The Alpha and Beta of Private Equity Investments[*]

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Abstract

This paper introduces a novel methodology to estimate abnormal performance and systematic risk of private equity from observable cash flows. The methodology is validated using Monte-Carlo simulations and is applied to a unique sample of 10,798 portfolio company investments by private equity funds. The results show that both venture capital and buyout investments have substantial loadings on the market factor and earn statistically significant positive abnormal returns before fees. The paper also provides the first comprehensive analysis of the differences in systematic risk and abnormal returns across time periods, exit routes, regions, industries, and across companies in different development stages.

Keywords: private equity, venture capital, abnormal return, systematic risk

JEL Classification: C51, G12, G23, G24

Despite the increasing importance of private equity as an asset class, relatively little is known on the abnormal performance and systematic risk of private equity investments. This gap is due to econometric problems and lack of comprehensive data. Estimating the risk and return characteristics of private equity investments is inherently difficult because of the illiquid nature of the asset class. For private equity investments one can typically only observe a stream of multiple cash flows but no intermediate market valuations. Thus, no time series of returns can be constructed which precludes the use of the standard econometric methods to estimate risk loadings and abnormal returns of the investments.

The first main contribution of this paper is to develop a novel econometric approach to estimate abnormal returns and systematic risk of private equity investments from a cross-section of observable investments cash flows. The methodology assumes that the returns of a private equity investment are generated by the standard market model, and that the dividends from the investment occur at a stochastic, yet increasing rate from its unobservable interim values until the investment finally liquidates.[1] Using a non-linear least-squares optimization, the methodology then estimates the systematic risk and abnormal returns of private equity by minimizing the distance between the model expected dividends and the cross-section of observed dividends over time. It is important to note that the estimation approach does not require any assumption on how intermediate capital outflows are re-invested by the investor. This assures that the method only measures systematic risk and abnormal returns of a pure private equity investment, and not that of a combined investment into private equity and other asset classes.

The estimation method is validated using detailed Monte-Carlo simulations. For the Monte-Carlo simulations, investment cash flows are generated assuming a given set of realistic model parameters and different non-standard one-period return specifications.

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[1] As shown, this assumption fits well with the typical bounded lifecycle of private equity investments, for which dividends are low in the beginning and increase over time, with deals and funds eventually liquidating.

As the estimation method relies on the fact that idiosyncratic shocks in investment returns average out across a large sample of investments, the simulation results show that the precision of the estimation method depends on the size of the idiosyncratic volatility and sample size. However, even for very large levels of idiosyncratic volatility and small samples, obtained parameter estimates are very close to the true values. This behavior underlines the statistical consistency of the presented estimation methodology.

The method is applied to a unique and comprehensive dataset containing the exact monthly (gross of fee) cash flows generated by a large number of portfolio company investments from private equity funds. The data come directly from a large general partner network and are less exposed to the self-reporting and survivorship biases that plague standard commercial private equity databases (see Harris et al. (2010) for a discussion). Overall, the data contains 10,798 (6,380 venture capital and 4,418 buyout) fully liquidated private equity investments from all regions worldwide and spans the period from 1980 to 2009. This is the largest sample yet used in the literature that estimates systematic risk and abnormal returns of private equity investments. Previous research (e.g., Driessen et al. (2012) and Ang et al. (2013)) often uses *fund* level data to estimate abnormal returns and risk loadings. The main advantage of using *deal* level data, as done here, is that this leads to substantially more independent observations and consequently greater statistical power. Additionally, this more disaggregated data allows to identify differences across shorter time periods, across industries, across regions, and across companies with different characteristics, such as their stage of development.

The estimation results show that both venture capital and buyout investments have substantial loadings on the market factor. For the venture capital segment, the estimated beta coefficient is statistically significant and equals 2.6 relative to the S&P 500 which is broadly consistent with previous evidence indicating a large systematic risk exposure of this segment. For the buyout segment, the results show a statistically significant beta coefficient of 2.2 relative to the S&P 500. This result is consistent with contemporaneous work of Axelson et al. (2013) who report beta coefficients of 2.2-2.4. Previous studies, including Driessen et al. (2012), Franzoni et al. (2012), and Ang et al. (2013), find markedly lower single-factor market model betas around 1.3. These low betas seem puzzling from a corporate finance theory perspective because they would suggest that the sharp increase in debt that is typically associated with leveraged buyout transactions coincides only with a surprisingly small increase in equity beta.

The results also provide evidence for a strong positive abnormal performance of both venture capital and buyout investments. For the venture capital segment, the alpha is statistically significant and equals 8.9% per annum when the S&P 500 is used as proxy for market returns. For the buyout segment, the results indicate a slightly lower outperformance of 7.0% annually. This outperformance is also statistically significant. These estimates reflect the average annual abnormal returns for companies receiving private equity financing, which may not directly translate to the abnormal returns earned by a private equity fund investor. The reason for this is that cash flows are measured before fees and carried interest are paid to the fund manager. A simple back-of-the-envelope calculation illustrates that the alphas net of fees and carried interest are markedly lower but still positive.

The data also allows studying abnormal returns and systematic risk over different time periods. For the venture segment, the results indicate negative alphas during the period before 1996, whereas the 1996-2000 period is characterized by high positive alphas which corresponds well with the technology and venture capital boom in the late 1990s. Post-2000 the alpha appears to have turned to zero. In the buyout segment, abnormal returns were moderately positive and relatively stable during 1980s and 1990s and then increased sharply in the 2000s. This sharp increase coincides with the well-known buyout boom in the mid 2000s.

The second main contribution of the paper is to provide a comprehensive analysis of the differences in risk and return characteristics across exit routes, regions, industries, and across companies with different characteristics, such as their stage of development in the venture segment. These results help to develop a more nuanced view of the abnormal returns and systematic risk of private equity investments than provided in previous studies and have important implications for the valuation of private companies. The most important findings are: (i) For venture investments the exposure to market risk decreases with the development stage of the investment. This is consistent with the theory of Berk et al. (2004) that growth opportunities, which are particularly relevant for the valuation of venture-backed companies at early development stages, typically include embedded options that add implicit leverage to the companies. (ii) Venture investments in the high growth sectors "Biotechnology" and "Information Technology" significantly outperform the S&P 500. However, as shown, the large risk-adjusted returns are mainly due to good industry timing of the venture capitalists and not due to sound company selection. (iii) Venture investments in continental Europe show similar abnormal returns when compared to their US counterparts but carry markedly lower market risk. Buyout investment in continental Europe, UK, and US show similar market risk and abnormal returns. (iv) Investments that are exited via IPOs generate extraordinarily large alphas and have surprisingly low betas. Additionally, the results indicate that risk-adjusted returns earned in sales exits are substantially larger for the venture investments.

The remainder of this paper is organized as follows. Section 1 discusses related literature. Section 2 describes the estimation approach and validates it using Monte-Carlo simulations. Section 3 describes the private equity data used for the empirical analysis. Section 4 presents the empirical results. The paper concludes with Section 5.

1 Related Literature

This paper is related to the literature that investigates abnormal returns and risk exposures of private equity investments. Important previous research in this area includes Cochrane (2005), Ewens (2009), Korteweg and Sorensen (2010), Driessen et al. (2012), Franzoni et al. (2012), Axelson et al. (2013), and Ang et al. (2013).

Cochrane (2005), Ewens (2009), and Korteweg and Sorensen (2010) assess the abnormal returns and loadings on the market factor of US venture-backed companies using round-to-round valuation data. Round-to-round valuation data has the advantage that a time-series of returns can be constructed and consequently standard regression methods can be applied to estimate risk loadings and abnormal returns. However, the drawback of this approach is that one has to deal with missing rounds and selection bias that arises from the problem that one can only observe valuations for companies that perform well and get additional financing or are being acquired. Correcting for this sort of selection bias is difficult and requires explicit assumptions on the shape of the probability distribution of returns and on the selection process.

In a second line of research, Axelson et al. (2013) and Franzoni et al. (2012) estimate abnormal returns and risk loadings with standard regression techniques by using either internal rates of return (IRRs) or modified internal rates of return (MIRRs).[2] A main concern regarding the approach of Axelson et al. (2013) are several methodological problems that arise when estimating betas from standard IRRs. First, for a given cash flow, the IRR may not exist and it may not be unique. Second, this performance measure implicitly assumes that all intermediate cash flows can be reinvested at the corresponding IRR. Third, when using investment IRRs that are measured over several years in regressions, it may be difficult to construct the appropriate market returns because the actually invested capital can vary heavily over time. Using MIRRs instead, as proposed in Franzoni et al. (2012), avoids some of these methodological issues but has the drawback that abnormal returns and risk loading are essentially estimated for a mixed private equity and stock market investment. This might introduce a bias into the estimated coefficients that is difficult to assess.[3]

The method presented here is not exposed to the aforementioned problems as it

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[2] The difference between the two measures is the underlying reinvestment hypothesis. While the IRR method assumes that intermediate cash flows from an investment are reinvested at the corresponding IRR, the MIRR method assumes that intermediate cash flows are reinvested into a stock market index.

[3] In particular, one might expect that this approach leads to downward biased estimates of systematic risk because the beta of a stock market investment will essentially be equal to one.

allows estimating abnormal returns and systematic risk directly from a cross-section of observable investment cash flows. Regarding methodology, the closest works are Driessen et al. (2012) and Ang et al. (2013) who present an approach that extends the standard internal rate of return (IRR) calculation to a dynamic setting in which they can solve for the abnormal returns and risk exposures using the Generalized Method of Moments (GMM). Similar to the method developed here, this approach requires only a cross-section of observable investment cash flows. The main difference is that this paper assumes a dividend process for private equity investments which allows estimating parameters by minimizing the distance between the model expected dividends and the cross-section of observed dividends over time. In contrast, Driessen et al. (2012) and Ang et al. (2013) identify parameters by using a net present value (NPV) framework. A major difficulty that arises from using this NPV framework are numerical issues because the resulting goal function is not globally convex.[4] This makes the implementation of this method challenging. In a Monte-Carlo simulation experiment, it is shown that the method presented here does not suffer from such numerical problems.

In terms of data, this study makes use of a high-quality research dataset that has also been used by Franzoni et al. (2012). The main difference is that Franzoni et al. (2012) only had access to the sub-sample of liquidated buyout investments while this study makes use of all liquidated buyout and venture capital deals. The overall dataset contains 10,798 private equity deals which is the largest sample yet used in the literature that estimates systematic risk and abnormal returns of private equity investments. This feature may lead to more precise parameter estimates and also allows for a detailed analysis of systematic risk and abnormal returns as a function of project characteristics. In particular, this is the first study that examines how abnormal returns and systematic risk differ across industries, regions, and exit routes.

In terms of empirical results, Axelson et al. (2013) report beta coefficients for the

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[4] In an earlier version of the paper, Driessen et al. (2008) also point out this problem and conclude that "Using this method is therefore problematic in practice, especially if one does not have good starting values for the optimization algorithm." (see Driessen et al. (2008), p.16.)

buyout segment of 2.2-2.4 which is consistent with the estimated 2.2 of this paper, while Driessen et al. (2012), Franzoni et al. (2012), and Ang et al. (2013) report markedly lower single-factor market model betas between 0.9 and 1.5. For the venture segment, beta coefficients reported by Korteweg and Sorensen (2010) and Driessen et al. (2012) both average to 2.8 which is close to the estimated beta of 2.6 given here. Cochrane (2005), Ewens (2009), and Ang et al. (2013) estimate somewhat lower betas for venture capital between 1.7 and 1.9. The alpha coefficients found in this study are significantly positive before fees in the venture and buyout segment which also is broadly consistent with previous evidence. For the buyout segment, Axelson et al. (2013) report annual alphas before fees of 8.3%-8.6% and Franzoni et al. (2012) report an annual alpha before fees of 9.3%. These results are close to the estimated before-fee alpha of 7.0% annually reported in this study for the overall sample of buyout investments. For the venture segment, the previous empirical evidence regarding the magnitude of alpha is less conclusive. Using round-to-round valuation data, Cochrane (2005), Ewens (2009), and Korteweg and Sorensen (2010) find very large before-fee alphas of over 30% annually. Using cash flow data, Ang et al. (2013) find after-fee alphas of 4.0%-5.0% annually. This latter result is largely consistent with the before-fee alpha of 8.9% annually reported in this study for the overall venture segment.[5]

2 Estimation Methodology

Estimating risk and return of private equity investments is complicated by the fact that observable market values are missing because of the illiquid nature of the asset class. Therefore, standard econometric methods to estimate risk and return fail in this situation as periodic returns cannot be calculated.[6]

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[5] In addition, note that Driessen et al. (2012) report large negative alphas of both buyout and venture capital funds before and after fees. However, this result is most likely due to incomplete records in the dataset they use (see also Stucke (2011) for a detailed discussion).

[6] Note that a return can only be computed if the cash flow stream consists of a single inflow at the start of the investment and a single outflow at the end. When there are multiple in- and outflows,

This section develops a novel estimation methodology that overcomes the problem of missing valuations as it relies only on the observable cash flows from private equity investments. In the first part, we define the assumptions underlying the estimation framework. Next, we present the estimation function and outline its derivation. We then provide evidence for the consistency of the derived methodology and test for potential biases of the estimates using a detailed Monte Carlo simulation experiment.

Note that our estimation methodology can be applied to both, investments in private equity funds by institutional investors, as well as investments in portfolio companies by the funds themselves. In the following, we use the term *investment* and note that this can refer likewise to a private equity *fund* or *portfolio company* investment.

2.1 Assumptions

The goal is to develop an approach that allows to estimate systematic risk and abnormal returns from a cross-section of private equity investment cash flows. Basis of the estimation is a sample of N private equity investments for which the the following four main assumptions hold:

Assumption 1 *(Value Dynamics) The dynamics of the value* $V_{i,t}$ *of investment* $i = 1, \dots, N$ *are given by*

$$V_{i,t} = V_{i,t-1}(1 + R_{i,t}) + \Delta T_{i,t} - \Delta D_{i,t}, \tag{1}$$

where $R_{i,t}$ *is the period-t return of investment i,* $\Delta T_{i,t}$ *denotes capital inflows (i.e., investments) that occur in period-t, and* $\Delta D_{i,t}$ *denotes capital outflows (i.e., dividends) that occur in period-t.*[7]

Specification (1) is straightforward. The first term, $V_{i,t-1}(1 + R_{i,t})$, states that the

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returns can only be computed at the cost of making an assumption on how intermediary cash flow are re-invested. An example for this is the internal rate of return.

[7] Note that by period-t we refer to the time period ranging from $t - 1$ to t.

change in value of a private equity investment is, at first, the result of the performance of the investment already in place. In addition, the second and third term show that the value is increased by capital inflows (i.e., investments) $\Delta T_{i,t}$ and decreased by capital outflows (i.e., dividends) $\Delta D_{i,t}$. Including $\Delta T_{i,t}$ and $\Delta D_{i,t}$ into Equation (1) takes into account that private equity investments typically involve several investment rounds and generate substantial intermediate dividends during their bounded lifecycle. As an investment is gradually exited, the dividends (whether in form of cash or marketable securities) are directly distributed to the investors. Therefore, dividends simply decrease the investment value and there is no need to impose any assumption about the reinvestment of cash flows.

Assumption 2 *(Dividends) Dividends or cash outflows $\Delta D_{i,t}$ of investment i in period-t occur at a non-negative rate from the beginning of period investment value, i.e.,*

$$\Delta D_{i,t} = \delta_{i,t} V_{i,t-1}, \tag{2}$$

where $\delta_{i,t}$ is the period-t dividend rate of investment i. The dividend rate $\delta_{i,t}$ is assumed to be a stochastic function of time given by

$$\delta_{i,t} = \left(\frac{\tau}{\tau_i} \delta + z_{i,t} \right) t, \tag{3}$$

where $\delta > 0$ is a common factor for all investments, τ_i is the investment duration of investment i, and τ is the average duration of all sample investments (i.e., $\tau = \frac{1}{N} \sum_{i=1}^{N} \tau_i$), and $z_{i,t}$ is an i.i.d. random variable with zero mean.

Equation (2) represents the standard approach in the literature to model the dynamics of dividend paying assets.[8] It states that a fraction $\delta_{i,t}$ of the beginning of period asset value $V_{i,t-1}$ of investment i is paid out in period-t. Equation (3) adds the assumption

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[8] See, for example, Björk (1998), Chapter 11.

that the rate $\delta_{i,t}$ is a stochastic function of time t. Taking expectations of (3) yields

$$E[\delta_{i,t}] = \frac{\tau}{\tau_i}\delta t, \tag{4}$$

which shows that the dividend rate increases in a linearly over time.[9] From an economic perspective, this specification is reasonable since it reflects well the typical lifecycle of private equity investments, where dividends are low at the beginning and increase over the bounded life of an investment as it gets gradually realized and eventually liquidated. Specification (3) also allows incorporating that investments with shorter investment durations typically pay out capital faster than investments with longer durations. This is achieved by scaling the common factor δ by τ/τ_i, where τ_i is the duration of investment i, and τ is the average duration of all sample investments, i.e., $\tau = \frac{1}{N}\sum_{i=1}^{N}\tau_i$. In addition, Equation (3) further accounts for the fact that the dividend rate of an investment can change over time by including the stochastic component $z_{i,t}$ to the specification.

Assumption 2 is important for the estimation methodology. It enables us to overcome the problem of missing market valuations of private equity investments. This holds because the equation gives a direct connection between the unobservable market values $V_{i,t-1}$ and the observable dividends $\Delta D_{i,t}$ of the investment. Moreover, the dividend rate δ provides information on the average speed at which investments are liquidated. In this context, the parameter δ can be seen as a measure of cash flow liquidity of private equity investments.

Substituting Equation (2) into (1), it follows that the value dynamics can be expressed by

$$V_{i,t} = V_{i,t-1}(1 + R_{i,t} - \delta_{i,t}) + \Delta T_{i,t}. \tag{5}$$

Assumption 3 *(Return Dynamics) The return $R_{i,t}$ of investment i in period-t is gen-*

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[9] Note that various linear and non-linear functions of time have been tested for the dividend rate in the empirical implementation. The linear function of time given in Equation (3) provided the best fit with the empirical data. The excellent fit of this specification is also illustrated in Section 4 by various measures of goodness-of-fit.

erated by a one-factor market model of the form

$$R_{i,t} = r_{f,t} + \alpha_i + \beta_i(R_{M,t} - r_{f,t}) + \epsilon_{i,t}, \tag{6}$$

for which $r_{f,t}$ *and* $R_{M,t}$ *are the period-*t *returns on the risk-free asset and on the market portfolio, respectively. Variable* $\epsilon_{i,t}$ *is an i.i.d. disturbance term with zero mean that is uncorrelated with the market returns for all* t*. In addition, it holds that disturbance terms of different investments are also uncorrelated.*

This is a standard specification used in the performance measurement literature. For a constant β_i over time, the intercept α_i equals Jensen's alpha, which directly measures a manager's selection or micro-forecasting ability (see Jensen (1968) and Fama (1972)). In the private equity literature, e.g. Cochrane (2005), Driessen et al. (2012), and Korteweg and Sorensen (2010) use similar specifications. We use this as a base case in our estimation and also extend it to multi-factor pricing models which include other macroeconomic variables.

Assumption 4 *(Cross-Sectional Restrictions) Investments can be categorized according to their strategy (i.e., the sub-asset class like venture capital or buyout), their geographic focus, etc. It is assumed that investments from a certain category have a similar exposure to systematic risk* β *and abnormal returns* α*.*

In order to being able to correctly estimate α and β from a cross-section of private equity fund cash flows, it is necessary to assume that all sample investments have similar risk and return characteristics. The assumption of a similar α and β is also used by, for example, Cochrane (2005) and Driessen et al. (2012). The economic justification behind this assumption is that the performance of a given investment type is subject to the same systematic risk together with an idiosyncratic component.

2.2 Estimation Function

In the following, the estimation function´is presented. Given is a sample of N investments for which Assumptions 1-4 hold. For each of the $i = 1, \ldots, N$ investments, we can observe a stream of periodic cash flows, i.e. periodic capital inflows $\Delta T_{i,1}, \ldots, \Delta T_{i,K}$ and dividends $\Delta D_{i,1}, \ldots, \Delta D_{i,K}$ over the total observation period of length K. In order to make investments of different sizes comparable, the capital inflows and dividends are thereby scaled on the basis the corresponding total invested capital. In addition, we can also observe the market return $R_{M,k}$ and the riskless rate $r_{f,k}$ in each period $k = 1, \ldots, K$. Under these specification, the parameters α, β and δ can be estimated using a *non-linear least squares* approach.

Theorem 2.1 *Given a sample of N investments and a total observation period of length K, model parameters α, β and δ can be estimated by*

$$\min_{\alpha,\beta,\delta} \sum_{k=1}^{K} \left(\Delta D_k - E[\Delta D_k]\right)^2, \tag{7}$$

where ΔD_k are the average dividends of the N sample investments in period-k, i.e.,

$$\Delta D_k = \frac{1}{N} \sum_{i=1}^{N} \Delta D_{i,k}, \tag{8}$$

and $E[\Delta D_k]$ are the expected dividends in period-k, given by

$$E[\Delta D_k] = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{k-1} \bar{\delta}_{i,k} \Delta T_{i,j} \prod_{s=j+1}^{k-1} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) - \bar{\delta}_{i,s}], \tag{9}$$

for the expected dividend rate $\bar{\delta}_{i,k} = \frac{\tau}{\tau_i} \delta k$.

A detailed derivation of this estimation function is provided in Appendix A.

The idea underlying this estimation is straightforward. We estimate parameters such that the distance between the empirically observed dividends (8) and the model expected dividends (9) is minimized. Given that the sample size N and the observation period K are sufficiently large, this generates asymptotically consistent estimates.[10] Based on this approach, the systematic risk β and the abnormal returns α of private equity investments can be estimated, even though intermediate market values of the investments are unobservable. In addition, this approach allows us to estimate the dividend rate δ, which can be interpreted as a measure of cash flow liquidity of the investments.

Note that the estimation approach in Theorem 2.1 does not require any assumption on how intermediate capital outflows are re-invested by the investor. That is, the estimation results are not affected by whether an investor re-invests capital outflows e.g. in a stock market index or government bonds. This assures that we only measure systematic risk and abnormal returns of a pure private equity investment, and not that of a combined investment into private equity and other asset classes.

2.3 Monte-Carlo Simulation

As argued above, our methodology generates asymptotically consistent estimates of the model parameters when the sample size N is sufficiently large. To validate the consistency of the derived methodology and to test for potential biases of the estimation, we conduct a detailed Monte Carlo simulation experiment in the following.

2.3.a Implementation

To match the type of data used in the empirical application below, deal level cash flows are simulated for the Monte-Carlo experiment. This requires some additional assumptions on the cash flow dynamics of private equity investments. We assume that all investments to be modeled have a total size given by C. Cumulated capital inflows of investment i up to time t are denoted by $T_{i,t}$, capital inflows in period-t are $\Delta T_{i,t}$. In the

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[10] Note that there is no closed-form solution to the non-linear least squares problem defined in (7). Instead, numerical algorithms are used to find the value of the parameters α, β and δ which minimize the objective.

following, we assume that capital inflows in each period-t occur at some constant and non-negative rate from the remaining capital at the beginning of the period, $C - T_{i,t-1}$. That is, the dynamics of the capital inflows can be described by

$$\Delta T_{i,t} = \gamma(C - T_{i,t-1}), \tag{10}$$

where $\gamma > 0$ denotes the constant investment rate that is assumed to be the same for all investments.[11]

In the following simulations, we choose model parameters such that they closely match the characteristics of venture capital investments. The reason for this choice is that model parameters of venture capital investments will typically be more difficult to estimate because they exhibit higher levels of idiosyncratic risk compared to buyout investments. Consistent with the sample venture capital investments used in the empirical analysis, we set $\gamma = 0.8$ per annum. This implies that 80 percent of the total investment size C is invested within the first year.

Dividends of the investments occur according to the specification (see Assumption 2)

$$\Delta D_{i,t} = \left(\frac{\tau}{\tau_i} \delta + z_{i,t} \right) t V_{i,t-1}, \tag{11}$$

For simplicity, we assume that all investments have identical durations τ (i.e., $\tau / \tau_i = 1$) and that variable $z_{i,t}$ is i.i.d. normal with zero mean and identical volatility σ_δ for all investments. Consistent with the estimation results for the venture capital investments below, we set $\delta = 0.18$ per annum. In the base case, we use $\sigma_\delta = 0$ and extend this to a stochastic setting later on.

The unobservable value dynamics of the investments are given by (see **Assumptions**

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[11] This deterministic modeling framework for the capital inflows of private equity investments is similar to the one developed by Takahashi and Alexander (2002). It could easily be extended to stochastic settings by assuming that γ follows some stochastic process. See, for example, Malherbe (2004).

1 and 3)

$$V_{i,t} = V_{i,t-1}[1 + r_f + \alpha + \beta(R_{M,t} - r_f) + \epsilon_{i,t}] + \Delta T_{i,t} - \Delta D_{i,t}, \tag{12}$$

for which the returns are generated by the standard market model, in which $R_{M,t}$ is the market return in period t and r_f is the risk-free rate.

Given the large idiosyncratic volatility of the returns of venture capital investments, assuming a normal distribution for $R_{M,t}$ and $\epsilon_{i,t}$ would generate returns that could fall below -100% with non-negligible probability. We therefore use shifted log-normal distributions for both variables (see Driessen et al. (2012)). For the market return $R_{M,t}$ we assume an i.i.d. shifted log-normal distribution over time, i.e., $R_{M,t} = \exp(X) + c_M$, where X is normally distributed with mean μ_M and variance σ_M^2, and c_M is a constant. Similarly, we assume $\epsilon_{i,t}$ is i.i.d. shifted log-normal across investments and over time. Finally, we assume that the risk-free rate is constant at $r_f = 0.05$ per annum.

For the market return, we match the S&P 500 total return index over the sample period from January 1980 till June 2009. We set $c_m = -0.2$ which is close to the minimum monthly return of -21.76% during this period. The mean μ_M and variance σ_M^2 of the shifted log-normal distributed market returns are calculated by

$$\mu_M = \ln\left[\frac{(E(R_{M,t}) - c_M)^2}{\sqrt{Var(R_{M,t}) + (E(R_{M,t}) - c_M)^2}}\right], \tag{13}$$

$$\sigma_M^2 = \ln\left[\frac{Var(R_{M,t})}{(E(R_{M,t}) - c_M)^2} + 1\right], \tag{14}$$

for which $E(R_{M,t})$ and $Var(R_{M,t})$ are the average and the variance of the arithmetic monthly S&P 500 returns, respectively. It turns out that $\mu_M = -1.58$ and $\sigma_M = 0.18$ per month.

For the idiosyncratic error $\epsilon_{i,t}$, parameter c_ϵ is fixed such that returns cannot fall below -100%, i.e., $r_f + \alpha + \beta(c_M - r_f) + c_\epsilon = -1$ holds. In the base case the 'true' alpha is set to zero and the 'true' beta is set to 2.5. This gives $c_\epsilon = -0.43$. Similar to above, the mean μ_ϵ and variance σ_ϵ^2 of the shifted log-normal distributed error terms can be calculated by [12]

$$\mu_\epsilon = \ln\left[\frac{c_\epsilon^2}{\sqrt{Var(\epsilon_{i,t}) + c_\epsilon^2}}\right], \tag{15}$$

$$\sigma_M^2 = \ln\left[\frac{Var(\epsilon_{i,t})}{c_\epsilon^2} + 1\right], \tag{16}$$

for which $Var(\epsilon_{i,t})$ is the idiosyncratic variance of the private equity investments. We set $Var(\epsilon_{i,t}) = 0.16$ per month, which closely matches the monthly idiosyncratic volatility of 41% as reported by Korteweg and Sorensen (2010) for venture capital deals. Using this result, we get $\mu_\epsilon = -1.17$ and $\sigma_\epsilon^2 = 0.80$.

Table I summarizes all model parameters for the base case of our Monte Carlo simulation experiment.

2.3.b Simulation Results

The simulation is carried out using 5,000 iterations, i.e. parameters are estimated from a sample of 5,000 investments.[13] The simulation is repeated 1,000 times and the mean, standard deviation, median, and interquartile range of the 1,000 estimated pairs of parameters is calculated.

Table II presents the estimation results from the Monte Carlo simulation. The results show that all parameters can be estimated with very high precision. In the Base Case in Panel A, the methodology estimates an average alpha of -0.01% per month, an average beta of 2.51, and an average delta of 0.18 across simulations. These numbers are very close or equal to the true parameters of alpha=0%, beta=2.5, and delta=0.18. Note that we have repeated the simulations using different pairs of the true parameters alpha, beta, and delta. In all cases, the precision of the resulting estimates remained at the

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[12] Note that $E(\epsilon_{i,t}) = 0$ holds by definition.

[13] This is similar to the sample sizes used in the empirical analysis.

same high level.

Moreover, Figure 1 indicates that our estimation approach does not suffer from any numerical problems. The figure shows the objective function space of parameters alpha and beta for the optimal value of delta. We can observe that the objective function exhibits a well defined and easy to determine minimum in the parameters alpha and beta.

The estimation methodology relies on the fact that idiosyncratic shocks in investment returns average out over large samples. Consequently, the precision of the estimation depends on the idiosyncratic volatility of the investments and on the size of the sample. Panel A shows how a change in the idiosyncratic volatility affects the accuracy of the estimates. We assume two different cases, Low and High, with an idiosyncratic volatility of 20% per month (i.e., about 69% p.a.) and 60% per month (i.e., about 208% p.a.), respectively. The results illustrate that the level of precision further increases when the idiosyncratic volatility is relatively lower, whereas the precision slightly decreases in case of larger idiosyncratic shocks. Nevertheless, given the very large idiosyncratic shocks in the high volatility case, the accuracy of the resulting estimates is still at a high level for all model parameters.

The last two columns in Panel A show how a change in the sample size affects the precision of the parameter estimates. As expected, the estimators converge towards the true values with increased precision as the sample size increases, whereas the precision slightly decreases as the sample size is reduced. This behavior provides further evidence of the statistical consistency of the estimation methodology.

In Panel B, we relax the assumption that the dividend rate is deterministic, and examine how different cross-dependencies between the dividend rate and other model variables affect the precision of the estimates. First of all, we make the dividend rate stochastic by setting $\sigma_\delta = 0.05$. In the Base Case, the dividend rate is independently distributed. In the next two columns, we add the property that the dividend rate is correlated with lagged market returns over the last six months. The results show that a positive correlation, i.e., higher proportional dividends following strong public markets, leads to slightly upward biased estimates of beta, whereas alpha and delta can still be estimated with very high precision. However, with less than 3%, the bias of the estimated beta is rather small given the relatively large correlation of 0.5.

In the last two columns, we analyze to what extent cross-dependencies between the dividend rate and the parameters alpha and beta can affect the precision of the estimation. We first create a cross-sectional variation in alpha among the simulated sample deals by assuming that alpha is normally distributed with a zero mean and a standard deviation of 8%. This specification generates a variation of alpha among the sample such that about 99% of the deals have alphas being in the interval between -20% and +20%. To incorporate a dependency between alpha and the dividend rate, we make the constant δ in equation (11) investment specific via the function $\delta_i = \exp(\alpha_i)\delta$. In this case, investments with a low (negative) abnormal returns α_i will distribute capital slower, which seems reasonable as private equity fund managers typically hold on to their less successful investments. The estimation results show that this specification does not have any material effect on the estimation precision. The average beta is slightly too low, but this bias seems negligible.

In the last column we examine the effect a cross-dependency between beta and the dividend rate. To incorporate a dependency between beta and the dividend rate, we replace the constant δ in Equation (11) via the function $\delta_i = (\beta/\beta_i)\,\delta$, for which β is the average beta of all sample deals and β_i is the beta of investment i. This specification results in a lower dividend rate of high beta investments, which seems reasonable due to the longer time needed to reestablish a sustainable capital structure and exit the investment. The beta of the sample investments is assumed to be normally distributed with a mean of 2.5 and a standard deviation of 0.4. This results in a variation of beta among the sample such that about 99% of the deals have a beta between 1.5 and 3.5. Again, the results show only a very small effect on the estimation precision and all estimates are close to the true values.

Overall, the simulations in Panel B confirm that, even though we assume independently distributed dividend rates in our derivation of the estimation methodology, the results do not exhibit any meaningful bias under more general specifications where the dividend rate is correlated with (lagged) market returns or depends on the parameters alpha and beta of the investments.

3 The Data

This section describes the data used for the empirical analysis. First, the data source and data collection process is outlined. Second, descriptive statistics of the data are presented.

3.1 Data Source

We study systematic risk and abnormal returns of private equity investments using a quality-research dataset of individual private equity deals. The unique dataset that contains monthly cash flows generated by private equity investments is provided by the Center for Private Equity Research (Cepres).

Cepres is a private consulting company that obtains data from private equity firms that participate in a general partner network. Private equity firms that participate in this network report monthly cash flows and investment details (e.g. industry, investment stage, etc.) for each deal they have made in the past. In return, they receive statistics such as risk-adjusted performance measures for their own investments and for the aggregate private equity market. It is important to stress that Cepres treats all information anonymously to meet the confidentiality requirements of the private equity industry and, as a result, the data providers have only little incentive towards a positive reporting bias. Particularly, no third parties are able to identify individual funds' or managers' performances. This improves data accuracy and representativeness, as it eliminates incentives to manipulate cash flows or cherry-pick past investments. In addition, this also minimizes survivorship bias because there is no direct incentive for poor performing private equity firms to cease to report, as holds for many other commercially available databases (see Harris et al. (2010) for a discussion).

The total database includes over 30,000 worldwide investments covering early- and later-stage venture capital, buyout and mezzanine investments. Earlier versions of this dataset have been used in several previous studies. A subset of this database covering mainly venture capital investments is used by Cumming et al. (2010), Cumming and Walz (2010), and Krohmer et al. (2009). Franzoni et al. (2012) use the subset of all liquidated buyout investments for their study on systematic liquidity risk. For the purpose of this study, Cepres granted access to all investments in the database as of September 2009, covering the full universe of venture capital and buyout deals. We restrict the following analysis to fully realized investments thereby avoiding any issues regarding the accuracy of the estimated net asset values (NAVs) of unrealized investments, or timing issues regarding when the NAVs were reported. The data set contains all monthly deal-level gross cash flows (before management fees, carried interest payments, or other fund-related costs) between the private equity firms and the portfolio companies. This feature allows precisely measuring the systematic risk and abnormal returns using the methodology developed above.

3.2 Descriptive Statistics

The total sample used for the empirical analysis contains 10,798 fully liquidated private equity deals with 81,989 monthly cash flows. This data set is the largest sample yet used in the literature that estimates systematic risk and abnormal returns of private equity investments. Roughly 59 percent (6,380) of the deals are venture capital while 41 percent (4,418) are buyout deals. Detailed descriptive statistics for the sample are provided in Table III.

The average size of the investments in the sample is $12.01 million, with venture capital investments being substantially smaller on average than buyout investments, $7.25 million versus $18.89 million. In addition, the high standard deviations reveal that there is a very large variation investment sizes among the samples.

The geographical split of investments shown in Table III indicates that the sample is characterized by a large proportion (57.09%) of investments in the US, consistent with earlier work showing the US has the dominant private equity market worldwide (see Armour and Cumming (2006)). However, there is a marked difference in the geographical split between the sample venture capital and buyout investments. More than two-thirds (72.51%) of the venture capital investments are in the US, whereas the buyout investments are more evenly distributed across the main investment regions: US 34.83%, UK 29.09%, and Europe (excluding UK) 25.31%.

Table III further shows that the sample investments represent a wide range of industries. Industries are aggregated into five sectors: "Biotechnology", "Consumer Industry", "Industrials", "Information Technology", and "Others/Unspecified". In line with expectations, the largest fraction of sample venture capital investments are in "Information Technology" (63.71%). Buyout investments are concentrated in the sectors "Consumer Industry" (40.63%) and "Industrials" (26.78%).

The statistics in Table III also allow insights into the investment durations and the exit routes chosen by the investment management firms. On average, it takes around 4.25 years (median 3.83 years) until an investment is fully exited. Venture capital investments are on average exited faster than buyout investments, 4.07 years versus 4.50 years. However, the large standard deviations reported imply that this difference is not statistically significant. The dominant exit route among the sample is Sales, which occur at a substantially higher frequency than IPOs in both investment segments. In addition, companies in the buyout segment have a greater likelihood to be successfully exited. This is reflected by the high percentage of write-off for venture capital investments (28.51%), which is almost three times the fraction of write-offs for buyout investments (10.30%).

The sample split by investment years is illustrated in Table IV and Figure 2.

4 Estimation Results

In this section we estimate the exposure of private equity to systematic risk and the corresponding abnormal returns. In the first part, we estimate beta coefficients and alpha residuals for the full samples of venture capital and buyout investments. Next, we analyze to which degree parameter estimates differ across stage sub-classes, exit routes, regions, time periods, and industries.

4.1 Benchmark Estimation Results

We analyze systematic risk and abnormal returns in the context of the one-factor market model described in Section 1. The S&P 500 total return index is used as a proxy for market returns, which allows examining the sensitivity of our sample investments to US stock markets. The one-month US Treasury Bill rate is employed as the risk-free rate in all subsequent estimations.

Panel A of Table V presents the estimation results, corresponding standard errors, and measures of goodness-of-fit of the estimation. The systematic risk estimates are 2.57 for venture capital investments and 2.25 for buyout investments. These results suggest that the sample investments have a large and statistically significant exposure to market risk. The estimated beta of the venture capital investments is largely consistent with previous research. Beta coefficients reported by Korteweg and Sorensen (2010) and Driessen et al. (2012)) both average to 2.8. Cochrane (2005), Ewens (2009), and Ang et al. (2013) find somewhat lower beta coefficients of venture capital investments which are, however, still consistently above one. From a theoretical perspective the large beta of venture capital investments can be explained by the fact that growth opportunities play an important role in the valuation of young entrepreneurial companies. Growth opportunities typically include embedded options, which add implicit leverage to this type of, usually, all-equity financed investments (see Berk et al. (2004) and Bernardo et al. (2007)). As a result, venture capital investments carry substantial systematic risk.

For the buyout segment, the estimated beta is consistent with Axelson et al. (2013) who report betas on the deal level ranging from 2.2 to 2.4. In contrast, Driessen et al. (2012), Franzoni et al. (2012), and Ang et al. (2013) report a market betas around 1.3. This is considerably less than the estimated coefficient of 2.25 given here. However, Driessen et al. (2012)) also suspect their estimation result and wonder how highly leveraged buyout investments can yield such a low market beta. The estimation result given here seems to be more reasonable for highly leveraged buyout transactions.

Standard corporate finance theory confirms this impression. Assuming a debt beta equal to zero, for simplicity, the unlevered beta β_u (or asset beta) of an investment can be calculated from the corresponding levered beta (or equity beta) β_l by the relationship

$$\beta_u = \frac{\beta_l}{1 + (1 - \tau_c)\frac{D}{E}}, \tag{17}$$

where τ_c is the corporate tax rate and D/E is the debt/equity ratio. By definition, the levered beta of an average stock market investment equals one. Given a typical capital structure of publicly-listed companies with 25% debt and 75% equity, and assuming a corporate tax rate of 30%, this yields an average unlevered beta of 0.74. For buyout transactions, Groh and Gottschalg (2009) and Axelson et al. (2013) report an average debt/equity ratio of around 3 (i.e., a capital structure with 75% debt and 25% equity). Assuming that buyout investments have average unlevered betas that are comparable to publicly traded stocks, this yields an equity beta for buyout transactions of 2.3. This is considerably larger than the beta of around 1.3 estimated in previous empirical studies. A number of this magnitude would suggest that the sharp increase in debt that is typically associated with buyout transactions coincides only with a surprisingly small increase in equity beta.

The results in Panel A of Table V also suggest that the sample private equity investments earn significantly positive abnormal returns (alpha). The estimated abnormal return is significantly positive at 8.9% per annum for the venture capital investments and significantly positive at 7.0% per annum for the buyout segment. Given these estimates for alpha and beta, Panel B of Table V reports the cost of capital and the market model implied expected returns of the investments. Using weighted averages of the monthly S&P 500 returns and of the monthly US Treasury Bill rates over the sample period, the cost of capital according to the CAPM is 15.7% per annum for the venture and 19.5% per annum for the buyout investments. Expected returns implied by the market model for the venture and buyout investments are 24.6% and 26.6% per annum, respectively. Note that these estimated expected returns are very close to the average sample internal rates of return (IRR) that are also reported in Panel B, which lends further confidence to the estimation results.

The estimation methodology also allows drawing inferences on the cash flows liquidity of the sample private equity investments via the estimated dividend rates delta. The coefficient delta equals 0.17 per annum for the buyout investments and 0.18 for the venture capital investments. For sample of buyout investments, this suggests that the expected dividends are approximately 17% in the first year, as measured relative to the overall invested capital. The solid lines in Figure 3 further illustrate the expected dividend flows that follow from Equation (9). Interestingly, these dynamics suggest that, despite being considered as highly illiquid investments, private equity deals show high average cash flow liquidity, even in the very early times of their lifetime. For comparison, the average annual dividend yield of the S&P 500 index stocks in the sample period was much lower at 3.11 percent annually.

Figure 3 also illustrates the excellent fit between the model expected dividends (solid lines) and the empirical observations (dotted lines). This close relationship is quantified by the two measures of the goodness-of-fit in Table 5. With a coefficient of determination, R^2, of 83.7% our model explains a very high degree of the variation in monthly dividends of all buyout investments. In addition, the root mean squared error, $RMSE$, is as low as 0.0053 per month. For venture capital investments, the measured R^2 is slightly lower at 75.1%. This slightly lower number is due to the higher level of idiosyncratic risk in the venture segment.

It is important to keep in mind that care must be taken when interpreting the reported estimates of alpha as measures of risk-adjusted investment returns of private equity *fund investors*. The estimates reflect the average annual abnormal returns for companies receiving private equity financing, which may not directly translate to the abnormal returns earned by a private equity fund investor. The reason for this is that gross cash flows on the deal level are used that are not adjusted for fixed management fees and performance related carried interest payments. A simple back-of-the-envelope calculation can illustrate the effect of these payments on alpha. For the calculation exercise, we use a typical fund lifetime of 10 years and employ the most common terms for private equity funds which are a 2 percent annual management fee, a carried interest level of 20 percent per annum, and a hurdle rate of 8 percent per annum (see Sahlman (1990), Metrick and Yasuda (2010) and Buchner and Wagner (2012)). For the buyout segment, the market model estimation results in Table V suggest an expected gross return of 26.6% annually. Given this level of expected return, management fee payments reduce alpha by approximately 2.8% p.a. and carried interest payments by an additional 3.2% p.a.[14] Overall, this reduces alpha of buyout investments before fees from a 7% p.a. to 1% p.a. after fees. Similarly, for the venture capital segment alpha decreases from a high 9% p.a. to a low 3.5% p.a.

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[14] Note that this a simple approximation that assumes that capital committed capital is steadily and fully invested over the lifetime and that fee and carry payments only reduce alpha but have no effect on beta. Let $E[R_F]$ denote expected gross returns, f the annual management fee, c the carried interest level per annum, h the hurdle rate per annum, and T the lifetime of the fund. The reduction in alpha caused by management fees is can be approximated by: $\Delta\alpha_{fee} = E[R_F] - [(1 - fT)^{1/T}(1 + E[R_F]) - 1]$. The reduction in alpha caused by carried interest payments can be approximated by: $\Delta\alpha_{carry} = [E[R_F] - \Delta\alpha_{fee} - h]c$.

4.2 Estimation Results Across Stage Sub-Classes

Recognizing that the samples span different stage sub-classes, we now present estimates with separate coefficients for investments in “Early-Stage” and “Later-Stage” for the venture segment and “Leveraged Buyout” and “Growth” investments for the buyout segment.[15]

In the venture segment, the results in Table VI suggest that early-stage investments earn slightly negative but statistically insignificant abnormal returns while later-stage investments show substantial and statistically significant positive abnormal returns. A possible reason for this marked difference is that it is more difficult to screen for attractive deals in the early-stage segment since early-stage investments in high-technology industries are typically associated with high levels of uncertainty about product/technology, market size, future customer adoption, quality of management, and so on (see, for example, Kaplan and Stromberg (2004)). Additionally, Figure 2(a) shows that a very large part of the sample early-stage investments were being started at the height of the internet bubble around the year 2000. During this period many investments were not properly screened and had to be written-off after the bubble burst. Consistent with this observation the data reveals a much larger fraction of write-off deals in the early-stage segment (33.33%) compared to the later-stage segment (18.75%). In addition, it is interesting to see that the exposure to market risk tends to decrease with the development stage of the investments, as indicated by the lower beta coefficient of later-stage investments. This is consistent with the theoretical models of Berk et al. (2004) and Bernardo et al. (2007) that show that the beta of growth opportunities is generally much larger than the beta of assets-in-place for entrepreneurial companies. Early-stage venture capital investments are primarily investments to develop young ideas or prototypes where growth options

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[15] Note that the following stage definitions are used: “Early-Stage” includes the categories “Seed”, “Start-Up” and “Early”; “Later-Stage” includes “Expansion” and “Later”; “Leveraged Buyout” includes the categories “LBO”, “MBO/MBI”, “Turnaround”, “Acquisition Financing”, “Public to Private”, “Spin-Off”, “Special Situations”, “Recapitalization” and “Secondary Trading”; “Growth” includes all “Growth” investments.

play a dominant role in the valuation. As the companies mature the value of the growth options decreases relative to the value of the assets-in-place, decreasing their overall exposure to market risk. Besides, the results also indicate that later-stage investments distribute capital faster, as indicated by the somewhat higher coefficient delta. This is in line with the general view that investments into young entrepreneurial companies require longer time periods until they deliver significant cash outflows for investors.

For the buyout segment, the results reveal a somewhat lower market risk of growth investments. This is consistent with lower average leverage ratios associated with these transactions (see, for example, Mooradian et al. (2013)). Growth capital is a type of buyout investment in relatively mature companies that are looking for capital to expand or restructure operations, enter new markets or finance a significant acquisition. The companies targeted by growth capital typically benefit from high levels of organic growth and profitability and are less reliant on debt capital to sustain themselves. For this reason, growth investments are typically characterized by low or no use of leverage. Additionally, the results show that both categories earn significantly positive abnormal returns. However, the alpha of the growth investments is roughly twice that of the leveraged buyout investments. As noted, growth capital investors target companies with rapid organic growth, which gives these investments a high upside return potential. To mitigate downside risk, these transactions typically involve low or no leverage, are senior to management's equity ownership, and have a full set of protective shareholder and governance provisions. The limited downside risk of growth investments is also confirmed in the data by their relatively moderate fraction of write-off deals (growth: 13.54%; leveraged buyout: 9.58%). This combination of high upside return potential with relatively limited downside seems to generate a very attractive risk/return relationship and result in high abnormal returns of the sample investments. In addition, the sample distribution given in Figure 2(a) shows that a large part of the growth investments were being started in the 2000s where large abnormal returns were easy to achieve due favorable market environment for buyout investments in the mid-2000s.

4.3 Estimation Results Across Exit Routes

Table VII compares the abnormal performance and market risk of the sample investments across different exit routes. We restrict the analysis to successful exits and consider the main exit routes: IPOs and Sales.

Regarding abnormal returns of Sales exits, the estimation results show that alpha is substantially lower for the buyout segment. This difference can possibly be explained by the strong trend towards so-called secondary buyouts (i.e., buyout transactions in which a private equity firm sells the company to another private equity firm) that exhibit modest returns according to Degeorge et al. (2013). In contrast, the most common sales channel in the venture segment are trade sales, i.e., transactions in which a private equity firm sells the company to a strategic investor. According to Cumming and MacIntosh (2003) venture capital deals exited by trade sales offer very attractive returns.

The results in Table VII also suggest that extraordinarily large abnormal returns can be earned in IPO exits. Venture capital investments that are exited via IPO show a stunning alpha of 62.6% p.a., which is more than six times the alpha of all sample venture capital deals and more than two times the alpha of deals exited by Sales. The alpha of buyout-backed IPOs is only slightly lower at 52.6%. Previous research also indicates that investments exited via IPOs generate high *absolute* returns when compared to other exit routes (see Gompers (1995) and Das et al. (2003)). The novel insight provided by the results in Table VII is that high absolute returns earned in IPO exits are a direct result of high abnormal returns and not of high levels of market risk of the investments. Indeed, the results even indicate that beta of deals that are exited by IPOs is surprisingly low when compared to the beta of all sample deals or to the beta of the subsample of deals that are exited by Sales.

A main reason for the high abnormal returns is that IPOs are primarily an exit route for the top-performing companies (see Schwienbacher and Giot (2007)). Additionally, they might also indicate some degree of overpricing of private equity-backed IPOs. Consistent with this conjecture, Ritter (1991) and Loughran and Ritter (1995) point out that investors cyclically overprice IPOs of young growth companies because they are overoptimistic about the future prospects of these firms. Megginson and Weiss (1991) present additional evidence indicating that overpricing might even be more pronounced for private equity-backed IPOs because these are typically associated with high underwriter prestige, high institutional holdings, and a low level of IPO underpricing compared to non venture-backed IPOs.[16] Additionally, Lerner (1994) examines the timing of initial public offerings and finds that venture capitalists take firms public at market peaks to coincide with hot-issue markets. That is, the large abnormal returns of IPO exits may partly also be a result of good exit market timing.

In order to test to which degree exit market timing affects abnormal returns, Table VII also shows estimation results including an investment specific bubble dummy equaling one if the deal is exited during the internet bubble (January 1998 to March 2000), and zero otherwise. As expected, estimation results reveal that abnormal returns of venture-backed IPOs were significantly higher during the internet bubble, as can be inferred from the statically significant coefficient for the bubble dummy. In contrast, abnormal returns of buyout-backed IPOs are not affected by the bubble. Interestingly, the results also suggest that the bubble had a particularly large and statistically significant effect on the abnormal returns of Sales exits of both venture capital and buyout investments.

4.4 Estimation Results Across Different Regions

Since the venture capital and buyout sample contains investments from all regions worldwide, the analysis now turns to the question whether there are systematic differences in risk and return characteristics of investments across different regions.

Table VIII illustrates estimation results across the main geographical regions. The

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[16] However, Brav and Gompers (1997) note that overpricing and associated long-run underperformance of IPOs is almost entirely concentrated in non-private equity-backed IPOs.

S&P 500 index is used as benchmark for deals of all regions in Panel A of the table, whereas Panel B shows estimation results with different benchmark indices being used for deals outside the US.[17] In the venture segment, deals done in US and Europe (ex UK) show relatively similar (and statistically significant) positive abnormal returns, but the beta coefficient of European deals is markedly lower. The lower beta of the European deals can most likely be attributed to the trend that European fund managers traditionally focused more on later-stage financing of lower technology industries (see Black and Gilson (1998) and Metrick and Yasuda (2011)).[18] Overall the results reveal that US deals do not outperform Europe (ex UK) deals on a risk-adjusted basis. At first sight, this contrasts with previous evidence from Hege et al. (2009) who conclude that European deals underperform their US peers over their sample period ranging from 1997 to 2003. This different result can, however, be explained by the fact that Hege et al. (2009) only look at *absolute* returns (as measured by the IRR). The results presented here also imply higher average *absolute* returns of US deals over the sample period, but suggest that this is due to a higher market risk and not due to higher abnormal returns of the deals.

Additionally, it is interesting to see that Rest of World deals show the largest outperformance in the venture segment, both with respect to the S&P 500 and to the MSCI World index used in Panel B. Note that estimation results for UK venture capital investments are not shown in the table because the overall number of deals (232) is too small to draw any reliable inferences.

In the buyout segment, estimation results with the S&P 500 index show large and statistically significant positive abnormal returns for US and UK, slightly lower abnormal returns for Europe (ex UK), and insignificant negative abnormal returns for Rest of

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[17] Refer to the description of Table VIII for more information on the benchmark indices being used for each region.

[18] These trends are also evident from the sample data. The fraction of later stage venture capital deals equals 32% for US compared to 40% for Europe (ex UK). Additionally, the fraction of investments into the high technology sectors “Information Technology” and “Biotechnology” in US equals 84% compared to 67% for Europe (ex UK).

World deals. This picture is slightly reversed when different benchmark indices are being used for each region. The results in Panel B suggest that deals done in Europe (ex UK) and UK show a similar outperformance that is slightly higher but not statistically different from that of deals done in US. Regarding market risk of the investments, the results reveal very similar beta coefficients of US and Europe (ex UK) deals. This suggests that there are relatively small differences in terms of deals characteristics and leverage ratios applied among the US and Europe. Consistent with this finding, Axelson et al. (2007) show that leverage ratios applied in the US and Europe exhibit similar patterns over time.

4.5 Estimation Results Across Different Time Periods

To identify cycles in alpha and beta, Table IX illustrates estimation results for different time periods. Consistent with industry and research practice deals are grouped according to their investment year, i.e., the year in which the deals were initiated.

For the venture segment, we consider three distinct periods: the pre-boom period from 1980 to 1995; the boom period from 1996 to 2000; the post-boom period from 2001 to 2005 (see Metrick and Yasuda (2011) for a similar definition).

The results in Panel A of Table IX reveals that the abnormal performance and systematic risk of venture capital investments vary considerably over time. For deals done in the pre-boom period, the results suggest a statistically significant underperformance of -6.4% p.a. with respect to the S&P 500 and a large market beta of 3.49. The large market beta can be attributed to the fact that venture capital investments in this pre-boom period were mainly concentrated in the early-stage segment (see Metrick and Yasuda (2011)). The underperformance in this period can be explained by the high competition for hot startups, many inexperienced venture capital fund managers entering the market, and unfavorable exit opportunities due to a cooling market for IPOs in the mid-1980s that completely collapsed after the stock market crash in 1987. In the period from 1996 to 2000, during the dot-com boom, the abnormal performance increased substantially to an annual alpha of 26.3%. This finding is consistent with anecdotal evidence that some of the most profitable deals were being made during this time period. The main reason for this large outperformance was a huge surge of interest in the nascent internet and other computer technologies that could easily be exited with high profits due to the flourishing IPO market at that time. The excellent exit opportunities during this time are also illustrated by the high level of delta. Interestingly, the beta during this period somewhat decreased to 2.56. This decrease can partly be explained by the trend towards later-stage investing as angel investors had largely replaced venture capital fund managers in the early-stage segment by the late 1990s. In addition, the growth options of many investments were far in-the-money during this boom period which also decreased beta because of lower average option leverage of the deals. With the NASDAQ crash and technology slump that started in March 2000 the entire venture capital industry virtually collapsed. Over the next years, many venture firms had been forced to write-off large proportions of their investments as valuations for startup technology companies crumbled away. Consistent with collapsing valuations and vanishing profitable exit opportunities the results in Table IX show that alpha decreased to an statistically insignificant -0.9% p.a. In contrast, beta was relatively similar compared to the previous bubble period.

The performance of buyout contrasts considerably with that of venture capital over time, as can be inferred from Panel B of Table IX. The results indicate that buyout investments consistently outperform the S&P 500 over the sample period, which confirms previous evidence presented by Higson and Stucke (2012) and Harris et al. (2013) based on IRR spreads and public market equivalents (PMEs). Deals done in the 1980s show a statistically significant outperformance of 4.3% p.a. and a large market beta of 2.01. The large market beta is consistent with high leverage ratios applied particularly in the second half of the 1980s (see Higson and Stucke (2012)). Investments in the 1990s show a relatively similar outperformance of 5.0% p.a. but a considerably lower beta of 1.35. The lower beta of the deals is, at least, partly due to lower average leverage ratios applied in the 1990s. Additionally, the sound economic conditions in the second half of the 1990s led to rising equity valuations and the ability to redeem high initial debt quickly which further decreased average leverage ratios over the deal lifetimes. Investments from 2000 onwards show a large abnormal return of 16.8% p.a. and a large market beta of 3.30. The likely reasons for this pattern are relatively low entry multiples until the mid-2000s, unprecedented levels of cheap debt, and leveraged recapitalizations of the investments. However, note that the sample employed includes only fully realized deals. Because buyout funds tend to hang on to their losers (see also Degeorge et al. (2013)), the estimated alpha from 2000 onwards might to some degree be upward biased.

4.6 Estimation Results Across Different Industries

Table X illustrates differences among the risk and return characteristics of investments made in different industries. The S&P 500 total return index is being used as proxy for market returns in Panels A and C to explore abnormal returns and systematic risk with respect to the overall stock market. Additionally, value-weighted returns on all NYSE, AMEX, and NASDAQ stocks of the corresponding industries are being used as proxies for market returns in Panels B and D to explore abnormal returns and systematic risk with respect to traded stocks of the same industry.[19]

Table X reveals interesting patterns regarding abnormal returns across industries. When the S&P 500 is being used as a proxy for market returns, the results indicate that venture capital investments in the high-growth sectors “Information Technology” and “Biotechnology” earn large positive and statistically significant abnormal returns, while venture capital investments in the lower-growth sectors “Consumer Industry” and “Industrials” underperformed the S&P 500. At first glance this is consistent with general view that venture capitalists possess special selection and monitoring skills that can best

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[19] Data obtained from the website of Kenneth R. French: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

be exploited with investments in high-growth sectors (see Sahlman (1990) and Kaiser and Westarp (2010)). However, this picture gets more refined when corresponding industry returns are being used as relevant benchmarks. The results in Panel B reveal that investments in "Information Technology" did not outperform traded stocks of the same industry on a risk-adjusted basis and that the outperformance of investments in "Biotechnology" drops substantially to 4.8% p.a. when compared to traded stocks of this sector. These striking differences in abnormal returns suggest that the large outperformance of investments in "Information Technology" and "Biotechnology" with respect to the S&P 500 is mainly a consequence of sound industry timing (i.e., venture capitalists invested into these sectors at times when they outperformed the overall stock market) and not of refined selection and monitoring skills. Some additional indication for the positive industry timing abilities of venture capitalists can also be gained by comparing the sample distribution by industries in Figure 2(b) with the industry performance cycles illustrated in Figure 4. The comparison reveals that the number of "Information Technology" deals increased substantially in the late 1990s when hitech stocks outperformed the S&P 500 index.

In the buyout segment, "Consumer Industry" investments do not show economically significant outperformance with respect to the S&P 500 or with respect to corresponding industry returns. In contrast, investments in "Industrials" significantly outperform the S&P 500 and corresponding traded stocks from the same industry. The outperformance with respect to industry returns is markedly lower which, again, implies some positive timing effects in this sector. Investments in "Information Technology" do not show statistically significant out- or underperformance, whereas "Biotechnology" investments show large and statistically significant outperformance with respect to the S&P 500 and to traded stocks of the same industry.

Regarding the exposure to market risk, venture capital and buyout investment show relatively similar patterns. The results imply a large exposure to overall market risk (as measured relative to the S&P 500) of investments in "Information Technology" and

"Consumer Industry", while investments in "Biotechnology" and buyout investments in "Industrials" show a low exposure. The large market betas of "Information Technology" and "Consumer Industry" investments is consistent with these sectors being highly cyclical in the sense that investment returns strongly depend on the overall state of the economy. The low beta of buyout investments in "Industrials" is most likely a direct result of a large fraction of deals being made in defensive sectors, such as Energy and Waste/Recycling (10%), Transportation (11%), Business Support Services (19%), and Materials (25%). Finally, the low market beta of investments in "Biotechnology" reflects that the risks of investments in this sector are mainly driven by technological uncertainty that represents diversifiable (i.e., unsystematic) risk.[20] In addition, previous studies point out that healthcare firms are primarily financed by equity because of long research and development cycles and relatively low scientific success rates that prevent high leverage ratios (see Harrington (2009)).

5 Summary and Conclusion

This paper develops a novel econometric methodology to estimate the risk and return characteristics of private equity investments from cash flow data. The methodology is validated using detailed Monte-Carlo simulations and is applied to a comprehensive sample of 10,798 fully liquidated private equity investments. The estimation results extend previous research in that they provide the first comprehensive analysis of the differences in systematic risk and abnormal returns across different time periods, exit routes, regions, industries, and across companies with different characteristics, such as their stage of development in the venture segment.

It should be acknowledged that although this paper focuses on private equity investments, the developed estimation methodology can also be used for other non-traded

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[20] For example, the technical uncertainty associated with success or failure during product development and approval of a new drug typically reflects unique risk that can be diversified.

alternative asset investments (e.g. infrastructure, real estate, and mezzanine) and for corporate investments in case that one can only observe a stream of cash flows but no market valuations.

A Appendix: Derivation of the Estimation Function

To derive the estimation approach presented in Theorem 2.1, first note that under Equation (2) (**Assumption 2**), expected dividends of an investment i in period-k are given by

$$E[\Delta D_{i,k}] = E[\delta_{i,k} V_{i,k-1}] = \bar{\delta}_{i,k} E[V_{i,k-1}], \tag{18}$$

with $\bar{\delta}_{i,k} \equiv E[\delta_{i,k}] = \frac{\tau}{\tau_i} \delta k$. The second equality in Equation (18) because it is implicitly assumed here that the dividend rate and value dynamics are uncorrelated.[21] As outlined above, one can only observe a stream of cash flows (i.e., capital inflows and dividends) for each investment i. Therefore, the value $V_{i,k-1}$ that enters the expectation on the right hand side of (18) is unobservable. However, it can be expressed in terms of observable variables. A starting point for this is the specification of the value dynamics of a private equity investment (**Assumption 1 and 2**) given by

$$V_{i,k} = V_{i,k-1}(1 + R_{i,k} - \delta_{i,k}) + \Delta T_{i,k}. \tag{19}$$

While the value here still enters the right hand side of the equation, it can be eliminated as follows. First, note that $V_{i,0} = 0$ holds for all investments, i.e., the value of all investments is zero when they are set-up and no capital inflow has yet occurred. Based on this condition, we can recursively solve Equation (19) for the value at any point in

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[21] The extend to which this assumption introduces a bias into the estimation methodology is analyzed in Section 2.3. Using Monte Carlo simulations we show that under more general specifications, assuming various cross-dependencies of the dividend rate, the precision of the estimates is largely unaffected.

time. It turns out:

$$
\begin{aligned}
V_{i,0} =& 0, && (20)\\
V_{i,1} =& \Delta T_{i,1}, && (21)\\
V_{i,2} =& V_{i,1}(1 + R_{i,2} - \delta_{i,2}) + \Delta T_{i,2} &&\\
=& \Delta T_{i,1}(1 + R_{i,2} - \delta_{i,2}) + \Delta T_{i,2}, && (22)\\
V_{i,3} =& V_{i,2}(1 + R_{i,3} - \delta_{i,3}) + \Delta T_{i,3} &&\\
=& \Delta T_{i,1}(1 + R_{i,2} - \delta_{i,2})(1 + R_{i,3} - \delta_{i,3}) &&\\
& + \Delta T_{i,2}(1 + R_{i,3} - \delta_{i,3}) + \Delta T_{i,3} && (23)\\
& \vdots &&\\
V_{i,k} =& \sum_{j=1}^{k} \Delta T_{i,j} \prod_{s=j+1}^{k} (1 + R_{i,s} - \delta_{i,s}). && (24)
\end{aligned}
$$

Substituting the return dynamics (**Assumption 3**) from Equation (6) into (24) yields

$$
V_{i,k} = \sum_{j=1}^{k} \Delta T_{i,j} \prod_{s=j+1}^{k} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) + \epsilon_{i,s} - \delta_{i,s}]. \tag{25}
$$

This equation states that the value of an investment i is the sum of compounded capital inflows, where compounding is carried out with the one-factor market model that is corrected for the periodical dividends. Taking expectations on both sides of Equation (25) gives

$$
E[V_{i,k}] = E\left\{ \sum_{j=1}^{k} \Delta T_{i,j} \prod_{s=j+1}^{k} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) - \bar{\delta}_{i,s}] \right\}. \tag{26}
$$

Note that the error term $\epsilon_{i,s}$ does no longer need to appear in the expectation on the right hand side of Equation (26). This follows as $\epsilon_{i,s}$ has zero expectation. Moreover, the error term $\epsilon_{i,s}$ and the market return $R_{M,t}$ are uncorrelated for all t and s and the expectations of cross-products of the form $\epsilon_{i,s}\epsilon_{i,t}$ (as well as higher-order cross-products) are equal to zero for $s \neq t$. Additionally, we have replaced $\delta_{i,s}$ by its expectation $\bar{\delta}_{i,s} = \frac{\tau}{\tau_i}\delta s$, which is again feasible under the assumption that the dividend rate is independently distributed.

Inserting (26) into Equation (18), the expected dividends of investment i in period-k can be represented by

$$E[\Delta D_{i,k}] = \bar{\delta}_{i,k} E\left\{\sum_{j=1}^{k-1} \Delta T_{i,j} \prod_{s=j+1}^{k-1} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) - \bar{\delta}_{i,s}]\right\}. \tag{27}$$

Using this specification, and assuming a sufficiently large sample of N investments that satisfy the cross-sectional restrictions given in **Assumption 4**, expected dividends of the sample investments can be approximated by averaging across all N investments, i.e.,

$$E[\Delta D_k] = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{k-1} \bar{\delta}_{i,k}\Delta T_{i,j} \prod_{s=j+1}^{k-1} [1 + r_{f,s} + \alpha + \beta(R_{M,s} - r_{f,s}) - \bar{\delta}_{i,s}]. \tag{28}$$

$E[\Delta D_k]$ gives the expected dividends of the sample investments predicted by the model for period-k. In addition, as the dividends of the sample investments can be observed directly, we can calculate the average sample dividends in each period-k, i.e.,

$$\Delta D_k = \frac{1}{N}\sum_{i=1}^{N} \Delta D_{i,k}. \tag{29}$$

$E[\Delta D_k]$ in Equation (28) gives the expected model dividends for period-k, whereas ΔD_k in Equation (29) gives the empirical average dividends of the sample investments for the same period. Given these two types of information, the idea is now to estimate the parameters α, β and δ by minimizing the distance between the empirical averages and model expectations over time. This can be done by a *non-linear least squares* estimation. For a total observation period of length K, the goal function for the estimation is then given by the optimization problem

$$\min_{\alpha,\beta,\delta} \sum_{k=1}^{K} \left(\Delta D_k - E[\Delta D_k]\right)^2, \tag{30}$$

where $E[\Delta D_k]$ and ΔD_k are given by Equation (28) and (29), respectively.

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Figures and Tables

Table I: Monte Carlo Simulation Parameters

This table reports the base case parameters used in the Monte Carlo simulation experiment. Model parameters represent monthly values, if not stated otherwise.

Parameter	Symbol	Value
Riskless rate (p.a.)	r_f	0.05
Alpha	α	0.00
Beta	β	2.50
Dividend rate (p.a.)	δ	0.18
Investment rate (p.a.)	γ	0.80
Minimum market return	c_M	-0.20
Lognormal mean market	μ_M	-1.58
Lognormal volatility market	σ_M	0.18
Minimum error term	c_ϵ	-0.43
Lognormal mean error term	μ_ϵ	-1.17
Lognormal volatility error term	σ_ϵ	0.80

Table II: Monte Carlo Simulation

This table presents the estimation results from the Monte Carlo simulation experiment. The simulation is carried out using 5,000 iterations, i.e., parameters are estimated from a sample of 5,000 investments. Investment returns are modeled by a single-factor market model, for which market returns and error terms are assumed to follow a shifted log-normal distribution. In the base case of Panel A, idiosyncratic volatility is matched to that of Korteweg and Sorensen (2010) at 40% per month. Idiosyncratic volatility is set to 20% per month and 60% per month in the lower and higher volatility case, respectively. The last two columns of Panel A show the results when the sample size is reduced to 1,000 and increased to 10,000 observations. In Panel B, we relax the assumption that the dividend rate is deterministic and analyze how different cross-dependencies between the dividend rate and other model variables affect estimation precision (see text for details). All simulations are repeated 1,000 times with the true parameters set to alpha=0, beta=2.5, and delta=0.18. The mean, median, standard deviation, and interquartile range are based on 1,000 sets of estimated parameters.

Panel A	True Model	Idiosyncratic Volatility			Sample Size	
	Parameters	Base Case	Low	High	1,000	10,000
Alpha						
mean	0.00%	-0.01%	0.00%	-0.02%	-0.02%	0.00%
median		-0.07%	0.00%	-0.28%	-0.19%	0.00%
std.		0.52%	0.08%	1.49%	1.04%	0.19%
interquartile		[-0.24%,0.11%]	[-0.05%,0.05%]	[-0.68%,0.31%]	[-0.50%,0.23%]	[-0.10%,0.08%]
Beta						
mean	2.50	2.51	2.50	2.56	2.52	2.50
median		2.52	2.50	2.51	2.51	2.50
std.		0.65	0.11	1.98	1.33	0.12
interquartile		[2.23,2.77]	[2.43,2.56]	[1.76,3.28]	[1.91,3.12]	[2.41,2.58]
Delta						
mean	0.18	0.18	0.18	0.19	0.18	0.18
median		0.18	0.18	0.19	0.18	0.18
std.		0.01	0.00	0.03	0.02	0.00
interquartile		[0.17,0.19]	[0.18,0.18]	[0.18,0.20]	[0.17,0.20]	[0.18,0.18]

Table II continued

Panel B	True Model	Dividend Rate Stochastic			Dividend Rate Dependent on...	
	Parameters	Base Case	Corr=0.25	Corr=0.5	Alpha	Beta
Alpha						
mean	0.00%	-0.03%	0.01%	0.03%	0.08%	0.08%
median		-0.07%	-0.07%	-0.05%	0.00%	-0.04%
std.		0.43%	0.59%	0.70%	0.60%	1.09%
interquartile		[-0.25%,0.14%]	[-0.24%,0.15%]	[-0.24%,0.17%]	[-0.18%,0.20%]	[-0.23%,0.21%]
Beta						
mean	2.50	2.53	2.55	2.57	2.43	2.49
median		2.52	2.54	2.56	2.47	2.50
std.		0.77	0.91	0.95	0.90	0.97
interquartile		[2.18,2.89]	[2.54,2.90]	[2.21,2.94]	[2.11,2.80]	[2.17,2.83]
Delta						
mean	0.18	0.18	0.18	0.18	0.18	0.18
median		0.18	0.18	0.18	0.18	0.18
std.		0.01	0.01	0.02	0.01	0.01
interquartile		[0.18,0.19]	[0.17,0.20]	[0.17,0.20]	[0.18,0.19]	[0.17,0.19]

Table III: Descriptive Statistics

This table shows descriptives for the investment data provided by CEPRES. The overall dataset includes 10,798 liquidated private equity investments that were started between 1980 and 2009. The follwing stage definitions are used: Venture capital (VC) represent the universe of all early- and later-stage venture investing. Buyout (BO) represent the universe of all growth and leveraged buyout investing.

	All Deals	VC Deals	BO Deals
Number of Observations			
absolute	10,798	6,380	4,418
relative	100.00%	59.09%	40.91%
Investment Size (in USD Mio.)			
mean	12.01	7.25	18.89
median	4.52	3.14	7.44
std.	74.85	90.49	42.33
Region			
US	57.09%	72.51%	34.83%
UK	14.05%	3.64%	29.09%
Europe (ex. UK)	19.86%	16.10%	25.31%
Rest of World	8.99%	7.75%	10.77%
Industry			
Industrials	15.43%	7.57%	26.78%
Consumer Industry	23.65%	11.90%	40.63%
Information Technology	45.24%	63.71%	18.56%
Biotechnology	11.99%	14.86%	7.85%
Others/Unspecified	3.69%	1.96%	6.18%
Exit Type			
IPO	12.55%	13.53%	11.14%
Sale/Merger	33.55%	29.51%	39.38%
Write-Off	21.06%	28.51%	10.30%
Unspecified	32.84%	28.45%	39.18%
Investment Duration (in Years)			
mean	4.25	4.07	4.50
median	3.83	3.67	4.08
std.	2.77	2.76	2.76

Table IV: Sample Distribution

This table shows the distribution of venture capital and buyout deals by investment year. We show numbers for all buyout and venture capital investments worldwide (All), as well as for the respective sub-samples of venture capital and buyout deals by geographic location of the companies.

	Venture Capital Deals					Buyout Deals				
	All	US	UK	Europe (ex UK)	Rest of World	All	US	UK	Europe (ex UK)	Rest of World
1980	0	0	0	0	0	0	0	0	0	0
1981	22	22	0	0	0	0	0	0	0	0
1982	32	32	0	0	0	3	2	0	0	1
1983	40	36	0	4	0	4	1	1	0	2
1984	50	44	0	6	0	13	9	2	1	1
1985	56	52	0	4	0	28	14	12	2	0
1986	88	78	0	10	0	48	31	13	3	1
1987	83	75	1	7	0	54	20	21	11	2
1988	124	114	1	6	3	85	46	25	10	4
1989	149	136	2	7	4	90	55	18	10	7
1990	134	113	4	15	2	139	56	40	31	12
1991	159	100	10	45	4	196	63	66	45	22
1992	162	127	8	22	5	223	71	97	35	20
1993	198	144	12	34	8	219	82	79	44	14
1994	223	153	7	52	11	371	116	142	84	29
1995	280	189	10	49	32	338	110	131	71	26
1996	377	275	11	63	28	363	107	133	103	20
1997	407	301	7	65	34	423	121	132	143	27
1998	564	413	12	91	48	355	127	95	98	35
1999	911	671	32	152	56	356	116	74	131	35
2000	1,196	752	64	243	137	393	159	65	100	69
2001	401	265	19	82	35	194	62	39	54	39
2002	226	161	6	27	32	172	49	34	53	36
2003	191	141	11	18	21	151	57	18	46	30
2004	171	127	7	15	22	97	30	23	24	20
2005	87	61	7	8	11	66	23	13	14	16
2006	36	33	0	1	2	31	8	12	3	8

Table IV continued

	Venture Capital Deals					Buyout Deals				
	All	US	UK	Europe (ex UK)	Rest of World	All	US	UK	Europe (ex UK)	Rest of World
2007	13	11	1	1	0	5	3	0	2	0
2008	0	0	0	0	0	1	1	0	0	0
2009	0	0	0	0	0	0	0	0	0	0
Total	6,380	4,626	232	1,027	495	4,418	1,539	1,285	1,118	476

Table V: Market Model Estimation Results

Panel A reports the estimated abnormal performance (Alpha p.a.), market risk (Beta Market), and dividend rate (Delta p.a.) using the one-factor market model. The S&P 500 total return index is used as proxy for market returns and the one-month US Treasury Bill rate is employed as the risk-free rate. Standard errors of the estimated coefficients are given in parentheses and are derived from the Hessian matrix of the estimates. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. Below each estimation, the root mean squared error (RMSE) and the coefficient of determination (R^2) are reported to indicate the goodness-of-fit of the estimation. In Panel B, market return and riskless rate give the weighted averages of the S&P 500 return and the US Treasury Bill rate over the sample period. Weighted averages are calculated by weighting monthly returns by the number of deals running in each month. The reported values are annualized by multiplying monthly estimates by 12. The risk premium is the product of the estimated beta coefficient times the average market return in excess of the average riskless rate. The cost of capital according to the CAPM is the sum of the average riskless rate plus the risk premium. Expected return according to the market model is the sum of the cost of capital plus the estimated alpha. For comparison, Panel B also reports the average IRRs of the sample investments.

	Venture Capital	Buyout
Panel A: Model Estimates		
Alpha (p.a.)	0.089***	0.070***
	(0.018)	(0.014)
Beta Market	2.567***	2.248***
	(0.204)	(0.127)
Delta (p.a.)	0.183***	0.173***
	(0.001)	(0.002)
No. Obs.	6,380	4,418
RMSE	0.0054	0.0053
R^2	75.10%	83.73%
Panel B: Cost of Capital and Expected Returns		
Market Return (p.a.)	0.086	0.109
Riskless Rate (p.a.)	0.040	0.041
Risk Premium (p.a.)	0.116	0.155
Cost of Capital (CAPM; p.a.)	0.157	0.195
Expected Return (Market Model; p.a.)	0.246	0.266
Average IRR (Sample; p.a.)	0.249	0.254

Table VI: Estimation Results Across Stages

This table reports estimation results for different stage specifications using the one-factor market model. The S&P 500 total return index is used as proxy for market returns and the one-month US Treasury Bill rate is employed as the risk-free rate. Standard errors of the estimated coefficients are given in parentheses and are derived from the Hessian matrix of the estimates. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. Below each estimation, the root mean squared error (RMSE) and the coefficient of determination (R^2) are reported to indicate the goodness-of-fit of the estimation.

	Venture Capital			Buyout		
	All	Early Stage	Later Stage	All	Leveraged Buyout	Growth
Alpha (p.a.)	0.089***	-0.022*	0.169***	0.070***	0.058***	0.119***
	(0.018)	(0.012)	(0.053)	(0.014)	(0.014)	(0.017)
Beta Market	2.567***	3.663***	1.871***	2.248***	2.357***	1.748***
	(0.204)	(0.128)	(0.666)	(0.127)	(0.125)	(0.199)
Delta (p.a.)	0.183***	0.166***	0.210***	0.173***	0.177***	0.155***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)
No. Obs.	6,380	4,284	2,096	4,418	3,613	805
RMSE	0.0054	0.0056	0.0062	0.0053	0.0053	0.0068
R^2	75.10%	70.93%	75.26%	83.73%	84.64%	69.15%

Table VII: Estimation Results Across Exit Routes

This table reports estimation results for different exit routes using the following one-factor market model specification: $R_{i,t} = r_{f,t} + (\alpha + \alpha_{Dummy} Dummy_i) + \beta_M(R_{M,t} - r_{f,t}) + \epsilon_{i,t}$, where $Dummy_i$ is an investment specific dummy variable that equals one if the deal is exited during the bubble period (January 1998 to March 2000), and zero otherwise. The S&P 500 total return index is used as proxy for market returns and the one-month US Treasury Bill rate is employed as the risk-free rate. Standard errors of the estimated coefficients are given in parentheses and are derived from the Hessian matrix of the estimates. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. Below each estimation, the root mean squared error (RMSE) and the coefficient of determination (R^2) are reported to indicate the goodness-of-fit of the estimation.

	Venture Capital				Buyout			
	IPO	IPO	Sales	Sales	IPO	IPO	Sales	Sales
Alpha (p.a.)	0.626***	0.412*	0.291*	-0.089***	0.526***	0.528***	0.090***	-0.018
	(0.095)	(0.241)	(0.150)	(0.021)	(0.037)	(0.090)	(0.010)	(0.033)
Alpha Dummy (p.a.)		0.372**		0.915***		-0.004		0.231***
		(0.189)		(0.031)		(0.568)		(0.037)
Beta Market	0.829	1.850	1.517	1.694***	0.527	0.517	2.319***	2.566***
	(0.972)	(1.391)	(2.006)	(0.112)	(0.328)	(1.943)	(0.085)	(0.154)
Delta (p.a.)	0.352***	0.406***	0.240***	0.466***	0.346***	0.347***	0.190***	0.198***
	(0.008)	(0.025)	(0.004)	(0.009)	(0.004)	(0.004)	(0.001)	(0.003)
No. Obs.	863	863	1,883	1,883	492	492	1,740	1,740
RMSE	0.0173	0.0173	0.0071	0.0067	0.0140	0.0140	0.0058	0.0059
R^2	72.25%	72.14%	75.17%	77.72%	73.42%	73.41%	84.14%	83.80%

Table VIII: Estimation Results Across Regions

This table reports estimation results for different regions using the one-factor market model. In Panel A, the S&P 500 total return index is used as proxy for market returns and the one-month US Treasury Bill rate is employed as the risk-free rate. In Panel B, different total return indices are used for different regions: S&P 500 for US; MSCI Europe ex UK for Europe (ex UK); MSCI UK for UK; MSCI World for Rest of World. Standard errors of the estimated coefficients are given in parentheses and are derived from the Hessian matrix of the estimates. Note that estimation results for UK venture capital investments are not shown because the overall number of deals (232) is too small to draw any reliable inferences. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. Below each estimation, the root mean squared error (RMSE) and the coefficient of determination (R^2) are reported to indicate the goodness-of-fit of the estimation.

	Venture Capital			Buyout			
	US	Europe (ex UK)	Rest of World	US	Europe (ex UK)	UK	Rest of World
Panel A: Benchmark Index S&P 500							
Alpha (p.a.)	0.116***	0.096***	0.153***	0.067***	0.041**	0.119***	-0.025
	(0.026)	(0.024)	(0.010)	(0.015)	(0.016)	(0.014)	(0.018)
Beta Market	2.493***	1.423***	2.270***	2.515***	2.800***	1.438***	2.934***
	(0.306)	(0.348)	(0.135)	(0.136)	(0.145)	(0.122)	(0.174)
Delta (p.a.)	0.198***	0.117***	0.178***	0.174***	0.170***	0.187***	0.175***
	(0.001)	(0.004)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)
No. Obs.	4,626	1,027	495	1,539	1,118	1,285	476
RMSE	0.0060	0.0056	0.0110	0.0078	0.0063	0.0052	0.0063
R^2	73.04%	64.81%	42.27%	72.46%	80.58%	84.07%	66.53%
Panel B: Different Benchmarks Indices							
Alpha (p.a.)	0.116***	0.099***	0.156***	0.067***	0.089***	0.091***	-0.037***
	(0.026)	(0.017)	(0.009)	(0.015)	(0.006)	(0.010)	(0.013)
Beta Market	2.493***	1.427***	2.230***	2.515***	2.865***	2.687***	3.280***
	(0.306)	(0.313)	(0.128)	(0.136)	(0.086)	(0.158)	(0.160)
Delta (p.a.)	0.198***	0.116***	0.177***	0.174***	0.147***	0.172***	0.173***
	(0.001)	(0.002)	(0.002)	(0.002)	(0.004)	(0.003)	(0.002)
No. Obs.	4,626	1,027	495	1,539	1,118	1,285	476
RMSE	0.0060	0.0057	0.0111	0.0078	0.0063	0.0051	0.0063
R^2	73.04%	64.25%	41.71%	72.46%	80.38%	84.41%	66.66%

Table IX: Estimation Results Over Time

This table reports estimation results for different periods using the one-factor market model. The S&P 500 total return index is used as proxy for market returns and the one-month US Treasury Bill rate is employed as the risk-free rate. Standard errors of the estimated coefficients are given in parentheses and are derived from the Hessian matrix of the estimates. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively.

Investment Years	Alpha (p.a.)	Beta Market	Delta (p.a.)	R^2	No. Obs.
Panel A: Venture Capital					
1980-1995	-0.064***	3.494***	0.102***	65.10%	1,800
	(0.013)	(0.102)	(0.002)		
1996-2000	0.263***	2.559***	0.232***	70.05%	3,455
	(0.004)	(0.107)	(0.004)		
2001-2005	-0.009	2.771***	0.101***	60.08%	1,076
	(0.018)	(0.409)	(0.002)		
Panel B: Buyout					
1980-1989	0.043***	2.010***	0.150***	57.67%	325
	(0.006)	(0.111)	(0.002)		
1990-1999	0.050***	1.350***	0.127***	83.23%	2,983
	(0.009)	(0.076)	(0.002)		
2000-2005	0.168***	3.301***	0.177***	81.11%	1,073
	(0.006)	(0.163)	(0.002)		

Table X: Estimation Results Across Industries

This table reports estimation results for different industries using the one-factor market model. The S&P 500 total return index is used as proxy for market returns in Panels A and C. A value-weighted return on all NYSE, AMEX, and NASDAQ stocks of the corresponding industry is used as proxy for market returns in Panels B and D (data obtained from `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). The one-month US Treasury Bill rate is employed as the risk-free rate. Standard errors of the estimated coefficients are given in parentheses and are derived from the Hessian matrix of the estimates. ***, ** and * denotes statistical significance at the 1%, 5% and 10% level, respectively. Below each estimation, the root mean squared error (RMSE) and the coefficient of determination (R^2) are reported to indicate the goodness-of-fit of the estimation.

	Information Technology	Biotechnology	Consumer Industry	Industrials
Panel A: Venture Capital Against S&P 500				
Alpha (p.a.)	0.080***	0.196**	-0.099***	-0.045***
	0.015	0.089	0.022	0.018
Beta	3.511***	0.805	3.185***	2.744***
	0.168	1.185	0.210	0.175
Delta	0.231***	0.128***	0.104***	0.082***
	0.002	0.003	0.002	0.002
No. Obs.	4,193	909	586	526
RMSE	0.0063	0.0073	0.0079	0.0077
R^2	69.16%	67.28%	54.95%	60.94%
Panel B: Venture Capital Against Industry Returns				
Alpha (p.a.)	0.001	0.048**	-0.006***	-0.051***
	0.015	0.019	0.002	0.010
Beta	3.930***	1.830	3.736***	3.773***
	0.257	0.302	0.073	0.172
Delta	0.204***	0.103***	0.097***	0.077***
	0.001	0.006	0.002	0.003
No. Obs.	4,193	909	586	526
RMSE	0.0062	0.0072	0.0078	0.0077
R^2	70.62%	67.81%	55.46%	61.07%

Table X continued

	Information Technology	Biotechnology	Consumer Industry	Industrials
Panel C: Buyout Against S&P 500				
Alpha (p.a.)	-0.023	0.221***	0.004	0.185***
	0.017	0.013	0.013	0.021
Beta	3.741***	1.493***	2.480***	1.110***
	0.148	0.117	0.118	0.211
Delta	0.209***	0.196***	0.141***	0.188***
	0.002	0.002	0.002	0.003
No. Obs.	905	352	1,579	1,272
RMSE	0.0081	0.011	0.0053	0.0059
R^2	71.08%	65.79%	81.27%	80.40%
Panel D: Buyout Against Industry Returns				
Alpha (p.a.)	0.040	0.218***	0.009***	0.052***
	0.043	0.019	0.003	0.014
Beta	2.668***	1.273***	4.375***	2.981***
	0.361	0.154	0.107	0.198
Delta	0.208***	0.206***	0.125***	0.165***
	0.003	0.002	0.002	0.004
No. Obs.	905	352	1,579	1,272
RMSE	0.0082	0.0112	0.0054	0.0056
R^2	70.51%	65.06%	80.89%	82.19%

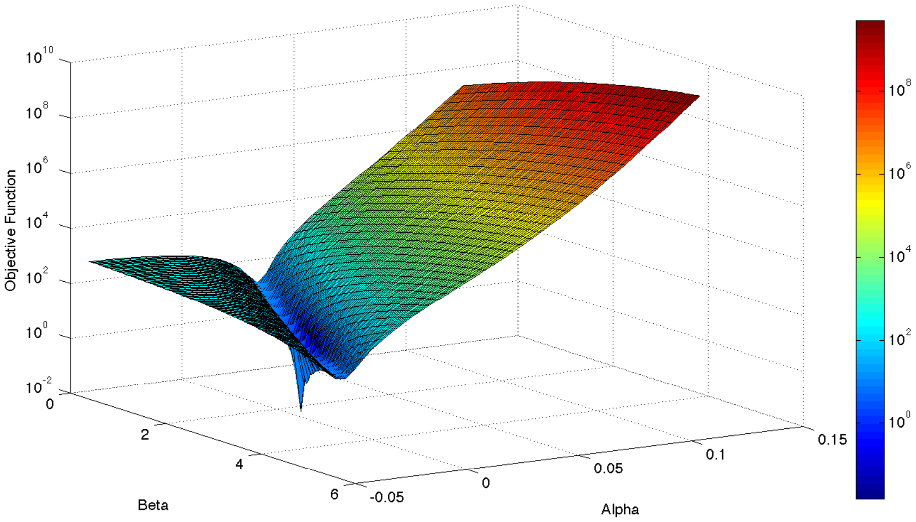
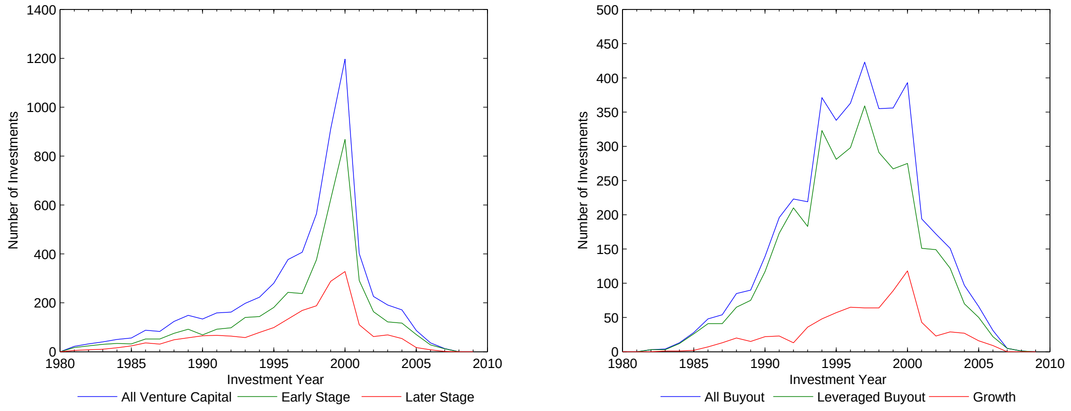
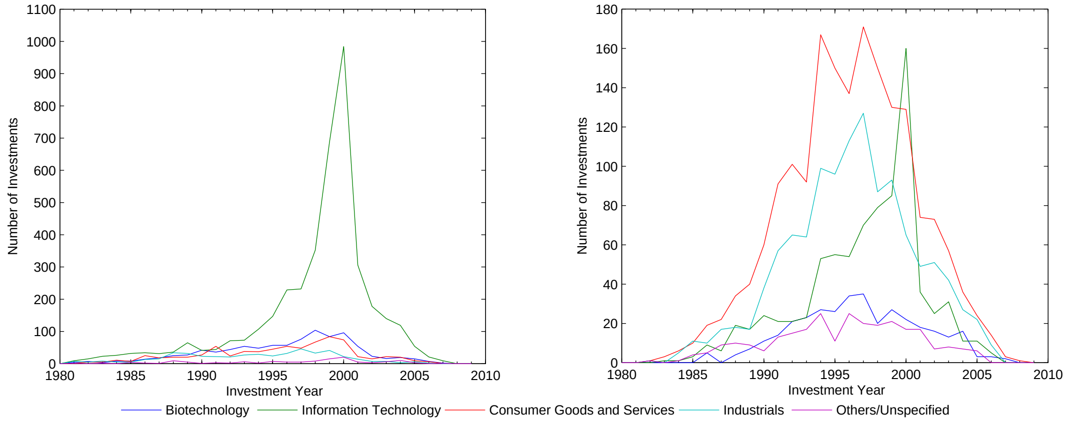


Figure 1: Objective Function Space of Parameters Alpha and Beta This figure illustrates the simulated objective function space of parameters alpha and beta. The objective function space is plotted for the optimal value of delta.



(a) Sample Distribution by Sub-Stages: Venture Capital (Left) and Buyout (Right)



(b) Sample Distribution by Industries: Venture Capital (Left) and Buyout (Right)

Figure 2: Sample Distribution of the Venture Capital and Buyout Deals by Sub-Stages and Industries This figure shows the distribution of venture capital and buyout deals by investment year. Part (a) illustrates the sample split by sub-stages and part (b) by industries.

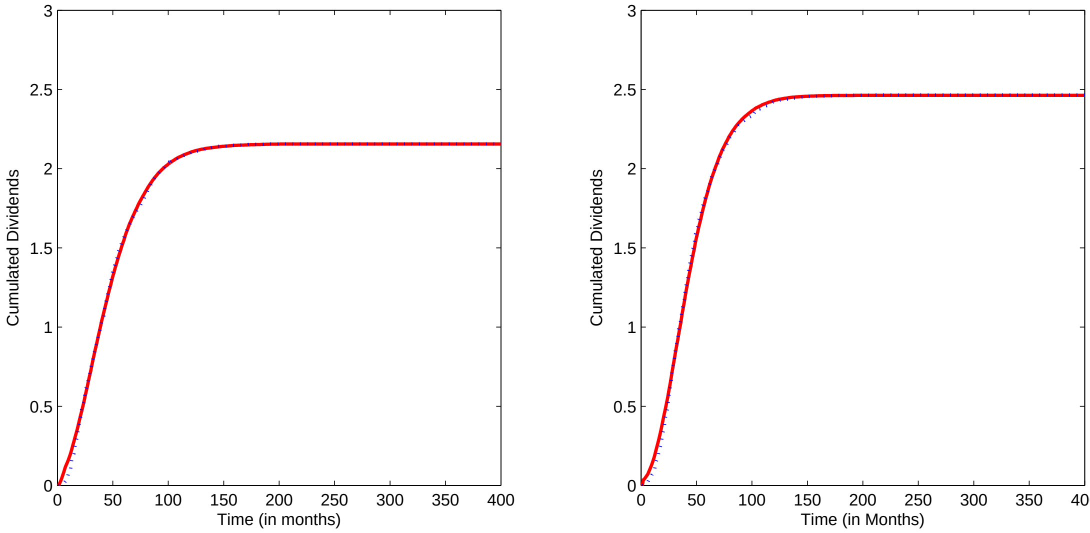


Figure 3: Goodness-of-Fit: Venture Capital (left) and Buyout (right) Model expectations of the dividends are plotted as compared to sample average dividends. Solid lines represent model expectations, dotted lines represent sample means.

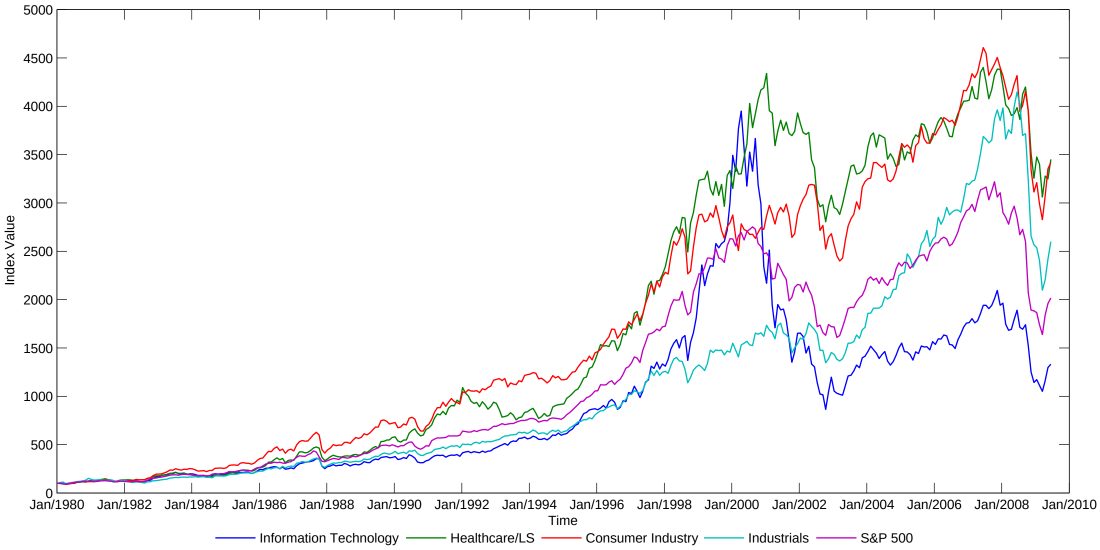


Figure 4: Industry and S&P 500 Index Performance Industry performance is measured using value-weighted returns on all NYSE, AMEX, and NASDAQ stocks of the corresponding industry.