$. We depict the key elements of the timeline in Figure 3. All agents are risk-neutral and the discount rate is 0%. An insider is equipped with wealth E_1. The insider has access to a set of investment opportunities (described in more detail further below) that require an investment of I_1 at $t = 1$. The amount $I_1 - E_1$ needs to be financed from outsiders at $t = 1$. As we are interested in exploring the effect of leverage on risk-taking, we assume that the insider provides equity funding while outsiders' financing comes in the form of a debt claim with notional F_2. If the value of the assets in $t = 2$ is not enough to meet the repayment, then the lenders have the right to seize the assets and liquidate them. Liquidation incurs fixed bankruptcy costs of $b > 0$.[11]

After the financing has happened at $t = 1$, the insider can choose between two investment options $c_1 \in \{s_1, r_1\}$ at $t = 1\frac{1}{2}$: first, a safe investment s_1 that pays off I_1 at $t = 2$ for sure. Second, a risky investment r_1 that returns $I_1 \pm \gamma$ with 50% probability each, i.e., the second project adds a mean-preserving spread to the safe project. We assume that $I_1 > \gamma$ so that the value of assets is positive. The parameter γ governs the degree of risk-shifting in our model. We assume that the project choice is non-contractible. It can be deduced at $t = 2$ from the asset value, but because the insider has limited liability, lenders cannot impose the strategy choice.

The insider can also choose not to raise financing at $t = 1$ and not to invest in $t = 1.5$ in which case her payoff is zero. For the mechanics of our model it is not important that the outside option is zero; all we require is that the investment options have a higher expected payoff than the outside option. Choosing an expected payoff of zero for the outside option serves as a convenient normalization and increases the ease of notation.

Taken together, there are four possible asset values at $t = 2$: $A_2 \in \{0, I_1 - \gamma, I_1, I_1 + \gamma\}$, refering to the outside option ($A_2 = 0$), the safe investment opportunity ($A_2 = I_1$) and the risky investment opportunity ($A_2 = I_1 \pm \gamma$). The value of the debt claim at $t = 2$ is

$$D_2 = \begin{cases} F_2 & A_2 \geq F_2 \\ A_2 - b & A_2 < F_2 \end{cases} \tag{1}$$

and the value of the equity claim at $t = 2$ is

$$E_2 = \begin{cases} A_2 - F_2 & A_2 \geq F_2 \\ 0 & A_2 < F_2. \end{cases} \tag{2}$$

We make the following assumption about the bankruptcy costs b:

[11] Our results are robust to using proportional bankruptcy costs. We use fixed costs for two reasons: First, fixed costs ease notational convenience and facilitate the interpretability of the results. Second, proportional bankruptcy costs imply somewhat counterintuitive comparative statics: conditional on default, a firm occurs lower deadweight costs of bankruptcy the worse its performance.

Assumption 1 *The cost of liquidating assets in bankruptcy is not too large:*

$$b < 2\gamma \tag{3}$$

If $b \geq 2\gamma$, then the insider could only issue safe debt or not obtain financing at all. This assumption thus ensures that inefficient risk taking is a possibility in the one-period model.

Note that the setting described above covers three interpretations:

1. Insider equipped with wealth E_1: A new firm is set-up with inside equity E_1 and outside debt D_1. The NPV of investment inside the firm is zero, while the outside option has a negative NPV. The negative NPV of the outside option simply serves as a convenient normalization; the alternative is to assume a positive NPV inside the firm and a zero NPV for the outside option.

2. Insider equipped with intangible capital E_1: A new firm is set-up with intangible capital E_1 provided by the entrepreneur (e.g. a patent). The entrepreneur needs outside financing of D_1 to produce an expected payoff of I_1. The outside option is zero because the entrepreneur's contribution to the firm consists of intangible capital which would be worthless without financing.

3. Debt structure in place: An existing firm with a capital structure of E_1 (equity) and D_1 (debt) needs to refinance its debt D_1. The assets in place cannot be liquidated. The outside option of zero is thus a consequence of the illiquidity of the assets in place.

3.1.2 One-period model: Equilibrium

At $t = 1$, the insider maximizes the expected value of her (equity) claim, $\mathbb{E}_1[E_2]$, by choosing the face value F_2 and an investment strategy c_1 subject to the choice being incentive compatible, and subject to the outsiders' participation constraint, $\mathbb{E}_1[D_2] \geq I_1 - E_1$. The following proposition gives the insider's optimal choice of F_2 and c_1 as a function of her equity at $t = 1$.

Proposition 1 *Given the value of the insider's equity at* $t = 1$, E_1, *there are three possible equilibrium outcomes:*

i) When $E_1 \geq \gamma$, *then the insider can issue safe debt. She is indifferent between choosing the safe or the risky strategy. The expected value of her equity at* $t = 1$ *is* E_1.

ii) When $\gamma > E_1 \geq \frac{1}{2}b$, *then the insider cannot issue safe debt. She chooses the risky strategy. The expected value of her equity at* $t = 1$ *is* $E_1 - \frac{1}{2}b$.

iii) When $E_1 < \frac{1}{2}b$, *the insider cannot obtain financing and the expected value of her equity at* $t = 1$ *is zero.*

When the insider has enough equity, $E_1 \geq \gamma$, losses cannot exceed the equity amount and safe debt can be issued. There is no default and because the safe and risky strategy have the same expected value, the insider is indifferent between them.

When the insider's wealth is less than γ, but larger than the expected costs of bankruptcy, $\frac{1}{2}b$, the insider cannot issue safe debt and chooses the risky strategy. The face value of debt is too large to make the safe strategy incentive compatible. The expected value of the insider's equity at $t = 2$ reflects the costs of having to compensate the lenders for the expected costs of bankruptcy. Assumption 1 ensures that $\gamma > \frac{1}{2}b$ and thus makes the risk-taking case possible.

When the insider's wealth is not enough to cover the expected costs of bankruptcy, she cannot raise outside financing. In this case, her payoff drops to the outside option, which by assumption is equal to zero.

Figure 4 plots the equilibrium outcome as a function of initial equity capital E_1. The results demonstrate the well-known fact: in a one-period model, risk-shifting occurs for high levels of leverage. If lenders rationally anticipate risk-shifting, then the expected costs of risk-shifting are borne by the equity holder in expectation.

The following corollary establishes the fact that the insider can strictly prefer the risky strategy over the safe strategy even if the safe strategy has a higher expected payoff than the risky strategy.

Corollary 1 *(***Higher expected payoff of the safe strategy***) Assume the same setup as in Proposition 1, but the safe strategy's payoff is $I_1 + \Delta$, with $\Delta \in [0, \frac{1}{2}\gamma - \frac{1}{4}b)$ being non-negative but not too large. The insider chooses the risky strategy whenever $E_1 \in [\frac{1}{2}b, \gamma - 2\Delta)$.*

Proposition 1 has established the fact that the insider may prefer the risky strategy over the safe strategy if both have the same expected payoff. Corollary 1 establishes that the insider may even prefer the risky strategy over the safe strategy if the safe strategy has a higher expected payoff. The length of the interval where risk-shifting occurs ($E_1 \in [\frac{1}{2}b, \gamma - 2\Delta)$) is decreasing in the expected excess payoff Δ of the safe strategy over the risky strategy.

3.2 Two period model with short-term debt

3.2.1 Set-up

The two-period model adds a second period, from $t = 0$ to $t = 1$, in which the same one-period model from the previous section plays out again. Figure 5 shows the timeline. At $t = 0$ the insider is endowed with equity E_0 and needs to invest I_0. The financing need is $I_0 - E_0$, which she finances by raising short-term debt with face value F_1. Depending on the choice of strategy, the asset value at $t = 1$ is $A_1 \in \{0, I_0 - \gamma, I_0, I_0 + \gamma\}$.

To connect the two periods, we assume that the assets in place A_1 cannot be liquidated. This implies that the insider cannot simply pay back the debt by liquidating (part of) the assets. Instead, the insider needs to refinance the first-period debt notional F_1 from new lenders. If the insider is unable to raise F_1 from new lenders then she defaults, lenders receive $\min(F_1, A_1 - b)$ and the insider receives $\max(0, A_1 - b - F_1)$.

Notes on the assumptions:

1. The assumption of short-term debt is exogenous. We consider long-term debt and an endogenous choice of debt maturity as extensions in Section 3.3.

2. We rule out renegotiation of first-period debt, i.e., if the insider cannot raise new debt after the first period then bankruptcy occurs. One way to rationalize this is to

assume short-lived debt holders, where the first generation finances the first period while the second generation finances the second period.

3. We assume that assets in place A_1 cannot be liquidated by the insider outside of bankruptcy. This assumption can be relaxed by assuming liquidation costs l. In this case, there are two trade-offs: first, if liquidation costs are smaller than bankruptcy costs ($l < b$), the insider might prefer liquidating assets to avoid bankruptcy. Second, if liquidation costs are smaller than expected agency costs from the second period (i.e., costs of risk-shifting), the insider can prefer liquidation to continuation.

3.2.2 Equilibrium

At $t = 0$, the insider maximizes the expected value of her equity claim, $\mathbb{E}_0[E_2(E_0)]$, which is a function of her initial equity E_0, by choosing debt face values (F_1, F_2) and an investment strategy in both periods (c_0, c_1) subject to the choice being incentive compatible, and subject to the outsiders' participation constraint. The following proposition gives the equilibrium outcome of the two-period model.

Proposition 2 *Given the value of the insider's equity at $t = 0$, E_0, and assuming $b < \frac{4}{3}\gamma$, there are four possible equilibrium outcomes:*

i) When $E_0 \geq 2\gamma$, then the insider can issue safe debt in both periods. She is indifferent between the safe and the risky strategy in both periods. The expected value of her equity at $t = 0$ is E_0.

ii) When $2\gamma > E_0 \geq \gamma$, the insider can issue safe debt in the first period and prefers the safe strategy in the first period. In the second period she can issue safe debt and is indifferent between the safe and the risky strategy. The expected value of her equity at $t = 0$ is E_0.

iii) When $\gamma > E_0 \geq \frac{3}{4}b$, then the insider cannot issue safe debt in the first period and chooses the risky strategy in the first period. If the risky strategy is unsuccessful, she defaults. If the risky strategy succeeds, the second period strategy depends on E_0 and she either perfers the risky strategy or is indifferent in the second period. The expected value of her equity at $t = 0$ is $E_0 - \frac{1}{2}b$ or $E_0 - \frac{3}{4}b$.

iv) When $E_0 < \frac{3}{4}b$, then the insider cannot obtain financing. The expected value of her equity at $t = 0$ is zero.

Notes:

1. In case iii), default always occurs when the first-period strategy is unsuccessful. Depending on the value of E_0, default may also occur when the first-period strategy is successful and the second-period strategy is unsuccessful. The first case gives rise to expected bankruptcy costs of $\frac{1}{2}b$, while the latter case gives rise to expected bankruptcy costs of $\frac{3}{4}b$, see the proof of Proposition 2 for details.

2. The restriction $b < \frac{4}{3}\gamma$ ensures that case iii) exists. If $b \geq \frac{4}{3}\gamma$, then risk-shifting is so costly that financing with risk-shifting (i.e., case iii)) is no longer possible. For $b \geq \frac{4}{3}\gamma$, case ii) is unaffected and case iv) occurs for $E_0 < \gamma$, see the proof of Proposition 2 for details.

Figure 6 illustrates the equilibrium outcomes as a function of the insider's initial equity at $t = 0$. If initial equity is abundant, $E_0 \geq 2\gamma$, then the insider can issue safe debt in both periods and she is indifferent between the safe and the risky strategy. This corresponds to case i) of Proposition 1 in the one period case, except that the initial equity has to be large enough to cover failure of the risky strategy in both periods.

When the level of initial equity is intermediate, $\gamma \leq E_0 < 2\gamma$ (i.e., the equity can cover failure in one period, but not in both), then the key result of our paper occurs. The insider actually prefers the safe strategy over the risky strategy. Doing so allows to issue safe debt again in the second period. The insider could risk-shift in the first period, but this is costly because if the project fails, she is no longer able to issue safe debt in the second period. Not being able to issue safe debt in the second period would imply that the insider engages in risk-shifting in the second period and the lenders would price in the expected costs of bankruptcy.

Case iii) of the two-period model is analogous to case ii) of the one period model. The insider's equity is not high enough to issue safe debt in the first period and inefficient risk-shifting occurs.

Case iv) of the two-period model in turn is analogous to case iii) of the one period model. The insider's equity is insufficient to cover the cost of risky debt that prices in the expected costs of bankruptcy. Note that in the two-period model, more initial equity is needed than in the one-period model. The insider factors in that even if she succeeds in the first period, her equity at $t = 1$ (after paying off the first-period lenders) may not be high enough to obtain financing with safe debt from second period lenders.

The following corollary establishes the fact that the insider can strictly prefer the risky strategy over the safe strategy even if the safe strategy has a higher expected payoff than the risky strategy.

Corollary 2 *(***Higher expected payoff of the risky strategy***) Assume the same setup as in Proposition 2, but the risky strategy's payoff in the first period is* $I_0 + \Delta \pm \gamma$*, with* $\Delta \in [0, \frac{1}{4}b)$ *being non-negative but not too large. The insider chooses the safe strategy in the first period whenever* $E_0 \in [\gamma + \Delta, 2\gamma - \Delta)$*.*[12]

Proposition 2 has established the fact that the insider may prefer the safe strategy over the risky strategy if both have the same expected payoff. Corollary 2 establishes that the insider may even prefer the safe strategy over the risky strategy if the risky strategy has a higher expected payoff. The length of the interval where the safe project is preferred over the risky project ($E_0 \in [\gamma, 2\gamma - \Delta)$) is decreasing in the expected excess payoff Δ of the risky strategy over the safe strategy. Corollary 2 therefore suggests that investment decisions are not only distorted for highly leveraged firms (who may prefer a risky project to a safe project even if the latter has a higher payoff, see Corollary 1), but also for medium leveraged firms (who may prefer a safe project to a risky project even if the latter has a higher payoff). This finding contributes to the low leverage puzzle, i.e., the empirical observation that firm leverage seems to be too low to be explained by standard frictions (Miller (1977), Graham (2000), Korteweg (2010)).

[12]Note that $\Delta \in [0, \frac{1}{4}b)$ together with Assumption 1 ensures that the interval for E_0 is non-empty.

3.3 Two period model with long-term debt

3.3.1 Set-up

The set-up is as in the two-period model in Section 3.2, with the exception that the maturity of the debt is two periods. Section 3.3.2 derives equilibrium outcomes for an exogenous debt maturity of two periods. Section 3.3.3 discusses the equilibrium outcome if debt maturity is endogenous, i.e. if the insider can choose between issuing one or two period debt.

3.3.2 Equilibrium

Proposition 3 *Given the value of the insider's equity at $t = 0$, E_0, and assuming $b < \gamma$, there are three possible equilibrium outcomes:*

i) When $E_0 \geq 2\gamma$, then the insider can issue safe long-term debt. She is indifferent between choosing the safe or the risky strategy in both periods. The expected value of her equity at $t = 0$ is E_0.

ii) When $2\gamma > E_0 \geq \frac{3}{4}b$, then the insider cannot issue safe debt. She chooses the risky strategy in both periods. The expected value of her equity at $t = 0$ is $E_0 - \frac{1}{4}b$ or $E_0 - \frac{3}{4}b$.

iii) When $E_0 < \frac{3}{4}b$, then the insider cannot obtain financing. The expected value of her equity at $t = 0$ is zero.

Notes:

1. In case ii), depending on the value of E_0, default may either occur only after two unsuccessful outcomes of the risky strategy, or if the risky strategy is unsuccessful in at least one of the two periods. The first case gives rise to expected bankruptcy costs of $\frac{1}{4}b$, while the latter case gives rise to expected bankruptcy costs of $\frac{3}{4}b$, see the proof of Proposition 3 for details.

2. If $b \geq \gamma$, then the threshold that separates cases ii) and iii) becomes $\frac{1}{2}\gamma + \frac{1}{4}b$ (instead of $\frac{3}{4}b$). See the proof of Proposition 3 for details.

Proposition 3 mimics the results from the one-period model: the insider has an incentive to risk-shift for low levels of equity ($E_0 < 2\gamma$) as long as she is able to raise outside debt ($E_0 > \frac{3}{4}b$). The insider bears the costs of risk-shifting, reflected in the terms $-\frac{1}{4}b$ and $-\frac{3}{4}b$ in case ii). In contrast to the one-period model, a higher insider wealth is needed to avoid risk-shifting ($E_0 > 2\gamma$ in the two-period model with long-term debt as opposed to $E_0 > \gamma$ in the one-period model).

3.3.3 Choice between short-term and long-term debt

The following table summarizes the equilibrium strategies in the first period and the resulting expected equity values as a function of the amount of initial insider wealth E_0 for short-term and long-term debt:

		2 x Short-period debt		Long-term debt	
Leverage	E_0	Strategy	$\mathbb{E}[E_2(E_0)]$	Strategy	$\mathbb{E}[E_2(E_0)]$
Low	$[2\gamma, \infty)$	Indifferent	E_0	Indifferent	E_0
Medium	$[\gamma, 2\gamma)$	Risk-averting	E_0	Risk-shifting	$E_0 - \frac{1}{4}b$
High	$[\frac{3}{4}b, \gamma)$	Risk-shifting	$E_0 - \frac{1}{2}b$ or $E_0 - \frac{3}{4}b$	Risk-shifting	$E_0 - \frac{1}{4}b$ or $E_0 - \frac{3}{4}b$
Very High	$[0, \frac{3}{4}b)$	No financing	0	No financing	0

Both long-term debt and short-term debt lead to the same expected equity value as long as the initial insider wealth is sufficient to cover two periods of unsuccessful investment with the risky strategy ($E_0 > 2\gamma$). This makes intuitive sense as the insider always fully bears the gains and losses of any strategy that she implements.

The crucial case arises if equity is large enough to cover one, but not two, periods of losses, i.e. $E_0 \in [\gamma, 2\gamma)$. In this case, the insider is strictly better off by issuing short-term debt and choosing the safe project in the first period. Thus, the insider has an incentive to issue short-term debt and avert risk in the first period.

When the insider's wealth is not sufficient to cover one period of losses, i.e. $E_0 < \gamma$, the insider either has an incentive to risk-shift (resulting in an expected equity value smaller than E_0) or is unable to obtain financing (resulting in an equity value of zero) both with short-term and with long-term debt.

Taken together, this implies that the key result from Proposition 2 carries over to a setting with endogenous debt maturity: a firm with a medium leverage $E_0 \in [\gamma, 2\gamma)$ has an incentive to issue short-term debt, avert risk and choose the safe strategy over the risky strategy.

3.4 Extension to N periods

Proposition 4 *Assume the model consists of N periods. In periods* $1, ..., N-1$*, the model contains the same payoffs and potential strategy choices as depicted in the first period of Figure 5. In period* N*, the payoffs and potential strategy choices are as depicted in the last period of Figure 5. Given the value of the insider's equity at* $t = n$*,* E_n*, and assuming* $b < \gamma$*, there are four possible equilibrium outcomes:*

i) When $E_n \geq 2\gamma$*, then the insider can issue safe debt in the next period. She is indifferent between the safe and the risky strategy in the next period. The expected value of her equity at* $t = n$ *is* E_n*.*

ii) When $2\gamma > E_n \geq \gamma$*, the insider can issue safe debt in the next period and prefers the safe strategy in all subsequent periods. The expected value of her equity at* $t = n$ *is* E_n*.*

iii) When $\gamma > E_n \geq \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$*, then the insider cannot issue safe debt in the next period and chooses the risky strategy in the next period. The expected value of her equity at* $t = n$ *is smaller than* E_n*.*

iv) When $E_n < \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$*, then the insider cannot obtain financing. The expected value of her equity at* $t = n$ *is zero.*

Notes:

1. Case ii) is an absorbing state, i.e., for $2\gamma > E_n \geq \gamma$, the insider chooses the safe strategy in all subsequent periods. Choosing the safe strategy preserves the opportunity to issue safe debt and thus avoids incurring (expected) bankruptcy costs.

2. The minimum amount of equity capital needed to be able to refinance is $E_n^{min} = \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$. Special cases include the one-period model ($n = N-1$) which yields

$E_{N-1}^{min} = \frac{1}{2}b$, and the two-period model ($n = N-2$) which yields ($E_{N-2}^{min} = \frac{3}{4}b$). The risk-shiting region $\gamma > E_n \geq \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$ decreases as n increases. Thus, the standard risk-shifting result from the one-period model applies to an increasingly smaller interval as the number of periods grows larger.

3. For large N and equity values slightly above b, the insider is trapped in a risk-shifting spiral: failure of the risky project implies bankruptcy, while success implies ending up in a region where agency conflicts are still too large to allow for the issuance of safe debt. The insider either defaults or remains in the risk-shifting region iii).

4 Conclusion

Risk-shifting à la Jensen and Meckling features prominently in corporate finance and banking. At first sight, the logic is intuitive. When debt is risky, borrowers have limited liability, and their investment choices are unobservable, then borrowers have an incentive to choose a risky investment strategy. If the risky strategy fails, the downside for borrowers is limited. If the risky strategy succeeds, they obtain a larger upside than under a safe investment strategy.

Risk-shifting features prominently in explanations of why high leverage is harmful. Risk-shifting is inefficient because lenders anticipate borrowers' incentives to choose risky investments and increase the cost of debt accordingly. The higher cost of debt lowers the value of borrowers' equity stake but there is nothing they can do about it because they cannot credibly commit not to risk-shift. Moreover, risk-shifting makes bankruptcy more likely, which increases the deadweight costs of bankruptcy.

Our paper shows that the risk-shifting logic is not robust in a dynamic model because it ignores the option value of being able to issue cheap, safe debt in the future. While it remains true that firms with high leverage engage in risk-shifting, firms with medium leverage actively avert risk (and firms with low leverage can always issue safe debt and hence, are not tempted to risk shift in the first place). Medium-leverage firms prefer safe investment strategies in early periods in order to avoid the costs of having to issue costly risky debt in future periods.

The logic to avert risk in a dynamic model relies entirely on the inefficiency of risk-shifting. In a static model, borrowers have to bear this inefficiency. In a dynamic model, the decision not to risk-shift today allows insiders to avoid the cost of inefficient risk-shifting tomorrow.

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Figures

Figure 1: One-period model / Example

This figure illustrates the payoff structure of the safe project and the risky project in our one-period example. Both projects require an investment of 80 and have zero net present value. The risky project adds a mean-preserving spread to the safe project. Probabilities for the up/down state for the risky project are 0.5 each.

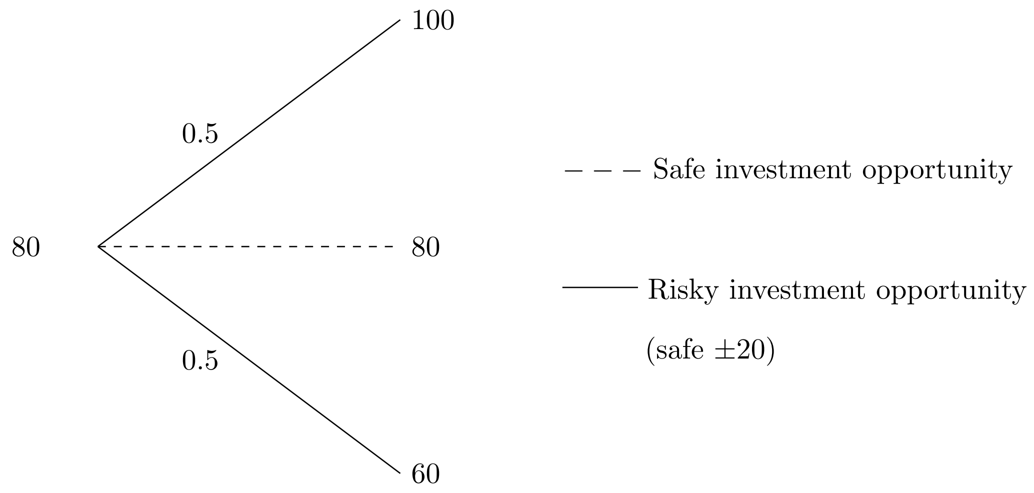
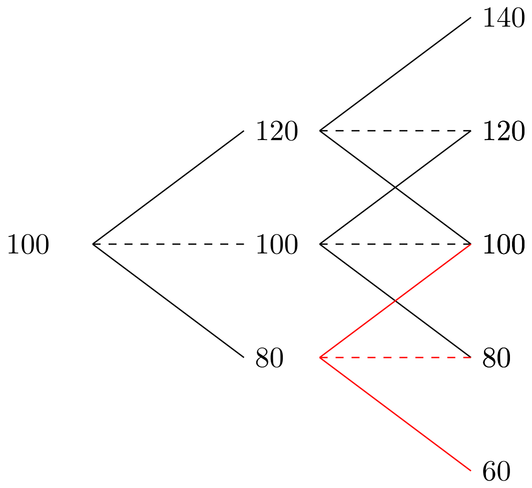
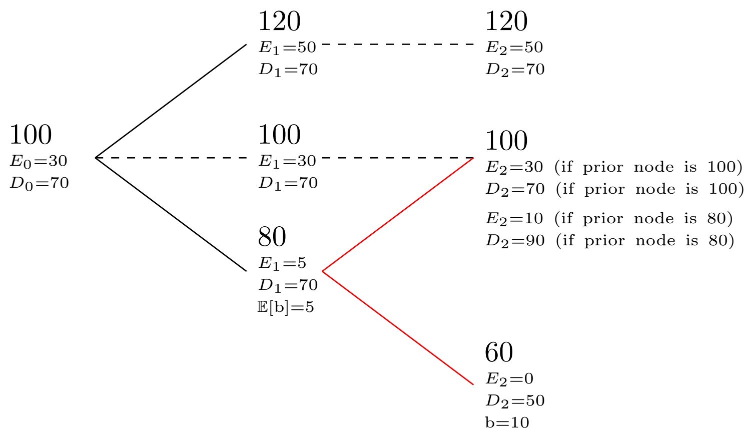


Figure 2: Two-period model / Example

This figure illustrates the payoff structure in our two-period example. Figure (a) depicts all possible outcome paths. Figure (b) depicts all possible paths for period 1 and only the equilibrium paths for period 2. Probabilities for the up/down state for the risky project are 0.5 each. The example assumes a fixed bankruptcy cost of $b = 10$, implying a payoff to lenders of $60 - 10 = 50$ in the down-state. Note that the red part of each figure is equivalent to the one-period game from Figure 1. In Figure (b), we assume without loss of generality that the equity-holder chooses the safe project if she is indifferent between the safe and the risky project.



(a) All paths. Red: One-period model from Figure 1.



(b) All paths for period 1, equilibrium paths for period 2 only. Red = One-period model from Figure 1. E_i and D_i refer to expected payoffs for equity and debt at time i in equilibrium.

Figure 3: One-period model / Timeline

This figure illustrates the timeline of events in the one-period model.

Financing

- Insider wealth E_1
- Financing $I_1 - E_1$ from outsiders via debt contract with notional F_2

Investment choice

- Strategy $c_1 \in \{s_1, r_1\}$, s_1 yields payoff I_1 (safe project), r_1 yields payoff $I_1 \pm \gamma$ (risky project)
- Outside option (no investment) yields payoff 0

Payoff

- Assets: $A_2 \in \{0, I_1 - \gamma, I_1, I_1 + \gamma\}$
- Equity: $E_2 = max(A_2 - F_2, 0)$
- Debt:

$$D_2 = \begin{cases} F_2 & A_2 \geq F_2, \\ A_2 - b & A_2 < F_2. \end{cases}$$

- Note: b = bankruptcy costs

Figure 4: One-period model / Equilibrium

This figure illustrates the equilibrium in the one-period model. The parameter γ denotes the risk-shifting parameter, $\mathbb{E}(BC) = \frac{1}{2}b$ denotes expected bankruptcy costs.

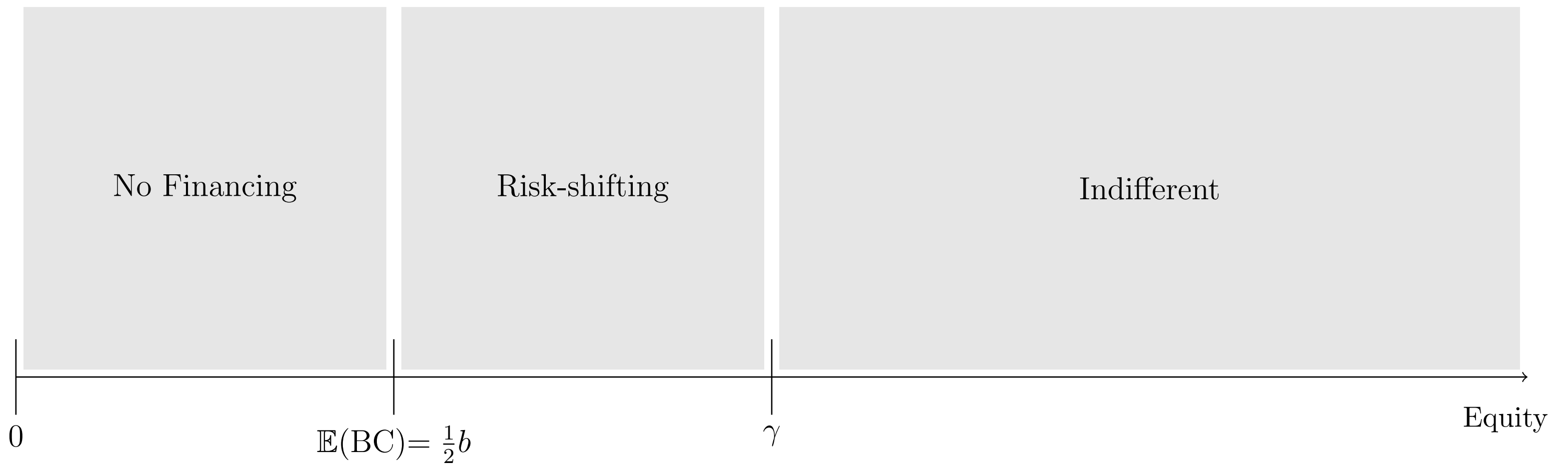


Figure 5: Two-period model / Timeline

This figure illustrates the timeline of events in the two-period model.

t=0 — Financing

- Insider wealth E_0
- Financing $I_0 - E_0$ from outsiders via debt contract with notional F_1

t=0.5 — Investment choice

- Strategy $c_0 \in \{s_0, r_0\}$, s_0 yields payoff I_0 (safe project), r_1 yields payoff $I_0 \pm \gamma$ (risky project)
- Outside option (no investment) yields payoff 0

t=1 — Value of assets

- Assets: $A_1 \in \{0, I_0 - \gamma, I_0, I_0 + \gamma\}$

Financing

- Refinancing of debt notional F_1 from outsiders via debt contract with notional F_2

Payoff in case that refinancing of debt notional F_1 not successful

- Equity: $E_1 = \max[A_1 - b - F_1, 0]$
- Debt: $D_1 = \min[A_1 - b, F_1]$

t=1.5 — Investment choice

- Strategy $c_1 \in \{s_1, r_1\}$, s_1 yields payoff I_1 (safe project), r_1 yields payoff $I_1 \pm \gamma$ (risky project)
- Outside option (no investment) yields payoff 0

t=2 — Payoff

- Assets: $A_2 \in \{0, I_1 - \gamma, I_1, I_1 + \gamma\}$
- Equity: $E_2 = \max[A_2 - F_2, 0]$
- Debt:

$$D_2 = \begin{cases} F_2 & A_2 \geq F_2, \\ A_2 - b & A_2 < F_2. \end{cases}$$

Figure 6: Two-period model / Equilibrium

This figure illustrates the equilibrium in the one-period and two-period model. The parameter γ denotes the risk-shifting parameter, $\mathbb{E}(BC)$ denotes expected bankruptcy costs in the one-period model when the insider is just able to receive outside financing ($E_0 = \frac{1}{2}b$), and $\mathbb{E}(BC') = \frac{3}{4}b$ denotes expected bankruptcy costs in the two-period model with risky debt when the insider is just able to receive outside financing ($E_0 = \frac{3}{4}b$). $\mathbb{E}(BC')$ differs from $\mathbb{E}(BC)$ because a firm with risky debt in the two-period model can go bankrupt in either $t = 1$ (with probability $\frac{1}{2}$) or in $t = 2$ (with probability $\frac{1}{4}$) and $\mathbb{E}(BC')$ captures the sum of the expected bankruptcy costs in $t = 1$ and $t = 2$.

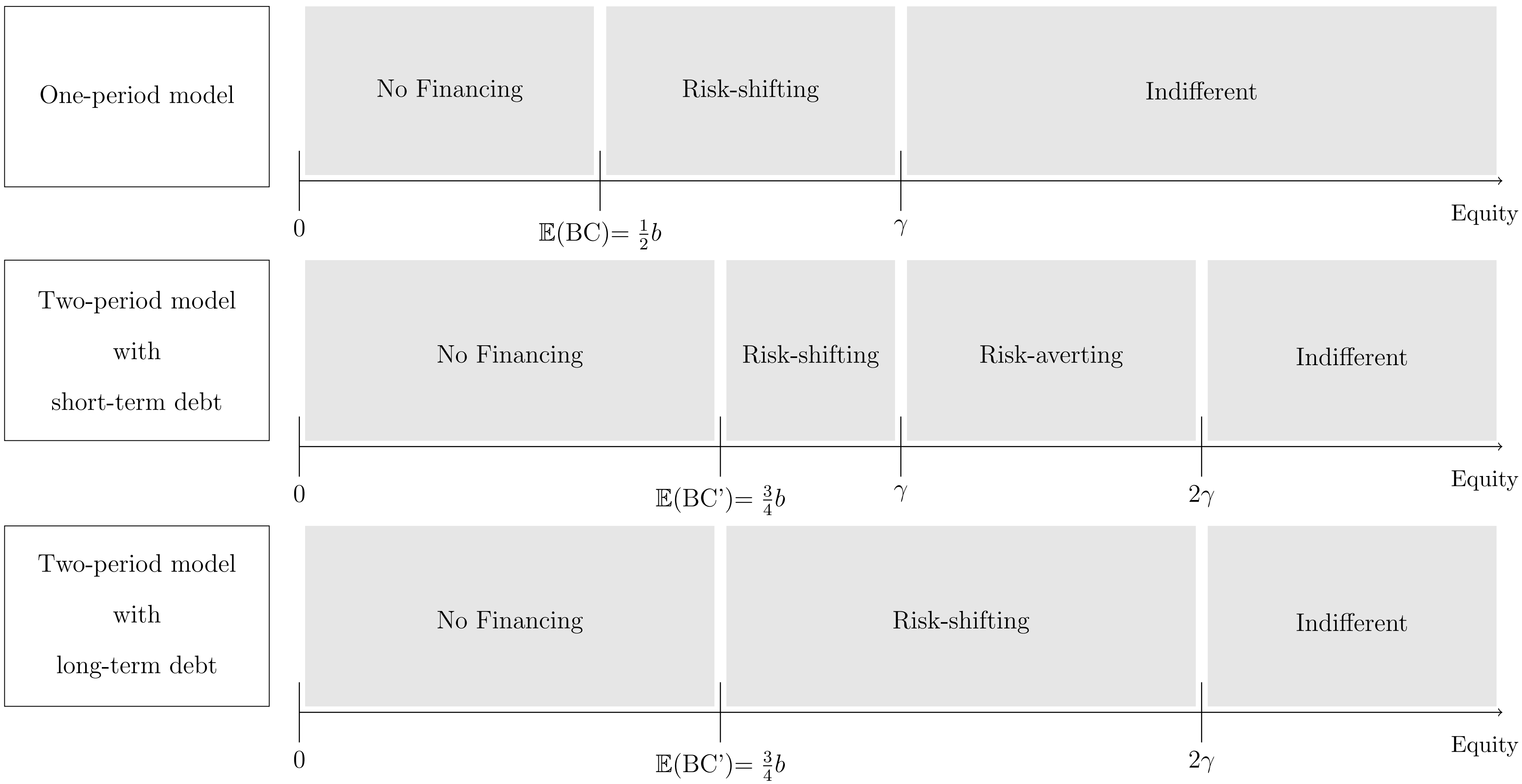


Table 1: Review of the empirical literature on leverage and risk-taking

This table provides an overview of empirical papers that analyze the relationship between leverage and risk-taking. In the first step, we select papers published in the three major finance journals (*Journal of Finance*, *Journal of Financial Economics*, *Review of Financial Studies*) that cite Jensen and Meckling (1976) and whose abstract discusses the relationship between leverage and risk-taking. We expand this list with papers from other journals that are frequently cited in the related literature in the papers identified in the first step.

Authors	Journal (Year)	Relation between leverage and risk-taking	Short summary
Panel A: General			
Haushalter	JF (2000)	Negative	Firms with higher leverage hedge more frequently
Eisdorfer	JF (2008)	Positive	Evidence of risk-shifting behavior in investment decisions of distressed firms
Pichler, Stomper, and Zulehner	RFS (2008)	Positive	Firms engange in risk-shifting by setting lower prices if debt is higher
Purnanandam	JFE (2008)	Non-monotonic	Moderately leveraged firms hedge more than low or high leveraged firms
Becker and Strömberg	RFS (2012)	Positive	Highly levered firms reduce risk when managers face less incentives to favor equity over debt
Gilje	RFS (2016)	Negative	Firms in the oil industry reduce investment risk when approaching financial distress
Aretz, Banerjee, and Pryshchepa	RF (2018)	Non-monotonic	Moderately, but not highly, leveraged firms increase risk-taking following a distress-risk-increasing hurricane
Panel B: Studies of distressed firm behavior			
Asquith, Gertner, and Scharfstein	QJE (1994)	Negative	Firms in financial distress decrease (asset) risk via asset sales
Andrade and Kaplan	JF (1998)	No relation	No evidence of risk-shifting when firms became financially (not economically) distressed
Panel C: Risk-shifting via defined-benefit corporate pension plans			
Cocco and Volpin	FAJ (2007)	Positive	Risk-shifting by leveraged firms in the asset mix of defined benefit pension plans
Rauh	RFS (2009)	Negative	Firms with weak credit ratings allocate a larger share of defined benefit pension fund assets to safer securities
Pedersen	JFQA (2019)	No relation	No evidence of risk-shifting in corporate pension plans following a shock to leverage
Panel D: CEO surveys			
Graham and Harvey	JFE (2001)	No relation	No relationship between leverage and risk-shifting behavior in survey of CEOs

Proofs

Proof of Proposition 1

We restrict our attention to a face value of debt smaller than the payoff after success of the risky project, i.e. $F_2 \leq I_1 + \gamma$, otherwise the insider is as well off by not investing.

Incentive constraint: The insider chooses the safe strategy iff the expected payoff from the safe strategy is larger or equal to the payoff from the risky strategy:

$$
\begin{aligned}
& \max[0, I_1 - F_2] && \geq \frac{1}{2}(I_1 + \gamma - F_2) + \frac{1}{2}\max[0, I_1 - \gamma - F_2] \\
\Leftrightarrow \quad & I_1 - \gamma - F_2 && \geq 0 \\
\Leftrightarrow \quad & F_2 && \leq I_1 - \gamma
\end{aligned} \tag{4}
$$

The risky strategy is always incentive compatible. Because the insider has a call option payoff, the risky strategy provides a higher payoff whenever the call option is valuable in at least one state of the world.

Participation constraint: If the insider issues debt with a low face value, $F_2 \leq I_1 - \gamma$, then default does not occur even under the risky strategy, and the outsiders' participation constraint is

$$F_2 \geq I_1 - E_1. \tag{5}$$

If the insider issues debt with a face value such that, $I_1 + \gamma > F_2 > I_1 - \gamma$, then the outsiders knows that the insider chooses the risky strategy, default occurs in the low state and the outsiders' participation constraint is

$$
\begin{aligned}
& \frac{1}{2}F_2 + \frac{1}{2}(I_1 - \gamma - b) \geq I_1 - E_1 \\
& \Leftrightarrow F_2 \geq I_1 - 2E_1 + \gamma + b.
\end{aligned} \tag{6}
$$

Equity value: Now consider the insider's optimal choice of face value F_2 subject to the outsiders' participation constraint and her own incentive compatible strategy choice. Given that the equity value decreases in the face value of debt, the insider has the incentive to reduce the face value of debt as much as possible. If the insider chooses safe debt, $I_1 - \gamma \geq F_2$, then the lowest possible face value of debt is given by the binding participation constraint (5), and the expected value of her equity is

$$\mathbb{E}_1[E_2] = I_1 - F_2 = I_1 - (I_1 - E_1) = E_1 \tag{7}$$

If she chooses risky debt, $I_1 + \gamma > F_2 > I_1 - \gamma$, then the face value of debt is given by the binding participation constraint (6), and the expected value of her equity is

$$\mathbb{E}_1[E_2] = \frac{1}{2}(I_1 + \gamma - F_2) + \frac{1}{2}0 = E_1 - \frac{1}{2}b. \tag{8}$$

Equilibrium strategies: To realize the maximum value $\mathbb{E}_1[E_2] = E_1$, it must be that $I_1 - \gamma \geq F_2 = I_1 - E_1$, or $E_1 \geq \gamma$. In this case, the insider can issue safe debt and she is indifferent between choosing the safe or the risky strategy.

If $E_1 < \gamma$, only risky debt $I_1 + \gamma > F_2 > I_1 - \gamma$ is possible with a face value of $F_2 = I_1 - 2E_1 + \gamma + b$. The value of equity with risky debt is given by (8).

When the value of equity $E_1 < \frac{1}{2}b$, then the participation constraint (6) requires $F_2 > I_1 + \gamma$. As this is higher than the payoff after success of the risky project, the insider cannot obtain financing and the value of equity is zero.

Proof of Corollary 1

The incentive constraint (4) becomes $\max[0, I_1 + \Delta - F_2] \geq \frac{1}{2}(I_1 + \gamma - F_2) + \frac{1}{2}\max[0, I_1 - \gamma - F_2] \Leftrightarrow F_2 \leq I_1 - \gamma + 2\Delta$. The participation constraints for safe debt (5) and risky debt (6) remain unchanged. The equity value becomes $\mathbb{E}[E_2] = E_1 + \Delta$ for the safe strategy and remains $\mathbb{E}[E_2] = E_1 - \frac{1}{2}b$ for the risky strategy.

In equilibrium, the safe strategy is choosen for $F_2 \leq I_1 - \gamma + 2\Delta$, which – using the participation constraint with the lowest possible face value $F_2 = I_1 - E_1$ – becomes $E_1 \geq \gamma - 2\Delta$. The risky strategy is choosen for $E_1 < \gamma - 2\Delta$ as long as the insider can obtain funding (governed by the same participation constraint (6) as in the proof of Proposition 1), i.e. for $E_1 \in [\frac{1}{2}b, \gamma - 2\Delta)$. The interval for E_1 is non-empty if $\frac{1}{2}b < \gamma - 2\Delta \Leftrightarrow \Delta < \frac{1}{2}\gamma - \frac{1}{4}b$.

Proof of Proposition 2

We introduce the following notation:

- $E_1(s_0)$ denotes the equity payoff after the first period if the safe strategy is choosen in the first period and the model hypothetically ends after the first period without liquidation costs, i.e. $E_1(s_0) = \max(I_0 - F_1, 0)$. $E_1(r_0^-)$, $E_1(r_0^+)$ denote the corresponding equity payoffs if the risky strategy is choosen in the first period and the risky strategy is unsuccessful/successful, i.e. $E_1(r_0^-) = \max(I_0 - \gamma - F_1, 0)$ and $E_1(r_0^+) = \max(I_0 + \gamma - F_1, 0)$.

- $E_2[E_1(s_0)]$ denotes the equity payoff in the second period if the safe strategy is choosen in the first period and the equilibrium strategy is choosen in the second period. $E_2[E_1(r_0^-)]$ and $E_2[E_1(r_0^+)]$ are defined accordingly. $E_2[E_1(r_0)] := \frac{1}{2}E_2[E_1(r_0^-)] + \frac{1}{2}E_2[E_1(r_0^+)]$ corresponds to the expected payoff if the risky strategy is choosen in the first period and the equilibrium strategy is choosen in the second period.

- $\mathbb{E}_0[E_2(E_0)]$ is the expected equity payoff under the equilibrium strategy given initial insider wealth of E_0.

The following lemma establishes that the safe strategy is not incentive compatible if the face value (F_1) exceeds the asset value in the down-state ($I_0 - \gamma$), i.e. if the limited liability feature of equity can be binding.

Lemma 1 *If $F_1 > I_0 - \gamma$, the safe strategy is not incentive compatible in the first period.*

If $F_1 > I_0$ the insider always defaults under the safe strategy and the safe strategy is therefore not incentive compatible. We can focus our attention on the case $F_1 \in (I_0 - \gamma, I_0]$.

- If the safe strategy is choosen in the first period, then $E_1(s_0) = I_0 - F_1 < \gamma$, implying either the risk-taking case from Proposition 1 (case ii)) or the non-financing case from Proposition 1 (case iii) in the second period, and thus $E_2[E_1(s_0)] < I_0 - F_1$.

- If the risky strategy is choosen in the first period, $E_1(r_0^+) = I_0+\gamma-F_1 \geq \gamma$, implying case i) of Proposition 1 in case of a first-period success and thus $E_2[E_1(r_0^+)] = I_0+\gamma-F_1$. By definition $E_1(r_0^-) \geq 0$ and thus, $E_2[E_1(r_0)] \geq \frac{1}{2}0+\frac{1}{2}(I_0+\gamma-F_1) > \frac{1}{2}2(I_0-F_1) = I_0-F_1$.

We therefore have $E_2[E_1(s_0)] < I_0-F_1$ and $E_2[E_1(r_0)] > I_0-F_1$ which proves our lemma.

In the following, we derive the participation constraint and the expected equity value.

Participation constraint and expected equity value

Case 1: $F_1 \leq I_0-\gamma$ (Safe debt in the first period)
Note that $F_1 \geq I_0-E_0$ because the face value of debt F_1 cannot be smaller than the debt funding required I_0-E_0. If $F_1 \leq I_0-\gamma$, it must be that $E_0 \geq \gamma$.

- *Case 1a: Safe strategy in the first period.* The safe strategy ensures that the insider ends up above the risk-shifting region from Proposition 1 after the first period. To see this formally, note that

$$E_1(s_0) = \max(I_0-F_1, 0) \geq \max(\gamma, 0) = \gamma \tag{9}$$

  This implies that second-period financing can be obtained, first-period debt is safe (i.e. the participation constraint becomes $F_1 = I_0-E_0$) and the equity payoff under the safe strategy is therefore

$$E_2[E_1(s_0)] = I_0-F_1 = E_0 \tag{10}$$

- *Case 1b: Risky strategy in the first period.* After success,

$$E_1(r_0^+) = I_0+\gamma-F_1 > 2\gamma, \tag{11}$$

  case i) of Proposition 1 applies and the value of equity is

$$E_2[E_1(r_0^+)] = I_0+\gamma-F_1. \tag{12}$$

  After failure,

$$E_1(r_0^-) = I_0-\gamma-F_1. \tag{13}$$

  We now have to distinguish two cases to determine $E_2[E_1(r_0^-)]$:

  - $I_0-F_1 \geq 2\gamma$: Case i) of Prop. 1 applies and

$$E_2[E_1(r_0^-)] = I_0-\gamma-F_1 \tag{14}$$

    Thus, $E_2[E_1(r_0)] = \frac{1}{2}E_2[E_1(r_0^-)]+\frac{1}{2}E_2[E_1(r_0^+)] = I_0-F_1$. Debt is always safe so that the participation constraint is $F_1 = I_0-E_0$ which implies $E_2[E_1(r_0)] = E_0$. Choosing the risky strategy in the first period therefore provides the same expected payoff to the insider as choosing the safe strategy in the first period.

  - $I_0-F_1 < 2\gamma$: Case ii) or iii) of Prop. 1 applies and

$$E_2[E_1(r_0^-)] < I_0-\gamma-F_1 \tag{15}$$

Thus, $E_2[E_1(r_0)] = \frac{1}{2}E_2[E_1(r_0^-)] + \frac{1}{2}E_2[E_1(r_0^+)] < I_0 - F_1$. Since $E_2[E_1(r_0)] < E_2[E_1(s_0)]$, choosing the risky strategy in the first period provides a lower expected payoff to the insider as choosing the safe strategy in the first period.

Taken together, for $F_1 \leq I_0 - \gamma$, the strategy choice and expected value of equity at $t = 0$ are:

$$\mathbb{E}_0[E_2(E_0)] = \begin{cases} E_0 & \text{if } E_0 \in [2\gamma, \infty) \qquad c_0 \in \{s_0, r_0\} \text{ (indifferent)} \\ E_0 & \text{if } E_0 \in [\gamma, 2\gamma) \qquad c_0 = s_0 \text{ (safe)} \end{cases} \tag{16}$$

Case 2: $F_1 > I_0 - \gamma$ (risky debt in the first period)

With $F_1 > I_0 - \gamma$ the insider will always choose the risky strategy according to Lemma 1. The minimal face value that satisfies the participation constraint is given by the one-period participation constraint for risky debt(6), replacing F_2 by F_1, I_1 by I_0, and E_1 by E_0, i.e. $F_1 = I_0 - 2E_0 + \gamma + b$. This implies that $E_0 < \gamma + \frac{1}{2}b$. After failure, the insider defaults, $E_1(r_0^-) = \max(I_0 - \gamma - F_1, 0) = 0$ and thus $E_2[E_1(r_0^-)] = 0$. After success $E_1(r_0^+) = I_0 + \gamma - F_1 = 2E_0 - b$ and we have to distinguish three cases:

- First, case i) of Proposition 1 applies when $E_1(r_0^+) \geq \gamma$, which is equivalent to $E_0 \geq \frac{1}{2}\gamma + \frac{1}{2}b$. In this case,

$$E_2[E_1(r_0^+)] = E_1(r_0^+) = 2E_0 - b$$

- Second, case ii) of Proposition 1 applies when $\gamma > E_1(r_0^+) \geq \frac{1}{2}b$, which is equivalent to $\frac{1}{2}\gamma + \frac{1}{2}b > E_0 \geq \frac{3}{4}b$. In this case,

$$E_2[E_1(r_0^+)] = E_1(r_0^+) - \frac{1}{2}b = 2E_0 - \frac{3}{2}b$$

  Note that this second case exists only if $b < 2\gamma$.

- Third, case iii) of Proposition 1 applies when $\frac{1}{2}b > E_1(r_0^+)$, which is equivalent to $\frac{3}{4}b > E_0$. In that case,

$$E_2[E_1(r_0^+)] = 0.$$

Combining the payoff after a failure of the risky strategy ($E_2[E_1(r_0^-)] = 0$) with the three payoffs after success of the risky strategy ($2E_0 - b$, $2E_0 - \frac{3}{2}b$, and 0) gives the following result: with $F_1 > I_0 - \gamma$, the risky strategy is implemented in the first period, $E_0 \leq \gamma + \frac{1}{2}b$ and the expected value of the insider's equity at $t = 0$ is:

$$\mathbb{E}_0[E_2(E_0)] = \begin{cases} E_0 - \frac{1}{2}b & \text{if } E_0 \in [\frac{1}{2}\gamma + \frac{1}{2}b, \gamma + \frac{1}{2}b) \\ E_0 - \frac{3}{4}b & \text{if } E_0 \in [\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{2}b) \\ 0 & \text{if } E_0 \in [0, \frac{3}{4}b) \end{cases} \tag{17}$$

Equilibrium strategies

The equilibrium strategies follow from three constraints. First, (16) and (17) provide the range of values for the initial insider wealth E_0 that make safe debt and risky debt feasible. Second, if both are feasible, the insider chooses whichever strategy yields the highest expected value $\mathbb{E}_0[E_2(E_0)]$. Third, one has to carefully consider the upper and lower bounds in (16) and (17) to make sure not to make statements about empty intervals. The following table provides the equilibrium strategies, the cases where default occurs, and the expected equity value. Subindex "1" denotes the first period, subindex "2" denotes the second period, a slash differentiates second-period strategies after a successful/non-successful risky first-period strategy, and ID_i denotes that the insider is indifferent between the safe and the risky strategy (i.e., indifferent between r_0 and s_0 or indifferent between r_1 and s_1).

	E_0				
	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{2}b)$	$[\frac{1}{2}\gamma + \frac{1}{2}b, \gamma)$	$[\gamma, 2\gamma)$	$[2\gamma, \infty)$
Strategy	No_0	r_0, r_1/No_1	r_0, $\mathrm{ID}_1/\mathrm{No}_1$	s_0, ID_1	ID_0, ID_1
Default	always	$\mathrm{down}_0 \vee \mathrm{up}_0,\mathrm{down}_1$	down_0	never	never
Equity value	0	$E_0 - \frac{3}{4}b$	$E_0 - \frac{1}{2}b$	E_0	E_0
Non-empty	always	$b < 2\gamma$	$b < \gamma$	always	always
$b \in (0, \gamma)$	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{2}b)$	$[\frac{1}{2}\gamma + \frac{1}{2}b, \gamma)$	$[\gamma, 2\gamma)$	$[2\gamma, \infty)$
$b \in [\gamma, \frac{4}{3}\gamma)$	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \gamma)$	———	$[\gamma, 2\gamma)$	$[2\gamma, \infty)$
$b \in [\frac{4}{3}\gamma, 2\gamma)$	$[0, \gamma)$	———	———	$[\gamma, 2\gamma)$	$[2\gamma, \infty)$

Proof of Corollary 2

We start with $F_1 \leq I_0 - \gamma$. Case 1a (safe strategy in the first period) of the proof of Proposition 2 is unaffected, thus $E_2[E_1(s_0)] = I_0 - F_1$ (see (10)). In case 1b (risky strategy in the first period), the value of equity after a first-period success is now $E_2[E_1(r_0^+)] = I_0 + \Delta + \gamma - F_1$. In case of a first-period failure of the risky strategy, we have to distinguish four cases:

- $I_0 - F_1 \geq 2\gamma - \Delta$: In this case, $E_1(r_0^-) = \max(I_0 + \Delta - \gamma - F_1, 0) \geq \gamma$. Thus, Case i) of Proposition 1 applies, $E_2[E_1(r_0^-)] = I_0 + \Delta - \gamma - F_1$ and $E_2[E_1(r_0)] = \frac{1}{2}E_2[E_1(r_0^-)] + \frac{1}{2}E_2[E_1(r_0^+)] = I_0 + \Delta - F_1$. This implies that $E_2[E_1(r_0)] > E_2[E_1(s_0)]$ so that the risky strategy is preferred over the safe strategy. Debt is safe, implying that this case applies for $E_0 \geq 2\gamma - \Delta$.

- $I_0 - F_1 \in [\gamma - \Delta + \frac{1}{2}b, 2\gamma - \Delta)$: In this case, $E_1(r_0^-) = \max(I_0 + \Delta - \gamma - F_1, 0) \in [\frac{1}{2}b, \gamma)$. Thus, Case ii) of Proposition 1 applies, $E_2[E_1(r_0^-)] = I_0 + \Delta - \gamma - F_1 - \frac{1}{2}b$ and $E_2[E_1(r_0)] = \frac{1}{2}E_2[E_1(r_0^-)] + \frac{1}{2}E_2[E_1(r_0^+)] = I_0 + \Delta - F_1 - \frac{1}{4}b$. For $\Delta < \frac{1}{4}b$, this implies that $E_2[E_1(r_0)] < E_2[E_1(s_0)]$ so that the safe strategy is preferred over the risky strategy. Debt is safe, implying that this case applies for $E_0 \in [\gamma - \Delta + \frac{1}{2}b, 2\gamma - \Delta)$.

- $I_0 - F_1 \in [\gamma + \Delta, \gamma - \Delta + \frac{1}{2}b)$:[13] In this case, $E_1(r_0^-) = \max(I_0 + \Delta - \gamma - F_1, 0) < \frac{1}{2}b$. Thus, Case iii) of Proposition 1 applies, $E_2[E_1(r_0^-)] = 0$, and $E_2[E_1(r_0)] = \frac{1}{2}E_2[E_1(r_0^-)] + \frac{1}{2}E_2[E_1(r_0^+)] = \frac{1}{2}(I_0 + \Delta + \gamma - F_1)$. For $\Delta + \gamma \leq I_0 - F_1$, this implies that $E_2[E_1(r_0)] \leq E_2[E_1(s_0)]$ so that the safe strategy is preferred over the risky strategy (for $\Delta + \gamma < I_0 - F_1$) or has the same expected payoff as the risky strategy (for $\Delta + \gamma = I_0 - F_1$). As a tiebreaker rule, we assume that the safe strategy is choosen. Debt is safe, implying that this case applies for $E_0 \in [\gamma + \Delta, \gamma - \Delta + \frac{1}{2}b)$.

[13] Note that by assumption, $\Delta < \frac{1}{4}b$ so that the interval is non-empty.

- $I_0 - F_1 \in [\gamma, \gamma + \Delta)$: In this case, $E_1(r_0^-) = \max(I_0 + \Delta - \gamma - F_1, 0) < \frac{1}{2}b$. Thus, Case iii) of Proposition 1 applies, $E_2[E_1(r_0^-)] = 0$, and $E_2[E_1(r_0)] = \frac{1}{2}E_2[E_1(r_0^-)] + \frac{1}{2}E_2[E_1(r_0^+)] = \frac{1}{2}(I_0+\Delta+\gamma-F_1)$. For $\Delta+\gamma > I_0-F_1$, this implies that $E_2[E_1(r_0)] > E_2[E_1(s_0)]$ so that the risky strategy is preferred over the safe strategy. Debt is risky, which, using the participation constraint for risky debt ($F_1 = I_0 - 2E_0 - \Delta + \gamma + b$) implies $E_0 \in [\gamma + \frac{1}{2}b - \frac{1}{2}\Delta, \gamma + \frac{1}{2}b)$. Since $\Delta < \frac{1}{4}b$, this case is dominated by choosing a lower face value of debt $I_0 - F_1 \in [\gamma+\Delta, \gamma-\Delta+\frac{1}{2}b)$ and choosing the safe strategy.

For $I_0 - F_1 < \gamma$, the risky strategy is prefered over the safe strategy for $\Delta = 0$ (see Proposition 2), so that it is also refered for $\Delta > 0$. Taken together, the first-period strategy choice and expected value of equity for $E_0 \in [\gamma, \infty)$ at $t = 0$ are:

$$\mathbb{E}_0[E_2(E_0)] = \begin{cases} E_0 & \text{if } E_0 \in [2\gamma - \Delta, \infty) & c_0 = r_0 \text{ (risky)} \\ E_0 & \text{if } E_0 \in [\gamma + \Delta, 2\gamma - \Delta) & c_0 = s_0 \text{ (safe)} \\ < E_0 & \text{if } E_0 \in [\gamma, \gamma + \Delta) & c_0 = r_0 \text{ (risky)} \end{cases} \tag{18}$$

Proof of Proposition 3

We restrict our attention to a face value of debt smaller than the payoff after success of the risky project, i.e. $F_2 \leq I_1 + \gamma$, otherwise the insider is as well off by not investing.

Incentive constraint
The insider can choose six strategies in the two-period model: four deterministic strategies (safe in both periods, risky in both periods, safe/risky, risky/safe), and two strategies that depend on the first-period outcome (risky in the first period and safe after success/risky otherwise, risky in the first period and risky after success/safe otherwise), resulting in payoffs $A_2 \in \{I_0 - 2\gamma, I_0 - \gamma, I_0, I_0 + \gamma, I_0 + 2\gamma\}$. Let p_i denote the probability of a payoff $A_2^i \in \{I_0 - 2\gamma, I_0 - \gamma, I_0, I_0 + \gamma, I_0 + 2\gamma\}$. The insider chooses the strategy that maximizes the payoff

$$\mathbb{E}_0[E_2] = \sum_i p_i \max[0, A_2^i - F_2] = \underbrace{\sum_i p_i(A_2^i - F_2)}_{I_0 - F_2} + \sum_i p_i \max[F_2 - A_2^i, 0] \tag{19}$$

The first sum in (19) is equal to $I_0 - F_2$ for all strategies and the IC constraint thus hinges on the last sum in (19). The last sum is zero for the safe/safe strategy in any case, but it is larger than zero for all other strategies as long as $F_2 > \min_i A_2^i = I_0 - 2\gamma$. Thus, the safe/safe strategy is only incentive-compatible iff

$$F_2 \leq I_0 - 2\gamma$$

If $F_2 > I_0 - 2\gamma$, it is straightforward to see that the strategy risky/risky maximizes the payoff (19). The strategy risky/risky results in a payoff of $I_0 - 2\gamma$ with probability $\frac{1}{4}$, I_0 with probability $\frac{1}{2}$, and $I_0 + 2\gamma$ with probability $\frac{1}{4}$.

Participation constraint and expected equity value
PC for safe debt: With safe debt ($F_2 \leq I_0 - 2\gamma$), the participation constraint is

$$F_2^{\text{safe}} \geq I_0 - E_0,$$

which, together with $F_2 \leq I_0 - 2\gamma$ implies $E_0 \geq 2\gamma$. The resulting equity value at $t = 0$ is

$$\mathbb{E}_0[E_2(E_0)] = I_0 - F_2^{\text{safe}} = E_0 \text{ if } E_0 \in [2\gamma, \infty) \tag{20}$$

PC for risky debt: With risky debt that defaults in the low state ($A_2 = I_0 - 2\gamma$) only, the participation constraint is

$$\begin{aligned} & \frac{3}{4}F_2^{\text{riskyI}} + \frac{1}{4}(I_0 - 2\gamma - b) \geq I_0 - E_0 \\ \Leftrightarrow \quad & F_2^{\text{riskyI}} \geq I_0 - \frac{4}{3}E_0 + \frac{2}{3}\gamma + \frac{1}{3}b \end{aligned}$$

For default to occur in the low state, but not in the medium state, we require $F_2^{\text{riskyI}} \leq I_0$ and $F_2^{\text{riskyI}} > I_0 - 2\gamma$ which implies $E_0 \in [\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma + \frac{1}{4}b)$. The resulting equity value at $t = 0$ is:

$$\begin{aligned} \mathbb{E}_0[E_2(E_0)] &= \frac{1}{4}(I_0 + 2\gamma - F_2^{\text{riskyI}}) + \frac{1}{2}(I_0 - F_2^{\text{riskyI}}) \\ &= E_1 - \frac{1}{4}b \text{ if } E_0 \in [\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma + \frac{1}{4}b) \end{aligned} \tag{21}$$

With risky debt that defaults in the low state ($A_2 = I_0 - 2\gamma$) and in the medium state ($A_2 = I_0$), the participation constraint is

$$\begin{aligned} & \frac{1}{4}F_2^{\text{riskyII}} + \frac{1}{2}(I_0 - b) + \frac{1}{4}(I_0 - 2\gamma - b) \geq I_0 - E_0 \\ \Leftrightarrow \quad & F_2^{\text{riskyII}} \geq I_0 - 4E_0 + 3b + 2\gamma \end{aligned}$$

For default to occur in the low and medium state, but not in the high state, we require $F_2^{\text{riskyII}} \leq I_0 + 2\gamma$ and $F_2^{\text{riskyII}} > I_0$ which implies $E_0 \in [\frac{3}{4}b, \frac{1}{2}\gamma + \frac{3}{4}b)$. The resulting equity value at $t = 0$ is:

$$\begin{aligned} \mathbb{E}_0[E_2(E_0)] &= \frac{1}{4}(I_0 + 2\gamma - F_2^{\text{riskyII}}) \\ &= E_0 - \frac{3}{4}b \text{ if } E_0 \in [\frac{3}{4}b, \frac{1}{2}\gamma + \frac{3}{4}b) \end{aligned} \tag{22}$$

Note that the upper bound of (22) is higher than the lower bound of (21), which suggests that there are values for E_0 where the insider can issue either debt with a face value that defaults in the low and medium state or debt with a face value that defaults in the low state only.

Equilibrium strategies
The equilibrium strategies follow from three constraints. First, (20), (21), and (22) provide the range of values for the initial insider wealth E_0 that make safe debt and the two versions of risky debt (default in the low state only, default in the low and medium state) feasible. Second, if several strategies are feasible, the insider chooses whichever strategy yields the highest expected value $\mathbb{E}_0[E_2(E_0)]$. Third, one has to carefully consider the upper and lower bounds in (20), (21), and (22) to make sure not to make statements about empty intervals. The following table provides the equilibrium strategies, the cases where default occurs, and the expected equity value. Subindex "1" denotes the first period, subindex "2" denotes the second period, and ID_i denotes that the insider is indifferent

between the safe and the risky strategy (i.e., indifferent between r_0 and s_0 or indifferent between r_1 and s_1).

	E_0			
	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{4}b)$	$[\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma)$	$[2\gamma, \infty)$
Strategy	No_0	r_0, r_1	r_0, r_1	ID_0, ID_1
Default	always	down_0, down_1 $\vee$ down_0, up_1 $\vee$ up_0, down_1	down_0, down_1	never
Equity value	0	$E_0 - \frac{3}{4}b$	$E_0 - \frac{1}{4}b$	E_0
Non-empty	Always	$b < \gamma$	$b < 6\gamma$	Always
$b \in (0, \gamma)$	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{4}b)$	$[\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma)$	$[2\gamma, \infty)$
$b \in (\gamma, 6\gamma)$	$[0, \frac{1}{2}\gamma + \frac{1}{4}b)$	———	$[\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma)$	$[2\gamma, \infty)$

Proof of Proposition 4

Our proof uses induction. The periods are labelled $1, ..., N$. We start by specifying a key relationship that holds in a one-period model ($n = N - 1$) and – as we will show below – holds more generally for periods $1, ..., N - 1$.

Relationship 1 *Given period-n-wealth E_n, the expected equity payoff under the equilibrium strategies for all subsequent periods (from n to N) admits the following functional form:*

$$\mathbb{E}_n[E_N(E_n)] = \begin{cases} E_n & E_n \geq \gamma \;\; (A) \\ f(E_n) & E_n < \gamma \;\; (B) \end{cases} \tag{23}$$

with

- *(B.1) $f(E_n) \geq 0$ (limited liability)*
- *(B.2) $f(E_n) < E_n \; \forall \gamma > E_n > 0$ (deadweight loss below γ)*
- *(B.3) $f(E_n) = 0 \Leftrightarrow E_n \leq E_n^{min}$ with $E_n^{min} \geq 0$ (zero iff below a threshold)*

In the one period model ($n = N - 1$), $f(E_n) = E_n - \frac{1}{2}b$ for $\gamma > E_n \geq \frac{1}{2}b$ and $f(E_n) = 0$ for $E_n \leq \frac{1}{2}b$ (see Proposition 1) and Relationship 1 is fulfilled. In the following, we (i) derive the equilibrium strategy for period $n - 1$ under the assumption that Relationship 1 holds for period n and (ii) show that Relationship 1 holds for period $n - 1$ if it holds for period n. By induction, this allows us to derive equilibrium strategies for all periods $1, ..., n - 1$.
For the following, it is important to note that debt that is due in period n is safe if $\mathbb{E}_n[E_N(E_n)] > 0$, or if $\mathbb{E}_n[E_N(E_n)] = 0$ but $I_n - b > F_n$ for all possible equilibrium realizations of I_n.

(i) Equilibrium strategies in period $n - 1$:
Step 1: $F_n \leq I_{n-1} - \gamma$

Step 1a: $F_n \leq I_{n-1} - 2\gamma$
If $F_n \leq I_{n-1} - 2\gamma$, then (A) implies $E_N[E_n(s_{n-1})] = E_N[E_n(r_{n-1})] = E_{n-1}$ under both the safe and the risky strategy. The insider is thus indifferent between the safe and the risky strategy. Debt is safe.

Step 1b: $F_n \in (I_{n-1} - 2\gamma, I_{n-1} - \gamma)$
If $F_n \in (I_{n-1} - 2\gamma, I_{n-1} - \gamma)$, the safe strategy yields

$$E_n(s_{n-1}) = I_{n-1} - F_n > \gamma \tag{24}$$

$$\overset{(A)}{\Rightarrow} E_N[E_n(s_{n-1})] = I_{n-1} - F_n \tag{25}$$

while the risky strategy yields

$$E_n(r^-_{n-1}) = I_{n-1} - \gamma - F_n \in (0, \gamma),\ E_n(r^+_{n-1}) = I_{n-1} + \gamma - F_n > 2\gamma \tag{26}$$

$$\overset{(A),(B.2)}{\Rightarrow} E_N[E_n(r_{n-1})] < \frac{1}{2}(I_{n-1} - \gamma - F_n) + \frac{1}{2}(I_{n-1} + \gamma - F_n) = I_{n-1} - F_n \tag{27}$$

As $E_N[E_n(s_{n-1})] = I_{n-1} - F_n$ under the safe strategy and $E_N[E_n(r_{n-1})] < I_{n-1} - F_n$ under the risky strategy, the insider chooses the safe strategy in period $n-1$. Debt is safe.

Step 1c: $F_n = I_{n-1} - \gamma$
If $F_n = I_{n-1} - \gamma$, $E_N[E_n(r_{n-1})] = E_N[E_n(s_{n-1})] = I_{n-1} - F_n$, the insider is indifferent between the safe and the risky project, but the lenders are strictly worse off if the insider chooses the risky project. We thus assume the insider chooses the strategy that maximizes overall firm value in this tie-breaker situation. We thus assume the insider chooses the safe strategy.[14] Debt is therefore safe.

Step 1d: Range of applicable period-$n-1$ equity values
As debt is safe for $F_n \leq I_{n-1} - \gamma$, the PC is $F_n = I_{n-1} - E_{n-1}$. The range of applicable period-$n-1$ equity values is therefore $I_{n-1} - E_{n-1} \leq I_{n-1} - \gamma \Leftrightarrow E_{n-1} \geq \gamma$. Taken together, for step 1 ($F_n \leq I_{n-1} - \gamma$) we get

$$\mathbb{E}_{n-1}[E_N(E_{n-1})] = \begin{cases} E_{n-1} & \text{if } E_{n-1} \in [2\gamma, \infty) \qquad c_{n-1} \in \{s_{n-1}, r_{n-1}\} \text{ (indifferent)} \\ E_{n-1} & \text{if } E_{n-1} \in [\gamma, 2\gamma) \qquad c_{n-1} = s_{n-1} \text{ (safe)} \end{cases} \tag{28}$$

Step 2: $F_n > I_{n-1} - \gamma$

Step 2a: $F_n \geq I_{n-1}$
The insider's payoff under the safe strategy is always zero. The insider therefore always chooses the risky strategy as long as the risky strategy's expected payoff is strictly larger than zero (we check this condition for the risky strategy in more detail below).

Step 2b: $F_n \in (I_{n-1} - \gamma, I_{n-1})$

[14]Note that this case is different from Step 1a ($F_n \leq I_{n-1} - 2\gamma$), where the insider is also indifferent between the safe and the risky strategy, but the lenders are indifferent as well.

If $F_n \in (I_{n-1} - \gamma, I_{n-1})$, the safe strategy yields

$$E_n(s_{n-1}) = I_{n-1} - F_n < \gamma \tag{29}$$

$$\overset{(B.2)}{\Rightarrow} E_N[E_n(s_{n-1})] < I_{n-1} - F_n \tag{30}$$

Under the risky strategy in period $n-1$[15]

$$E_n(r_{n-1}^-) = 0,\ E_n(r_{n-1}^+) = I_{n-1} + \gamma - F_n > \gamma \tag{31}$$

$$\overset{(A),(B.3)}{\Rightarrow} E_N[E_n(r_{n-1})] = \frac{1}{2}0 + \frac{1}{2}(I_{n-1} + \gamma - F_n) > I_{n-1} - F_n \tag{32}$$

As $E_N[E_n(s_{n-1})] < I_{n-1} - F_n$ under the safe strategy and $E_N[E_n(r_{n-1})] > I_{n-1} - F_n$ under the risky strategy, the insider chooses the risky strategy in period $n-1$.

Step 2c: Range of applicable period-$n-1$ equity values
Upper limit: Debt is risky because only the risky strategy is incentive-compatible and $F_n > I_{n-1} - \gamma$. Using $F_n > I_{n-1} - \gamma$ and the PC for risky debt $\frac{1}{2}F_n + \frac{1}{2}(I_{n-1} - \gamma - b) = I_{n-1} - E_{n-1} \Leftrightarrow F_n = I_{n-1} - 2E_{n-1} + \gamma + b$, the upper limit of applicable period-$n-1$ equity values is $I_{n-1} - 2E_{n-1} + \gamma + b > I_{n-1} - \gamma \Leftrightarrow E_{n-1} < \gamma + \frac{1}{2}b$.
Lower limit: To be incentive-compatible, the risky strategy needs to have a strictly positive expected payoff, which – using (B.1) and (B.3) — implies $I_n + \gamma - F_n > E_n^{min}$. Using the PC for risky debt ($F_n = I_{n-1} - 2E_{n-1} + \gamma + b$) yields

$$2E_{n-1} - b > E_n^{min} \Leftrightarrow E_{n-1} > \frac{1}{2}(E_n^{min} + b). \tag{33}$$

Note that $E_N^{min} = 0$ and $E_{N-1}^{min} = \frac{1}{2}b$ (consistent with Proposition 1). More generally, it is straightforward to see that (33) implies

$$E_n = \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b. \tag{34}$$

For $F_n > I_{n-1} - \gamma$, the risky strategy always results in an expected payoff lower than E_{n-1}. To see this, note that (B.1) and (B.2) imply that $E_N[E_n(r_{n-1})] = \frac{1}{2}0 + \frac{1}{2}E_N[E_n(I_{n-1} + \gamma - F_n)] = \frac{1}{2}E_N[2E_{n-1} - b)] \leq E_{n-1} - \frac{1}{2}b$. Taken together, for $F_n > I_{n-1} - \gamma$

$$\mathbb{E}_{n-1}[E_N(E_{n-1})] = \begin{cases} \in (0, E_{n-1} - \frac{1}{2}b] & \text{if } E_{n-1} \in [E_{n-1}^{min}, \gamma + \frac{1}{2}b) \quad c_{n-1} = r_{n-1} \text{ (risky)} \\ 0 & \text{if } E_{n-1} \in [0, E_{n-1}^{min}) \quad c_{n-1} = r_{n-1} \text{ (no financing)} \end{cases} \tag{35}$$

Step 3: Equilibrium strategies
The equilibrium strategies follow from (28) and (35). If both safe and risky debt is possible (i.e., for $E_{n-1} \in [\gamma, \gamma + \frac{1}{2}b)$), then the insider will choose the face value that maximizes

[15] The last inequality uses $F_n > I_{n-1} - \gamma \Leftrightarrow \gamma > I_{n-1} - F_n$

equity value:

$$\mathbb{E}_{n-1}[E_N(E_{n-1})] = \begin{cases} E_{n-1} & \text{if } E_{n-1} \in [2\gamma, \infty) & c_{n-1} \in \{s_{n-1}, r_{n-1}\} \text{ (indifferent)} \\ E_{n-1} & \text{if } E_{n-1} \in [\gamma, 2\gamma) & c_{n-1} = s_{n-1} \text{ (safe)} \\ \in (0, E_{n-1} - \frac{1}{2}b] & \text{if } E_{n-1} \in [E_{n-1}^{min}, \gamma) & c_{n-1} = r_{n-1} \text{ (risky)} \\ 0 & \text{if } E_{n-1} \in [0, E_{n-1}^{min}) & c_{n-1} = r_{n-1} \text{ (no financing)} \end{cases} \tag{36}$$

(ii) Relationship 1 also holds for period $n-1$:
It is straighforward to see that (36) fulfills the Relationship 1 too. By complete induction, (36) thus provides equilibrium strategies for all periods and payoffs for all periods 1 to $n-1$.