Adopting Solvency III ?

1 Introduction

Solvency I and II were designed to enhance insurers' solvency on the premise that it would be too low from a societal viewpoint in the absence of public regulation. Although opposed by the insurance industry because of its extra cost (IQ, 2011), several countries outside the European Union consider adopting the concept of risk-based capital and with it, potentially of *Solvency II* [Sukpaiboonwat, Piputsitel and Punyasavatsut (2014); SwissRe (2015); with C-ROSS, China is pursuing a policy of its own (Liu, 2015)]. However, *Solvency I* (and to a smaller extent *Solvency II*) might well induce insurers to take on more rather than less risky asset and liability positions (Zweifel, 2014). This raises the question of whether these countries would be well advised to aim for *Solvency II* rather than directly for *Solvency III*, which is likely to become reality in a not-to-far future (van Hulle, 2016). Indeed, Singapore is already using banking regulation as the benchmark on the premise that *Solvency III* will be modelled after *Basel III* in the same way as *Solvency II* was modelled after *Basel II* (Ng and Cheung, ca. 2015).

This contribution leads to the conclusion that planned *Solvency III* is likely to achieve a higher solvency level for weakly capitalized insurers, contrary to its predecessors, provided it copies two crucial provisions of *Basel III* . This encouraging finding derives from a model that depicts an insurance company in the process of its sequential decision making (see Figure 1). In period 1, exogenous changes in expected returns and volatility on the capital market ($d\bar{\mu}, d\bar{\sigma}$, dubbed 'interest rate risk' in *Basel III*) impinge on its underwriting and investment departments which are assumed to form one division ('the Division' henceforth) for simplicity. A typical cause could be investments made in the previous period that turn out to have a lower rate of return or higher volatility than expected. In period 2, the Division adjusts the company's solvency level by $dS^*/d\bar{\mu}$ and $dS^*/d\bar{\sigma}$, respectively in ways predicted by maximization of its risk-adjusted rate of return on capital and comparative-static analysis. The changed solvency level in turn acts like an exogenous change causing the Division to internally develop an endogenous perceived efficiency frontier (EPEF) in $(\hat{\mu}, \hat{\sigma})$-space on which senior management chooses the optimum in period 3, taking into account its degree of risk aversion.

However, this EPEF is modified by solvency regulation. It will be argued that both *Solvency I* and *II* neglected the fact that the relationship between solvency capital and solvency depends on exogenous changes in expected returns on invested capital and their volatility in the capital

Figure 1. Timeline of the model

0 — The Division in charge of underwriting and investment maximizes the risk-adjusted rate of return R by optimally selecting solvency S (see text for definitions)

1

Shocks $d\bar{\mu}$, $d\bar{\sigma}$ occur

2 — The Division adjusts solvency level by $dS*$

3 — Adjustment $dS*$ moves the Division along the endogenous ($\hat{\mu}, \hat{\sigma}$)-frontier which is modified by *Solvency I*, *II*, and *III* regulation; senior management of insurer selects optimum on ($\hat{\mu}, \hat{\sigma}$)-frontier

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market. Through their neglect of parameters of importance to insurers themselves, both *Solvency I* and *II* have the unexpected consequence of increasing the slope of the insurer's EPEF, thus inducing senior management to opt for higher expected returns and higher volatility. This deficiency may be corrected by *Solvency III* provided it is modelled after *Basel III*, which explicitly requires banks to take account of developments in the capital markets (dubbed 'interest rate risk') in the formulation of their business strategies designed to ensure solvency [Basel Committee on Banking Supervision (2016), Principle 5]. In addition, *Basel III* stipulates a maximum leverage ratio, which if adopted by *Solvency III* will mean an increased amount of solvency capital for most insurers. This increase is shown to reduce the slope of the EPEF for weakly capitalized insurance companies (while increasing it for highly capitalized start-ups with small premium volumes). At least in the first case, senior management is predicted to opt for less volatility. For these reasons, *Solvency III* is likely to attain its objective of enhancing solvency where it is crucially important, constituting a decisive improvement over *Solvency I* and *Solvency II,* although at an increased cost to insurers (and ultimately, policyholders).

This paper is structured as follows. Section 2 contains a review of the pertinent literature to conclude that solvency regulation indeed may serve to avoid negative externalities. In Section 3, the insurer's amalgamated division responsible for underwriting and investment (the Division) is modelled as choosing a solvency level that maximizes the company's risk-adjusted rate of return on capital (period 0; see Figure 1 again). On the one hand, solvency serves to lower the cost of refinancing; on the other, it ties capital that would have other, more lucrative uses. In Section 4, this optimum is disturbed by exogenous shocks in terms of expected returns and volatility in the market environment, causing the Division to adjust the insurer's solvency level (period 1). These adjustments are derived using comparative-static analysis; however, there can be only one adjustment of solvency. This adjustment acts in a way similar to an exogenous change in Section 5, where comparative statics are performed 'in reverse' to derive the slope of an endogenous perceived efficiency frontier (EPEF) in period 2. In Section 6, senior management chooses a point on the EPEF, taking account of its degree of risk aversion (period 3). The regulations imposed by *Solvency I* and *II* are introduced as parameter restrictions in Section 7 which increase the slope of the insurer's EPEF in $(\hat{\mu},\hat{\sigma})$-space, thus inducing senior management to opt for more rather than less volatility. These parameter restrictions are likely to be voided by *Solvency III* if it copies *Basel III* in the same way as *Solvency I* and *II* have copied *Basel I* and *II* designed for banks. In addition, the increased capital requirement is shown to reduce the slope of the EPEF for weakly capitalized insurance companies, which is conducive to a choice of reduced volatility compared to the no-regulation benchmark. A summary and conclusions follow in Section 8.

2 Literature review

The solvency regulation of insurers has traditionally been justified by the external costs of insolvency (Cummins, 1988). This view was challenged by the proponents of the Capital Asset Pricing Model, who emphasized that for well-diversified investors, the solvency of a particular insurance company does not constitute a reasonable objective. By way of contrast, for little-diversified investors (among them, policyholders of the insurer), the companytree's overall risk is relevant, which importantly includes the risk of insolvency [for the case of banks, see Goldberg and Hudgins (1996), Park and Peristiani (1998), Jordan (2000), Goldberg and Hudgins (2002)]. Option Pricing Theory in turn shows that due to their limited liability, shareholders of the insurer in fact have a put option that is written by the other stakeholders (notably policyholders as creditors) of the insurer [Cummins and Phillips (2001), Zweifel and Eisen (2011), ch. 6.3].

When a solvency risk materializes, internal costs are borne by the insurer's shareholders, who see the value of their shares drop to zero; due to reputation effects, the insurer is unlikely to be in business again [in analogy to Stulz (1996) for banks]. Yet intervention by public policy is usually justified by the external costs of insolvency, consisting of three components. While an insolvency is unlikely to trigger a run on insurance companies, it causes consumers to lose at least part of their assets in the event of loss due to lack of insurance coverage [Bauer and Ryser (2004)]. Second, the insolvent insurer may drive other companies into insolvency through coinsurance contracts (Furfine, 2003). Third, owners and policyholders may re-evaluate the estimated risk of insolvency. In response to the revised estimate, they demand a higher rate of return from their insurers, driving up their cost of refinancing. According to Rees, Gravelle and Wambach (1999), these external costs could be avoided if regulatory authorities established full transparency w.r.t. the risk of insolvency because the value of the insurance company becomes maximum if this risk is zero. However, Boulès and Henriet (2009) argue that this holds only if the shareholders commit to recapitalize the company, a condition they deem unlikely to be satisfied.

At present, the solvency capital requirement of *Solvency II* [based on the Value at Risk (*VaR*) criterion] implies a risk of insolvency risk of 0.5 percent over a one-year horizon [Institute and Faculty of Actuaries (2016), p. 10]. This high safety level does not come without cost: Capital is diverted from uses with high expected rate of return, to the detriment not only of shareholders but also holders of participating policies; adopting the information technology for monitoring the *VaR* measure and (re-)adjusting asset-liability management to it generates cost of compliance; and senior management has to devote time and effort to regulatory affairs rather than to product development in the interest of policyholders. Indeed, this (non-recurrent) cost has been estimated at more than £ 200 mn. (USD 248 mn.) for each of the major insurance companies of the UK (Actuarial Post, 2018). To put this into perspective, Legal& General, the number two company (for which data rare readily available) wrote £ 1.2 bn. gross premiums in 2016 (relbanks.com, 2017). Therefore, implementation of *Solvency II* may claim one-sixth or even more of one year's worth of premium income. Yet infosys (2017) claims that *Solvency II* will reduce the total cost of ownership through improved risk management.

For a comprehensive evaluation of *Solvency III* , one would have to estimate its marginal cost as well as benefit. While this is beyond the scope of this paper, the present contribution still adds to the literature in three ways. First, it takes formally into account that insurers have a business model that differs from that of banks. Contrary to banks, insurers derive profit from two activities, risk underwriting and capital investment, a difference that has important implications for solvency regulation (Zweifel, 2015). Second, the paper clearly distinguishes between earlier *Solvency I* and *II* and planned *Solvency III* regulation, showing that the latter may indeed induce one type of insurance company to attain a higher level of solvency. The condition is that *Solvency III* will be modelled after *Basel III* in the same was as *Solvency I* and *II* were modelled after *Basel I* and *II* for banks. The third distinguishing feature of this paper is its emphasis on dynamics in the following way. Whereas earlier contributions focused on optima or [in the case of Repullo (2004)] equilibria, here the insurer's path of adjustment from one optimum to the next is analyzed. Adjustment to exogenous shocks will be shown to be conditioned by regulation of the *Solvency I* to *III* type.

3 The Division selects the optimal solvency level (Period 0)

For simplicity, the two main activities of an insurance company, risk underwriting and capital investment, are amalgamated into a single division (the Division henceforth). Thus, let this Division maximize the risk-adjusted rate of return on capital (*R*) through its choice of solvency *S*. It is assumed to act in a risk-neutral manner; otherwise, a strongly risk-averse employee would reject a risk that another one with more risk appetite accepts, resulting in inconsistencies that senior management needs to avoid. Risk aversion enters in period 3 when senior management selects its preferred position on the internal $(\hat{\mu}, \hat{\sigma})$-efficiency frontier generated by the Division (as argued below, there is no capital market line). A higher level of solvency enables the company (and hence the Division) to obtain more funds through higher premium income (Epermanis and Harrington, 2006).

The use of a $(\hat{\mu}, \hat{\sigma})$-efficiency frontier can be criticized because these two parameters suffice to characterize a distribution only in the case of normality whereas returns to underwriting and particularly investment are known to exhibit skewness and curtosis. However, being of fourth order, curtosis necessarily adds to variance, which is frequently true of skewness as well. Solvency is negatively related to the variance of portfolio returns if defined as a shortfall probability as in *Solvency I* and *II* (Institute and Faculty of Actuaries, 2016). In this way, the level of solvency constitutes a decision variable both for the insurer and the regulatory authority.

In the case of insurance companies, Cummins and Sommer (1996) have shown that a higher level of solvency serves to increase demand and hence premium income P[1]. Assuming decreasing marginal returns as usual, one has

[1] Sections 4 and 5 draw on Zweifel (2014) with eq. (4) corrected; this is necessary to make the paper self-contained.

$$P = P(\cdot, S), \quad \text{with } \frac{\partial}{\partial S} P(\cdot, S) > 0 \text{ and } \frac{\partial^2}{\partial S^2} P(\cdot, S) < 0; \tag{1}$$

the arguments other than S are discussed in Section 7 below. The amount of solvency capital $C > 0$ increases with the targeted solvency level S and α, the parameter reflecting regulatory capital requirements,

$$\frac{\partial}{\partial S} C(\cdot, S; \alpha) > 0, \ \frac{\partial^2 C(\cdot, S; \alpha)}{\partial S^2} > 0; \frac{\partial}{\partial \alpha} C(\cdot, S; \alpha) = 1 \text{ for simplicity.} \tag{2}$$

The risk-adjusted rate of return R depends on profits from capital investment and risk underwriting. As to profits from investment activity, they have two components. The first is denoted by $r_G C$ in eq. (3) below. Solvency capital C (which is equated to capital for simplicity) must predominantly be invested in guild-edged securities (mainly government bonds) at a rate of return r_G. Note that this is not a risk-free interest rate. The financial crisis of 2007 has shown that such a rate does not really exist [see e. g. Moody's Investor Service (2010) for a survey of sovereign defaults]; accordingly, there is no capital market line complementing the efficiency frontier depicted in Figure 2 (see Section 7 below). The second component is $k\bar{\mu} \cdot P(\cdot, S)$, i.e. premium income carried over from the previous period (the time difference is neglected for simplicity) which is not matched by insurance claims yet[2]. This makes funds available for investment according to the so-called funds-generating factor k (Cummins and Phillips, 2001). The higher k, the longer the lag between premiums received and claims paid. These funds can be invested at the rate of return $\bar{\mu}$ prevailing on the capital market.

[2]The time difference between premiums written and claims paid can be substantial in life insurance in this case, the length of period 0 has to be adjusted accordingly.

The insurer also derives profit from risk underwriting, which is simply given by the difference between premium income *P*(;*S*) and losses paid *L*. Assuming *L* to be exogenous and abstracting from operating costs and taxes, *R* can therefore be expressed as follows,

$$
\begin{aligned}
R &= \frac{r_G \cdot C(\cdot, S, \alpha) + k\overline{\mu} \cdot P(\cdot, S) + P(\cdot, S) - L}{C(\cdot, S; \alpha)} \\
&= r_G + \frac{(1+k\overline{\mu})P(\cdot, S) - L}{C(\cdot, S; \alpha)}.
\end{aligned} \qquad (3)
$$

Maximization of *R* w.r.t. solvency *S* leads to the first-order condition (4) for optimal solvency. Here, $e(P,S) := (\partial P/\partial S)(S/P) > 0$ and $e(P,C) := (\partial P/\partial C)(C/P) > 0$ denote the elasticity of premium income and solvency capital w.r.t. the solvency level, respectively:

$$
\begin{aligned}
\frac{dR}{dS} &= \frac{(1+k\overline{\mu})[\partial P/\partial S \cdot C - (P-L)\cdot \partial C/\partial S]}{C^2} = 0, \text{ hence} \\
&= \partial P/\partial S \cdot \frac{S}{P} - P \cdot \partial C/\partial S \frac{S}{C \cdot P} + L \cdot \partial C/\partial S \frac{S}{C \cdot P} \\
&= e(P,S) - e(C,S)(1 - L/P) = 0
\end{aligned} \qquad (4)
$$

By eqs. (1) and (2), both elasticities are positive while $L/P < 1$, justifying neglect of boundary solutions ($S^* = 0$ in particular).

Equation (4) can be interpreted as follows. Since $e(P,S) > 0$, the first term represents the marginal benefit of increased solvency in percentage terms. The Division needs to weigh this marginal return of solvency against its marginal cost, which is given by $e(C,S) > 0$ reflecting the capital needed for a higher solvency level, weighted by the contribution to profit additional underwriting would make.

However, the elasticities $e(P,S)$ and $e(P,C)$ depend not only on solvency S but also on the changing conditions on the capital market reflected by exogenous shocks $d\bar{\mu}$ and $d\bar{\sigma}$, respectively (see assumptions A6 and A7 of Table A.1). This implies that the optimal adjustment to an exogenous change will not be given once and for all but importantly varies with parameters not yet specified, in particular the risk-return profile inherited from the past. Solvency regulation that fails to reflect this variability runs the risk of creating perverse incentives, as will become clear in Section 7.1.

4 The Division adjusts solvency to exogenous shocks (Period 1)

During the first period, exogenous shocks impinging on rates of return $(d\bar{\mu})$ and volatility of returns $(d\bar{\sigma})$ occur. To derive the optimal adjustments of the solvency level, the assumptions listed in Table A.1 of Appendix A are introduced.

As shown in Appendix B, optimal adjustment of the solvency level S^* to a shock $d\bar{\mu} > 0$ in expected returns is given by (apart from $-1/H$, with $H := \partial^2 R / \partial S^2 < 0$ as the second-order condition),

$$\frac{\partial^2 R}{\partial S \partial \bar{\mu}} = S\left\{\underset{(+)}{\frac{\partial^2 P}{\partial S \partial \bar{\mu}}} \cdot \frac{1}{P} - \underset{(+)}{\frac{\partial P}{\partial S}} \frac{\partial P / \partial \bar{\mu}}{P^2} - \left[\underset{(-)}{\frac{\partial^2 C}{\partial S \partial \bar{\mu}}} \cdot \frac{1}{C} + \underset{(+)}{\frac{\partial C}{\partial S}} \underset{(+)}{\frac{\partial C / \partial \bar{\mu}}{C^2}}\right](1 - L/P) + \frac{\partial C}{\partial S} \underset{(+)}{\frac{L \cdot \partial P / \partial \bar{\mu}}{C \cdot P^2}}\right\}$$

and hence

$$\frac{dS^*}{d\bar{\mu}} \to 0 \text{ if } S \to 0;$$

$$\frac{dS^*}{d\bar{\mu}} > 0 \text{ if } C \to 0 \text{ since } 1/C^2 \to 0 \text{ faster than } 1/C \to 0; \qquad (5)$$

$$\frac{dS^*}{d\bar{\mu}} \gtrless 0 \text{ otherwise.}$$

Here (and henceforth), any impact of $d\bar{\mu}$ on H is neglected on the grounds that it must be minor lest H change sign, transforming a maximum into a minimum. The results stated in (5) are intuitive. In a situation where the solvency level is very low to begin with, its adjustment in response to increased returns in the capital market does not matter. However, with a large solvency capital C, the opportunity cost of an increased solvency level is small, leading the Division to propose an increase to senior management with the aim of boosting premium income. Since most insurers have excess solvency capital [Nakada et al. (1999)], $dS^*/d\bar{\mu} > 0$ is considered the normal response.

Now consider a shock $d\bar{\sigma} > 0$ (again, details are given in Appendix B)

$$\frac{\partial^2 R}{\partial S \partial \bar{\sigma}} = S\left\{ \underset{(+)}{\frac{\partial^2 P}{\partial S \partial \bar{\sigma}}} \cdot \frac{1}{P} - \underset{(+)}{\frac{\partial P}{\partial S}} \frac{\partial P/\partial \bar{\sigma}}{P^2} - \left[\underset{(-)}{\frac{\partial^2 C}{\partial S \partial \bar{\mu}}} \cdot \frac{1}{C} + \underset{(+)}{\frac{\partial C}{\partial S}} \underset{(+)}{\frac{\partial C/\partial \bar{\sigma}}{C^2}} \right] (1 - L/P) + \frac{\partial C}{\partial S} \underset{(+)}{\frac{L \cdot \partial P/\partial \bar{\sigma}}{C \cdot P^2}} \right\}$$

implying

$$\frac{dS^*}{d\bar{\sigma}} \to 0 \text{ if } S \to 0;$$

$$\frac{dS^*}{d\bar{\sigma}} > 0 \text{ if } C \to 0 \text{ since } 1/C^2 \to \infty \text{ faster than } 1/C \to \infty; \tag{6}$$

$$\frac{dS^*}{d\bar{\sigma}} \gtrless 0 \text{ otherwise.}$$

As will be argued below eq. (7), $dS^*/d\bar{\sigma} > 0$ can be considered the normal response.

5 The Division derives an endogenous perceived efficiency frontier (Period 2)

In the second period, the insurer inherits a net adjustment of solvency $dS*$ from the first period, $dS*$ being the result of responses to the shocks ($d\overline{\mu}, d\overline{\sigma}$) that occurred in the first period. The Division now proceeds to adjust $\hat{\mu}$ and $\hat{\sigma}$, the endogenous components of μ and σ, respectively. Optimal adjustments are described by eqs. (5) and (6), with $dS*$ assuming the role of an exogenous shock. Therefore, comparative statics can now be performed in reverse to derive optimal endogenous adjustments $d\hat{\mu}/dS$ and $d\hat{\sigma}/dS$, respectively. The Division effects these changes by reshuffling assets and liabilities, creating an endogenous efficiency frontier (EPEF) with slope $d\hat{\mu}/d\hat{\sigma}$. Frictional costs of adjustment (including a possible loss of tax benefits) are neglected; they would shift the EPEF in Figure 2 down, without however affecting its slope and hence the findings of Section 7 in a major way. The slope of this frontier, on which senior management chooses the optimum in period 3 (see Section 6.1), can be obtained by dividing (6) by (5), yielding

$$\left.\frac{d\hat{\mu}}{d\hat{\sigma}}\right|_{S*} = \frac{\overset{(+)}{\frac{\partial^2 P}{\partial S \partial \overline{\sigma}}} \cdot \frac{1}{P} - \overset{(+)}{\frac{\partial P}{\partial S}} \, \frac{\overset{(+)}{\partial P / \partial \overline{\sigma}}}{P^2} - \left[\overset{(+)}{\frac{\partial^2 C}{\partial S \partial \overline{\sigma}}} \cdot \frac{1}{C} - \frac{\partial P}{\partial S} \, \frac{\overset{(+)}{\partial C / \partial \overline{\sigma}}}{C^2} \right] (1 - L/P) + \frac{\partial C}{\partial S} \, \frac{L \cdot \overset{(+)}{\partial P / \partial \overline{\sigma}}}{C \cdot P^2}}{\overset{(+)}{\frac{\partial^2 P}{\partial S \partial \overline{\mu}}} \cdot \frac{1}{P} - \overset{(+)}{\frac{\partial P}{\partial S}} \, \frac{\overset{(+)}{\partial P / \partial \overline{\mu}}}{P^2} - \left[\overset{(+)}{\frac{\partial^2 C}{\partial S \partial \overline{\mu}}} \cdot \frac{1}{C} - \frac{\partial P}{\partial S} \, \frac{\overset{(+)}{\partial C / \partial \overline{\mu}}}{C^2} \right] (1 - L/P) + \frac{\partial C}{\partial S} \, \frac{L \cdot \overset{(+)}{\partial P / \partial \overline{\mu}}}{C \cdot P^2}} > 0. \tag{7}$$

In principle, the sign of eq. (7) is indeterminate, even if normally its denominator is positive [see the comment below eq. (5)]. However, daily experience of investors in the capital market suggests that the slope of the efficiency frontier in (μ, σ)-space is positive. Therefore, a positive sign is assumed for eq. (7) in the following. Since the normal response makes its denominator positive, its numerator must be positive as well, an implication that will be of relevance in Section 7.2 below. A crucial result to be noted already at this point is that the slope defined in eq. (7) depends not only on easily observable parameters {C, P, S} and first-order effects the regulator likely is aware of {$\partial P/\partial\overline{\mu}, \partial P/\partial\overline{\sigma}$} but also the terms $\partial^2 P/\partial S \partial \overline{\mu}$ and

$\partial^2 P / \partial S \partial \overline{\sigma}$ which indicate that the relationship between premium income and solvency depends on conditions on the capital market (see assumptions A8 and A9 in Table A.1 again).

Figure 2 shows three endogenous efficiency frontiers (EPEFs; minimum variance points are not shown to preserve space). Note that μ, $\overline{\mu}$, and $\hat{\mu}$ as well as σ, $\overline{\sigma}$, and $\hat{\sigma}$ are depicted on the same axis, reflecting the assumption that e.g. a low first-period value of $\overline{\sigma}$ tends to translate into a low second-period $\hat{\sigma}$. The first EPEF (labeled '*S**') holds prior to the influence of regulation. The two other frontiers (labeled '*I,II*; *dα*>0&*C*→0' and '*III*¸ *dα*>0&*C*→∞' respectively) are modified by *Solvency I,II* and planned *Solvency III* regulation in ways to be discussed in Section 7 below.

6 Senior management opts for a point on the frontier (Period 3)

In the third period, senior management opts for the optimum on the EPEF, in accordance with its risk preferences. This is the view of *Basel III* [which is likely to characterize planned *Solvency III* as well, see Van Hulle (2016], where the regulator states, "*Principle 3: The bank's risk appetite for IRRBB [Interest Rate Risk in the Banking Book] should be articulated in terms of the risk to both economic value and earnings. Banks must implement policy limits that target maintaining IRRBB exposures consistent with their risk appetite.*". However, this task can be delegated to senior management: "*Principle 2: The governing body of each bank is responsible for oversight of the IRRBB management framework, and the bank's risk appetite for IRRBB. Monitoring and management of IRRBB may be delegated body by the governing to senior management, expert individuals or an asset and liability management committee* …" (Basel Committee on Banking Supervision, 2016).

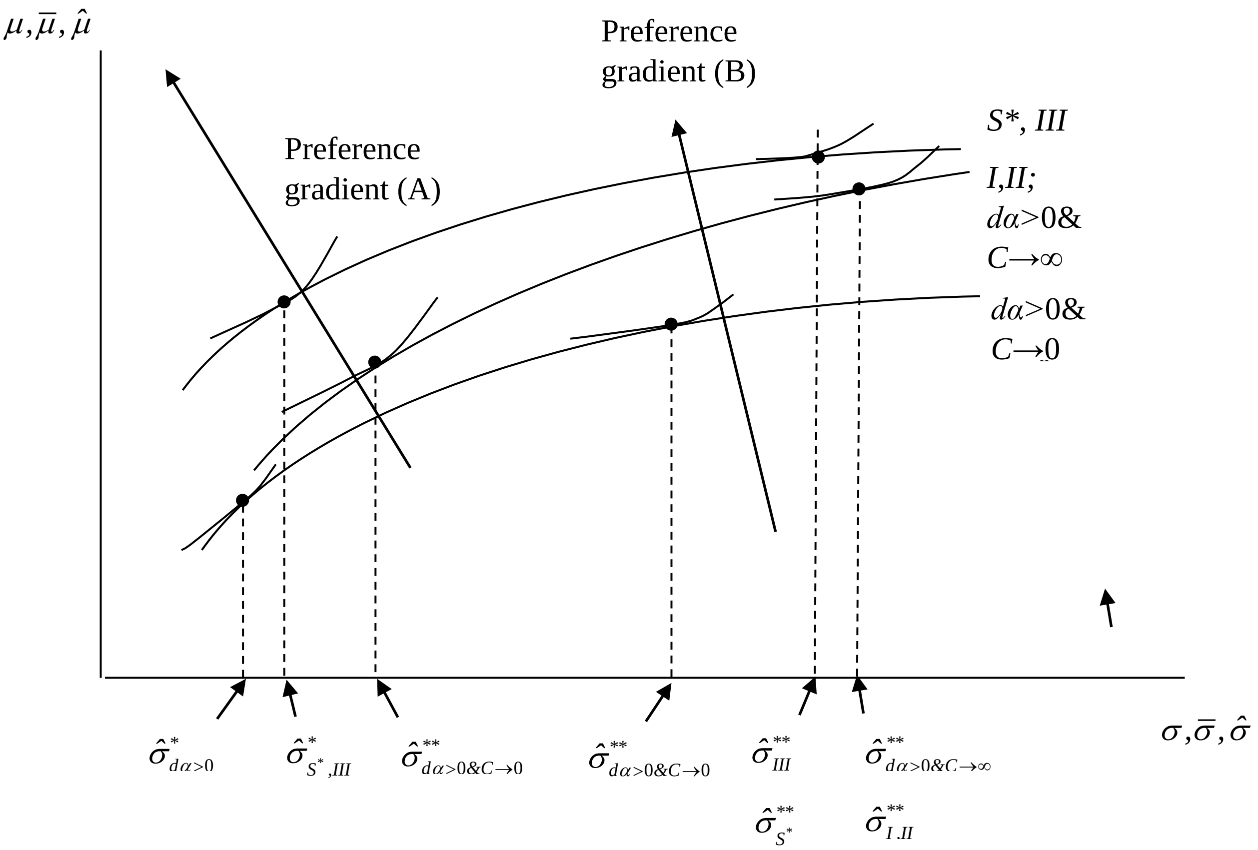
The 'risk appetite' is reflected by two sets of indifference curves in Figure 2, reflecting a strongly risk-averse insurer (A) and weakly risk-averse one (B). For type A, $\left\{\mu^*_{S^*}, \sigma^*_{S^*}\right\}$ reflects the optimal expected rate of return on the insurer's capital combined with the volatility of these returns prior to the imposition of solvency regulation.

7 Solvency regulation affects the endogenous perceived efficiency frontier EPEF

7.1 Solvency I and II

Solvency I stipulates capital requirements as a function of risk-weighted assets and separately for off-balance sheet positions in the same way as for banks (Basel Committee on Banking Supervision, 1988). Its focus is on the relationship between solvency and capital. By defining four asset classes with fixed weights, *Solvency I* imposes a fixed relationship between solvency capital *C* and solvency *S*. It therefore does not allow insurers to react to changes in market

Figure 2. Endogenous perceived efficiency frontiers in (μ,σ)-space



conditions affecting the risk characteristics of assets. In terms of the model, this neglect amounts to the restrictions

$$\frac{\partial^2 C}{\partial S \partial \overline{\mu}} = 0, \quad \frac{\partial^2 C}{\partial S \partial \overline{\sigma}} = 0. \tag{8}$$

Therefore, both the relationship between solvency and risk capital and between solvency and premium income are seen as being independent of conditions prevailing in the capital market, in contradistinction with assumptions A6 to A9 (see Table A.1 again). Inserting these restrictions in eq. (7), one immediately sees that the numerator increases while denominator decreases due to the deletion of $\partial^2 C / \partial S \partial \overline{\mu}$. The restrictions (8) thus result in a steeper slope of the endogenous efficiency frontier (subscript *I* denoting *Solvency I*),

$$\left.\frac{d\hat{\mu}}{d\hat{\sigma}}\right|_{I} > \left.\frac{d\hat{\mu}}{d\hat{\sigma}}\right|_{S^*}. \tag{9}$$

As to *Solvency II*, it allows a choice of approach for the calculation of capital requirements, viz. the Standardized Approach and the Internal Ratings-Based Approach (Basel Committee on Banking Supervision, 2004). Whilst the first approach is based on *Basel I*, the second lets banks and insurance companies choose their probability of default, their percentage loss at default, and the maturity of their credits and liabilities. Large institutions with average and below-average risks mostly prefer the Internal Ratings-Based approach to save on capital despite its higher cost of implementation. In terms of the model, *Solvency II* permits insurers to take all elements of eqs. (5) and (7), respectively into account, which amounts to a lifting of the restrictions stated in eq. (8) as long as the constraint regarding the solvency level is not binding. However, the situation where the solvency requirement is binding while the relationship between capital and solvency is viewed as fixed (as predicated by $\partial^2 C / \partial S \partial \overline{\mu} = 0$ and $\partial^2 C / \partial S \partial \overline{\sigma} = 0$) may have been more common than envisaged. As stated by Benink and Benston (2005), "*Although the CEOs and directors of insurers may not deliberately hold an insufficiently high level of capital necessary to avoid insolvency, they may be lulled into believing that they are adequately capitalized if they adhere to the Basel Committee's models (which they are unlikely to understand*" (p. 308). Therefore, the difference between *Solvency I* and *II* is neglected here for simplicity [for more detail on the difference between *Solvency I* and *II*, see Zweifel (2014)].

In Figure 2, the *Solvency I,II* frontier therefore runs steeper than the original *S** frontier, approaching but without crossing it for high values of μ because regulation cannot increase the insurer's feasible set.

One might argue that the insurer can choose to act in accordance with parameters it knows to be of importance, contrary to the regulator's decision rule. This would amount to neglecting the restrictions stated in (8). However, as emphasized by Power (2004, ch. 7), managers are responsibility-averse, leading them to use regulatory decision rules as a convenient justification of their actions. For example, let there be a second-period upward adjustment in solvency indicating that the company should move away from the origin on the efficiency frontier. If its senior management were to move along *S** with its low slope, it could be criticized by the regulator for taking on an excessive amount of risk. By adopting the restrictions (8) and accepting the steeper *Solvency I,II* efficiency frontier as the relevant one, senior management can shift responsibility to the regulator. On the assumptions of homothetic preferences, it is predicted to be less conservative regardless of its risk appetite due to *Solvency I* and *II* regulation (types A and B in Figure 2; see e.g. the shift from σ^*_{S*} to σ^*_I and from σ^{**}_{S*} to σ^{**}_I, respectively).

7.2 Solvency III

After the implementation of *Basel II*, both the International Monetary Fund (IMF) and the European Central Bank (ECB) engaged in stress testing of banks, simulating the type of exogenous shocks posited in this paper. Evidently, both the IMF and the ECB were concerned that banks may have been exposed to excessive risk in spite of (possibly because of, as argued here) *Basel*-type solvency regulation. Insights of this type have led to *Basel III* regulation, to be implemented by 2019. It seeks to increase solvency by more solvency capital of which a greater part is to be equity [Basel Committee on Banking Supervision (2011)]. Although *Basel III* may still undergo changes before its implementation, these requirements are likely to stand (Laas and Siegel, 2017). The expectation is that before long, *Solvency III* will follow for insurance companies, who will also be subjected to higher capital requirements (Van Hulle, 2016).

To see the effect of a higher capital requirement $(d\alpha > 0)$ on the EPEF, define $N > 0$ as the numerator and $\Delta > 0$ as the denominator of eq. (7) and consider its derivative w.r.t. to α [recalling that $\partial C / \partial \alpha = 1$ and assumptions A2, A3, A6, and A7 of Table A.1 and eqs. (5) and (7)],

$$
\begin{aligned}
\frac{\partial}{\partial \alpha}\left[\left.\frac{d\hat{\mu}}{d\hat{\sigma}}\right|_{S^*}\right] \approx & \left\{-\left[-\frac{\partial^2 C}{\partial S \partial \overline{\sigma}} \cdot \frac{1}{C^2} - \frac{\partial P}{\partial S} \cdot \frac{-\partial C / \partial \overline{\sigma}}{C^4}\right](1 - L/P) - \frac{\partial C}{\partial S} \cdot \frac{L \cdot \partial P / \partial \overline{\sigma}}{C^2 \cdot P^2}\right\} \cdot \Delta \\
& - N\left\{\left[-\underset{(-)}{\frac{\partial^2 C}{\partial S \partial \overline{\mu}}} \cdot \frac{1}{C^2} - \frac{\partial P}{\partial S} \cdot \frac{-\partial C / \partial \overline{\mu}}{C^4}\right](1 - L/P) - \frac{\partial C}{\partial S} \cdot \frac{L \cdot \partial P / \partial \overline{\mu}}{C^2 \cdot P^2}\right\} \\
& \approx \Delta\left\{\left[\underset{(+)}{\frac{\partial^2 C}{\partial S \partial \overline{\sigma}}} - \underset{(+)}{\frac{\partial P}{\partial S}} \cdot \underset{(+)}{\frac{\partial C / \partial \overline{\sigma}}{C^2}}\right](1 - L/P) + \underset{(+)}{\frac{\partial C}{\partial S}} \cdot \underset{(+)}{\frac{L \cdot \partial P / \partial \overline{\sigma}}{P^2}}\right\} \\
& + N\left\{\left[\underset{(-)}{\frac{\partial^2 C}{\partial S \partial \overline{\mu}}} - \underset{(+)}{\frac{\partial P}{\partial S}} \cdot \underset{(+)}{\frac{\partial C / \partial \overline{\mu}}{C^2}}\right](1 - L/P) - \underset{(+)}{\frac{\partial C}{\partial S}} \cdot \underset{(-)}{\frac{L \cdot \partial P / \partial \overline{\mu}}{P^2}}\right\} \\
& < 0 \quad \text{if } C \to 0; \\
& \gtrless 0 \quad \text{if } C \to \infty. \qquad (10)
\end{aligned}
$$

Therefore, the slope of the EPEF decreases if the insurance company has little solvency capital initially. In analogy with the discussion in Section 5, this means that a more stringent capital requirement induces this type of insurers to reduce their risk exposure. However, the predicted response of insurers with plenty of initial solvency capital is ambiguous; it may well be a steepening of its EPEF if its premium volume is small ($P \to 0$), as would be typical for a start-up who has the backing of an established mother company. This case is represented by the EPEF labeled '*III*, *d*α>0&→∞ ', drawn to coincide with the one labeled '*I*,*II* 'for simplicity. Evidently, senior management is induced to take on more risk.

The open question remains whether the comparator is the EPEF labeled '*I,II*' or the original one labelled '*S**'. Indeed, the comparator is the original EPEF because *Basel III* (serving as the model for planned *Solvency III*) recognizes that some of the parameters appearing in equation (7) are not fixed but reflect responses to shocks in the capital market. Specifically, the Basel Committee on Banking Supervision (2016) stipulates "*Principle 5: In measuring IRRBB, key behavioral and modelling assumptions should be fully understood, conceptually sound and documented. Such assumptions should be rigorously tested and aligned with the banks' business strategies.*". Evidently, insurers would be required to develop best estimates of all the parameters entering the determination of $dS^* / d\overline{\mu}$ and $dS^* / d\overline{\sigma}$ in eqs. (5) and (7) reflecting 'the insurer's business strategies'. Since these two equations define the slope of the EPEF, this implies that insurance companies can now determine their EPEF without any parameter restrictions of the type stated in (10). In terms of Figure 2, *Solvency III* may re-establish the original EPEF denoted by S^* as the point of departure, from where the shifts to the one labeled '*III*, *d*α>0&→0' and '*III*, *d*α>0&→∞', respectively occur. With the 'deformation' caused by *Solvency I* and *II* suppressed, properly implemented *Solvency III* regulation thus will induce at least insurers with little solvency capital to move to less volatile asset-liability positions and increased solvency. However, this desired effect will be absent in the case of highly capitalized insurance companies; to the contrary, they are predicted to take on a more risky position compared to the no-regulation benchmark, similar to the effect of *Solvency I* and *II*.

8 Summary and conclusion

The basic hypothesis of this paper states that an insurer's amalgamated underwriting and investment divisions (the Division) seeks to attain a solvency level that balances the advantage of lower refinancing cost against the disadvantage of tying capital that would yield higher returns in other uses. However, this solvency level is too low from a societal point of view because it neglects the fact that insolvency causes substantial external costs. In a simple model of insurer behavior, the Division maximizes the risk-adjusted rate of return on capital in period 0. A higher level of solvency (defined in terms of *VaR*, in accordance with *Solvency II*) lowers the cost of refinancing but causes returns forgone by tying extra capital. In period 1, exogenous changes in expected returns and in the volatility of returns on the capital market occur. These changes induce adjustments during period 2, predicted by comparative-static analysis. In period 3, previous adjustment acts like an exogenous change, triggering a reallocation of assets and liabilities. These adjustments, derived from 'reverse comparative statics', define the slope of a perceived endogenous efficiency frontier (EPEF) in $(\hat{\mu}, \hat{\sigma})$-space prior to solvency regulation.

However, this slope depends importantly on the fact that the relationship between risk capital and solvency varies with exogenous changes in expected returns and volatility occurring in the capital market. The regulations imposed by *Solvency I* are shown to neglect this dependence on market conditions, causing a modification of the regulated insurer's EPEF. This modification may well induce senior management to take a more risky position than it would absent regulation. The implications of *Solvency II* are the same for insurers who initially just attain the prescribed solvency level [detailed in Zweifel (2014)]. Provided planned *Solvency III* will be modeled after *Basel III*, it will correct these deficiencies by asking insurers to account for developments in capital markets (called 'interest rate risk' in *Basel III*) in their business strategies designed to ensure solvency (Principle 5). In addition, imposing a maximum leverage ratio amounts to a higher capital requirement *ceteris paribus*, which is shown to reduce the slope of an insurer's EPEF who has little solvency capital and hence is likely to induce senior management to adopt a less risky position. However, a highly capitalized startup with a small premium volume is predicted to take on more risk compared to the no-regulation benchmark in a way similar to *Solvency I* and *II*.

This analysis is subject to a number of limitations. First, the behavioral model might be too simplistic; insurers possibly pursue other objectives than just maximizing their risk-adjusted rate of return. Second, the (μ, σ)- approach adopted in this paper is compatible with equilibrium in the capital market only if expectations are homogenous and quoted prices are available for all assets. Third, interpreting *Basel III* (especially Principle 5) in terms of the parameters of the model developed in this article could be inappropriate; the regulators may have used a different theoretical background.

Yet, the following insights are likely to be robust. Regulation of the *Solvency* type is commonly justified by the need to strengthen insurers' ability to bear losses thanks to additional capital of 'good quality' permitting them to continue their activity. The analysis performed in this paper shows planned *Solvency III* to differ from its predecessors: If modeled after *Basel III* [as envisaged by the authorities in Singapore (Ng and Cheung, ca. 2015)], it is indeed likely to enhance the level of solvency at least for weakly capitalized insurance companies and can therefore be recommended for adoption by countries outside the European Union.

APPENDIX A

Table A1. Assumptions of the model

A1:	$\mu = \overline{\mu} + \hat{\mu}$; $\sigma = \overline{\sigma} + \hat{\sigma}$	Returns and volatility (μ, σ) are additive in an exogenous ($\overline{\mu}, \overline{\sigma}$) component determined on the capital market and an endogenous one.
A2:	$\partial C / \partial \overline{\mu} < 0$	The higher returns on the capital market, the less risk capital is needed to attain a given solvency level. A positive shock on returns makes positive net values of the company more likely, therefore reducing the need for risk capital.
A3:	$\partial C / \partial \overline{\sigma} > 0$	The higher volatility on the capital market, the more risk capital is needed to attain a given solvency level. Positive net values of the insurer are less likely, and this must be counteracted by more risk capital.
A4:	$\frac{\partial P}{\partial \overline{\mu}} < 0$	The (present value of) premium income depends negatively on the rate of return attainable on the capital market because policyholders now have more favorable investment alternatives.
A5:	$\frac{\partial P}{\partial \overline{\sigma}} > 0$	The (present value of) premium income depends positively on the volatility of returns on the capital market because the insurer now offers a comparatively safe investment alternative to risk-averse policyholders.
A6:	$\frac{\partial^2 C}{\partial S \partial \overline{\mu}} < 0$	A higher solvency level calls for more risk capital but to a lesser degree if higher market returns prevail, making positive net values of the company more likely.
A7:	$\frac{\partial^2 C}{\partial S \partial \overline{\sigma}} > 0$	A higher solvency level calls for more risk capital, especially when market volatility is high, making positive net values of the company less likely.

A8: $$\frac{\partial^2 P}{\partial S \partial \overline{\mu}} > 0$$

While a higher rate of return on the capital market depresses premium income (see A4), this effect weakens if the insurer offers a high level of solvency,

A9: $$\frac{\partial^2 P}{\partial S \partial \overline{\sigma}} > 0$$

Higher volatility on the capital market serves to increase premium income (see A5); this effect is reinforced if the solvency levels is high.

APPENDIX B

First, consider a shock $d\bar{\mu}$ disturbing the first-order condition (4). The comparative static equation reads,

$$\frac{\partial^2 R}{\partial S^2} dS^* + \frac{\partial^2 R}{\partial S \partial \bar{\mu}} d\bar{\mu} = 0. \qquad \text{(B.1)}$$

Since $\partial^2 R / \partial S^2 < 0$ in the neighborhood of a maximum, $\mathrm{sgn}\left[\partial^2 R / \partial S \partial \bar{\mu}\right]$ determines $\mathrm{sgn}\left[dS^* / d\bar{\mu}\right]$. Differentiating eq. (4) w.r.t. $\bar{\mu}$, one has

$$\frac{\partial^2 R}{\partial S \partial \bar{\mu}} = \frac{\partial e(P,S)}{\partial \bar{\mu}} - \frac{\partial}{\partial \bar{\mu}}\left[e(C,S)(1-L/P)\right] = \frac{\partial}{\partial \bar{\mu}}\left[\frac{\partial P}{\partial S} \cdot \frac{S}{P}\right] - \frac{\partial}{\partial \bar{\mu}}\left[\frac{\partial C}{\partial S} \cdot \frac{S}{C}(1-L/P)\right]$$

$$= S\left\{\underset{(+)}{\frac{\partial^2 P}{\partial S \partial \bar{\mu}}} \cdot \frac{1}{P} - \underset{(+)}{\frac{\partial P}{\partial S}} \frac{\partial P / \partial \bar{\mu}}{P^2} - \underset{(+)}{\left[\frac{\partial^2 C}{\partial S \partial \bar{\mu}} \cdot \frac{1}{C} + \underset{(+)}{\frac{\partial C}{\partial S}} \frac{\partial C / \partial \bar{\mu}}{C^2}\right]}(1-L/P) + \frac{\partial C}{\partial S} \underset{(+)}{\frac{L \cdot \partial P / \partial \bar{\mu}}{C \cdot P^2}}\right\} \qquad \text{(B.2)}$$

The signs are based on assumptions A9, A4, and A2 as well as eqs. (1) and (2). This is expression (5) of the text.

Now consider $d\bar{\sigma} > 0$. In full analogy to (B.1), one obtains from eq. (4),

$$\frac{\partial^2 R}{\partial S \partial \bar{\sigma}} = \frac{\partial e(P,S)}{\partial \bar{\sigma}} - \frac{\partial}{\partial \bar{\sigma}}\left[e(C,S)(1-L/P)\right] = \frac{\partial}{\partial \bar{\sigma}}\left[\frac{\partial P}{\partial S}\cdot\frac{S}{P}\right] - \frac{\partial}{\partial \bar{\sigma}}\left[\frac{\partial C}{\partial S}\cdot\frac{S}{C}(1-L/P)\right]$$

$$= S\left\{\underset{(+)}{\frac{\partial^2 P}{\partial S \partial \bar{\sigma}}}\cdot\frac{1}{P} - \underset{(+)}{\frac{\partial P}{\partial S}}\frac{\partial P/\partial \bar{\sigma}}{P^2} - \underset{(+)}{\left[\frac{\partial^2 C}{\partial S \partial \bar{\mu}}\cdot\frac{1}{C} + \underset{(+)}{\frac{\partial C}{\partial S}}\frac{\partial C/\partial \bar{\sigma}}{C^2}\right]}(1-L/P) + \frac{\partial C}{\partial S}\underset{(+)}{\frac{L\cdot\partial P/\partial \bar{\sigma}}{C\cdot P^2}}\right\} \quad \text{(B.3)}$$

The signs are based on assumptions A9, A5, and A3 of Table A.1 as well as eqs. (1) and (2) of the text. This is expression (6) of the text.

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