

Long-Term Portfolio Management with a Structural Macroeconomic Model

Abstract

This paper aims to investigate long-term portfolio management in a fully structural macro-financial framework. First, we estimate a Dynamic Stochastic General Equilibrium (DSGE) model that describes the dynamics of the U.S. economy and financial markets. In addition to the typical macro-economic variables, the model includes financial variables such as firm market values, dividend payments, and long-term government bond returns. The model generates long-term forecasts of key variables, which are used for the dynamic asset allocation of long-term institutional investors. We show that the DSGE model outperforms an unrestricted VAR model in long-term portfolio allocation.

Keywords: Long-Term Asset Management, Dynamic Allocation, DSGE Model.

JEL Classification: C11, E44, G11.

1 Introduction

Given the population's ever-increasing life span, institutional investors and pension funds have an increasing social responsibility to allocate their funds in an optimal manner. The allocation must be made on a long-term basis and the institutional investor must account for the expected evolution of financial markets. The theoretical literature provides some guidance on how such allocations may be performed. If asset returns are independently and identically distributed (i.i.d.) and investor preferences do not change over time, then a simple buy-and-hold strategy is optimal and the optimal portfolio weights are the same across different investment horizons (Samuelson, 1969). Merton (1969, 1971) discusses multiperiod portfolio allocation under the assumption that the distribution of returns fluctuates over time. In such a case, it is optimal for the investor to hedge against adverse movements of expected returns. In addition, if the investment opportunity set depends on certain state variables, the investor should invest in a hedging portfolio based on these state variables, and thus, the long-term portfolio will differ from a buy-and-hold portfolio.

The predictability of asset returns plays a major role in the context of long-run portfolio allocation. Brennan, Schwartz, and Lagnado (1997) empirically analyze the impact of myopic versus dynamic portfolio choice. The authors find that, because of mean reversion in stock and bond returns, an investor with a long horizon will place a larger fraction of her wealth in stocks and bonds compared with an investor with a short horizon. Barberis (2000) reports that, even after parameter uncertainty is taken into account, sufficient stock return predictability remains to enable investors to allocate more to stocks. Campbell and Viceira (1999, 2001, 2002) and Campbell, Chan, and Viceira (2003) investigate several aspects of long-term investment in a VAR model with asset returns predictability. They observe that a long-term investor with a dynamic portfolio strategy should prefer stocks to cash and nominal long-term bonds because, in their model, the intertemporal hedging demand is positive for stocks and negative for nominal long-term bonds. Sangvinatsos and Wachter (2005), on the other hand, use an affine term structure model to investigate bond return predictability. They show that allowing for a time-varying bond

risk premium results in a high hedging demand for long-term bonds. This high hedging demand arises because investing in long-term bonds allows the investor to hedge against a decrease in the bond premium. Kojien, Nijman, and Werker (2010) use a model with a factor structure to evaluate the importance of the bond premium for a life cycle investor with short sales and borrowing constraints. They find that, with a time-varying bond risk premium, the hedging demand is negative for stocks and positive for bonds, although the magnitude of the effects is limited by the restrictions on short selling.

Most of the empirical work on long-term asset allocation has used reduced-form models. For instance, Barberis (2000), Campbell and Viceira (1999, 2001, 2002), and Campbell, Chan, and Viceira (2003) use a VAR(1) model to estimate the dynamics of asset returns. The VAR specification is an appealing approach because long-term forecasting can be performed in a straightforward manner. However, with this approach, the parameters that are estimated in the model are not structural (or deep) parameters and thus are likely to be affected by changes in government decisions and monetary policy, as highlighted by Lucas (1976). To address this issue, we consider a micro-founded Dynamic Stochastic General Equilibrium (DSGE) model with real and nominal rigidities, which is similar to the models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003, 2007). This approach performs well in reproducing U.S. macro data dynamics.

Several papers demonstrate that macro variables are useful for predicting bond and stock returns. See, e.g., Cochrane (1991), Lettau and Ludvigson (2001), Ang and Piazzesi (2003), Santos and Veronesi (2006), and Cooper and Priestley (2008). We therefore expect a DSGE model to capture such predictability. Because stock returns depend on dividends and thus on firm's earnings, we describe how the firm generates dividends with a model of firm's revenues and expenses (wages, investment, and taxes), following Jermann (1998) and Boldrin, Christiano, and Fisher (2001). We extend the model described by Alpanda (2013) to allow for the interaction between the business cycle and the stock market. This model builds on the work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003, 2007). We further introduce government bonds as a financing instrument

for the government and as a way for households to transfer wealth from one period to another. DSGE models have been widely used to describe term structure dynamics because the pricing kernel can be implicitly derived from macro models (Ang and Piazzesi, 2003; Hördahl, Tristani, and Vestin, 2008; Wu, 2006; Bekaert, Cho, and Moreno, 2010). A well-known limitation of DSGE macro-finance models, however, is their inability to generate time-varying risk premia (Rudebusch, Sack, and Swanson, 2007). To address this issue, we further introduce portfolio adjustment frictions (or rebalancing costs), which can be interpreted as dynamic bond and stock risk premia in the log-linearized model. This approach has two main advantages: First, it provides the DSGE model with some flexibility, so that it is able to reproduce the dynamics of bond and stock returns relatively well. Second, it allows us to investigate how changes in the risk premia will affect the hedging demands and the optimal allocation of the long-term investor.

Our DSGE macro-finance model includes ten observable variables that are based on U.S. data that were gathered over the period from 1955 to 2010. We estimate the model with Bayesian techniques, which allow us to address the complexity of the model and the relatively small sample of data that are available for the estimation. Given our focus on long-term investment, we then evaluate the ability of the DSGE model to predict the evolution of financial returns in the long term. This model performs very well in predicting bond returns in the long term, whereas forecasts of stock returns are less accurate. We then investigate the term structure of risk of the various assets across different investment horizons. We find that, in the DSGE model, the annualized volatility of stock returns decreases with the investment horizon, consistently with the mean reversion in stock returns. Annualized volatility also decreases with the horizon for rolled long-term bonds, but it increases with the horizon for short-term bonds. These patterns are very stable over time.

We then study the optimal dynamic allocation for a long-term institutional investor, adopting the approach of Campbell, Chan, and Viceira (2003). However, instead of using a VAR model to predict asset returns, we rely on our full-fledged macro-finance model. In this approach, the optimal weights are linear functions of the state variables, i.e., the

main macro variables. The constant component corresponds to the myopic allocation, whereas the components that are linear with respect to the state variables correspond to the hedging demands of the investor. Thus, the investment rule is set at the beginning of the investment process, and the portfolio weights are updated according to the evolution of the state variables. We find high hedging demands for bonds because of the large correlation between bond returns and changes in the bond premium. Hedging demands are also positive for stocks, although they decrease as risk aversion increases. This finding explains why an investor with low risk aversion holds large fractions of her wealth in stocks and bonds, whereas a very risk-averse investor favors bonds.

Finally, we evaluate the (out-of-sample) ex-post performances of investment strategies based on the DSGE model, which we compare with an unrestricted VAR(1) model. One important finding is that both types of models have a similar forecasting ability for macro variables; however, the DSGE model strongly outperforms the VAR model in forecasting financial returns. As long-term bonds are very good hedges against changes in the bond premium in the DSGE model, they are associated with high positive hedging demands. As a result, the DSGE portfolio is typically long bonds and short stocks. In contrast, long-term bonds do not help with hedging the bond premium in the VAR model, so the VAR portfolio typically comprises long stocks and short bonds. In addition, for all levels of risk aversion and investment horizons, the DSGE model exhibits higher expected returns and Sharpe ratios than the VAR model. We interpret this result to occur because of the better ability of the DSGE model to describe the dynamics of the stock and bond premia.

The remainder of the paper is organized as follows. In Section 2, we describe the theoretical DSGE model, which we will use to forecast future financial returns. In Section 3, we present the data and parameter estimates and evaluate the ability of the model to predict future financial returns, with a particular focus on out-of-sample predictability. In Section 4, we investigate the optimal dynamic asset allocation from a long-term perspective. In Section 5, we evaluate the out-of-sample performance of a long-term investor

using a DSGE model and compare this performance with that of an investor using a VAR model. The final conclusions are presented in Section 6.

2 Model

This section briefly describes the macro-financial model that is considered in this paper. The model is based on the workhorse model developed by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) and incorporates some financial aspects proposed by Alpanda (2013). We follow the presentation of the model proposed by Smets and Wouters (2007) and Alpanda (2013) and mostly focus on the new aspects of our model in this framework, i.e., the introduction of a complete term structure and portfolio adjustment frictions.

2.1 Labor Intermediaries

Labor intermediaries hire the labor services of households, aggregate them, and offer a composite labor service, h_t , to intermediate good producers. The labor service supplied by household j is denoted by $h_t^s(j)$. The composite labor service is aggregated by using a Dixit-Stiglitz aggregator (Dixit and Stiglitz, 1977):

$$h_t = \left[\int_0^1 h_t^s(j)^{(\Psi_t-1)/\Psi_t} dj \right]^{\Psi_t/(\Psi_t-1)}, \quad (1)$$

where $\psi_t = \Psi_t/(\Psi_t-1)$ is a wage markup shock.¹ At the steady state, $\psi = \Psi/(\Psi-1)$ is the gross markup of real wages received by households over the marginal rate of substitution between consumption and leisure. The aggregate labor services are then sold to the intermediate good producers. Maximization of the labor intermediaries' profit, $W_t h_t - \int_0^1 W_t(j) h_t^s(j) dj$, gives the labor demand curve of household j :

$$h_t^s(j) = \left[\frac{W_t(j)}{W_t} \right]^{-\Psi_t} h_t, \quad (2)$$

¹The dynamics of the shocks and risk premia are fully described in Section 2.8.

where W_t is the aggregate nominal wage.

2.2 Households

The economy is populated with infinitely lived households. The population of each household is denoted by N_t , which grows as $N_{t+1} = \eta N_t$. As in Fuhrer (2000), households have consumption habits, such that their utility depends on their current consumption relative to the past aggregate consumption. The habit level of consumption is defined as ζC_{t-1} , where C_{t-1} is the past level of aggregate consumption and ζ is the habit parameter. Each household j maximizes the expected utility defined over surplus consumption, $C_t(j) - \zeta C_{t-1}$, and labor supply, $h_t^s(j)$:

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} U_t(j) N_t, \quad (3)$$

with the following period utility:

$$U_t(j) = v_t \left[\frac{(C_t(j) - \zeta C_{t-1})^{1-\sigma_C}}{1-\sigma_C} \right] \exp \left(-\xi \frac{1-\sigma_C}{1+\sigma_L} (h_t^s(j))^{1+\sigma_L} \right), \quad (4)$$

where β is the time discount factor ($\beta < 1$), v_t is the preference shock that affects the discount rate, σ_C is the inverse of the elasticity of intertemporal substitution (and relative risk aversion), σ_L is the inverse of the elasticity of the labor supply, and ξ is a level parameter, which is set such that labor equals 1 at the steady state of the model.

Households own the intermediate good firms and trade shares of these firms. Household j holds $S_t(i, j)$ shares of intermediate firm i and receives $D_t(i)$ as per-share dividends. The value of a share of firm i is $V_t(i)$. In contrast to Smets and Wouters (2007) and Alpanda (2013), we also allow households to carry a portfolio of nominal zero-coupon government bonds with remaining maturities ranging from 1 to K periods. Household j holds $Q_t^{(k)}(j)$ bonds of maturity k at time t , which pay 1 dollar at the end of period $t+k-1$. The price of such a bond is $B_t^{(k)}$, where $B_t^{(0)} = 1$. This extension to a model in which households can hold stocks and bonds is an important contribution of our paper,

as it will allow us to analyze long-term investment strategies in cash, bonds, and stocks, along the lines of Campbell, Chan, and Viceira (2003), within a DSGE framework.

An important issue in the construction of macro-finance models in a DSGE framework is that, once log-linearized, DSGE models are unable to generate time-varying risk premia. This problem has been addressed in several papers, following Rudebusch, Sack, and Swanson (2007), by investigating alternative ways of introducing time variability in the risk premia. For instance, Hördahl, Tristani, and Vestin (2006) and Rudebusch and Swanson (2008) consider higher-order approximations of the DSGE model; van Binsbergen et al. (2012) and Rudebusch and Swanson (2012) introduce Epstein and Zin (1989) recursive preferences; Guvenen (2009) and De Graeve et al. (2010) allow for heterogenous agents. In this paper, we follow the approach proposed by Marzo, Soderström, and Zagaglia (2008) and Falagiarda and Marzo (2012), who describe bond market segmentation through portfolio adjustment frictions.² Given these frictions, which we also interpret as rebalancing costs, households have a preference for holding bonds of different maturities, resulting in nonzero demands for the various maturities.

We denote the rebalancing costs for the equity and bond holdings by $\Phi_{s,t}$ and $\Phi_{b,t}^{(k)}$, $k = 1, \dots, K$, respectively, where $\Phi_{b,t}^{(1)}$ is normalized to 0. As we will show later, these rebalancing costs can be interpreted as time-varying risk premia for the various risky assets. The budget constraint of household j in period t is:

$$\begin{aligned}
& N_t C_t(j) + \sum_{k=1}^{K-1} (1 + \Phi_{b,t}^{(k)}) \frac{B_t^{(k)}}{P_t} (Q_t^{(k)}(j) - Q_{t-1}^{(k+1)}(j)) + (1 + \Phi_{b,t}^{(K)}) \frac{B_t^{(K)}}{P_t} Q_t^{(K)}(j) \\
& + (1 + \Phi_{s,t}) \int_0^1 \frac{V_t(i)}{P_t} (S_t(i, j) - S_{t-1}(i, j)) di + \Phi_{w,t}(j) \leq (1 - \tau_h) \frac{W_t(j)}{P_t} N_t h_t^s(j) \\
& + (1 - \tau_d) \int_0^1 \frac{D_t(i)}{P_t} S_{t-1}(i, j) di + \frac{Q_{t-1}^{(1)}(j)}{P_t} - \frac{T_t}{P_t}, \tag{5}
\end{aligned}$$

²The justification for such adjustment frictions can be found in the “preferred habitat” theory (Modigliani and Sutch, 1966, 1967; Vayanos and Vila, 2009; Guibaud, Nosbusch, and Vayanos, 2008). Alpanda (2013) also introduces a time-varying risk premium for risky stocks; however, the shock affects the short-term bond holdings such that it cannot be extended to the case of several risky assets. See also Andrés, López-Salido, and Nelson (2004) and De Graeve et al. (2010). Buss and Dumas (2012) propose an equilibrium model in which financial trade entails deadweight transaction costs.

where P_t is the aggregate price, τ_h is the tax on labor, τ_d is the tax on dividend income, and T_t is a lump-sum tax (see McGrattan and Prescott, 2005). As in Rotemberg (1982) and Chugh (2006), wage stickiness is introduced in the form of a quadratic adjustment cost, $\Phi_{w,t}(j)$, defined as follows:

$$\Phi_{w,t}(j) = \frac{\kappa_w}{2} (\Psi - 1)(1 - \tau_h) \left[\frac{W_t(j)/W_{t-1}(j)}{(\pi\gamma)(\pi_{t-1}/\pi)^{\eta_w}} - 1 \right]^2 \frac{W_t}{P_t} N_t h_t,$$

where κ_w is the cost-of-adjustment parameter; η_w is the indexation parameter of wage adjustments to past aggregate inflation, denoted by $\pi_{t-1} = P_{t-1}/P_{t-2}$; and π is the steady-state inflation rate. Household j maximizes expected utility (3) subject to the sequence of budget constraints (5) for $t = \tau, \dots, \infty$. We denote by Λ_t the Lagrange multiplier with respect to the budget constraint at t .

2.3 Final Good Producers

Final good producers purchase goods from intermediate firms, aggregate them, and sell the final good to consumers. The composite final good is aggregated by using a Dixit-Stiglitz aggregator:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\Theta_t-1)/\Theta_t} di \right]^{\Theta_t/(\Theta_t-1)},$$

where $\theta_t = \Theta_t/(\Theta_t - 1)$ is a price markup shock. At the steady state, $\theta = \Theta/(\Theta - 1)$ is the price gross markup over the marginal cost. Maximization of the final producers' profit, $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$, yields the demand curve for the intermediate goods:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\Theta_t} Y_t. \quad (6)$$

2.4 Intermediate Good Producers

Intermediate good producers own capital stock and are price setters in the goods market. The production function of intermediate good producer i is:

$$Y_t(i) = z_t [u_t(i) K_{t-1}(i)]^\alpha [A_t N_t h_t(i)]^{1-\alpha} - (\eta\gamma)^t f,$$

where z_t is the aggregate technology shock, $K_t(i)$ is the capital owned by firm i , $u_t(i)$ is the utilization rate of capital, $N_t h_t(i)$ is the amount of labor that is used in the production of intermediate good i , A_t is the trend of productivity growth ($A_t = \gamma^t$), and $(\eta\gamma)^t f$ is the fixed cost of production.³ Parameter α represents the share of capital in production.

Capital accumulation is given by:

$$K_t(i) = (1 - \delta)K_{t-1}(i) + \left[1 - \frac{\kappa_I}{2} \left(\frac{I_t(i)}{(\eta\gamma)I_{t-1}(i)} - 1 \right)^2 \right] z_t^I I_t(i),$$

where $I_t(i)$ is the investment of firm i , z_t^I is an investment-specific technology shock, δ is the depreciation rate of capital, and κ_I is the cost-of-investment-adjustment parameter. The term in squared brackets captures the cost of investment adjustment.

Dividends that are paid out to shareholders are equal to the residual of the total revenue after payments for wages, investments, price adjustment costs, and taxes are subtracted:

$$\begin{aligned} \frac{D_t(i)}{P_t} &= (1 - \tau_s) \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} N_t h_t(i) - I_t(i) - \Phi_{p,t}(i) \\ &\quad - \tau_y \left[(1 - \tau_s) \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} N_t h_t(i) - \delta_a K_{t-1}(i) \right] + (\eta\gamma)^t \Phi_{d,t}, \end{aligned}$$

where τ_s and τ_y are proportional taxes on sales and income, respectively, and δ_a is the accounting depreciation rate. Regarding wages, we introduce price stickiness in the form of quadratic adjustment cost (Rotemberg, 1982, and Chugh, 2006). The quadratic cost of price adjustment, $\Phi_{p,t}$, is defined as follows:

$$\Phi_{p,t}(i) = \frac{\kappa_p}{2} (\Theta - 1) (1 - \tau_s) (1 - \tau_y) \left[\frac{P_t(i)/P_{t-1}(i)}{\pi(\pi_{t-1}/\pi)^{\eta_p}} - 1 \right]^2 Y_t,$$

where κ_p is the cost-of-adjustment parameter and η_p is the price indexation parameter. $(\eta\gamma)^t \Phi_{d,t}$ is an exogenous transfer from the government to firms, where the stationary component $\Phi_{d,t}$ represents a dividend shock.

³The fixed cost is set such that intermediate good producers make no economic profit in the long run.

The objective of intermediate good producers is to maximize after-tax dividends:

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \frac{\Lambda_t}{\Lambda_\tau} (1 - \tau_d) \left[\frac{D_t(i)}{P_t} - \frac{\kappa_u}{1 + \chi} (u_t(i)^{1+\chi} - 1) K_{t-1}(i) \right],$$

where κ_u is a scale parameter ensuring that the utilization rate equals 1 at the steady state and χ is a capacity utilization elasticity parameter. The last term measures the cost of capital utilization.

2.5 Government and the Central Bank

As in Smets and Wouters (2007), government expenditure is defined as $G_t = (\eta\gamma)^t g_t$, the stochastic component of which, g_t , responds to productivity innovations. At time t , the government issues new bonds with maturities $k = 1, \dots, K$ and reimburses the bonds issued $k = 1, \dots, K$ periods beforehand. The budget constraint in period t is given by:

$$\begin{aligned} G_t + (\eta\gamma)^t \Phi_{d,t} + \sum_{k=1}^K \frac{Q_{t-k}^{(k)}}{P_t} &= \frac{T_t}{P_t} + \tau_h \frac{W_t}{P_t} N_t h_t + \tau_d \frac{D_t}{P_t} s_{t-1} + \tau_s Y_t \\ &+ \tau_y \left[(1 - \tau_s) Y_t - \frac{W_t}{P_t} N_t h_t - \delta_a K_{t-1} \right] + \sum_{k=1}^K \frac{B_t^{(k)} Q_t^{(k)}}{P_t}. \end{aligned}$$

We assume that the lump-sum tax, T_t , paid by households reacts to the debt level to avoid an explosive path of debt.

The central bank's Taylor rule gives the dynamics of the one-period gross nominal interest rate:

$$\frac{R_{1,t}}{R_1} = \left(\frac{R_{1,t-1}}{R_1} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\bar{\pi}_t} \right)^{a_\pi} \left(\frac{Y_t}{Y_t^n} \right)^{a_y} \left(\frac{Y_t/Y_{t-1}}{Y_t^n/Y_{t-1}^n} \right)^{a_g} \right]^{1-\rho_r} \epsilon_{r,t},$$

where R_1 is the steady-state level of the nominal policy rate, ρ_r is the interest rate smoothing parameter, and a_π , a_y , and a_g are the Taylor rule's weights. Y_t^n is the natural rate of output, which is defined as the level of output that would prevail under flexible prices in the absence of cost-push shocks.⁴ $\epsilon_{r,t}$ is the monetary policy shock, and $\bar{\pi}_t$ is the

⁴The natural rate of output is computed in a model-consistent manner by solving the model with flexible prices and wages. See Smets and Wouters (2003).

time-varying inflation rate targeted by the central bank, which is described as an AR(1) process:

$$\log \bar{\pi}_t = \rho_{\bar{\pi}} \log \bar{\pi}_{t-1} + \varsigma_{\bar{\pi}} \eta_{\bar{\pi},t}. \quad (7)$$

2.6 Market-Clearing Conditions

At equilibrium, all the markets clear, which results in the following relations:

- Goods market-clearing condition:

$$N_t C_t + I_t + G_t = Y_t - \Phi_{w,t} - \Phi_{p,t} - \Phi_{ac,t},$$

where $\Phi_{ac,t}$ denotes the sum of the adjustment costs for bonds and stocks paid by households, as they represent a resource cost.

- Labor services market-clearing condition:

$$h_t = \int_0^1 h_t(i) di = \left[\int_0^1 h_t^s(j)^{1/\psi_t} dj \right]^{\psi_t}.$$

- Bond market-clearing condition:

$$\bar{Q}_t^{(k)} = \int_0^1 Q_t^{(k)}(j) dj, \quad \forall k = 1, \dots, K,$$

where $\bar{Q}_t^{(k)} = \sum_{\tau=0}^{K-k} Q_{t-\tau}^{(\tau+k)}$ denotes the number of government bonds with maturity k available at date t to households.

- Equity market-clearing condition:

$$S_t(i) = \int_0^1 S_t(i, j) dj, \quad \forall i.$$

Equilibrium is attained when all agents maximize their objective functions and all markets clear. We assume a symmetric equilibrium, where all the households and all the intermediate good producers have the same characteristics.

2.7 Financial Asset Returns

At the symmetric equilibrium, the first-order condition with respect to consumption yields the following marginal utility of consumption (Lagrange multiplier with respect to the household budget constraint):

$$\Lambda_t = v_t(C_t - \zeta C_{t-1})^{-\sigma_C} \exp\left(-\xi \frac{1 - \sigma_C}{1 + \sigma_L} (h_t^s)^{1 + \sigma_L}\right).$$

Under complete markets and in the absence of arbitrage opportunities, the first-order conditions with respect to asset holdings yield the pricing equations for the financial assets. For a one-period bond, we have:

$$1 = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_{1,t+1}}{\pi_{t+1}} \right],$$

where $R_{1,t+1} = 1/B_t^{(1)}$ is the one-period gross nominal interest rate set by the central bank for the period between t and $t + 1$.

For longer-maturity bonds, we have:

$$1 = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{(1 + \Phi_{b,t+1}^{(k-1)}) R_{b,t+1}^{(k)}}{(1 + \Phi_{b,t}^{(k)}) \pi_{t+1}} \right], \text{ for } k = 2, \dots, K,$$

where $R_{b,t+1}^{(k)} = B_{t+1}^{(k-1)}/B_t^{(k)}$ denotes the gross nominal holding period return of the bond of maturity k held between t and $t + 1$. We also define $Y_{b,t}^{(k)} = (B_t^{(k)})^{1/k}$ the gross nominal yield to maturity of a k -period bond issued at time t .

For a given stock i , the pricing equation is (we omit the exponent i in the following expressions):

$$1 = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{1}{\pi_{t+1}} \frac{(1 + \Phi_{s,t+1})V_{t+1} + (1 - \tau_d)D_{t+1}}{(1 + \Phi_{s,t})V_t} \right].$$

We define $R_{s,t+1} = (V_{t+1} + (1 - \tau_d)D_{t+1})/V_t$ as the gross nominal return of a stock held between t and $t + 1$.

Finally, we define real returns as $\rho_{1,t+1} = R_{1,t+1}/\pi_{t+1}$ for the risk-free asset, $\rho_{b,t+1}^{(k)} = R_{b,t+1}^{(k)}/\pi_{t+1}$ for government bonds, and $\rho_{s,t+1} = R_{s,t+1}/\pi_{t+1}$ for stocks.

2.8 Shocks and Log-Linearization

Consistent with the approach promoted by Smets and Wouters (2007) and Alpana (2013), the model is driven by several shocks: the shocks to preferences (v_t), the wage markup (ψ_t), the price markup (θ_t), technology (z_t), investment-specific technology (z_t^I), dividends ($\Phi_{d,t}$), government spending (g_t), and monetary policy ($\epsilon_{r,t}$). These shocks have the following ARMA(1,1) dynamics:

$$\begin{aligned}
\log v_t &= \rho_v \log v_{t-1} + \eta_{v,t} - \varsigma_v \eta_{v,t-1}, \\
\log \psi_t &= (1 - \rho_\psi) \log \psi + \rho_\psi \log \psi_{t-1} + \eta_{\psi,t} - \varsigma_\psi \eta_{\psi,t-1}, \\
\log \theta_t &= (1 - \rho_\theta) \log \theta + \rho_\theta \log \theta_{t-1} + \eta_{\theta,t} - \varsigma_\theta \eta_{\theta,t-1}, \\
\log z_t &= \rho_z \log z_{t-1} + \eta_{z,t} - \varsigma_z \eta_{z,t-1}, \\
\log z_t^I &= \rho_I \log z_{t-1}^I + \eta_{I,t} - \varsigma_I \eta_{I,t-1}, \\
\Phi_{d,t} &= \rho_d \Phi_{d,t-1} + \eta_{d,t} - \varsigma_d \eta_{d,t-1}, \\
\log g_t &= (1 - \rho_g) \log g + \rho_g \log g_{t-1} + \rho_{g,z} \eta_{z,t} + \eta_{g,t} - \varsigma_g \eta_{g,t-1}, \\
\log \epsilon_{r,t} &= \rho_r \log \epsilon_{r,t-1} + \eta_{r,t} - \varsigma_r \eta_{r,t-1},
\end{aligned}$$

where η indicates mutually uncorrelated i.i.d. shocks. Regarding rebalancing costs, we adopt an agnostic view and assume ARMA(1,1) processes:

$$\begin{aligned}
\phi_{b,t}^{(k)} &= \log(1 + \Phi_{b,t+1}^{(k-1)}) - \log(1 + \Phi_{b,t}^{(k)}) = \rho_b^{(k)} \phi_{b,t-1}^{(k)} + \eta_{b,t}^{(k)} - \varsigma_b^{(k)} \eta_{b,t-1}^{(k)}, \\
\phi_{s,t} &= \log(1 + \Phi_{s,t+1}) - \log(1 + \Phi_{s,t}) = \rho_s \phi_{s,t-1} + \eta_{s,t} - \varsigma_s \eta_{s,t-1}.
\end{aligned}$$

The economy grows at a constant rate $\eta\gamma$, where η is the average growth of the population and γ is the average growth of per-capita output. Lower-case letters with tilde denote stationary variables, obtained by dividing each level variable (upper-case letters) by its deterministic trend, e.g., $\tilde{x}_t = X_t/(\eta\gamma)^t$. Next, after detrending, all variables are log-linearized around their steady state (\check{x}) and then denoted by $\hat{x}_t = (\tilde{x}_t - \check{x})/\check{x} \approx \log \tilde{x}_t - \log \check{x} \equiv x_t - \bar{x}$. The resulting log-linearized model is described in Appendix A.

As the asset pricing conditions in the appendix show, expected returns on risky assets are given by:

$$\begin{aligned} E_t \hat{\rho}_{b,t+1}^{(k)} &= E_t \hat{\rho}_{f,t+1}^{(k)} + \hat{\phi}_{b,t}^{(k)}, \\ E_t \hat{\rho}_{s,t+1} &= E_t \hat{\rho}_{f,t+1} + \hat{\phi}_{s,t}. \end{aligned}$$

We notice from these equations that $\hat{\phi}_{b,t}^{(k)}$ and $\hat{\phi}_{s,t}$ can be interpreted as time-varying risk premia on bonds and stocks, respectively. These dynamics of the risk premia should capture the second-order properties of the DSGE model that are lost in the log-linearization of the model. See Hördahl, Oreste, and Vestin (2008).⁵

3 Data and Estimation

3.1 Data

We use 10 observable variables to estimate the model: per-capita real GDP (GDP_t), per-capita real consumption ($CONS_t$), per-capita real investment (INV_t), per-capita labor hours (HRS_t), the real wage rate ($WAGE_t$), the GDP deflator (P_t), the per-capita real market value of nonfinancial firms (CAP_t), the per-capita real dividends of nonfinancial firms (DIV_t), the federal funds rate (FFR_t), and the long-term Treasury bond interest

⁵Alternatively, we could allow the risk premia to depend on some macro shocks. For instance, the bond risk premium could be driven by the shock on government expenditure and the stock risk premium could be driven by the technology shock. We leave the investigation of more general specifications for further research.

rate (LTR_t). The measurement equation, $x_t = \bar{x} + \hat{x}_t$, is given by:

$$\begin{bmatrix} \Delta \log GDP_t \\ \Delta \log CONS_t \\ \Delta \log INV_t \\ \log HRS_t \\ \Delta \log WAGE_t \\ \Delta \log P_t \\ \Delta \log CAP_t \\ \Delta \log DIV_t \\ \log(1 + FFR_t) \\ \log(1 + LTR_t) \end{bmatrix} = \begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta \iota_t \\ h_t \\ \Delta wp_t \\ \pi_t \\ \Delta vp_t \\ \Delta dp_t \\ r_{1,t} \\ y_{b,t} \end{bmatrix} = \begin{bmatrix} \log \gamma \\ \log \gamma \\ \log \gamma \\ \log h \\ \log \gamma \\ \log \bar{\pi} \\ \log \gamma \\ \log \gamma \\ \log R \\ \log R \end{bmatrix} + \begin{bmatrix} \Delta \hat{y}_t \\ \Delta \hat{c}_t \\ \Delta \hat{\iota}_t \\ \hat{h}_t \\ \Delta \hat{w}p_t \\ \hat{\pi}_t \\ \Delta \hat{v}p_t \\ \Delta \hat{d}p_t \\ \hat{r}_{1,t} \\ \hat{y}_{b,t} \end{bmatrix}. \quad (8)$$

In our study, the long-term bond is the 10-year Treasury bond. Although long-term bonds were described as zero-coupon bonds in the previous section, the common investment vehicles are coupon bonds. Thus, we adapt the definition of the bond holding-period return, $r_{b,t}$, to be consistent with that of coupon bonds.

For stocks, we consider the stock market as a whole instead of individual stocks. The stock market return is computed from the value of equity and dividends available from the aggregate balance sheet and flow of funds data. It is worth mentioning that, following Alpanda (2013), our dividend series includes net buybacks, which allows us to take into account corporate finance issues in a more realistic framework. Instead of paying dividends, firms often prefer to buy back their own shares as a way of distributing cash to shareholders. Further details regarding the data are provided in Appendix B.

Figure 1 displays the data that are used to estimate the model, and **Table 1** provides basic statistics for the observable variables. The sample that is used for the benchmark estimation comprises data from the 1955-2010 period (224 quarterly observations). The figure shows that most of the observable variables are clearly stationary, although hours (h), inflation (π), the federal funds rate (r_1), and the 10-year yield to maturity (y_b) show some persistence. As shown in the table, output growth mainly results from consumption,

whereas investment growth is lower, on average, than consumption growth. By contrast, most of the output volatility is derived from real investment. Regarding asset returns, we note that the average inflation is lower than the average short-term rate and the average long-term bond interest rate (3.44% per year for inflation versus 5.36% for the short-term rate and 6.20% for the long-term rate). Finally, the average real stock return is 6.60% per year.

3.2 Parametrization and Estimation

Some of the parameters of the model are calibrated when their values can be deduced from accounting data, fiscal data, or long-term trend data. For this calibration, we follow the approach of Alpanda (2013) by computing the parameters over the sample period, 1955-2010. Their values are reported in **Table 2**. The average real growth is equal to 3.1% per year (population growth, η , of 1.4% and per-capita output growth, γ , of 1.7%). The average annual inflation and short-term rates are 3.4% and 5.4%, respectively. Tax rates and shares are estimated by using accounting data to match steady-state relations over the postwar period.

To facilitate comparison with previous work, we redefine some parameters. The capacity utilization elasticity is defined as $\chi^e = \chi/(1 + \chi)$. Similarly, we rescale the adjustment costs as follows:

$$\kappa_p = \frac{10(\theta - 1) + 1}{(1 - \kappa_p^e)(1 - \tilde{\beta}\kappa_p^e)}\kappa_p^e, \quad \text{and} \quad \kappa_w = \frac{10(\psi - 1) + 1}{(1 - \kappa_w^e)(1 - \tilde{\beta}\kappa_w^e)}\kappa_w^e.$$

We also set $\xi = ((1 - \tau_h)\bar{w}p/\bar{y})(\psi(1 - \zeta/\gamma)\bar{c}/\bar{y})$ and $f = \bar{y}(\theta - 1)$ to normalize the labor supply and intermediate good producers' profit in the long run.

The model is estimated by using Bayesian methodology.⁶ The dynamic system is mapped to a state-space representation for the set of observable variables. The Kalman filter is then used to evaluate the likelihood of the observed variables and to form the posterior distribution of the structural parameters by combining the likelihood function

⁶For the estimation of DSGE models, Schorfheide (2003), Fernandez-Villaverde and Rubio-Ramirez (2004), and Smets and Wouters (2007), among others, propose the use of Bayesian methodology.

with a joint density characterizing some prior beliefs. Given the specification of the model, the posterior distribution cannot be recovered analytically but can be evaluated numerically by using a Markov Chain Monte-Carlo sampling approach. More specifically, we rely on the Metropolis-Hastings (MH) algorithm to obtain random draws from the posterior distribution of the parameters.⁷

Because Bayesian estimation requires some priors on the parameters, we begin with a description of these priors, reported in **Table 3**. Our priors are rather similar to, although generally less restrictive than, those adopted in previous studies (in particular, Smets and Wouters, 2003; Jondeau and Sahuc, 2008) and closely match the priors selected by Alpanda (2013). We assume a Beta distribution for the following parameters bounded between zero and one: the habit persistence parameter (ζ), the degree of price and wage indexation (η_p and η_w), the smoothing parameter in the monetary policy rule (ρ_r), the adjustment cost parameters (κ_w^e , κ_p^e , and κ_I^e), and the autoregressive and moving-average parameters for shocks. The inverse of the consumption elasticity of substitution (σ_C), the inverse of the elasticity of labor supply (σ_L), the price and wage markups (θ and ψ), and the Taylor rule parameters (a_π , a_y , and a_g) have a normal prior. All priors on the shock variances have an inverse gamma distribution.

The mean and confidence interval of the posterior distribution of the parameters are also reported in the table. Regarding household behavior, our estimate of the inverse of the consumption elasticity of substitution (σ_C) is 1, while the inverse of the elasticity of labor disutility (σ_L) is approximately 2.5. The habit persistence parameter ζ is estimated to be 0.94. Regarding the behavior of firms, we find that wage indexation is significantly larger than price indexation ($\eta_w = 0.46$ versus $\eta_p = 0.19$). The adjustment costs for prices and wages are estimated to be similar, with values of approximately 0.85. In the reaction function, the long-run impact of inflation and the output gap on the short-term interest rate is approximately 1.2 and 0.1, respectively. Our parameter estimates are broadly in line with the estimates reported by Alpanda (2013), which are shown in the last column

⁷We simulate two blocks of 250,000 random draws. The first 100,000 observations are discarded to eliminate any dependence on the initial values. The mode and Hessian of the posterior distribution evaluated at the mode are used to initialize the MH algorithm.

of our table for convenience.⁸ The new parameters correspond to the dynamics of the inflation target and the bond and stock premia.

The evolution of the bond and stock premia are displayed in **Figure 2**. As the figure shows, the bond premium is more persistent than the stock premium ($\rho_b = 0.958$ vs. $\rho_s = 0.816$). In contrast, the stock premium is more volatile than the bond premium (the variance of the premia is 4.5% and 16.3% per year for bonds and stocks, respectively). **Table 4** reports the correlation between the financial asset risk premia and the financial asset returns for the initial estimation and allocation samples. We observe that the correlations have changed over time, indicating that the hedging properties of the various assets have also changed over time. Over the 1955-89 subperiod, cash is a good hedge against stock risk but a bad hedge against short-term interest-rate risk and bond risk. In contrast, bonds and stocks are good hedges against short-term interest-rate risk as bond and stock returns increase when the short-term premium is negative. Over the recent period from 1990 to 2010, we also notice that the correlation between bond returns and the bond premium and that between stock returns and the stock premium are more negative. This result indicates that bonds are good hedges against bond risk and that stocks are good hedges against stock risk. As a consequence, we expect that when the bond risk increases, the bond hedging demand increases and that when the stock risk increases, the stock hedging demand increases. Correlations with a similar order of magnitude have been reported in Sangvinatsos and Wachter (2005) for an affine term structure model.

Two potential issues with the estimation of the DSGE model over such a long sample period are nonstationarity and parameter instability. Some of the autoregressive parameters are found to be large and close to one, suggesting highly persistent dynamics. Such is the case in particular for the investment shock (ρ_I) and the government shock (ρ_g), which clearly reflect near-to-unit-root behavior. We also investigate the stability of the parameter estimates over time by reestimating the model with a rolling window of 35 years. We find that some parameters (such as adjustment costs) have an increasing trend, whereas

⁸The model and sample period in Smets and Wouters (2007) differ from ours. In particular, their wage and inflation dynamics are modeled within a staggered price setting à la Calvo (1983), whereas we introduce stickiness through quadratic adjustment costs à la Rotemberg (1982). For these reasons, our parameter estimates cannot be directly compared with those reported in Smets and Wouters (2007).

other parameters (such as the inflation parameter in the reaction function) have a negative trend. This issue is partly due to the Federal Reserves adoption of a nonborrowed reserve operating procedure during the 1979-1982 period.

The variability of the parameters over time clearly indicates that the allocation experiment must be performed in real time and that it certainly cannot be performed by using the same sample period for the estimation and allocation. We therefore investigate the forecasting ability and allocation performance of both models over the 1990-2010 period, which is essentially stationary. To obtain relevant measures in the out-of-sample investigation, we restrict the investment horizon to a maximum of 10 years, with 35-year rolling windows for the parameter estimation.

3.3 Forecasts

To evaluate the strengths and weaknesses of the DSGE model in long-term asset allocation, we first consider the ability of the model to predict the variables of interest over long horizons. To do so, we first estimate the DSGE model from 1955Q1 to 1989Q4 and forecast all the variables over horizons ranging from one quarter to 10 years. We then roll the sample by one quarter, reestimate the model from 1955Q2 to 1990Q1, and forecast all the variables over the same horizons. We continue this procedure until we reach the last window, 1975Q4 to 2010Q3, for which we forecast for the next quarter only.

Forecasting is straightforward in a DSGE model because such a model has a backward-looking state-space representation, which can be ultimately written as a restricted VAR(1):

$$\hat{s}_{t+1} = G\hat{s}_t + H\eta_{t+1}, \quad (9)$$

where \hat{s}_t is the set of state variables, which includes the (demeaned) observables, the shocks, and the future expected variables, and η_t is the set of structural innovations. We assume that η_t is normally distributed with mean 0 and covariance matrix Σ_η . Thus, the observables can be recovered through measurement equation (8):

$$x_t = \Phi_0 + \Phi_1\hat{s}_t.$$

The matrix of parameters Φ_0 contains the long-term value of the observables. Matrix Φ_1 is simply a selection matrix because all the (demeaned) observables are also state variables. The distribution of x_{t+k} is simply:

$$x_{t+k} \sim N(\mu_{x,t}(k), \Sigma_x(k)),$$

where $\mu_{x,t}(k) = \Phi_0 + \Phi_1 G^k \hat{s}_t$, $\Sigma_x(k) = \Phi_1 \Sigma_s(k) \Phi_1'$, and $\Sigma_s(k) = D + GDG' + \dots + G^{k-1}D(G^{k-1})'$ denotes the k -period ahead covariance matrix of the state variables with $D = H\Sigma_\eta H'$.

The root mean square error (RMSE) is computed for the variables in levels. If a variable is already in levels, we simply compare the expectation, $\mu_{x,t}(k)$, to the ex-post observed variable, x_{t+k} . If the variable is in differences, we compare the cumulative expectation, $\sum_{i=1}^k \mu_{x,t}(i)$, to the ex-post observed variable in levels, $\sum_{i=1}^k x_{t+i}$.

Table 5 reports the RMSE of the variables of interest for the DSGE model. The RMSE is low for real output, consumption, hours, and real wages, even for long horizons. Even at the 10-year horizon, the RMSE is below 8%. This result suggests that the mean reversion of these variables plays a major role. The forecast error is larger for investment because this series is much more volatile. Even in this case, mean reversion plays an important role, as the RMSE decreases over long horizons.

Another important result is that the RMSE for cumulative inflation increases dramatically with the horizon. Even if the forecast error is low over short horizons (1.7% for two years), it increases to 17.5% at the 10-year horizon. This finding suggests a lack of mean reversion in the inflation rate dynamics. This feature has been proposed in previous work (Bekaert, Cho, and Moreno, 2010, for instance) and is usually captured by a random walk inflation target in the monetary policy rule. In the model, we allow for an autoregressive target and find the persistence parameter to be equal to $\rho_{\bar{\pi}} = 0.85$. As the value is less than unity, this parameter precludes an explosive dynamic. This approach works well for up to five years but seems to be insufficient for longer horizons.

Regarding financial variables, we notice that the RMSE of the nominal short-term interest rate is only slightly larger than the RMSE of inflation over all horizons. This

result suggests that monetary policy is rather well described by a Taylor-type rule, which provides an anchor for the short-term rate to the output gap and inflation gap. The forecast error for the nominal long-term rate is low in the short run and increases in line with the short-term rate for long horizons. We also observe that the RMSE of the real bond holding-period return is rather low, as it does not exceed 10% for the 10-year horizon. This result suggests that both inflation risk and real interest-rate risk are important for a long-term investor (Campbell and Viceira, 2002).

Finally, the RMSE is rather large for a firm's financial variables: the forecast error at the two-year horizon is larger than 20% for the real market value of equity and 40% for real dividends. For real cumulative stock returns, the RMSE is as high as 24%. These results are not particularly surprising given the high uncertainty surrounding stock return forecasts. Even if the model can be considered to provide good overall forecasts, the current version of the model does not account for stock market bubbles. Interestingly, we observe that the RMSE stabilizes at approximately 35% after five years for the market value of equity, dividends, and real stock returns. This finding is consistent with the substantial mean reversion indicated by Siegel (1994), Barberis (2000), and Campbell and Viceira (2002), among others.

3.4 Term Structure of Risks

To investigate the implications of the predictability of financial returns in a DSGE model from an investor perspective and in greater depth, we now consider the relative levels of risk of financial assets at various horizons. As demonstrated by Siegel (1994) and Campbell and Viceira (2002), an important implication of stock return predictability in the long run is that the annualized standard deviation of stock returns tends to decrease as the horizon increases. In contrast, the volatility of bond returns tends to increase with the horizon.⁹

⁹The decrease in stock return volatility in the long run has been recently questioned by Pastor and Stambaugh (2009), who argue that the negative effect of mean reversion on long-run volatility is more than offset by the uncertainty about future expected returns and parameter uncertainty.

We consider three asset classes: cash, bonds, and stocks. Investment in bonds is based on a “rolled” strategy in constant-maturity bonds: the bond portfolio is rebalanced in each period to keep the maturity constant over time. The investor buys a 10-year bond at the beginning of the quarter and sells it at the end of the quarter. Then, at the beginning of the next quarter, she buys a (possibly new) 10-year bond for one quarter. The bond position is, therefore, assumed to be rolled over time with quarterly rebalancing.¹⁰ We denote the vector of log returns by $z_{t+1} = (\rho_{1,t+1}, \rho_{b,t+1}, \rho_{s,t+1})'$, where $\rho_{1,t+1}$ is the ex-post real one-period interest rate, $\rho_{b,t+1}$ is the ex-post real holding-period return for a 10-year bond, and $\rho_{s,t+1}$ is the ex-post real stock return. The distribution of the k -period cumulative return, $Z_t[k] = z_{t+1} + \dots + z_{t+k}$, is given by:

$$Z_t[k] \sim N(\mu_{Z,t}[k], \Sigma_Z[k]),$$

where

$$\begin{aligned} \mu_{Z,t}[k] &= k\tilde{\Phi}_0 + \tilde{\Phi}_1(G + \dots + G^k)\hat{s}_t, \\ \Sigma_Z[k] &= \tilde{\Phi}_1 D \tilde{\Phi}'_1 + \tilde{\Phi}_1(I + G)D(I + G)'\tilde{\Phi}'_1 + \dots \\ &\quad + \tilde{\Phi}_1(I + G + \dots + G^{k-1})D(I + G + \dots + G^{k-1})'\tilde{\Phi}'_1. \end{aligned}$$

The term $\tilde{\Phi}_0 = H_z \Phi_0$ contains the long-term values of the real log returns, $\tilde{\Phi}_1 = H_z \Phi_1$, and H_z is the selection matrix that selects the vector of real log returns, z_t , from the observables. Finally, we compute the annualized variance of the k -period cumulative return as $(1/k)\Sigma_Z[k]$.

Figure 3 plots the annualized standard deviations of real returns for short-term bonds, long-term bonds, and stocks for investment horizons up to 120 quarters, i.e., 30 years. The figure clearly shows that the risk of rolled long-term bonds and stocks decreases with the investment horizon. The annualized standard deviation of rolled bonds decreases from 8.5% for a one-quarter horizon to less than 6% for a 30-year horizon. Similarly,

¹⁰Alternatively, the investor could buy a k -year bond and holds it until its maturity at date $t + k$ (“variable-maturity” strategy). From a dynamic allocation perspective, the rolled strategy makes more sense, whereas the variable-maturity strategy would be more relevant to a buy-and-hold strategy. For this reason, we focus on the rolled strategy in the following section.

the volatility of stock returns is highest at the shortest horizon (one quarter) and then continuously decreases from 14.8% to 12% over a 30-year horizon. From an investment strategy perspective, this result implies that rolled bonds and stocks are relatively safer over longer horizons; thus, a longer-horizon investor should allocate relatively more to bonds and stocks than a short-horizon investor (Barberis, 2000). Regarding the short-term bond, we find that the risk continuously increases with the investment horizon, from 1% to 5%. This pattern reflects the increase in inflation risk for long horizons and therefore weak mean reversion in inflation. This result is broadly consistent with the evidence reported by Campbell and Viceira (2002) for a different period.

4 Optimal Dynamic Strategy

We now investigate the optimal dynamic investment policy of a long-term institutional investor who uses a DSGE model to predict future asset returns. Dynamic strategies take into account changes in investment opportunities, such that the portfolio is, in principle, rebalanced in an optimal manner at specified regular intervals. Because the investor wishes to hedge the portfolio against adverse changes in the investment set, the strategy gives rise to intertemporal hedging demands (Merton, 1973).

One difficulty encountered with dynamic strategies is that, generally, no closed-form solution exists. See Kim and Omberg (1996) and Wachter (2002) for exact analytical solutions to continuous-time intertemporal portfolio-choice problems and Bodie, Detemple, and Rindisbacher (2009) and Wachter (2010) for recent surveys on dynamic allocation. In discrete time, some solutions to this problem based on numerical techniques (Barberis, 2000; Lynch, 2001) or analytical approximations of the solution (Campbell and Viceira, 1999, 2001, 2002; Campbell, Chan, and Viceira, 2003) have been proposed. For instance, Barberis (2000) considers the case of a single asset with no intermediate consumption and simulates the path of the state variables over the investment horizon by using a discretization scheme.

We consider an institutional investor with Epstein and Zin (1989) recursive preferences and follow the approximate analytical solution provided by Campbell and Viceira (1999,

2001) and Campbell, Chan, and Viceira (2003). The institutional investor can be, for instance, an insurance company, a pension fund, a sovereign wealth fund, an endowment, or a charity, with a long-term horizon and a regular payout to be made. This framework is well suited for evaluating dynamic investment strategies. Preferences are described as:

$$U(O_t, E_t[U_{t+1}]) = [(1 - \beta)O_t^{(1-\gamma)/\theta} + \beta(E_t[U_{t+1}^{1-\gamma}])^{1/\theta}]^{\theta/(1-\gamma)},$$

where O_t as the net outflow that has to be made by the investor in every period, $\gamma > 0$ is the relative risk aversion coefficient, $\psi > 0$ is the elasticity of intertemporal substitution, and $\theta = (1-\gamma)/(1-\psi^{-1})$. Epstein-Zin utility simplifies to the power utility when $\gamma = \psi^{-1}$ and to the log-utility when $\gamma = \psi^{-1} = 1$. The intertemporal budget constraint is:

$$W_{t+1} = (W_t - O_t)\rho_{p,t+1},$$

where $\rho_{p,t+1} = \rho_{1,t+1} + \alpha_{b,t}(\rho_{b,t+1} - \rho_{1,t+1}) + \alpha_{s,t}(\rho_{s,t+1} - \rho_{1,t+1})$ is the real portfolio return, with $\alpha_{b,t}$ and $\alpha_{s,t}$ denoting the fraction of wealth invested in bonds and stocks, respectively.¹¹

Under such preferences, Campbell and Viceira (1999) provide an approximate optimal solution based on the log-linear approximation of the Euler equation and the intertemporal budget constraint. Then, the dynamic optimization problem is solved and the optimal portfolio and outflow are shown to be linear in the state variables, \hat{s}_t :

$$\alpha_t = A_0 + A_1\hat{s}_t, \tag{10}$$

$$o_t - w_t = b_0 + B_1'\hat{s}_t + \hat{s}_t'B_2\hat{s}_t. \tag{11}$$

The optimal portfolio rule has two components: a myopic component, which corresponds to the one-period optimal allocation, and an intertemporal hedging portfolio, which accounts for movements in expected returns. Campbell, Chan, and Viceira (2003) demon-

¹¹As Figure 3 shows, the one-period return $\rho_{1,t+1}$ is not risk free at long horizons, but, for convenience, it is still used to define excess returns in this expression.

strate that the coefficient matrices A_0 and A_1 for the optimal portfolio are:

$$\begin{aligned} A_0 &= \frac{1}{\gamma} \Sigma_{xx}^{-1} \left[H_x \Phi_0 + \frac{1}{2} \sigma_x^2 + (1 - \gamma) \sigma_{1x} \right] + \left(1 - \frac{1}{\gamma} \right) \Sigma_{xx}^{-1} \left(-\frac{\Lambda_0}{1 - \psi} \right), \\ A_1 &= \frac{1}{\gamma} \Sigma_{xx}^{-1} H_x \Phi_1 + \left(1 - \frac{1}{\gamma} \right) \Sigma_{xx}^{-1} \left(-\frac{\Lambda_1}{1 - \psi} \right), \end{aligned}$$

where H_x is the selection matrix that selects the vector of excess returns $(\rho_{b,t+1} - \rho_{1,t+1}, \rho_{s,t+1} - \rho_{1,t+1})$ from the state vector, Σ_{xx} is the covariance matrix of excess returns, $\sigma_x^2 = \text{diag}(\Sigma_{xx})$ is the vector of variances of excess returns, and σ_{1x} is the covariance between the one-period rate and the excess returns. Vectors Λ_0 and Λ_1 are related to the covariance of the assets and the outflow growth. They are defined in the appendix of Campbell, Chan, and Viceira (2003).

The first terms in A_0 and A_1 correspond to the myopic component, which is the solution to the mean-variance allocation problem when excess returns are predictable. The first term of A_1 takes the prediction of excess returns into account as long as $\Phi_1 \neq 0$. The second terms in A_0 and A_1 correspond to the intertemporal hedging demands. They appear in the optimal portfolio weights as long as $\gamma \neq 1$ and depend on the ability of the assets to hedge the investor against the deterioration of consumption growth. When $\gamma = \psi = 1$ (log-utility), the hedging demands vanish. Campbell, Chan, and Viceira (2003) show that ψ is essentially irrelevant for the determination of the optimal portfolio rule, although it directly drives the optimal outflow rule. In our experiments, we indeed observe that for a given value of γ , altering ψ does not affect the optimal allocation rule. In the empirical application, we follow Campbell, Chan, and Viceira (2003) and focus on the case $\psi = 1$. In this case, the choice of the outflow c_t is myopic because the outflow-wealth ratio $o_t - w_t$ is constant, whereas the optimal portfolio rule is not myopic. The solution would be fully myopic in the log-utility case only.

As a first step of our analysis of the optimal strategy of a DSGE investor, we consider the total demands for stocks and bonds and their myopic and hedging components. **Figure 4** shows that the optimal demands are very similar when the model is estimated up to 1999Q4 or up to 2010Q4. Thus, even if the parameters of the DSGE model vary over

time, this variability does not significantly alter the investment strategy of the investor. The figure shows that the allocation to stocks is a concave function of risk tolerance ($1/\gamma$), as in Campbell, Chan, and Viceira (2003), with a very similar decomposition in its myopic and hedging components. The hedging demand is always positive and is the highest for intermediate levels of risk tolerance: it is as high as 50% for risk aversion levels between 2 and 4, and then decreases to nearly 0 for infinitely risk averse investors. Regarding the allocation to bonds, a different picture from that presented in Campbell, Chan, and Viceira (2003) emerges: hedging demands increase as risk aversion increases. For an infinitely risk-averse investor, the optimal portfolio is almost entirely composed of bonds, with a hedging demand for bonds of 95%.

To investigate the role of the risk premia in the optimal allocation in greater detail, we now consider data for the last date in the sample. In the last quarter of 2010, the DSGE model estimates a stock premium of 7% and a bond premium of -0.5% per year. These numbers correspond to the premia reported in Figure 2 (1.75% and -0.12% per quarter for stock and bond premia, respectively). **Figure 5** (Panel A) corresponds to deviations from the current stock premium (from 7% to 0% and 10%) when the bond premium is at its current value (-0.5%). As the figure shows, as the stock premium increases, the allocation to stocks also increases. The highest hedging demand is 25% when the stock premium is 0% and 50% when the premium increases to 10% (in both cases, for $\gamma = 2$). We also notice that the allocation to bonds decreases from 30% to 11% when the stock premium increases from 0% to 10%. This result is consistent with the positive correlation of bond returns with the stock premium and indicates that bonds are poor hedges against stock risk.

Panel B corresponds to deviations from the current bond premium (from -0.5% to -2% and 2%) when the stock premium is at its current value (7%). Interestingly, changes in portfolio weights are much more contrasted than for a change in the stock premium. For a negative premium, we find that the allocation to bonds becomes negative for low levels of risk aversion but that it may be positive and relatively high for highly risk-averse investors. Similar results are reported by Campbell, Chan, and Viceira (2003) for stocks

and Sangvinatsos and Wachter (2005) for bonds in the case of long-term investors. As bond returns are negatively correlated with the bond premium, a negative bond premium induces a negative bond holding, which can be as high as -20% for $\gamma = 2$. However, long-term investors know that the bond premium is positive, on average, and thus expect the risk premium to return to its long-run value in the future. Therefore, risk-averse investors still hold long positions in bonds (for $\gamma > 5$). As the bond premium increases, the allocation to bonds increases accordingly. For a premium of 2% , the hedging demand is as high as 100% for large risk aversion parameters. Regarding the allocation to stocks, we find that, as the bond premium increases, the fraction of wealth invested in stocks increase, although this effect is relatively limited.

5 Out-of-sample Performances

We compare the optimal allocation performance of a DSGE investor with that of an investor using an unrestricted VAR(1) model to predict future asset returns, as in Campbell, Chan, and Viceira (2003). The VAR model is composed of the following variables, with the same notations as in equation (8): real GDP growth (Δy_t), real consumption growth (Δc_t), real investment growth (Δi_t), hours growth (Δh_t), wage inflation (Δwp_t), price inflation (π_t), the federal funds rate ($r_{1,t}$), long-term versus short-term spread ($sp_t = y_{b,t} - r_{1,t}$), long-term holding-period excess returns ($x_{b,t} = r_{b,t} - r_{1,t}$, where $r_{b,t} = \log(1 + R_{b,t})$), real stock returns ($\rho_{s,t}$), and the log dividend-price ratio ($dpr_t = vp_t - dp_t$).¹² The DSGE model can be viewed as a restricted VAR(1) process, which incorporates all the restrictions imposed by the macro-financial mechanisms of the model. Thus, we expect the unrestricted VAR to perform better than the DSGE model in the short run because it provides the best fit for the next period. The VAR(1) model could still outperform the DSGE model in the long run if the economic restrictions turn out to be irrelevant. However, if the economic restrictions that are imposed in the

¹²Some differences exist in the selection of the variables between the VAR model and the DSGE model: namely, hours are introduced in differences to prevent nonstationarity of the system; similarly, the long-term rate and holding-period returns on government bonds are in excess to the risk-free rate; finally, the log dividend-price ratio is introduced following the approach of Barberis (2000).

DSGE model are relevant, the VAR model may not perform as well as the DSGE model. The parameter estimates of the VAR model are reported in **Table 6**. The table reveals that some predictability in bond and stock returns exists at a quarterly frequency, with adjusted R^2 values equal to 20% and 15%, respectively.

The RMSE of the variables of interest for the VAR model are reported in **Table 7**. Comparison of the VAR model with the DSGE model reveals that the models have a similar performance in forecasting macro variables, even over long horizons. However, the DSGE model performs better in forecasting financial returns over long horizons. The VAR model performs relatively poorly in forecasting real stock returns in the long term. Over two- and five-year horizons, the VAR model performs as well as the DSGE model. However, over the 10-year horizon, the RMSE for the VAR model increases to 42% (against 36.8% for the DSGE model). The better performance of the DSGE model is even more pronounced for real bond returns. The RMSE of the DSGE model only slowly increases as the horizon increases (from 6% for two years to 8% for 10 years), while the RMSE of the VAR model dramatically increases from 8% for two years to 22% for 10 years. This empirical evidence is consistent with the notion that imposing economic restrictions improves stock return predictions. See, for instance, Campbell and Thompson (2008), Cochrane (2008), and Ferreira and Santa-Clara (2011), who develop this argument in a regression context. In our context, although the unrestricted VAR model performs as well as the DSGE model in the short run, it clearly underperforms the DSGE model in the long run with respect to forecasting real bond holding-period returns and stock returns.¹³

These results are confirmed by the term structure of risks that is estimated using the VAR model, as shown in **Figure 6** for 2010Q4. We find that in contrast to the results for the DSGE model, the annualized volatility of stocks does not decrease with the investment horizon in the VAR model, indicating that mean reversion in stock returns is weaker in the VAR model. The volatility of rolled long-term bonds exhibits some mean reversion, but it decreases much more slowly in the VAR model than in the DSGE

¹³Campbell and Thompson (2008) also show that even a relatively small improvement in prediction can result in meaningful utility gains for investors.

model. If we focus specifically on a medium term such as 10 years, which will be our benchmark for the performance evaluation, the picture is rather different between the DSGE and the VAR models. The annualized volatilities of cash, bonds, and stocks are 4%, 4.75%, and 12.5%, respectively, for the DSGE model, and 4%, 6.75%, and 16.5%, respectively, for the VAR model. Interestingly, we find that, in the model with economic restrictions, annualized volatilities decrease with the investment horizon, as suggested by Siegel (1994) and Campbell and Viceira (2002). In contrast, in the unrestricted model, annualized volatilities do not decrease, as established by Pastor and Stambaugh (2009). These results clearly confirm that imposing economic restrictions in a forecasting model is beneficial for long-term investors. We now turn to this issue.

We now evaluate the out-of-sample performance of the DSGE and VAR models from a long-term investment perspective. Even if investors are supposed to be infinitely long lived, the out-of-sample performance of the optimal portfolio allocation can be evaluated for finite horizons only. Over our full sample, we use the first 35 years (1955-89) for the first estimation of the (DSGE and VAR) models and the sample for the period from 1990 to 2010 for the performance evaluation. We proceed as follows: for the cohort of investors who begin to invest in 1990Q1, we estimate the model over the 1955-1989 period and determine the optimal rule (Equation (10)) by using available data only. Then, every quarter from 1990Q1 to 2010Q3, we compute the optimal weights of their portfolios conditional on the available state variables. Finally, once asset returns are observed, we compute the ex-post portfolio return, from which we evaluate the out-of-sample performance of the investment rule. For the 1990Q1 cohort, we have 83 optimal portfolio weights and therefore 83 ex-post portfolio returns. We then move to the next cohort of investors, who begin to invest in 1990Q2, and proceed similarly. For the last cohort (2010Q3), only one optimal allocation and one ex-post portfolio return are available. As the various cohorts use different estimates of the models, the investment rules and, consequently, the ex-post performances differ from one cohort to the other.

In **Table 8** (Panels A and B), we report the average portfolio weights for stocks and bonds for the DSGE and VAR models. For instance, for the 5-year horizon, we average

the optimal portfolio weights of the cohorts from 1990Q1 to 2005Q3. In doing so, we avoid overreliance on a specific period, as we average the behavior of different cohorts over time. Over a short horizon, the optimal DSGE portfolio has positive weights, on average, for both stocks and bonds. For longer horizons, however, the this portfolio is short in stocks. In contrast, the optimal VAR portfolio has a large fraction invested in stocks and a small fraction (even negative for a long horizon) invested in bonds. This clear difference between the optimal DSGE and VAR portfolios can be explained as follows. As noted above, the DSGE model provides much better forecasts of long-term bond returns than the VAR model; thus, investors' perceived risk on long-term bonds is much lower. Although the DSGE model is also better at reducing the uncertainty surrounding stock returns than the VAR model, the decrease in risk is relatively less pronounced. As a consequence, the myopic demand is positive for bonds and negative for stocks for the DSGE investor and negative for bonds and positive for stocks for the VAR investor. In addition, as discussed in the previous section, in the DSGE model, the large hedging demands are high for long-term bonds, owing to their ability to hedge changes in the bond risk premium, and relatively limited for stocks. In fact, the hedging demand for bonds is almost entirely financed by a negative hedging demand for cash, which has a large positive correlation with the bond premium and does help with hedging this source of risk.

The difference in portfolio weights between the DSGE and VAR investors can be visualized in **Figure 7**, which shows the average portfolio weights for a given date across all the cohorts that invest on that date (with $\gamma = 20$). For instance, the weights for 2000Q1 correspond to the average of the weights for all the cohorts from 1990Q1 to 2000Q1 (assuming equal weights across cohorts). As Panel A clearly shows, the DSGE investor always has a large fraction of wealth invested in long-term bonds. The investment in stocks is more countercyclical: the weight on stocks decreases before the Internet bubble burst and before the subprime crisis. These episodes correspond to periods with a negative stock premium, as confirmed by Figure 2. The evolution of the hedging demands indicates that the demands are high and positive for bonds and high and negative for

cash. This result indicates that, in the DSGE model, investing in bonds is a good hedge against changes in the bond risk premium. As Panel B shows, such a hedge is much less pronounced for The VAR investor. The VAR model generates hedging (and total) demands for bonds that are close to zero or even negative over certain periods of time, whereas long-term investors are long in stocks over the entire sample period. The VAR investor is not countercyclical in the allocation to stocks. In particular, she does not reduce the allocation to stocks before the subprime crisis.

The out-of-sample performances of the dynamic strategies are also reported in Table 8. As can be seen in Panel C, the annualized (real) return of the DSGE portfolio is positive for all horizons and all levels of risk aversion. For an horizon of 10 years, the average real return is approximately 9.3% per year for low risk aversion ($\gamma = 5$), and this value decreases to 5% for $\gamma = 10$ and 3% for $\gamma = 20$. The annualized volatility is also relatively high, particularly for low risk aversion and long horizons (Panel D). The Sharpe ratio ranges between 0.3 and 0.6, a relatively high range of values given the limited investment set (Panel E).¹⁴

Turning to the VAR portfolio, we notice that the annualized return is low, and even negative for low risk aversion and long horizon. This poor performance primarily occurs because the VAR portfolio is long in stocks at the beginning of the subprime crisis, whereas the DSGE portfolio has zero exposure to the stock market. A similar, although less penalizing, situation occurred at the eve of the Internet bubble burst, when the DSGE portfolio is short in stocks, whereas the VAR portfolio has zero exposure to the stock market. As the annualized volatility is also relatively high for the VAR portfolio, similar to the DSGE portfolio volatility, the Sharpe ratio of the VAR portfolio is found to be very low for low and medium levels of risk aversion.

The main explanation for the performance difference between the DSGE and VAR models is that the former is able to forecast long-term returns very well whereas the

¹⁴As returns are measured quarterly and the investment horizons are from 2 to 10 years, we must compute the Sharpe ratio carefully. We follow Lo's (2002) approach and recognize that returns are not i.i.d. The Sharpe ratio for a k -year horizon is therefore defined as: $SR(k) = SR(1)\eta(k)$, where $SR(1) = (\bar{\rho}_p - \bar{\rho}_1)/\sigma_p$ denotes the 1-quarter Sharpe ratio and $\eta(k) = k/(k+2 \sum_{i=1}^{k-1} (k-i)\varphi_i)^{1/2}$ is the scale factor that corrects for the fact that returns are not i.i.d. The approach involves $\varphi_i = Cov(\rho_{p,t}, \rho_{p,t-i})/\sigma_p^2$ the i -th order serial correlation of real portfolio return, and $\sigma_p^2 = V(\rho_{p,t})$.

latter is not. The optimal allocations of the VAR model are broadly consistent with those reported by Campbell, Chan, and Viceira (2003), who find portfolios that are long in stocks and short in bonds. In contrast, our DSGE portfolios are more in line with the results obtained by Sangvinatsos and Wachter (2005), who allow for a better description of the bond premium. A similar result is also reported by Kojien, Nijman, and Werker (2010) but with much more limited hedging demands, owing to short sales restrictions.¹⁵

6 Conclusion

In this paper, we investigate the ability of a fully structural DSGE model to provide forecasts of future asset returns over long horizons. The model describes the demand and production sides of the economy to obtain the dynamics of the short-term interest rate (set by the central bank), the long-term Treasury bond return, and the stock market return. We also introduce rebalancing costs for risky financial assets, which allow us to generate time-varying risk premia for bonds and stocks. The model shows good performance in forecasting bond and stock returns over long horizons.

From a long-term allocation perspective, we find that the optimal portfolio should be invested in bonds and stocks. For high risk aversion, however, the hedging demands for bonds are much higher than the hedging demands for stocks, resulting in portfolios that are mostly invested in bonds. In our out-of-sample allocation exercise, we find that risk-averse investors will indeed mostly hold bonds. In contrast, when an unrestricted VAR model is used to forecast returns, the optimal portfolio will favor stocks.

The DSGE model clearly outperforms the unrestricted VAR model for long-term allocation. The main reason for this difference in performance is that the VAR model does not forecast bond and stock returns well over the long term. Therefore, the out-of-sample dynamic allocation experiment over the 1990-2010 period definitely favors the

¹⁵Another possible explanation for the overperformance of the DSGE allocation relative to the VAR allocation is that the DSGE model likely mitigates estimation error, which is a well-known major issue in portfolio management. In our context, the unrestricted VAR model involves the estimation of 187 parameters (autoregressive terms and the covariance matrix), whereas the DSGE model involves 48 parameters only. Empirical evidence clearly suggests that imposing theoretical restrictions is beneficial (mitigation of estimation error) rather than detrimental (loss of information).

DSGE model. The Sharpe ratios for the DSGE model are approximately 0.3–0.5 above those for the Sharpe ratios. All these findings suggest that the use of a structural macro model, by imposing long-run restrictions on financial returns, may be of value for long-term investors, such as insurance companies, pension funds, and sovereign wealth funds.

A Log-linearized Model

- Consumption demand (IS curve):

$$\begin{aligned}\hat{c}_t = & \frac{\zeta/\gamma}{1 + \zeta/\gamma} \hat{c}_{t-1} - \frac{1}{1 + \zeta/\gamma} E_t \hat{c}_{t+1} + \frac{(\sigma_C - 1)(1 - \zeta/\gamma)\xi}{\sigma_C(1 + \zeta/\gamma)} (\hat{h}_t - E_t \hat{h}_{t+1}) \\ & - \frac{(1 - \zeta/\gamma)}{\sigma_C(1 + \zeta/\gamma)} (E_t \hat{\rho}_{s,t+1} - (1 - \rho_v) \hat{v}_t - \varsigma \eta_{v,t})\end{aligned}$$

- Investment demand:

$$\hat{l}_t = \frac{1}{1 + \tilde{\beta}} \hat{l}_{t-1} + \frac{\tilde{\beta}}{1 + \tilde{\beta}} E_t \hat{l}_{t+1} + \frac{1}{(1 + \tilde{\beta})\kappa_I} (\hat{q}_t + \hat{z}_t^I)$$

where $\tilde{\beta} = \beta\eta\gamma^{1-\sigma_C}$.

- Tobin's q:

$$\hat{q}_t = (1 - \delta) \frac{\tilde{\beta}}{\eta\gamma} E_t \hat{q}_{t+1} + \left(1 - (1 - \delta) \frac{\tilde{\beta}}{\eta\gamma} \right) E_t \hat{\rho}_{k,t+1} - E_t \hat{\rho}_{s,t+1}$$

- Shadow rental rate of capital:

$$\hat{\rho}_{k,t} = -(\hat{u}_t + \hat{k}_{t-1} - \hat{h}_t) + \hat{w}p_t$$

- Price inflation (Phillips curve):

$$\hat{\pi}_t = \frac{\eta_p}{1 + \tilde{\beta}\eta_p} \hat{\pi}_{t-1} + \frac{\tilde{\beta}}{1 + \tilde{\beta}\eta_p} E_t \hat{\pi}_{t+1} - \frac{1}{(1 + \tilde{\beta}\eta_p)\kappa_p} \left(\hat{z}_t - \alpha \hat{\rho}_{k,t} - (1 - \alpha) \hat{w}p_t - \hat{\theta}_t \right)$$

- Wage inflation:

$$\hat{\pi}_t^w = \eta_w \hat{\pi}_{t-1}^w + \tilde{\beta} (E_t \hat{\pi}_{t+1}^w - \eta_w \hat{\pi}_t) - \frac{1}{\kappa_w} \left[\hat{w}p_t - \left(\sigma_L \hat{h}_t + \frac{\hat{c}_t - (\zeta/\gamma) \hat{c}_{t-1}}{1 - \zeta/\gamma} \right) - \hat{\psi}_t \right]$$

where $\hat{\pi}_t^w = \hat{w}p_t - \hat{w}p_{t-1} + \hat{\pi}_t$.

- Capital accumulation:

$$\hat{k}_t = \frac{1 - \delta}{\eta\gamma} \hat{k}_{t-1} + \left(1 - \frac{1 - \delta}{\eta\gamma} \right) \hat{l}_t + \left(1 - \frac{1 - \delta}{\eta\gamma} \right) (1 + \tilde{\beta}) \kappa_I \hat{z}_t^I$$

- Production function:

$$\hat{y}_t = \theta \left[\hat{z}_t + \alpha(\hat{u}_t + \hat{k}_{t-1}) + (1 - \alpha)\hat{h}_t \right]$$

- Capital utilization:

$$\hat{u}_t = \frac{1}{\chi} \hat{\rho}_{k,t}$$

- Real dividends:

$$\frac{\bar{d}p}{\bar{y}} \hat{d}p_t = (1 - \tau_y)(1 - \tau_s) \left[\hat{y}_t - (1 - \alpha)(\hat{w}p_t + \hat{h}_t) \right] - \frac{\bar{l}}{\bar{y}} \hat{l}_t + \tau_y \delta_a \frac{\bar{k}}{\bar{y}} \hat{k}_{t-1} + \hat{\Phi}_{d,t}$$

- Taylor rule of the central bank:

$$\hat{r}_{f,t} = \rho_r \hat{r}_{f,t-1} + (1 - \rho_r) \left[a_\pi (\hat{\pi}_t - \hat{\pi}_t) + a_y (\hat{y}_t - \hat{y}_t^n) + a_g \left((\hat{y}_t - \hat{y}_t^n) - (\hat{y}_{t-1} - \hat{y}_{t-1}^n) \right) \right] + \hat{\epsilon}_{r,t}$$

- Inflation target:

$$\hat{\pi}_t = \rho_\pi \hat{\pi}_{t-1} + \varsigma_\pi \eta_{\pi,t}$$

- Nominal short-term return (asset pricing condition):

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + E_t \hat{\rho}_{1,t+1}$$

- Real long-term bond return (asset pricing condition):

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + E_t \hat{\rho}_{b,t+1}^{(k)} - \hat{\phi}_{b,t}^{(k)}$$

- Real stock return (asset pricing condition):

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + E_t \hat{\rho}_{s,t+1} - \hat{\phi}_{s,t}$$

where $\hat{\rho}_{s,t} = \tilde{\beta} \hat{v}p_t + (1 - \tilde{\beta}) \hat{d}p_t - \hat{v}p_{t-1}$ (dividend discount model).

- Goods market clearing condition:

$$\frac{\bar{c}}{\bar{y}} \hat{c}_t + \frac{\bar{l}}{\bar{y}} \hat{l}_t + \frac{\bar{g}}{\bar{y}} \hat{g}_t = \hat{y}_t$$

- Shocks:

$$\begin{aligned}
\hat{v}_t &= \rho_v \hat{v}_{t-1} + \eta_{v,t} - \varsigma_v \eta_{v,t-1} \\
\hat{\psi}_t &= \rho_\psi \hat{\psi}_{t-1} + \eta_{\psi,t} - \varsigma_\psi \eta_{\psi,t-1} \\
\hat{\theta}_t &= \rho_\theta \hat{\theta}_{t-1} + \eta_{\theta,t} - \varsigma_\theta \eta_{\theta,t-1} \\
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \eta_{z,t} - \varsigma_z \eta_{z,t-1} \\
\hat{z}_t^I &= \rho_i \hat{z}_{t-1}^I + \eta_{i,t} - \varsigma_i \eta_{i,t-1} \\
\hat{\Phi}_{d,t} &= \rho_d \hat{\Phi}_{d,t-1} + \eta_{d,t} - \varsigma_d \eta_{d,t-1} \\
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \rho_{g,z} \eta_{z,t} + \eta_{g,t} - \varsigma_g \eta_{g,t-1} \\
\hat{c}_{r,t} &= \rho_r \hat{c}_{r,t} + \eta_{r,t} - \varsigma_r \eta_{r,t-1} \\
\hat{\phi}_{s,t} &= \rho_{\phi,s} \hat{\phi}_{s,t-1} + \eta_{\phi,s,t} - \varsigma_{\phi,s} \eta_{\phi,s,t-1} \\
\hat{\phi}_{b,t}^{(k)} &= \rho_{\phi,b}^{(k)} \hat{\phi}_{b,t-1}^{(k)} + \eta_{\phi,b,t}^{(k)} - \varsigma_{\phi,b}^{(k)} \eta_{\phi,b,t-1}^{(k)}.
\end{aligned}$$

B Data

Per-capita variables are computed using the U.S. population over 16 years of age. Real variables are computed using the GDP deflator (NIPA Table 1.1.9). Observable variables are defined as follows:

- Per-capita real GDP ($\Delta \log GDP_t$) is the log-difference in GDP adjusted by population and inflation (from NIPA Table 1.15, line 1).
- Per-capita real consumption ($\Delta \log CONS_t$) is the log-difference in consumption adjusted by population and inflation (from NIPA Table 1.15, line 2).
- Per-capita real investment ($\Delta \log INV_t$) is the log-difference in investment adjusted by population and inflation (from NIPA Table 1.15, line 7).
- Per-capita labor hours ($\log HRS_t$) is the log-difference in total labor hours adjusted by population. Labor hours are defined as total employment multiplied by the average workweek duration (from Current Employment Statistics).

- Real wage rate ($\Delta \log WAGE_t$) is the log-difference in hourly compensation rate in nonfarm business adjusted for inflation (from the Bureau of Labor Statistics, BLS Series id: PRS85006103).
- Inflation ($\Delta \log P_t$) is the log-difference in the GDP deflator.
- Per-capita real market value of nonfinancial firms ($\Delta \log CAP_t$) is computed as liabilities (line 21) minus financial assets (line 6) plus the market value of equities outstanding (line 35) per capita and in real terms (from Flow of Funds Account of the United States, Table B.102).
- Per-capita real dividends of nonfinancial firms ($\Delta \log DIV_t$). The series includes net buybacks and net financial acquisitions, minus the net increase in financial liabilities.
- The federal funds rate (FFR_t) (from the FRED database).
- The long-term interest rate (LTR_t) is the 10-year Treasury constant maturity yield (from the FRED database).

We compute the holding period return of the k -period bond between t and $t + 1$ as follows:

$$r_{b,t+1}^{(k)} = D_t^{(k)} y_{b,t}^{(k)} - (D_t^{(k)} - 1) y_{b,t+1}^{(k-1)}, \quad (12)$$

where $y_{b,t}^{(k)} = \log(1 + LTR_t^{(k)})$ and $D_t^{(k)}$ is Macaulay's duration defined as:

$$D_t^{(k)} = \frac{1 - e^{-ky_{b,t}^{(k)}}}{1 - e^{-y_{b,t}^{(k)}}} \simeq \frac{1 - (1 + LTR_t^{(k)})^{-k}}{1 - (1 + LTR_t^{(k)})^{-1}}. \quad (13)$$

We use the approximation $y_{b,t+1}^{(k-1)} = y_{b,t}^{(k)}$ for large k .

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Table 1: Summary statistics on observable variables

Variable		Annual. mean ($\times 100$)	Annual. std dev. ($\times 100$)	Persistence
Panel A: Macro variables				
Change in real GDP	Δy_t	1.68	3.70	0.32
Change in real consumption	Δc_t	1.88	2.92	0.25
Change in real investment	Δi_t	1.42	18.96	0.21
Hours	h_t	–	12.20	0.96
Change in real wages	Δwp_t	1.62	2.39	0.06
GDP inflation	π_t	3.44	2.36	0.84
Panel B: Financial variables				
Nominal short-term rate	$r_{1,t}$	5.34	3.21	0.96
Nominal long-term rate	$y_{b,t}$	6.19	2.47	0.97
Nominal bond holding period return	$r_{b,t}$	6.01	15.44	-0.03
Real bond holding period return	$\rho_{b,t}$	2.56	15.64	0.01
Change in real market value of equity	Δvp_t	2.44	30.16	0.05
Change in real dividends	Δdp_t	2.48	46.80	0.17
Real stock return	$\rho_{s,t}$	6.60	30.26	0.06

Note: The table reports the annualized mean, annualized standard deviation, and persistence parameter (first-order autocorrelation) of the observable variables. The data is quarterly from 1955 to 2010, for a total of 224 observations.

Table 2: Value of the calibrated parameters

	Symbol	Value
Average quarterly growth of the population	η	1.0035
Average quarterly growth of per-capita output	γ	1.0042
Trend inflation factor	π	1.0086
Average (gross) nominal interest rate	R_1	1.0134
Adjusted time-discount factor	$\tilde{\beta}$	0.9935
Capital share parameter	α	28.37%
Depreciation rate of capital	δ	1.28%
Accounting depreciation rate	δ_a	1.40%
Tax rate on firm income	τ_y	31.5%
Tax rate on dividend income	τ_d	21.5%
Tax rate on sales	τ_s	8.9%
Tax rate on labor income	τ_h	35%
Share of consumption	\bar{c}/\bar{y}	65.0%
Share of government expenditure	\bar{g}/\bar{y}	19.1%
Share of investment	\bar{i}/\bar{y}	16.0%
Share of dividends	$\bar{d}p/\bar{y}$	5.2%
Share of labor compensation in total income	$\bar{w}p/\bar{y}$	65.3%

Note: The table reports the value of the calibrated parameters. These values are drawn from Alpanda (2013). The calibration is based on the data available over the 1955-2010 period.

Table 3a: Parameter estimates of the DSGE model

		Prior			Posterior distribution			Alpanda
		Dist.	Par.1	Par.2	Mean	5%	95%	(2013)
ζ	Habit	beta	0.7	0.1	0.940	0.921	0.959	0.954
σ_C	Consumption elasticity	norm	1.5	0.37	1.014	0.917	1.106	0.995
σ_L	Labor supply elasticity	norm	2	0.75	2.587	1.619	3.511	2.497
ψ	Wage Mark-up	norm	1.5	0.12	1.614	1.434	1.799	1.609
θ	Price Mark-up	norm	1.5	0.12	1.698	1.580	1.814	1.707
χ^e	Utilization elasticity	beta	0.5	0.15	0.213	0.141	0.283	0.187
κ_w^e	Adjustment cost - Wage	beta	0.5	0.1	0.861	0.815	0.908	0.863
κ_p^e	Adjustment cost - Price	beta	0.6	0.1	0.815	0.771	0.859	0.782
κ_I	Adjustment cost - Invest.	norm	4	1.5	3.137	1.889	4.316	4.473
η_w	Wage indexation	beta	0.5	0.15	0.459	0.279	0.649	0.518
η_p	Price indexation	beta	0.5	0.15	0.193	0.084	0.297	0.200
ρ_r	Taylor - smoothing	beta	0.75	0.1	0.791	0.737	0.845	0.819
a_π	Taylor - inflation	norm	1.5	0.25	1.249	0.961	1.534	1.449
a_y	Taylor - output gap	norm	0.12	0.05	0.118	0.066	0.168	0.071
a_g	Taylor - output growth	norm	0.12	0.05	0.210	0.141	0.281	0.236
ρ_v	AR term. Consumption	beta	0.5	0.2	0.643	0.438	0.846	0.511
ρ_ψ	AR term. Wage mark-up	beta	0.6	0.2	0.870	0.767	0.964	0.864
ρ_θ	AR term. Price mark-up	beta	0.5	0.2	0.936	0.904	0.970	0.941
ρ_z	AR term. Productivity	beta	0.5	0.2	0.969	0.952	0.988	0.974
ρ_I	AR term. Investment	beta	0.6	0.2	0.998	0.998	0.998	0.996
ρ_g	AR term. Government	beta	0.5	0.2	0.984	0.976	0.992	0.985
ρ_r	AR term. Monetary	beta	0.5	0.2	0.502	0.253	0.749	0.579
ρ_d	AR term. Dividend	beta	0.5	0.2	0.952	0.931	0.973	0.947
$\rho_{g,z}$	Cross-corr. Gvt-Prod.	beta	0.5	0.2	0.675	0.445	0.923	0.685
ρ_b	AR term. Bond risk	beta	0.5	0.2	0.958	0.935	0.982	–
ρ_s	AR term. Stock risk	beta	0.5	0.2	0.816	0.777	0.852	0.824
$\rho_{\bar{\pi}}$	AR term. Target inflation	beta	0.5	0.2	0.853	0.736	0.952	–
ς_v	MA term. Consumption	beta	0.5	0.2	0.488	0.263	0.697	0.460
ς_ψ	MA term. Wage mark-up	beta	0.5	0.2	0.815	0.677	0.948	0.802
ς_θ	MA term. Price mark-up	beta	0.5	0.2	0.895	0.839	0.954	0.893
ς_z	MA term. Productivity	beta	0.5	0.2	0.070	0.012	0.121	0.063
ς_I	MA term. Investment	beta	0.5	0.2	0.884	0.815	0.951	0.927
ς_g	MA term. Government	beta	0.5	0.2	0.047	0.007	0.082	0.048
ς_r	MA term. Monetary	beta	0.5	0.2	0.566	0.233	0.899	0.428
ς_d	MA term. Dividend	beta	0.5	0.2	0.069	0.013	0.119	0.069
ς_b	MA term. Bond risk	beta	0.5	0.2	0.458	0.195	0.702	–
ς_s	MA term. Stock risk	beta	0.5	0.2	0.192	0.039	0.335	0.184

Table 3b: Estimates of the standard deviation of shocks ($\times 100$)

		Priors			Posterior distribution			Alpanda
		Dist.	Par.1	Par.2	Mean	5%	95%	(2013)
η_v	Consumption	invg	0.005	Inf	10.63	7.60	13.51	0.31 ^(*)
η_w	Wage	invg	0.005	Inf	0.49	0.43	0.55	0.49
η_θ	Price	invg	0.005	Inf	0.21	0.18	0.24	0.17
η_z	Productivity	invg	0.005	Inf	0.49	0.45	0.53	0.49
η_I	Investment	invg	0.005	Inf	1.55	1.29	1.81	1.64
η_g	Government	invg	0.005	Inf	1.93	1.78	2.08	1.93
η_r	Monetary	invg	0.005	Inf	0.17	0.15	0.19	0.20
η_d	Dividend	invg	0.005	Inf	0.79	0.72	0.85	0.79
η_b	Bond premium	invg	0.005	Inf	0.53	0.29	0.76	–
η_s	Stock premium	invg	0.005	Inf	1.37	1.01	1.71	1.42
$\eta_{\bar{\pi}}$	Target inflation	invg	0.005	Inf	0.28	0.17	0.39	–

Note: The table reports the information about the prior and posterior distributions of the parameters. For the prior distribution, the table indicates the class of distribution and its two characteristic parameters. For the posterior distribution, the table reports the mean and the 5%–95% confidence interval. The acronyms “beta”, “norm.”, and “invg” stand for the beta, the normal, and the inverse gamma distributions. ^(*) We have rescaled consumption shock for readability purpose, so that its standard deviation is not comparable to the one reported by Alpanda (2003).

Table 4: Correlations between asset returns and risk premia in the DSGE model

	Short-term interest rate premium	Bond risk premium	Stock risk premium
Panel A: 1955-1989			
Cash return	0.397	0.671	-0.487
Bond return	-0.280	-0.093	0.143
Stock return	-0.215	-0.094	-0.162
Panel B: 1990-2010			
Cash return	0.279	0.306	-0.617
Bond return	0.326	-0.380	0.104
Stock return	-0.043	0.091	-0.448

Note: The table reports the correlations between asset returns and risk premia in the DSGE model, over the subsamples 1955-89 and 1990-2010.

Table 5: RMSE – DSGE model

	Forecast horizon		
	2 years	5 years	10 years
Panel A: Macro variables			
Output	3.43	5.41	3.15
Consumption	2.73	4.75	7.42
Investment	18.97	33.01	21.62
Hours	2.88	3.71	3.07
Real wages	3.64	7.26	6.73
Inflation (cumulative)	1.72	6.17	17.52
Panel B: Financial variables			
Short-term rate (nominal)	2.73	7.73	18.19
Long-term rate (nominal)	1.51	4.96	14.46
Bond holding period return (real)	6.09	6.29	8.16
Market value of equity (real)	24.71	33.20	37.08
Dividends (real)	42.45	47.34	34.78
Stock returns (real)	24.19	33.38	36.76
Nb of observations	76	64	44

Note: The table reports the percent RMSE (root mean square error) for the observable variables over the 2, 5, and 10-year horizons in the DSGE model.

Table 6: Parameter estimates of the unrestricted VAR(1) model

	Δy_t	Δc_t	$\Delta \iota_t$	Δh_t	Δwp_t	π_t	$r_{1,t}$	$sp_{b,t}$	$x_{b,t}$	$\rho_{s,t}$	dpr_t	R^2
Panel A: Parameter estimates												
Δy_{t+1}	-0.26 (1.29)	0.37 (2.18)	0.04 (1.14)	0.15 (1.02)	0.05 (0.32)	0.00 (0.02)	-0.13 (1.05)	0.70 (3.01)	-0.06 (3.65)	0.02 (1.62)	0.00 (1.67)	0.35
Δc_{t+1}	0.04 (0.25)	-0.10 (0.76)	0.02 (0.70)	0.01 (0.10)	0.03 (0.24)	-0.23 (2.02)	0.06 (0.56)	0.73 (3.89)	-0.05 (3.94)	0.02 (2.48)	0.00 (1.47)	0.35
$\Delta \iota_{t+1}$	-2.10 (2.04)	3.29 (3.86)	0.22 (1.38)	1.30 (1.80)	0.45 (0.62)	1.50 (2.10)	-0.77 (1.20)	2.92 (2.51)	-0.23 (2.72)	0.06 (1.33)	-0.01 (0.48)	0.34
Δh_{t+1}	-0.22 (1.62)	0.42 (3.74)	0.05 (2.26)	0.09 (0.97)	0.00 (0.05)	-0.02 (0.25)	0.07 (0.79)	0.59 (3.86)	-0.05 (4.41)	0.00 (0.48)	0.00 (1.24)	0.44
Δwp_{t+1}	0.18 (1.26)	-0.17 (1.43)	-0.03 (1.19)	0.13 (1.27)	0.26 (2.59)	0.03 (0.33)	-0.17 (1.89)	-0.22 (1.36)	0.01 (0.78)	0.01 (2.22)	0.00 (1.05)	0.15
π_{t+1}	-0.13 (1.77)	0.16 (2.53)	0.01 (0.98)	0.04 (0.84)	0.12 (2.32)	0.86 (16.5)	0.04 (0.91)	-0.11 (1.34)	-0.01 (1.65)	0.00 (0.76)	0.00 (0.46)	0.82
$r_{1,t+1}$	-0.02 (0.45)	0.09 (2.05)	0.00 (0.22)	0.11 (2.84)	0.06 (1.66)	0.13 (3.47)	0.95 (29.0)	0.12 (1.95)	-0.03 (6.20)	0.00 (0.35)	0.00 (0.42)	0.94
$sp_{b,t+1}$	0.02 (0.33)	-0.09 (2.05)	0.00 (0.22)	-0.10 (2.72)	-0.06 (1.65)	-0.13 (3.51)	0.04 (1.35)	0.92 (15.7)	-0.01 (2.95)	0.00 (0.38)	0.00 (0.36)	0.77
$x_{b,t+1}$	-0.15 (0.15)	-0.02 (0.03)	-0.01 (0.09)	-0.93 (1.34)	0.49 (0.71)	-2.37 (3.47)	2.32 (3.80)	4.36 (3.92)	-0.18 (2.26)	-0.09 (2.03)	0.01 (1.10)	0.20
$\rho_{s,t+1}$	3.32 (1.80)	-2.83 (1.85)	-0.38 (1.35)	-2.87 (2.22)	-1.41 (1.10)	-2.11 (1.65)	0.11 (0.10)	2.33 (1.12)	0.32 (2.12)	-0.03 (0.32)	0.02 (0.61)	0.15
dpr_{t+1}	-2.31 (0.91)	1.52 (0.72)	0.17 (0.43)	1.54 (0.86)	-1.20 (0.67)	3.13 (1.76)	-0.31 (0.19)	6.32 (2.20)	-0.70 (3.39)	0.08 (0.68)	0.97 (27.6)	0.86
Panel B: Correlation matrix of residuals												
Δy_t	1	0.58	0.72	0.48	0.00	-0.22	0.05	-0.03	-0.04	0.01	0.37	
Δc_t	-	1	0.03	0.30	0.19	-0.29	0.03	-0.02	0.00	0.08	0.07	
$\Delta \iota_t$	-	-	1	0.40	-0.07	-0.02	0.07	-0.06	-0.01	-0.10	0.34	
Δh_t	-	-	-	1	-0.36	-0.02	0.17	-0.16	-0.09	0.00	0.13	
Δwp_t	-	-	-	-	1	-0.27	-0.06	0.06	0.11	0.03	-0.16	
π_t	-	-	-	-	-	1	0.09	-0.10	-0.01	-0.09	-0.01	
$r_{1,t}$	-	-	-	-	-	-	1	-0.98	-0.04	-0.09	-0.06	
$sp_{b,t}$	-	-	-	-	-	-	-	1	0.06	0.09	0.07	
$x_{b,t}$	-	-	-	-	-	-	-	-	1	-0.13	-0.02	
$\rho_{s,t}$	-	-	-	-	-	-	-	-	-	1	-0.58	
dpr_t	-	-	-	-	-	-	-	-	-	-	1	

Note: The table reports the parameter estimates (Panel A) and the correlation matrix of residuals (Panel B) for the unrestricted VAR(1) model. Numbers in parentheses represent the t-stat of the parameter estimates. We use the notations $sp_{b,t} = y_{b,t} - r_{1,t}$, $x_{b,t} = r_{b,t} - r_{1,t}$, and $dpr_t = dp_t - vp_t$.

Table 7: RMSE – Unrestricted VAR(1) model

	Forecast horizon		
	2 years	5 years	10 years
Panel A: Macro variables			
Output	3.28	5.25	6.03
Consumption	2.98	4.95	5.65
Investment	16.42	24.01	32.90
Hours	2.73	3.91	6.23
Real wages	2.96	5.72	4.31
Inflation (cumulative)	2.69	7.74	18.15
Panel B: Financial variables			
Short-term rate (nominal)	2.94	9.60	22.18
Long-term rate (nominal)	1.48	5.75	16.75
Bond holding period return (real)	8.33	11.56	22.30
Dividend - equity ratio	33.47	42.72	49.80
Stock returns (real)	21.72	33.91	42.36
Nb of observations	76	64	44

Note: The table reports the percent RMSE (root mean square error) for the observable variables over the 2, 5, and 10-year horizons in the unrestricted VAR(1) model.

Table 8: Out-of-sample performance of dynamic strategies

Risk aversion (γ)	DSGE model			Unrestricted VAR(1) model		
	Investment horizon			Investment horizon		
	2 years	5 years	10 years	2 years	5 years	10 years
Panel A: Average weight for stocks						
5	0.39	-0.36	-1.38	1.82	1.98	1.83
10	0.24	-0.13	-0.64	0.94	1.02	0.90
20	0.14	-0.04	-0.29	0.47	0.50	0.43
50	0.08	0.01	-0.09	0.17	0.18	0.14
Panel B: Average weight for bonds						
5	1.99	2.12	2.14	-0.46	-1.20	-1.51
10	1.52	1.58	1.57	0.10	-0.37	-0.66
20	1.20	1.22	1.19	0.45	0.15	-0.06
50	0.97	0.96	0.92	0.69	0.50	0.38
Panel C: Annualized real return						
5	7.81	8.15	9.29	0.39	-0.30	-0.47
10	4.37	4.50	5.00	0.07	-0.20	-0.03
20	2.69	2.75	3.02	0.44	0.28	0.41
50	1.67	1.68	1.83	0.71	0.62	0.73
Panel D: Annualized volatility						
5	37.70	38.47	41.11	39.59	38.77	34.77
10	20.19	20.62	22.14	21.90	20.88	17.69
20	10.96	11.11	11.82	11.34	10.65	8.79
50	5.55	5.49	5.60	5.20	4.79	4.03
Panel E: Sharpe ratio						
5	0.38	0.47	0.34	0.02	0.00	0.01
10	0.41	0.50	0.33	0.01	0.00	0.04
20	0.47	0.54	0.39	0.09	0.07	0.14
50	0.58	0.64	0.60	0.26	0.26	0.34

Note: The table reports the average optimal weight for stocks and bonds and statistics on ex-post performances of the dynamic investment strategies based on the DSGE and the unrestricted VAR(1) models. Return and volatility are in annualized percent.

Figure 1: Observable data

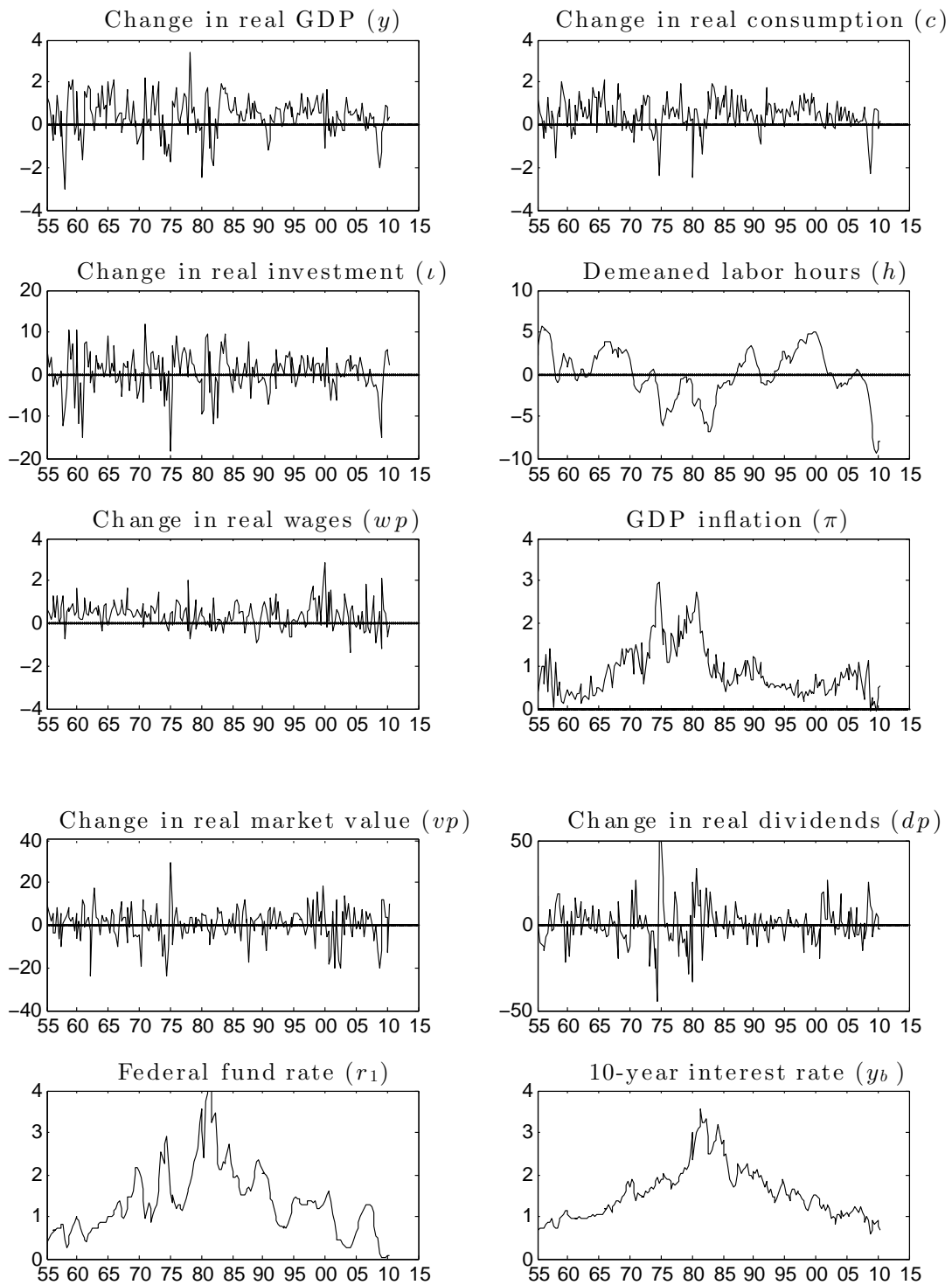


Figure 2: Bond and stock premia

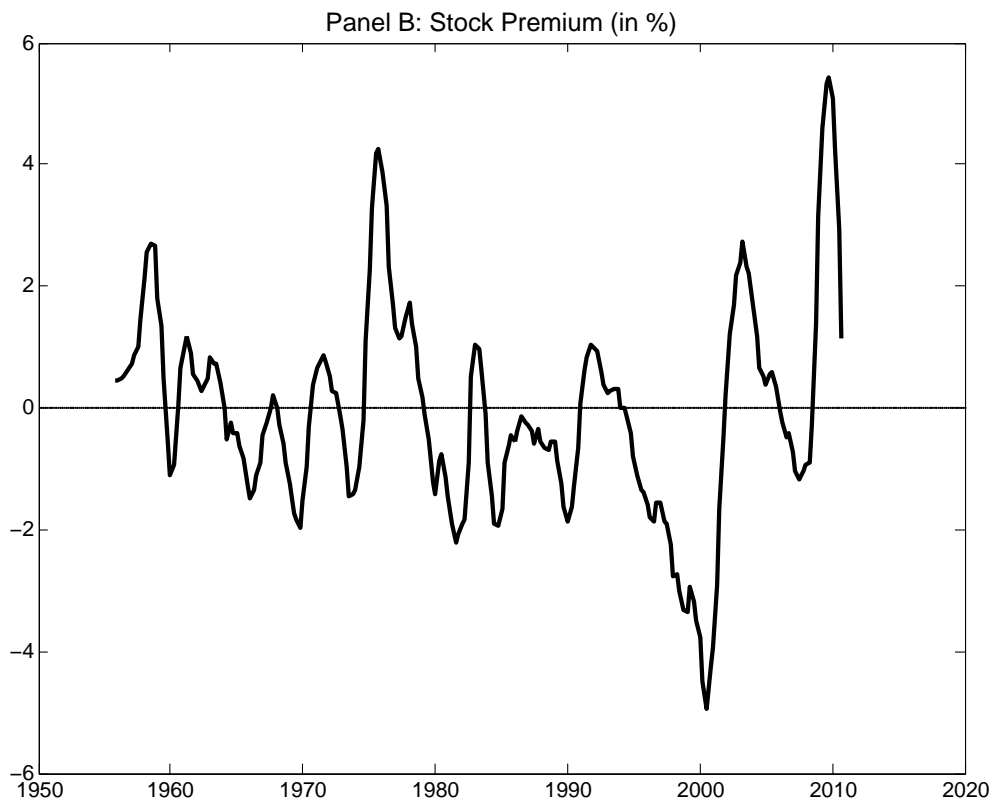
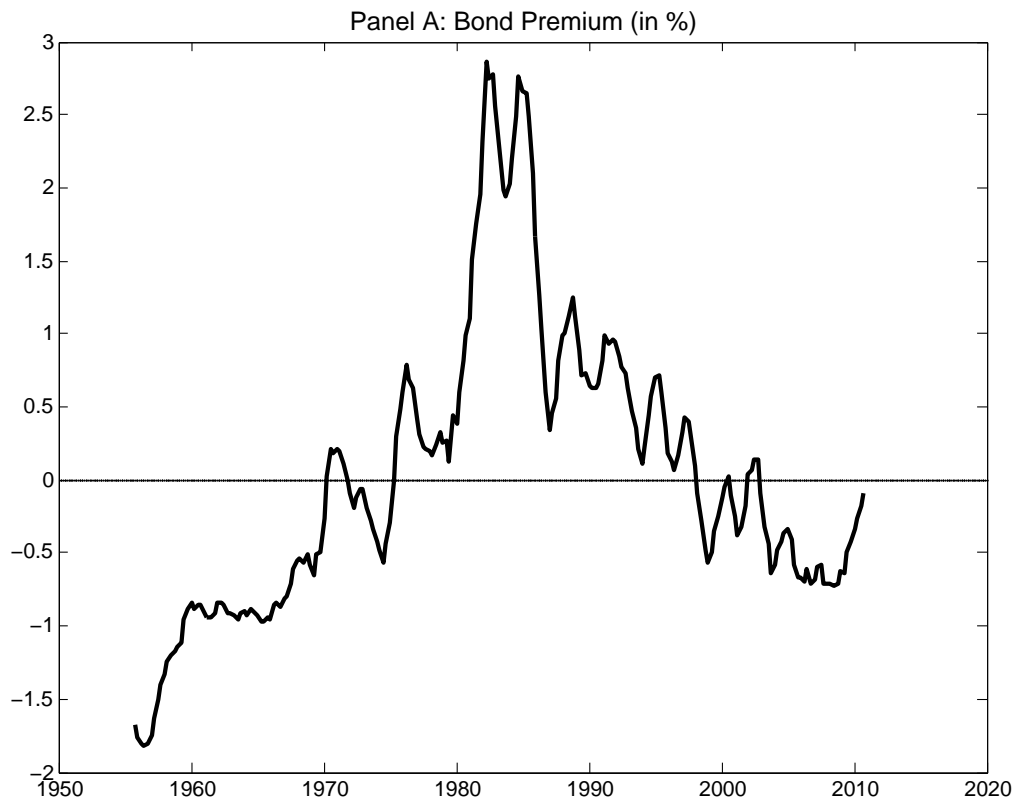


Figure 3: Annualized volatility of real returns – DSGE model (as of 2010Q4)

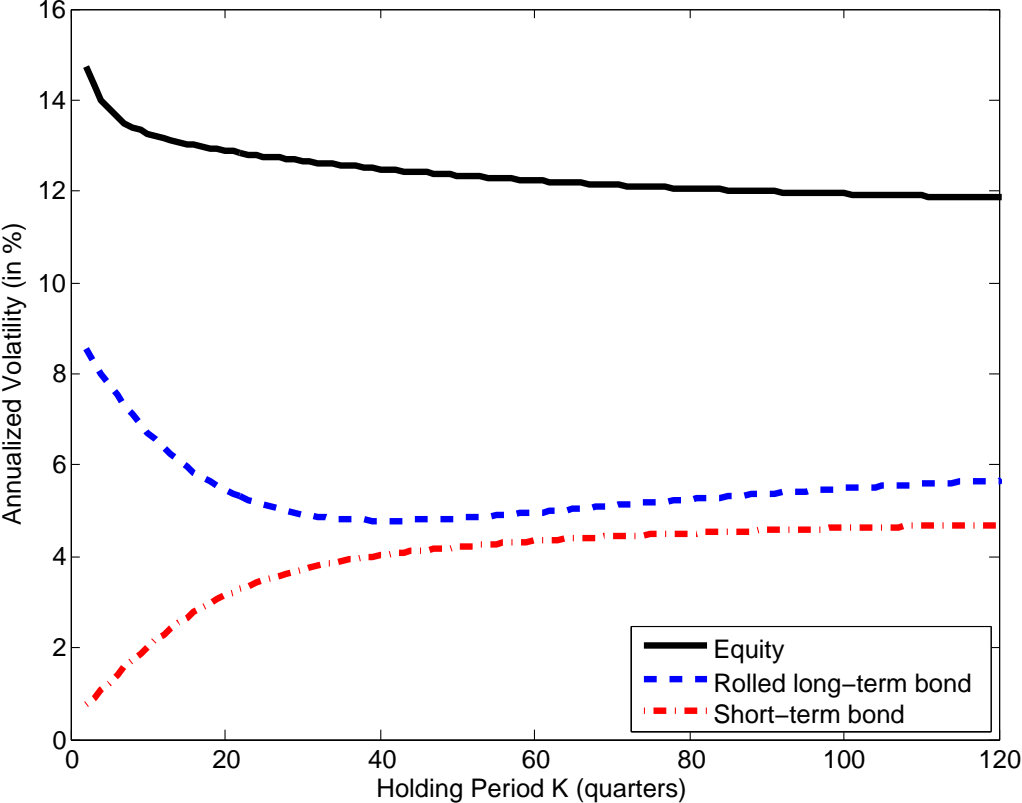
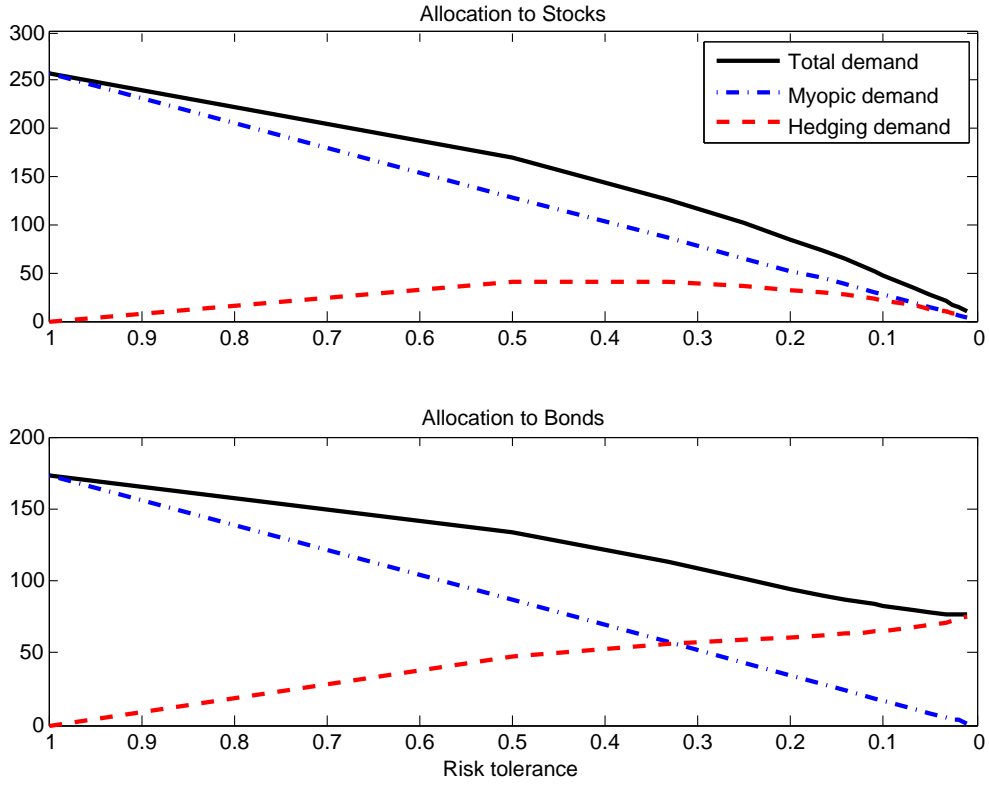


Figure 4: Optimal portfolio weights

Panel A: Allocation at the end of 1999



Panel B: Allocation at the end of 2010

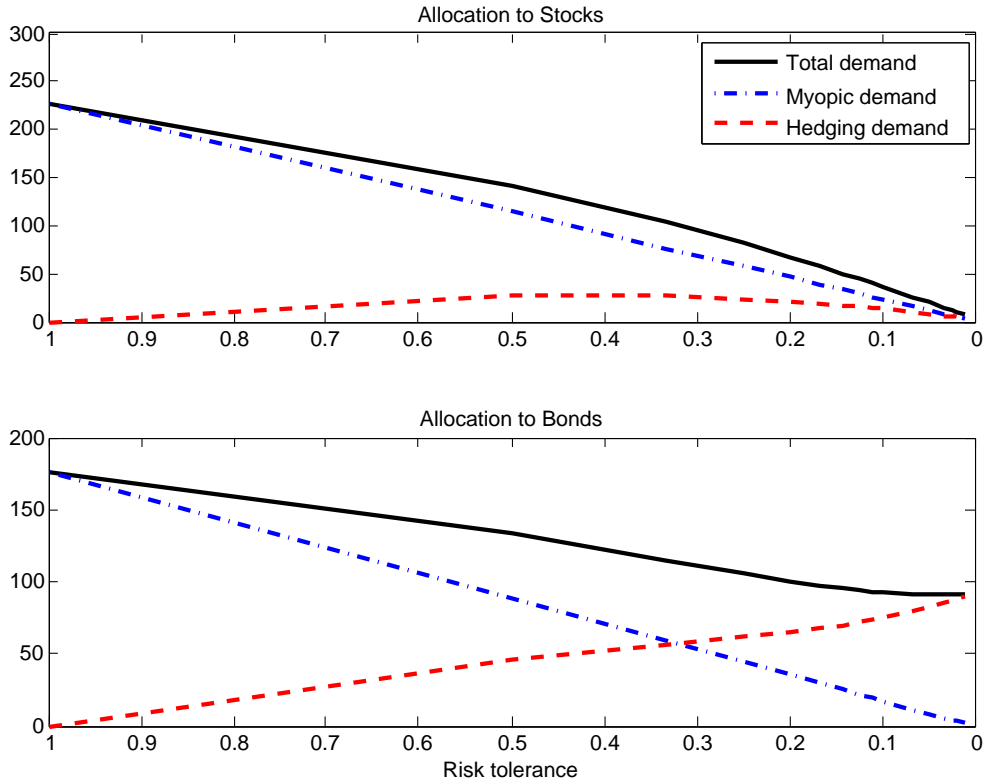
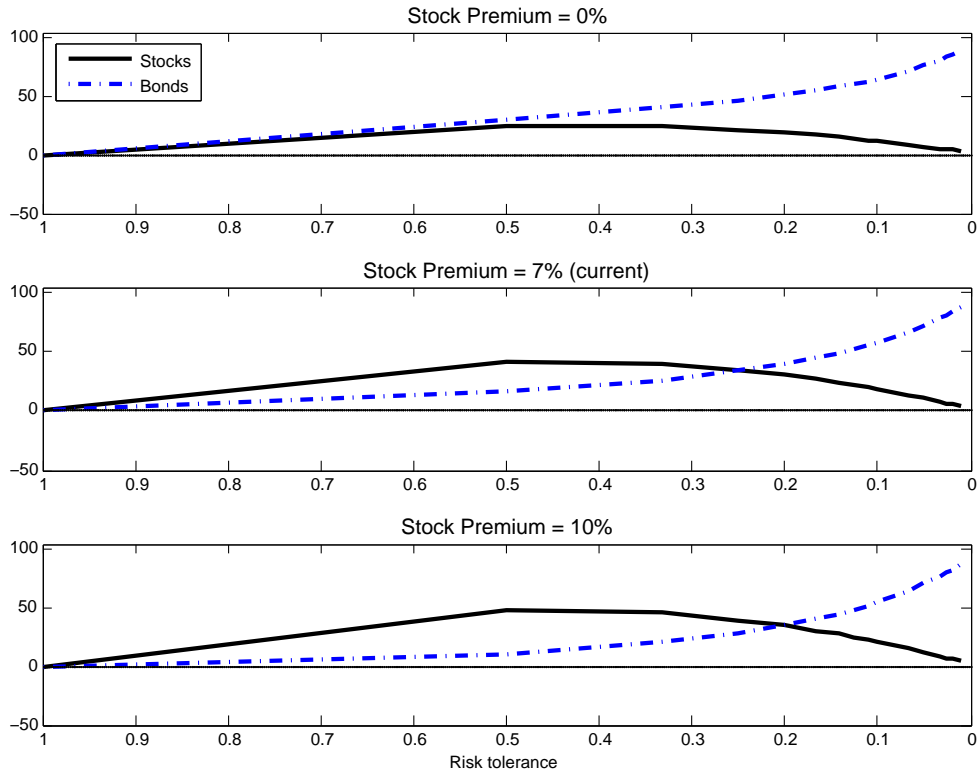


Figure 5: Hedging demands (allocation at end of 2010)

Panel A: Change in the stock premium (bond premium = -0.5%)



Panel B: Change in the bond premium (stock premium = 7%)

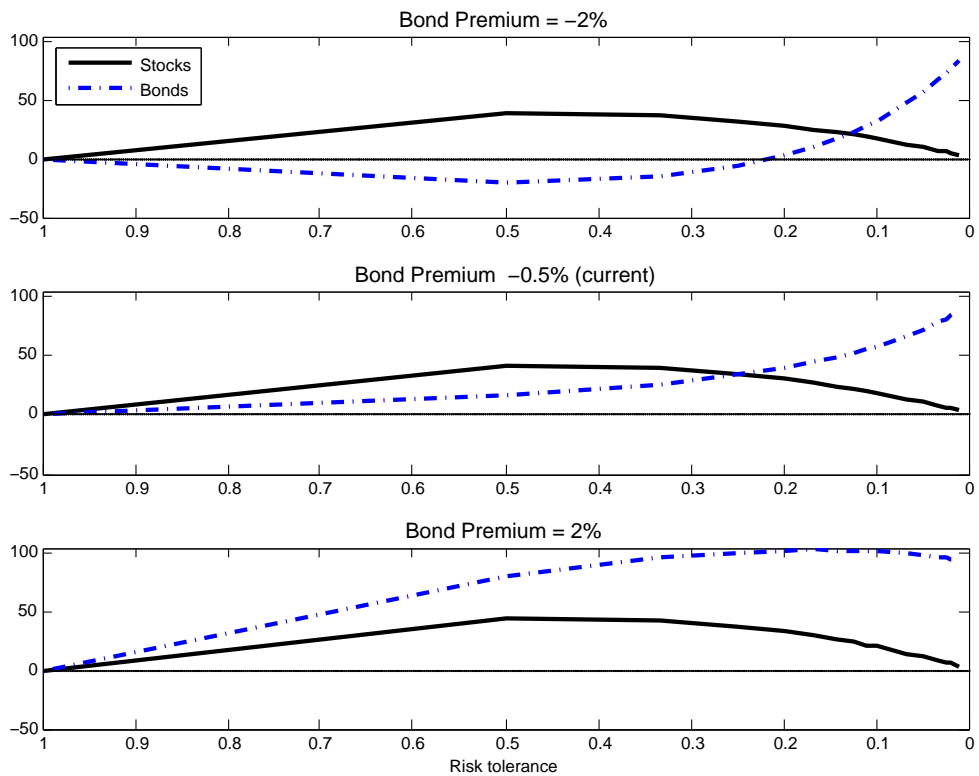


Figure 6: Annualized volatility of real returns – VAR(1) model (as of 2010Q4)

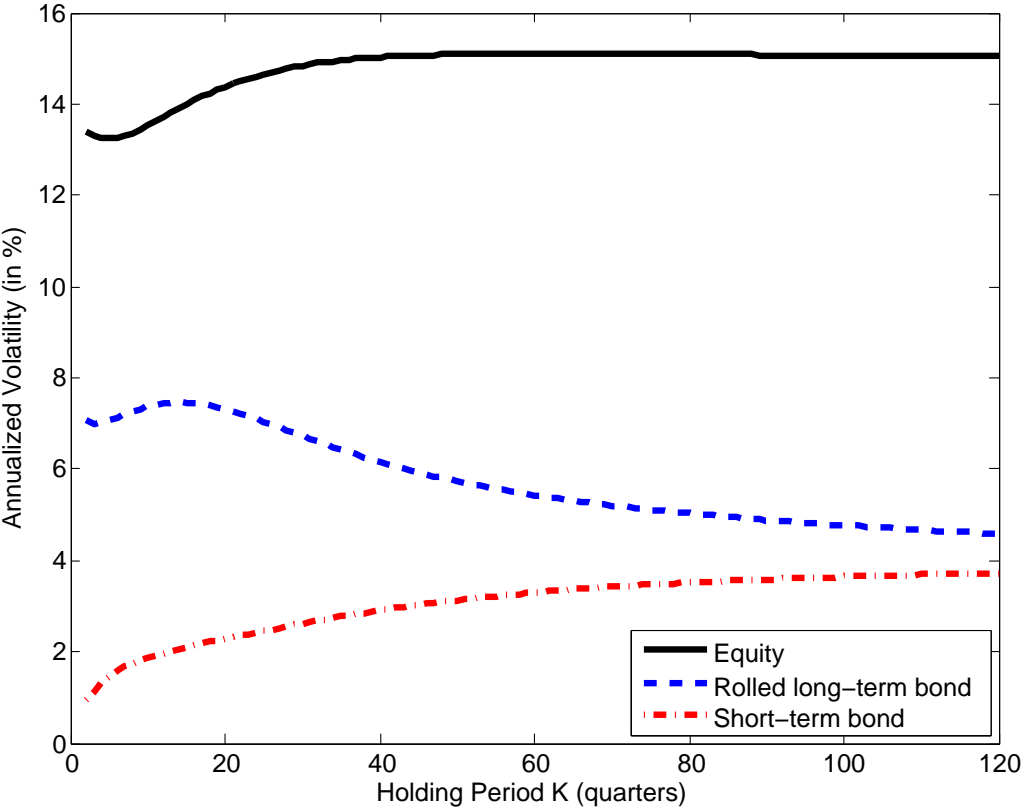
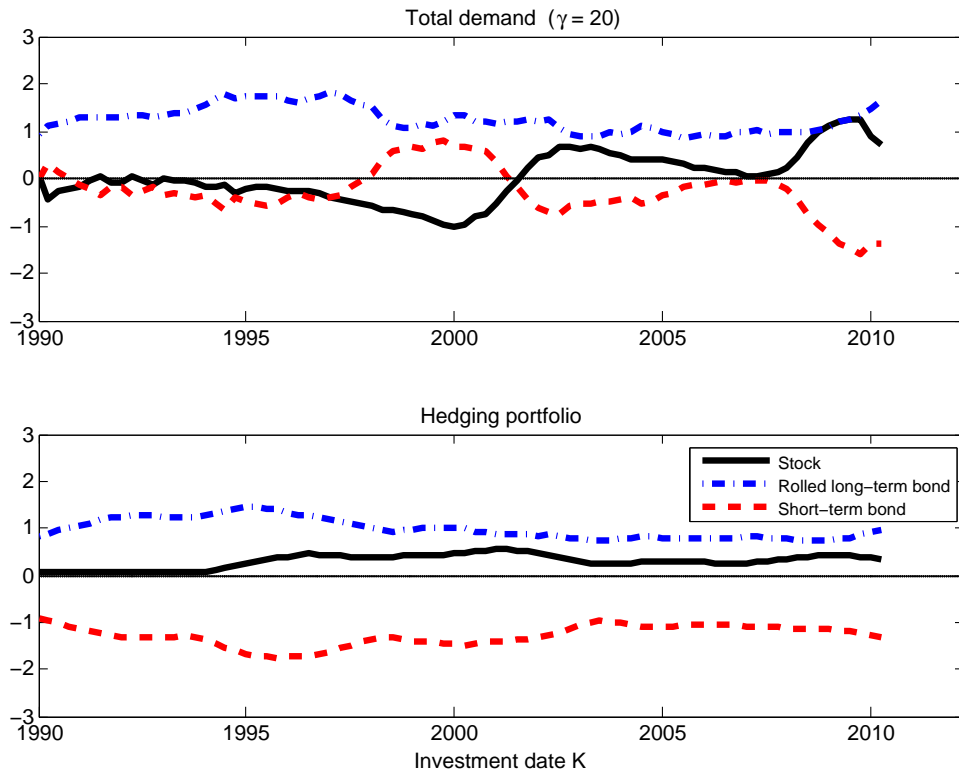


Figure 7: Average portfolio weight for a 10-year horizon

Panel A: Investor using DSGE model



Panel B: Investor using VAR model

