Collusion, Incentives and Reputation: The role of Experts in Corporate Governance*

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Abstract

We demonstrate that CEOs with higher incremental agency costs and experts (like audit firms) with imprecise signals have strong tendencies to manipulate information jointly. To deter collusion between them, firms must design both incentive contracting and elicit the expert’s reputation for honesty using optimal and probabilistic contract renewals. The expert’s skills to obtain precise information and her reputation for honesty are positively correlated. Reputable experts enjoy larger life-time earnings, have longer-term relationships with firms and contribute to corporate governance of their clients by facilitating truthful information disclosures at lower costs and helping timely restructuring of unprofitable projects.

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Introduction

Recent waves of corporate scandals have put expert intermediaries, such as accounting and consulting firms, corporate advisories and investment banks, directly under the media spotlight. For example, The New York Times gave an account of the top 10 accounting scandals and cover-ups of the last decade and the list includes names such as Enron, AIG, Fannie Mae and Freddie Mac and Satyam.¹ Many investment bankers and corporate advisories had their reputation smeared for failures to conduct due diligence before M&A deals or IPOs. Hewlett-Packard, for example, found out an accounting cover-up of $8.8 billion just after its acquisition of Autonomy for which it had paid, unwittingly, $11.8 billion.² Facebook’s IPO was another example; its share price went down by 52 per cent immediately after it was revealed that the company’s revenues had been grossly overstated at the time of the IPO.³

A common element present in all the examples cited above and in many others was the alleged cover-up of bad news or inflation of true value by a collusive alliance of the CEOs of companies and their expert advisory firms (audit firms, investment bankers etc.). This paper addresses problems of joint manipulation of private information by these advisory firms and CEO of their client firms which hurt shareholders’ interests. Specifically, the paper aims to answer the following questions: To what extent can corporate governance mechanisms such as incentives contracting contain such collusion intended to manipulate information? Can reputation concerns of the advisory firms alleviate the problem? If they do, how one can empirically differentiate between reputable and non reputable experts? Given these advisory firms’ extensive involvements in activities, ranging from disclosures to M and A and IPOs, these questions are of utter importance.

¹See FT "Corporate Year in Review 2012", 12/28/2012 for this and other recent scandals.
This paper examines these questions in a setting where a firm hires a CEO to run a project and an expert for advisory services. The CEO adds value to the firm by contributing effort to the project’s success. The expert obtains valuable information about this project’s profitability at the on-going stage and makes recommendation to the CEO whether the firm should continue as a going concern or not.\textsuperscript{4} This information is private, soft and liable to be mis-reported, especially to the interests of the CEO, who can bribe an expert to suppress unfavorable news.\textsuperscript{5} We find that two critical factors determine the extent to which contracting and reputation mechanism can contain the collusion problem. One, the CEO’s marginal private costs of delivering a successful project; two, precision of the expert’s signal that captures her ability.\textsuperscript{6}

The main findings of the paper are: (a) a firm can costlessly deter collusion by designing incentive contracts if the precision of expert’s information, exceeds a threshold value; (b) this critical value increases with the CEO’s private cost, implying that a CEO with higher agency costs could persuade an otherwise honest expert to manipulate information. (c) Reputation concerns of the expert mitigate collusion but the problem does not disappear even when expert and firm interact repeatedly, moreover, (d) reputation mechanism, which requires probabilistic renewal of the expert’s contracts as a part of

\textsuperscript{4}Thus the expert’s role is, for example, similar to the external auditor’s report on “going concern” which contains informed opinion stating whether a firm would continue or be liquidated. See DeFond, et.al.,(2002), Willenborg and McKeown (2009) and Li (2009).

\textsuperscript{5}The form of ”bribes” can take different forms. See Bebchuk and Fried (2003) for CEO-Board nexus and design of compensations schemes, Liu and Ritter (2009) for spinning shares to CEOs by investment bankers in IPOs or Ritter’s (2009) study on forensic finance. Demski (2003) provides many such examples of favoritisms between client firms and experts.

\textsuperscript{6}The CEO’s private cost of ”effort” stand as a metaphor for agency problems ranging from empire-building to initiating riskier projects. See Tirole (2006), chapter 2 for detailed discussions.
dynamic incentive scheme, may dissuade the firms to hire an honest expert. Occasional firing of an expert is also costly for firms as her specialized information is valuable.

This paper also predicts that reputable experts have more frequent renewal of contracts, earn larger life-time income and facilitate disclosures of information. These observable indicators help empirically distinguish them from the non reputable experts, even when their skill levels are not directly observed.

The following core message thus sums up this paper’s contribution: The abler experts contribute to corporate governance by enforcing both reputation mechanism and incentive compensations at lower costs and help their clients truthfully disclose relevant information which restructures bad projects timely.

For an intuition of the core message, we note that the collusion problem, in this paper, originates from multiple usage of the expert’s information. First, the expert’s signal guides the firm to take an informed decision whether to continue or to liquidate the project. Second, this information can be incorporated in the compensation scheme to lower the costs of incentivizing the CEO. Especially, a bad signal indicates shirking by the CEO and prescribes liquidation of the project without payment that acts as a form of punishment.

This dual usage of expert’s information, one for project evaluation and the other for designing CEO compensations, gives the CEO incentives to bribe the expert to hide or manipulate the bad signal and mis-report as a good one. This makes the project continue and, if successful, by chance, the CEO can share his good fortunes with the expert. The higher the CEO’s incremental agency costs, the larger the incentive payments to him in case of success, and the greater his willingness to bribe the expert.\footnote{This source of collusion conforms to the empirical findings by Agarwal and Chadha (2005), Burns and Kedia (2006), Efendi et. al (2007), and Feng et. al. (2010), who all document a strong link between aggressive incentive schemes of CEO/CFO and corporate frauds such as manipulation of earnings.}

On the other hand, the higher the ability of an expert, the more informative is her signal.
Thus, an abler expert knows that a project is less likely to succeed if she receives a bad news/signal. Hence, the present value of the CEO’s bribery to the expert will be smaller in such circumstances and she will have greater incentives to make truthful disclosures of signal. Therefore, the CEO with higher (lower) agency costs and the expert of lesser (higher) ability are more (less) likely to collude upon information.

This paper is related to the literature that discusses the impact of corporate governance on compensation of the management and the manipulation of information. Almazan, Banerji and Demotta (2008), Axelson and Baliga (2009), Hermalin and Wiesbach (2012), Goldman and Slezak (2006), Eisfeldt and Rampini (2008), Inderst and Mueller (2010), and Levitt and Snyder (1997) among others have discussed links between information manipulation and CEO compensations but without reference to expert firms, which, however, are prevalently involved in many important corporate decisions. While these papers may have an expert firm at the background, our paper puts it at the focus, examining its role in information acquisition and manipulation, and moreover, the extent to which the reputation concerns of the expert firm discipline it, which is not addressed in the extant literature.

Our paper is related to the literature on collusion in a principal-supervisor-agent structure, pioneered by Tirole (1986). Differently from this literature, the source of collusion in our paper lies in conflict between the multiple usages of the expert’s information and we consider the expert’s reputation as well.

The paper is organized as follows: in section 2, we show how the firm can contain collusion with corporate governance in terms of contracting in a static framework. In section 3, extending the basic model to repeated interactions between the firm and the expert, we analyze the reputation of the expert. Section 4 concludes.

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2 The Basic Model: Contracting against Collusion

In the model, the owners of a firm hire a CEO to run a project and an expert for advisory services. All the agents are risk neutral. Both the CEO and the expert enjoy limited liability so that the payments to them cannot be negative. The project could be liquidated before its completion, if in the midway the expert receives a bad signal about its prospect. The liquidation generates a deterministic cash flow $L$. If it is not liquidated, the project has an uncertain return $R \in \{y, 0\}$.

There are three relevant dates in the model, with $t = 0$ for contracting, $t = 1$ for the expert’s evaluation and the interim decision (to liquidate or to continue the project), and $t = 2$ for the realization of the outcome of the project if it is continued at $t = 1$. Events are illustrated in Figure 1.

At $t = 0$, the firm offers contingent contracts (elaborated below) to the CEO and the expert. The CEO then chooses either to shirk or to exert effort at a private cost $B$. This choice is his private information. If he exerts effort, the project will reach a good (or viable) state at $t = 1$ with probability $q$, and a bad (or inviable) state with probability $1 - q$. If he shirks, the probability of the project reaching the good state lowers to $q - \Delta$, with $\Delta > 0$. If the project is in the good state and is not liquidated at $t = 1$, it will succeed (namely yielding $y$) with probability $s_g < 1$. The probability of success in the bad state is $s_b$. We assume $s_b y < L < s_g y$, $L < (qs_g + (1 - q)s_b)y$, and that the firm prefers to incentivize the CEO to exert effort. Note that $B/\Delta$ measures the incremental agency cost of the CEO, as the higher is it, the more difficult to incentivize him to exert effort.

At $t = 1$, the expert evaluates the state of the project and reports her evaluation to the firm. At a cost of $B_x$, she receives a private and noisy signal which takes either a high or low value, representing good news or bad news respectively. If the signal is low, then the continuation value of the project is less than its liquidation value so that
the shareholders’ point of view, it is optimal to liquidate the project. The reverse is true when the expert receives a high signal. We assume that the service of the expert is essential for collecting and interpreting the signal; and that the expert’s signal, though soft in nature (namely, distortable to the shareholders), is perceived by the CEO from his experience of running the project.⁹ If the CEO finds that reporting the truth by the expert will cause his payment reduced, he may offer the expert a payment to have her make a false report in his favor.

Formally, let $\tilde{m} = h, l$ denote the high or low signal, respectively. Conditional on the state of the project, $\tilde{m}$ is distributed as follows:

$$\Pr(\tilde{m} = h|\text{Good}) = \Pr(\tilde{m} = l|\text{Bad}) = \lambda > 0.50$$

So $\lambda$ measures the informativeness of the signal of the expert. In this sense, $\lambda$ also measures her ability to get a precise knowledge about the project’s prospect.

Upon the report by the expert, the firm decides whether to liquidate or continue the project, with liquidation yielding value $L$. And we assume:

$$p_hy > L > p_ly$$

(1)

where $p_h$ ($p_l$) is the posterior probability of success conditional on $\tilde{m} = h$ ($\tilde{m} = l$). That is, $p_h = e_h/q_h; p_l = e_l/q_l$, where $e_h \equiv q\lambda s_g + (1 - q)(1 - \lambda)s_b$ is the ex ante probability that $\tilde{m} = h$ and the project succeeds (if not liquidated midway), $q_h \equiv q\lambda + (1 - q)(1 - \lambda)$ is the ex ante probability $\tilde{m} = h$, and similarly for $e_l$ and $q_l \equiv q(1 - \lambda) + (1 - q)\lambda$. Certainly, $q_h + q_l = 1$.

⁹The expert’s role here thus resembles the external auditor’s report ”on going concern” which contains her informed opinion, based on the material information, regarding a project would continue as a”going concern” or ought to be liquidated. There is a substantial literature in Auditing that discuss the issue.
Assumption (1) requires the expert’s signal is informative enough:

\[
\lambda > \max \left( \frac{1}{2}, \frac{(1-q) (\frac{L_y}{s_y} - s_b)}{q(s_g - \frac{L_y}{y}) + (1-q) (\frac{L_y}{s_y} - s_b)}, \frac{q(s_g - \frac{L_y}{y})}{q(s_g - \frac{L_y}{y}) + (1-q) (\frac{L_y}{s_y} - s_b)} \right)
\]

(2)

At \( t = 2 \), the cash flow of the continued project, either \( y \) or \( 0 \), is realized and publicly observed. The expert and the CEO are paid according to the contracts drawn up at \( t = 0 \) and the residual goes to the shareholders.\(^{10}\)

A contract to the CEO is represented by \( \{w_l, w_h, w_f\} \), where \( w_l \) is the payment to him when the expert reports \( \tilde{m} = l \) and thus the project is liquidated and \( w_h (w_f) \) is the payment to him when she reports \( \tilde{m} = h \) and the continued project succeeds (fails). Similarly, a contract to the expert is represented by \( \{x_h, x_f, x_l\} \).

### 2.1 The NPV of the Expert

The expert’s information improves the interim decision. Without her service, the expected value of the project is \( (qs_g + (1-q)s_b) y \). With her service, the project is continued only when she gets a high signal, which is with probability \( q_h \), and then it succeeds with probability \( p_h \), while it is liquidated when she gets a low signal, which occurs with probability \( 1 - q_h \). Hence, the expert’s contribution to the social value of the project (namely the firm’s value plus the CEO’s), \( \Omega \), is:

\[
[q_h \cdot p_h y + (1-q_h)L] - (qs_g + (1-q)s_b)y,
\]

which can be written more succinctly as

\[
\Omega = q(1 - \lambda) (L - s_g y) + (1 - q) \lambda (L - s_b y).
\]

Intuitively, the expert makes difference by recommending liquidation upon receiving bad news. She may receive the bad news in the good state or in the bad state. In the former case, liquidation, compared to continuation, is an error and causes a loss

\(^{10}\)We confine ourselves to the case where the expert’s signal is truthfully reported; thus the firm will liquidate the project at \( t = 1 \) if \( \tilde{m} = l \) is reported, and continue it if \( \tilde{m} = h \) is reported – then it either succeeds or fails. That is without loss of generality by revelation principle.
of value $L - s_g y$, which is represented by the first term. In the latter case, liquidation saves value $L - s_h y$, which is captured by the second term.

The private cost of her service is $B_x$. The paper assumes the service generates a positive value, that is, $\Omega - B_x > 0$.

### 2.2 The Contracting Problem of the Firm

The contracts with the CEO and the expert designed by the firm at $t = 0$ must induce (a) the CEO to exert effort at $t = 0$, (b) the expert to truthfully disclose her information at $t = 1$, and (c) the expert and the CEO not to collude to distort the disclosure at $t = 1$. The optimal contract minimizes the expected costs of compensating the CEO and the expert\(^{11}\) subject to incentive compatibility, truth-telling, collusion proofness, limited liability and participation constraints detailed below.

The firm’s optimization problem is thus: \(^{12}\)

$$\min_{\{w_h, w_f, w_l, x_h, x_f, x_l\}} e_h(w_h + x_h) + (q_h - e_h)(w_f + x_f) + q_l(w_l + x_l),$$

subject to

(a) CEO moral hazard constraint:

$$e_h w_h + (q_h - e_h)w_f + q_l w_l \geq B + [e_h - \Delta\{\lambda s_g - (1 - \lambda) s_h\}]w_h +$$

$$[(q_h - e_h) - \Delta\{\lambda (1 - s_g) - (1 - \lambda)(1 - s_h)\}]w_f + [q_l - \Delta(2\lambda - 1)]w_l;$$

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\(^{11}\)An important question here is how the CEO is hired and who hires him? For example, if the expert happens to be a board member, then the optimization problem is to maximize their joint surplus, subject to relevant constraints. Here, we assume that at a majority of the board members are honest and care about the shareholders’ interests. That is, our objective is to investigate whether a section of powerful insiders and outsiders can impose costs by colluding, even when the rest of the board is honest. We discuss more on the choice of an expert below after proposition 2.

\(^{12}\)Recalling $e_h \equiv q \lambda s_g + (1 - q)(1 - \lambda)s_h$ the ex ante probability of the event that $\bar{m} = h$ and the project succeeds, $q_h \equiv q \lambda + (1 - q)(1 - \lambda)$ the ex ante probability of the event $\bar{m} = h$, $q_h - e_h$ is the ex ante probability that $\bar{m} = h$ and the project fails; and $q_l = 1 - q_h$ is the probability that $\bar{m} = l$. 

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(b) Adverse selection constraints of the expert:

\[ p_h x_h + (1 - p_h) x_f \geq x_l \]  \hspace{1cm} (4)

\[ x_l \geq p_f x_h + (1 - p_f) x_f; \]  \hspace{1cm} (5)

(c) Collusion proofness constraints:

\[ p_h (x_h + w_h) + (1 - p_h) (x_f + w_f) \geq x_l + w_l \]  \hspace{1cm} (6)

\[ x_l + w_l \geq p_f (x_h + w_h) + (1 - p_f) (x_f + w_f) \]  \hspace{1cm} (7)

(d) Limited liability constraints: \( w_h \geq 0, w_f \geq 0, w_l \geq 0, x_h \geq 0, x_f \geq 0 \) and \( x_l \geq 0 \).

(e) The participation constraint (PC) constraint of the expert:

\[ e_h x_h + (q_h - e_h) x_f + q_l x_l \geq B \]  \hspace{1cm} (8)

(f) The (PC) constraint of the CEO:

\[ e_h w_h + (q_h - e_h) w_f + q_l w_l \geq B. \]

The moral hazard constraint of (3) stipulates that a CEO’s expected pay-off from working harder must exceed his pay-off from shirking.

Conditions (4) ensures that the expert alone does not gain by mis-reporting \( \tilde{m} = h \) as \( \tilde{m} = l \). Telling the truth leads to the project being continued, which then succeeds with probability \( p_h \) and fails with probability \( 1 - p_h \), and thus the expert gets \( p_h x_h + (1 - p_h) x_f \). She would obtain \( x_l \) by falsely stating \( \tilde{m} = l \) and getting the project liquidated. Similarly, (5) ensures no gain for her to mis-report \( \tilde{m} = l \) as \( \tilde{m} = h \).

Condition (6) is parallel to (4) and ensures that the CEO and the expert jointly do not gain by mis-reporting \( \tilde{m} = h \) as \( \tilde{m} = l \). If \( \tilde{m} = h \) is truthfully reported, thus the project continued, together they expect to get \( p_h (x_h + w_h) + (1 - p_h) (x_f + w_f) \), whereas
they get \( x_t + w_t \) by fraudulently reporting \( \tilde{m} = l \). Similarly, Condition (7) ensures no gain for the two together to collusively mis-report \( \tilde{m} = l \).

Finally, the last two constraints ensure that both the CEO and the expert receive no less than their respective reservation pay-off.

Note that the incentive constraint of the CEO, (3), together with the limited liability conditions implies that the CEO receives no less than \( B \), so that the participation constraint (PC) of the constraint of the CEO is never binding and thus will be ignored hereafter.

Before solving the contracting problem where the CEO and the expert can collude perfectly, we go to the benchmark case where they cannot collude at all, for two important reasons. First, a comparison between the two cases will pinpoint the source and magnitude of costs incurred by the shareholders due to collusion in the static framework. Secondly, in section 3, we will examine whether a firm can implement this ”low cost, collusion free” contracts in repeated interactions where the expert has incentives to build up reputation for honesty.

### 2.3 The Benchmark Case: No Collusion

The solution to optimal contracting problem in this section is obtained without the collusion proofness constraints, namely (6) and (7), and keeping the rest unchanged. We summarize the main results in the proposition 1 below, where superscript 'ncx' stands for "non-collusive expert".

**Proposition 1** If expert does not collude with the CEO, then,

(i) the contract to the CEO is: \( w_{h}^{ncx} = \frac{B/\Delta}{\lambda s_{y} - (1-\lambda)s_{l}} \), \( w_{i}^{ncx} = w_{j}^{ncx} = 0 \), and any contract to the expert is optimal so long as it satisfies adverse selection constraints and she receives her outside option \( B_x \).
(ii) the expected cost of compensation to the CEO is $C^{ncx} = \frac{B}{\Delta} [q + \frac{(1-\lambda)s_b}{[\lambda s_g - (1-\lambda)s_b]}]$, and that to the expert is $B_x$. The gain of the firm from the expert’s service, $\Gamma^{ncx}$, equals $(\Omega - B_x) + (C^n - C^{ncx})$, where $C^n$ is the cost of compensation to the CEO in the absence of the expert and $w^n_i = B_x$, $w^n_f = 0$, $C^n = \frac{B}{\Delta} (q + \frac{s_b}{s_g - s_b})$. And $C^n > C^{ncx}$.

**Proof.** See Appendix A. ■

Result (i) follows the *maximum incentive principle*, which, in the context of our model, means that the CEO is paid only when both the signal is high and the project succeeds. Especially, $w^{ncx}_i = 0$, that is, the CEO receives no severance payment when the project fails or gets liquidated.

Result (ii) says that the signal of the non-collusive expert creates value in two channels. First, it is used to improve the interim decision, which adds value $\Omega - B_x$. Second, this information, being indicative of the CEO’s effort, is used to reduce the cost of incentivizing the CEO by $C^n - C^{ncx}$.

In the remainder of this section, we put the collusion proofness conditions back and investigate under what circumstances and in what forms the prospect of collusion may corrode the value of the firm measured by the amount that owners receive after making payments to the CEO.

### 2.4 The Optimal Contracts Under Collusion

The proposition below summarizes the link between expert’s ability and deterrence of collusion and their links with the private costs of CEO. The superscript ’x’ stands for ”collusive expert”. To state the proposition, define $\lambda^*$ as the root of

$$
\frac{B/\Delta}{B_x} = \frac{(p_h - p_l) [\lambda s_g - (1-\lambda)s_b]}{p_h p_l} := \psi(\lambda),
$$

13The principle states that the agent shall receive a positive payment only when all the informative signals display the values indicating he has chosen high effort. For exposition of this principle, see Laffont and Martimont (2003) and Bolton and Dwatripont (2005)
if this root exists within \((1/2, 1)\),\(^{14}\) otherwise, let \(\lambda^* = 1\).

**Proposition 2** (i) If and only if the ability of the expert \((\lambda)\) is above threshold \(\lambda^*\) then collusion can be costlessly contained with the following contracts: \(\{w_i^x = w_i^f = 0, w_h^x = \frac{B/\Delta}{\lambda s_g - (1-\lambda)s_b}\}\) and \(\{x_f = 0, x_i = B_x, x_h = B_x/p_h\}\).

On the other hand, if \(\lambda < \lambda^*\), collusion destroys the firm’s value. Compared to the case of no collusion, the firm pays a higher incentive wage to the CEO \((w_h^x > w_{h}^{ncx})\) and the CEO also gets a severance payment upon the liquidation of the project \((w_i^x > 0)\).

The optimal contract to the expert is \(\{x_f = 0, x_i = B_x, x_h = B_x/p_h\}\).

(ii) \(\lambda^*\) increases with \(B/\Delta\).

(iii) Both the incentive wage \((w_h^x)\) and the severance payment \((w_i^x)\) to the CEO decreases with the ability of the expert \((\lambda)\).

**Proof.** See Appendix A. \(\blacksquare\)

The Proposition 2 highlights the importance of the expert’s ability \((\lambda)\) and the incremental agency costs \((B/\Delta)\) of the CEO in relation to the collusion problem. First, result (i) asserts that collusion destroys shareholders’ value, only if the expert’s ability falls below a threshold. Second, result (ii) shows that this threshold increases with the agency costs of the CEO. Therefore, the larger the agency costs of the CEO’s effort, the more difficult it is to contain the collusion via incentive contracting. An expert who does her job faithfully when working with a CEO with lower agency costs may engage collusion with another CEO with higher agency costs. Both patterns recur in the next section where the expert is in a long-run relationship with the firm.

An intuition behind both patterns is as follows: Primarily due to incentive and agency concerns, the joint pay-offs to the CEO and the expert tend to be greater when the project continues than in the circumstances when it fails and gets dissolved.

\(^{14}\)It uniquely exists if \(\psi(1) \geq \frac{B/\Delta}{s_x}\): \(\psi(\lambda) = (1/p_l - 1/p_h)(\lambda s_g - (1-\lambda)s_b)\) increases with \(\lambda\) because \(p_l\) (\(p_h\)) decreases (increases) with \(\lambda\); and that \(\psi(1/2) = 0\) because \(p_l = p_h\) at \(\lambda = 1/2\).
Hence, together they might want to collude to suppress the bad news which recommend liquidation of the project. Moreover, the joint expected payoffs to the collusive alliance from the continuation of the project tends to be even larger when the agency costs of the CEO \((B/\Delta)\) are higher or the expert’s signal \((\lambda)\) is less informative and it occurs due to the following reasons: If a CEO has bigger private interests to pursue (i.e. larger \(B/\Delta\)), a firm has to pay even a larger amount of incentive payments \((w_h)\) to motivate him to work harder on the project. This makes such CEOs more prone to hide bad news because they can obtain the payment only if the project is continued and is successful.\(^{15}\) On the other hand, if the expert’s signal is weaker, then the probability of success, conditional on bad news, \((p_t)\) tends to be larger, due to her statistical errors in predictions of events. Put differently, if the expert cannot foresee the project’s future precisely, the bad news may not look so bad because of her incompetence!.

Result (i) also shows that the incremental payment to the CEO attributed to the collusion takes two forms. One is the severance payment (i.e. \(w_l^x > 0 = w_l^{ncx}\)) and other is greater incentive payments i.e. \((w_h^x > w_h^{ncx} > 0)\) to the CEO. The severance payment is made to encourage the collusive alliance to disclose bad news. This payment, however, dilutes the incentives of the CEO to exert effort. To elicit his effort, as a result, the firm has to raise the incentive payments \((w_h^x > w_h^{ncx})\).

The form of the optimal contract with the expert given in result (i) is pinned down by the consideration of encouraging her to report the bad news. Her payment upon the report of the bad news, \(x_t\), should be increased as much as possible, up to the point where she does not mis-report the good news as the bad news, that is, \(x_t = p_h x_h\).

\(^{15}\)The result that larger magnitude of incentive payments are associated with a higher likelihood of the CEO colluding with the expert is consistent with the empirical findings, by, for example, Agarwal and Chadha (2005) for establishing the link between accounting scandals and stock-based compensations, and Bergstresser and Philippon (2006), Burns and Kedia (2006), and Efendi et. al. (2007) for studies on earning misstatements and its relation to the CEO stockholding.
Result (iii) asserts that an expert of higher ability helps the firm to reduce the payments to the CEO. This reduction is driven by two effects. One, she can provide more precise information so that the firm has a more accurate idea of the CEO’s effort choice. The other, as we emphasized before, an expert of higher ability is lesser prone to being captured by the CEO and colluding with him. The second effect is, as far as we know, the first time pointed out by this paper.

Altogether, Proposition 2 has interesting implications for the situations where the CEO, rather than the board, has a strong say in the selection of an expert. The proposition clearly demonstrates a tension between the shareholders and the CEO over their preferred ability level of the expert, measured by $\lambda$. The shareholders prefer an expert with highest possible $\lambda$. The payments to the CEO decrease with the ability of the expert, which in turn, makes shareholders’ and the CEO’s interests diametrically opposite when it comes to the choice of selection of the expert. Hence, if a board is completely captured by its CEO’s vested interests, then, the hired expert’s ability ($\lambda$) will be such that it exactly satisfies $C^x - C^n = \Omega - B_x$, i.e., the CEO demands a remuneration which is equals to (a) expected payment to him in the absence of expert ($C^n$) plus all the net incremental value ($\Omega - B_x$) generated by the expert.

The upshot of the Proposition 2 is: if $\lambda \geq \lambda^*$, the collusion problem can be contained by proper contracting without destructing any of the firm’s value; If $\lambda < \lambda^*$, the firm has to incur extra costs to contain the collusion problem in a static environment. The question is whether the prospect of a long term relationship with the firm can make expert so honest that she does not collude with the CEO at all? That is, can expert’s concerns for honesty in a long term relationship help the firm incentivize the CEO and make information disclosures at a lower cost in each period indicated by Proposition 1? \footnote{A large number of empirical studies show that the relationships between firms and experts (audit firms, investment bankers etc.) are often long lived; see, Burch, Nanda and Warther (2005), Fang...} We consider these questions in the next section.
3 Building a Reputation for Honesty by the Expert

In this section, we extend the static model of section 2 into a dynamic setting to examine the extent to which the firm can resort to the expert’s concerns for a reputation for honesty to contain the collusion problem without incurring extra payments (outlined in Proposition 2) to the CEO and the expert. Specifically, we address the following questions:

1. When does the concern for reputation motivate the expert not to collude with the CEO when both interact on a repeated basis?

2. Does a firm always have the incentives to hire a reputed expert or is it costlier for the firm to implement the reputation mechanism even when the expert is willing to be honest?

3. Finally, if both reputable and non reputable experts co-exist in the market, how does one distinguish them empirically on the basis of observable indicators like life-time income, duration of contracts etc.?

To discover the reputation mechanism we let the interaction between the expert and the firm described in the previous section to be infinitely repeated. Now the firm has all the contracting tools that we have studied in the previous section and in addition to those, it has one more set of tools to ensure the truth-telling the expert: the probabilities of renewing the business relationship with the expert, based on (i) the expert’s reporting of the news and (ii) the actual realized outcome, whenever it is observed by the firm. If these probabilities are properly designed, then the expert’s chance of getting re-hired in the future is smaller, if she mis-reports her signals. This prospect for loss of future business may induce the expert to build up the reputation for honesty. Thus reputation mechanism that elicits the expert’s honesty in our paper consists of (a) payments to the CEO and expert in the current period contingent on observed outcome and (b) the...
probability of expert's renewal of contracts in the next period.

Thus, in this section, there are an infinite number of time periods where each period is a replica of the set up in the earlier section. In each period, the firm has a new project and hires a new CEO to run it, while the firm and the expert live forever, both with a discount factor of $\beta \in (0, 1)$. The intra-period timeline in each period is the same as described in the figure 1(a) in last section: it consists of three dates; 0, 1 and 2; at $t = 0$, the CEO of the period faces the same choice of effort as it was in the basic model; at $t = 1$, the expert, if hired in this period, evaluates the project and reports her recommendation to the firm which then makes the interim decision. And, at $t = 2$, the project, if continued at $t = 1$, realizes its outcome, success or failure. In a given period, the expert may not be hired, because the firm now decides renew her contracts probabilistically. Depending on whether the expert is hired in the period or not, we call this period is in state 1 or 0.

In a period of state 0, the expert is not hired and gets 0. In the absence of the expert, there is no interim assessment of the project and no interim decision is made as for its continuation. The project is always carried out to $t = 2$ and ex ante it succeeds with probability $q s_g + (1 - q) s_b$. Thus, the firm gets from the current period $(q s_g + (1 - q) s_b) y - C^n$, where $C^n$ is the expected payment to the CEO if the firm

---

17 We thus abstract the dynamic interactions between the CEO and the firm. We do this for two reasons. First, this dynamic interactions have been extensively studied (see Noe and Rebello (2012), Edmas et. al. (2012) among others). Second, the abstraction enables us to focus on the reputation concern of the expert firm, which, to our best knowledge, has not been much studied in the literature on governance. Thus our analysis of the dynamic interactions between the expert and the firm complements the literature on the dynamic relationship between the firm and the CEO.

18 If the expert still gets something in state 0, then the punishment of the firm not hiring her is weaker. Therefore, it is even harder to give her incentives to report the truth, that is, even harder for her to establish the reputation. Our result is strengthened.
does not hire the expert. In the next period, the firm decides to hire the expert with probability \( z_0 \). That is, the next period will be in \textit{the state 1} with probability \( z_0 \) and will remain in the \textit{state 0} with probability \( 1 - z_0 \).

In a period of \textit{state 1}, the expert is hired. What occurs now is exactly the same as in the static framework of the basic model. At date 0, the firm offers contracts \( \{w_h, w_l\} \) to the CEO of the period and \( \{x_h, x_l\} \) to the expert (as \( x_f = w_f = 0 \)). At date 1, the expert receives a private signal, \( \tilde{m} = h \) or \( l \), and communicates to the CEO. The expert decides whether to report the true or false value of the signal. If reporting the truth reduces the CEO’s payoff, he may offer the expert a payment for manipulating and reporting a false signal. Conditional on the expert’s report, the firm decides to continue the project or to liquidate it. If it is liquidated, the firm pays \( w_l \) to the CEO and \( x_l \) to the expert, and it rehires the expert in the next period with probability \( z_l \). If the project gets continued and succeeds, the firm pays \( w_h \) to the CEO and \( x_h \) to the expert, and it rehires the expert in the next period with probability \( z_h \). And if the project fails, the firm pays nothing to the CEO or to the expert and it rehires the expert with probability \( z_f \).

The figures 2(a) and 2(b) respectively illustrate events in a period of \textit{state 0} and that of \textit{state 1} and describe inter-relationship between them.

While deciding whether to report the truth, especially the bad news, the expert faces a trade-off between the current gains from accepting the bribery/payments/favors from the CEO versus the losses of future business with the firm as the likelihood of her being hired will diminish with her lying. This happens because the probability of a project’s success tends to be smaller if the expert receives a bad signal but manipulates it and recommends to firm for its continuation. To avoid this potential loss of future

\footnote{The case in the absence of the expert is analyzed in the proof of Proposition 1 and is reproduced here for the reader’s convenience: \( w_h^n = \frac{B/\Delta}{s_q - s_h}, w_l^n = w_f^n = 0, C^n = \frac{B}{\delta}(q + \frac{s_h}{s_q - s_h}). \)
business, the expert may want to build up a reputation for honesty. We define reputation equilibria as the ones in which the expert, because of the reputation concern, reports the truth. In these equilibria, the reputation mechanism contains the collusion problem *at a no extra payment to the CEO or the expert*. Below, we first establish that the reputation mechanism works if and only if the expert’s signal processing ability *is above a threshold*.

### 3.1 Reputation Infeasible to the Expert of Lesser Ability

To find out the conditions under which the expert can build the reputation for honesty, we first describe the the pay-off to all parties that feature in reputation equilibrium. As the firm expects the expert not to collude with the CEO, then the contract it offers the CEO is given by Proposition 1 where no collusion takes place. The payments are: $w_{ncx}^h = \frac{B/\Delta}{s_g - (1-\lambda)s_h}$ and $w_{ncx}^l = 0$. (The superscript ’ncx’ stands for non collusive expert). The firm also offers a contract to the expert which most strongly encourages the truthful disclosure of bad news. From the proposition 2, we know that such contracts are: $x_h = B_x/p_h$ and $x_l = B_x$. The firm liquidates the project upon the report of $m = l$ and continues it upon that of $m = h$.

In addition to these static contracts, the firm also decides to renew expert’s contracts with probabilities given by $\{z_0, z_l, z_s, z_f\}$, which form the essence of reputation mechanism that prevent the expert from accepting bribes from the CEO. Let $V_1$ denote the equilibrium continuation value of the expert at the start of a period in *state 1* and let $V_0$ denote her continuation value at the start of a period in *state 0*, and let $\Pi_1$ and $\Pi_0$ be the respective counterparts for the firm.

Suppose that the expert has received $m = h$ in the current period. If she reports it truthfully, then the project is continued. With probability $p_h$, it succeeds, and she is paid with $x_h = B_x/p_h$ this period and will be rehired with probability $z_s$ in the
next period; and with probability $1 - p_h$, the project fails and she is paid nothing this period and will be rehired with probability $z_f$. Therefore, overall, with probability of $p_h z_s + (1 - p_h) z_f$, she will be re-hired, and hence her expected payoff is:

$$p_h \cdot B_x / p_h + \beta \{ [p_h z_s + (1 - p_h) z_f] V_1 + [1 - p_h z_s - (1 - p_h) z_f] V_0 \}.$$  

Alternatively, if she reports, dishonestly, that $\tilde{m} = l$ (which is not in the interest of the CEO who will thus not bribe for it), then the project is liquidated and she is paid with $x_l = B_x$ in this period and will be re-hired with probability $z_l$. Therefore, the expert’s expected payoff is:

$$p_h \cdot B_x + \beta \{ z_l V_1 + (1 - z_l) V_0 \}.$$  

The incentive compatibility (IC hereafter) constraint in the case of $\tilde{m} = h$ commands that this payoff is no larger than that from reporting the truth, or equivalently:

$$[p_h z_s + (1 - p_h) z_f - z_l] [V_1 - V_0] \geq 0. \quad (9)$$

Now suppose the expert receives $\tilde{m} = l$. If she reports it truthfully, her expected payoff, as calculated above, is

$$B_x + \beta \{ z_l V_1 + (1 - z_l) V_0 \}.$$  

However, in this case, the CEO has an interest to manipulate and distort her report. If the truth, $\tilde{m} = l$, is reported, the project is liquidated and he gets nothing ($w_{l_{nx}} = 0$), while if $\tilde{m} = h$ is reported, the project is continued and succeeds with probability $p_l$, in which case he gets $w_{h_{nx}}$. The CEO is willing to pay an amount of $T \leq w_{h_{nx}}$ ex post to bribe her to report $\tilde{m} = h$. If the expert accepts the bribery and reports $\tilde{m} = h$, she obtains $T + x_h = T + B_x / p_h$ upon the success of the project this period and will be re-hired with probability $p_l z_s + (1 - p_l) z_f$, because the project succeeds with probability $p_l$ and fails with probability $1 - p_l$. Hence, on mis-reporting, her expected payoff is:

$$p_l (T + B_x / p_h) + \beta \{ [p_l z_s + (1 - p_l) z_f] V_1 + [1 - p_l z_s - (1 - p_l) z_f] V_0 \}.$$
The IC constraint in the case of $\tilde{m} = l$ commands that this payoff is no bigger than that of reporting the truth for any $T \leq w_{h}^{ncx}$, or equivalently:

$$\beta[z_l - p_l z_s - (1 - p_l)z_f][V_1 - V_0] \geq p_l w_{h}^{ncx} - (p_h - p_l)B_z/p_h.$$ (10)

There exist reputation equilibria, in which the expert does not collude with the CEO due to the reputation concern, if and only if the two IC constraints above are satisfied for a profile of $(z_0, z_l, z_s, z_f)$. The proposition below states that this profile does not exist – thus neither do the reputation equilibria – if the ability of the expert, measured by $\lambda$, is low enough.

**Proposition 3 (i):** The expert cannot build the reputation for honesty, if her ability is below a critical level of $\lambda^{**} \in (1/2, 1]$. Thus, for $\lambda \leq \lambda^{**}$, there exists no $(z_0, z_l, z_s, z_f)$ that satisfies both IC constraints, (9) and (10), for any $\beta < 1$.

(ii): $\lambda^{**} < \lambda^{*}$, where $\lambda^{*}$ is the threshold given in the static contracts by Proposition 2, implying that concerns for reputation improve the outcome than in the static environment discussed in the earlier section.

(iii): $\lambda^{**}$ increases with $B/\Delta$. Thus, the larger the agency costs of the CEOs, the higher the threshold required for establishing reputation.

**Proof.** See Appendix B. ■

Result (i) demonstrates that not every expert can establish reputation no matter how patient she is (i.e. how much her $\beta$ is close to 1); those experts whose ability is at the low end (below $\lambda^{**}$) cannot. An intuition for the result is that an expert of low ability only obtains very imprecise information. Too often, thus, she will err in predictions (namely report good news in the bad state and bad news in the good state), and thereby be punished by non renewal of contracts. Hence, her future payoff will tend to be smaller and not enough to outweigh what she might gain by
accepting the CEO’s bribery payment. However, according to result (ii), the concern of reputation always helps the firm, as the threshold for containing the collusion problem goes down in the long-run relationship. Later we will show that if \( \lambda > \lambda^{**} \), the reputation equilibria exist as \( \beta \) approaches 1. Therefore, for an expert with ability \( \lambda \in (\lambda^{**}, \lambda^{*}) \), (a threshold larger than for establishing reputation but still fall below the reporting of truth due static contracting (proposition 1)), the collusion problem can be contained by the reputation mechanism and incentive contracting together, but not solely by the latter. Moreover, result (iii) indicates that the same patterns as observed in the basic model. That is: the larger the agency costs of CEO (measured by a high \( B/\Delta \)), or the lesser the ability of the expert (measured by \( \lambda \)), the more likely \( \lambda \leq \lambda^{**} \), and hence the more likely the collusion problem destructs values.

### 3.2 The Reputation Equilibria: Will a firm always choose Reputable expert? Case with \( \lambda > \lambda^{**} \)

In this section, we characterize (a) the reputation equilibria and then (b) discuss under which circumstances the firm chooses to enforce the reputation mechanism and may hire a reputable expert to contain the collusion problem. To find out the equilibrium strategy of the firm and derive the equilibrium payoff profile \((\Pi_1, \Pi_0)\), we apply the “self-generating” approach developed by Abreu, Pearce, and Stachetti (1986) and Myerson (1991) and explain the resulting mechanism as follows: In any given period, the firm is either in the state 0 or is in the state 1. If the current period is in the state 1, so that the expert is hired, and she truthfully reports her signal, then this period the firm obtains from the non-collusive expert, the value \( v^{ncx} \), as given in the Proposition 1 (ii). In the next period it rehires the expert with probability, as shown above, \( q_h z_h + (1-q_h) z_l \), where \( z_h \equiv p_h z_s + (1-p_h) z_f \) is the rehiring probability when \( \tilde{m} = h \) and \( q_h \equiv q \lambda + (1-q)(1-\lambda) \) is the probability of \( \tilde{m} = h \). The firm’s present value at the start of a period of state 1
is thus:

\[ v^{ncx} + \beta \{ [q_h z_h + (1 - q_h) z_f] \Pi_1 + [1 - q_h z_h - (1 - q_h) z_f] \Pi_0 \}. \]

On the other hand, if the current period is in state 0, that is, when the expert is not hired, the firm gets 0 from the expert for this period and rehires her with probability \( z_0 \) in the next period. Therefore, its present value is:

\[ \beta [z_0 \Pi_1 + (1 - z_0) \Pi_0]. \]

According to Abreu, Pearce, and Stachetti (1986) and Myerson (1991), \((\Pi_1, \Pi_0)\) is a profile of equilibrium strategies if and only if

\[ \begin{align*}
\Pi_1 &= \max_{\{z_l, z_s, z_f\}} v^{ncx} + \beta [Q \Pi_1 + (1 - Q) \Pi_0], \text{s.t. (9), (10)}; \\
\Pi_0 &= \max_{z_0} \beta [z_0 \Pi_1 + (1 - z_0) \Pi_0], \text{s.t. (9), (10)},
\end{align*} \]

where \( Q(z_l, z_s, z_f) \equiv q_h p_h z_s + q_h (1 - p_h) z_f + (1 - q_h) z_l \), that is, \( Q \) denotes the probability of the expert being rehired next period if she is hired this period.

Let \( T^* \equiv \frac{p_l}{\lambda p_l - (1 - \lambda) p_h} \cdot B / \Delta - \frac{p_h - p_l}{p_h} \cdot B_x \) and \( \rho = \frac{(p_h - p_l) B_x}{T^*} - (1 - p_h) \)

**Proposition 4 (i):** If \( \lambda \in (\lambda^*, \lambda^*) \), then, there are equilibria in which the firm makes no extra payments (compared to what it pays to the non-collusive expert), and the expert reports the truth because of her concern for the reputation of being honest. In all these equilibria, the respective values of the firm and the expert in the periods when the expert is hired are:

\[ \begin{align*}
\Pi_1 &= \frac{v^{ncx}}{1 - \beta \mu} \text{ and } V_1 = \frac{B_x}{1 - \beta \mu}, \\
\end{align*} \]

where \( \mu \equiv 1 - \frac{(1 - p_h) T^*}{(p_h - p_l) B_x} \). \( \mu < 1 \) if \( \lambda < \lambda^* \) and \( \mu = 1 \) at \( \lambda < \lambda^* \). and the values in the periods when the expert is not hired vary with \( z_0 \):

\[ \begin{align*}
\Pi_0 &= \phi \Pi_1 \text{ and } V_0 = \phi V_1, \\
\end{align*} \]

where \( \phi = \frac{\beta z_0}{1 - \beta (1 - z_0)} < 1. \)
(ii) In the most efficient equilibrium, where $\Pi_0$ and $V_0$ take the highest value, the rehiring probabilities are:

\[
\begin{align*}
  z_0 &= \min(\rho - \frac{1 - \beta}{\beta}, 1); \\
  z_f &= \max(0, 1 - \frac{1}{\rho}); \\
  z_l &= p_h + (1 - p_h)z_f; \\
  z_s &= 1,
\end{align*}
\]

(where $\rho = \frac{(1-p_h)\mu}{1-\mu}$). These probabilities satisfy $1 = z_s > z_l > z_f$ and all increase with $\lambda$ for $\lambda < \lambda^*$ and all equal to 1 at $\lambda = \lambda^*$. The values to the firm and to the expert (namely, $\Pi_1$, $\Pi_0$, $V_1$ and $V_0$) all increases with $\lambda$.

**Proof.** See Appendix B. □

For $\lambda \in (\lambda^{**}, \lambda^*)$, i.e., where the precision of the expert’s signal falls between thresholds that contain collusion due incentive contracting ($\lambda^*$) and expert’s considerations for honesty ($\lambda^{**}$), the firm need to employ both tools of incentive contracting and reputational mechanism outlined in the proposition 3. The experts whose information processing skills fall between these intermediate levels will collude and suppress bad news in one shot interaction with the firm unless extra payments are made to them (proposition 2). In the dynamic setting, by contrast, result (i) here says that the expert will report the truth and reject the CEO’s bribery offer without any extra payments to the group by the firm. This is precisely the reputation effect, namely, the concern about the loss of future business with the firm will prompt the expert to be honest. Hence, in order to deter collusion, the firm has to employ both incentive contracting (outlined in the proposition 1) and the expert’s rehiring probabilities (as shown in the proposition 4).

The firm can deter collusion in two perfectly substitutable ways to punish the ex-
pert. The first one is to reduce $z_f$, and the second one is to reduce $z_0$, namely, by either reducing the chance of rehiring the expert if she reports the good news but the projects turns out a failure, or prolonging the time spell of not hiring her if she is being punished.\(^{20}\)

The firm value, however, gets reduced by $\Pi_1 = \frac{v^{ncx}}{1-\beta} \mu$ compared to $v^{ncx}/(1-\beta)$, which is the value of the firm if the expert is hired in every period and does not collude with the CEO. As the Proposition 2 shows, the expert will collude unless the firm makes extra payments whenever $\lambda < \lambda^*$. In order to elicit honest and truthful information from the expert without extra payments, the firm must punish the expert by committing to not hiring her with some probability (namely, $z_f, z_0, \text{and } z_l$ are all less than 1). This in turn, reduces both the firm and expert’s long term income and these losses are captured by the additional discount factor $\mu < 1$.

The higher the ability of the expert, the smaller these losses to both firm and the expert. This result is obtained from two combined effects: One, $v^{ncx}$ increases with $\lambda$, namely, the expert of higher ability adds a higher value in each period when she is hired. This effect is present in the static model. The other, additional effect, captured by $\mu$, stems from reputational considerations in the dynamic model. When they interact repeatedly, the experts with higher abilities will be rehired by firms with greater probabilities and thus her services to the firm would be seldom lost.

The re-hiring probabilities capture non renewal of contracts as a means to discipline the expert and it always occurs as a part of the dynamic incentive scheme except when the expert’s prediction for success coincides with actual outcome, i.e, $z_s = 1$. Namely, reporting good news followed by an actual success, always leads to renewal of the business relationship with certainty, i.e. "nothing succeeds like a success". The

\[^{20}\text{If the expert is not hired this period, the expected number of periods in which she will be hired is}\]

$$\sum_{n \geq 1} n \times z_0(1-z_0)^{n-1} = 1/z_0.$$  If $z_0$ is reduced, then the expert expects not to rehired for a longer time.
expert’s contract is only probabilistically renewed when she reports bad news \((z_l < 1)\)
and her reporting of good news followed by an actual failure \((z_f < 1)\). However, both
punishments tend to be smaller for the experts with a higher ability as the rehiring
probabilities in the most efficient equilibrium increase with \(\lambda\). Therefore, the expert
of higher ability expects a longer relationship with the firm. As she is more likely to
build up the reputation for honesty, this paper also predicts that reputable expert-firms
enjoy a more durable relationship with the client firms. In addition, their clients tend
to disclose bad news more frequently and often restructure loss making projects.

If \(\lambda > \lambda^*\), the reputation mechanism can work to contain the collusion problem
because experts are willing to build up a reputation for honesty. However, as our
analysis shows that hiring a reputable expert creates the following trade-off for firms: An
honest expert saves a firm’s expected compensation costs (as shown in the proposition
1). However, in order to elicit honest behavior from the expert, a firm has to fire expert
occasionally as given by the probabilities in the proposition 4 (ii), which hurts the firm
because NPV of expertise is positive. Since all such firing probabilities decrease with \(\lambda\),
the probability of separation is lower for a highly able expert. Hence, a firm may be
reluctant to enforce the mechanism of reputation, if the information processing ability
is below a threshold level, as shown by the following proposition.

**Proposition 5** There exists a \(\lambda^{***}\), satisfying \(1 > \lambda^{***} > \lambda^*\) such that the firm prefers
to resort to the reputation mechanism to contain the collusion problem only if \(\lambda \geq \lambda^{***}\);
otherwise, it repeats the static contract with expert.

Intuitively, for \(\lambda \in (\lambda^*, \lambda^*)\), the advantage of using the reputation mechanism is
that for the current period, the firm gets more from the non-collusive expert than from
the collusive expert: \(v^{ncx} - v^x = C^x - C^{ncx} > 0\) (see Proposition 2). The disadvantage,
however, is that the former arrangement commands the firm not to rehire the expert
on some contingencies in order to honor the IC constraints. On the other hand, by resorting to static contracts, the firm incurs higher expected costs of compensation but obtains the expert’s advisory service each period. But note that the firm may get this service from either the expert of the last period or another, similar expert from the market. Thus, the non-reputable expert is substitutable over time. Therefore, we predict that their business relationship may feature a high turnover rate. See Srinivasan (2005) and Yermack (2004) for supportive empirical evidence for the tainted and non-tainted board of directors.

The proposition directly implies that only the expert firms of sufficiently high calibre are observed to possess the reputation for honesty. If \( \lambda \leq \lambda^{**} \), the expert firms cannot build the reputation. While they can for \( \lambda \in (\lambda^{**}, \lambda^{***}) \), the client firm opts out of the reputation mechanism and will prefer not to develop a long-run relationship with them. Only the experts whose ability belongs to the interval, \( \lambda \in (\lambda^*, \lambda^{***}) \), the firm is willing to enforce reputation mechanism together with incentive contracting. If in real life only a small number of expert firms exhibit a high calibre (which is an empirical question beyond the paper’s scope), then our paper suggests that the market of reputable experts is thin. Indeed, we do not observe many highly reputable brand-names in the circle of financial expertise services. For example, we only see four internationally reputed auditing firms; and according to the reputation index for commercial banks constructed by Ross (2010), only three banks occupied top positions from 2000 to 2009. See Rau (2000) for a similar observation regarding investment bankers and underwriters engaged in M&A. The figure A below thus captures the summary of these results that unify static incentive contracting and dynamic reputational framework discussed in the paper.
4 Conclusion

Firms extensively use the services of various types of expertise intermediaries, ranging from information disclosures, auditing, to advice on M&As and IPOs. In recent times, the outbreak of scandals involving expert firms has raised serious concerns regarding their functioning in corporate governance. On this issue, this paper investigates their roles pertaining to information provision, incentives and collusion, and delivers two core messages. One, the agency problem of a firm’s management and the low ability of its advisory firm not only hurt it individually on their own, but together make the collusion problem more likely to bite and more costly to be checked. The other, the abler advisory firms are more faithful in reporting their findings to the client firms: they are more easily (namely at lower costs) aligned to follow the client firms’ interest by proper contracting in a one-off relationship; and they are more likely restrained by the concern for a reputation of being honest in a long-run relationship. To summarize, an advisory firm of higher ability adds more value to the client firm not only because their information are more precise and thus bears a higher informational value, but also because they are less prone to being captivated by the management of the client firm.

The paper has discussed the CEO-expert nexus by assuming that they are matched somehow by some exogenous process. However, firms and experts interact in a market where experts with varying skill levels match with firms with heterogeneous characteristics. Our analysis thus far has ignored this aspect of endogenous matching and its impact on collusion. Second, our paper has treated the information alliance as almost a single entity without any friction in between. This approach seems reasonable when the communication between the expert and the CEO is perfect or the expert does not involve hidden effort in producing the signal. It remains to be seen how our results change once we introduce these elements of frictions between the collusive parties. In future, we would like to extend our research along these lines.
Figure 1: Timing of events with private signal in the static model

Contracts with the CEO and the Expert are drawn.
The CEO chooses the effort.
The state, G or B, is realized, but unobserved.
The expert obtains her private message.
The expert and the CEO possibly engage into collusion.
The expert reports her message. Based on the message, the project is either liquidated or carried on.
The outcome of the project, success or failure, is realized and publically observed.
The CEO and the expert are paid according to the contracts drawn at $t = 0$. 

Figure A
The renewals of contracts to expert leads to regeneration of State 1. If in state 1 intra period timeline follows Figure 1. All renewals of contracts, lead to State 0.
Appendix A

The Proof of Proposition 1:

Noting that the participation constraint (PC) of the CEO is never binding and removing the collusion proof constraints, namely (6) and (7), the optimization problem thus becomes:

\[
\min_{\{w_h, w_f, w_l, x_h, x_f, x_l\}} e_h(w_h + x_h) + (q_h - e_h)(w_f + x_f) + q_l(w_l + x_l),
\]

(13)

s.t. (3), (4), (5), (8), and the nonnegative constraints written in detail on pages 10 and 11, the optimization programme becomes:

Observation: Though the objective function aims to minimize joint expected payments to both the expert and the CEO, in effect, the problem (without collusion proof constraints) can be split into two separate optimization problem, one for the expert’s payment and other for the CEO’s as established below.
Note: \( \min\{e_h(w_h + x_h) + (q_h - e_h)(w_f + x_f) + q_l(w_l + x_l)\} \geq \min\{e_h w_h + (q_h - e_h)w_f + q_l w_l\} + \min\{e_h x_h + (q_h - e_h)x_f + q_l x_l\} \geq \min\{e_h w_h + (q_h - e_h)w_f + q_l w_l\} + B_x, \)

where the second “\( \geq \)” is because of (8), the participation constraint (PC hereafter) of the expert. Therefore, we will have solved the optimization problem, if we (a) find a contract with the expert, \( \{x_h, x_f, x_l\} \), that honors all of the relevant constraints and makes (8) binding; and (b) find a contract with the CEO, \( \{w_h, w_f, w_l\} \), that attains \( \min\{e_h w_h + (q_h - e_h)w_f + q_l w_l\} \) subject to all the relevant constraints. This can be done because the objective function is additively separable in each agent’s contingent payments, and constraints relevant to the CEO and those of experts are independent. They intertwine through the collusion proof constraints (to be taken up in the next proposition).

Task (a) is achieved in the following way: There is actually a continuum of contracts that pass the above standard, for example, \( \{x_h = \frac{B_x}{p_h}, x_f = 0, x_l = B_x\} \). The expected payment to expert is \( B_x \).

For (b): the constraints relevant to \( \{w_h, w_f, w_l\} \) are the non-negative constraints and (3), which is equivalent to

\[
[\lambda s_g - (1 - \lambda) s_b] w_h + [(\lambda(1 - s_g) - (1 - \lambda)(1 - s_b)] w_f - (2\lambda - 1) w_l \geq B/\Delta. \tag{14}
\]

The task becomes to find a contract \( \{w_h, w_f, w_l\} \) that minimizes the expected payment to the CEO subject to (14) and the nonnegative constraints. The solution is given below.

**Claim A**: \( w_l = 0, w_f = 0, \) and \( w_h = \frac{B/\Delta}{\lambda s_g - (1 - \lambda) s_b} \).

**Proof**: First, \( w_l = 0 \): A positive \( w_l \) both worsens (i.e. increases) the objective and tightens constraint (14) because \( 2\lambda - 1 > 0 \).

Second, for the same reason, \( w_f = 0 \) if \( \lambda(1 - s_g) - (1 - \lambda)(1 - s_b) < 0 \). Otherwise,
task (b) is equivalent to solving:

\[
\min_{w_h, w_f} \{ e_h w_h + (q_h - e_h)w_f \}, \text{ s.t. } (14) \text{ and } w_h, w_f \geq 0
\]

The solution to the problem is \( w_h = \frac{B/\Delta}{\lambda s_g - (1-\lambda) s_b} \) and \( w_f = 0 \), because \( \frac{e_h}{\lambda s_g - (1-\lambda) s_b} < \frac{q_h - e_h}{(1-s_g) - (1-\lambda)(1-s_b)} \), which is equivalent to: \( e_h \left[ (\lambda (1 - s_g) - (1 - \lambda) (1 - s_b) \right] < [\lambda s_g - (1 - \lambda) s_b] q_h \Rightarrow e_h \left[ (\lambda - (1 - \lambda) \right] < [\lambda s_g - (1 - \lambda) s_b] q_h \Rightarrow (q \lambda s_g + (1-q)(1-\lambda) s_b)(2\lambda - 1) < [\lambda s_g - (1 - \lambda) s_b] q_h \Rightarrow (1-\lambda) s_b((1-q)(2\lambda - 1) + q_h) < \lambda s_g |q_h - q(2\lambda - 1)| q_h \equiv q \lambda + (1-q)(1-\lambda) \Rightarrow (1-\lambda) s_b < (1-\lambda) \lambda s_g. \]

Therefore the solution to task (b) is \( w_h = \frac{B/\Delta}{\lambda s_g - (1-\lambda) s_b} \) and \( w_f = w_t = 0 \). With these wages, we find the expected payment to the CEO is \( C^{nc} = B \lambda q + \frac{(1-\lambda) s_b}{\lambda s_g - (1-\lambda) s_b} \) and the expected payment to expert is \( B_x \).

On the other hand, if the firm does not hire the expert, then the project is always continued at \( t = 1 \) and the firm’s problem is to minimize the payment to the CEO subject to giving him incentives to exert effort. Thus the firm’s optimization problem is

\[
\min_{\{w_h, w_f, x_h, x_f, q_t\}} \{ q s_g + (1-q) s_b \right] w_h + q (1-s_g + (1-q)(1-s_b))w_f \text{ subject to } (s_g - s_b)(w_h - w_f) \geq B/\Delta \text{ and } w_h \geq 0, w_f \geq 0 \]

The solution to this problem is given by \( w_h = \frac{B/\Delta}{s_g - s_b} \) and \( w_f = 0 \), by which the cost of compensation is \( C^n = \frac{B}{\Delta} q + \frac{s_b}{s_g - s_b} \). Hence, \( C^{nc} - C^n = -\frac{(2\lambda - 1)s_g s_b}{[\lambda s_g - (1-\lambda) s_b] s_g - s_b} < 0 \).

Q.E.D

**The Proof of Proposition 2:**

With the collusion proof constraints (6) and (7) returned, the optimization problem becomes:

\[
\min_{\{w_h, w_f, x_h, x_f, x_t\}} e_h (w_h + x_h) + (q_h - e_h)(w_f + x_f) + q_t (w_t + x_t), \tag{15}
\]

s.t. (3) through to (8), and the nonnegative constraints as detailed in page 10-11.
For (i): compared to problem (13) above, two more collusion proof constraints, one for high signal, (6) and the other for low signal, (7) are added to the current problem, which means the minimum value should increase weakly. Therefore, if we find a contract \( \{w_h, w_f, w_l, x_h, x_f, x_l\} \) that satisfies all of the constraints and attain the same minimum value as that of problem (13); that contract must be a solution to the current problem.

So consider \( \{w_h = \frac{B/\Delta}{x_{s_y} - (1-\lambda)s_h}, w_f = w_l = 0\} \), the optimal contract to the CEO derived above, and \( \{x_h = \frac{B_x}{p_h}, x_f = 0, x_l = B_x\} \), an optimal contract to the expert given above. The contract attains the same value for the objective function as that of problem (13). It also satisfies all the constraints if \( \frac{B/\Delta}{B_x} \leq \psi(\lambda) \), holds true. Therefore, if the condition holds, the contract characterized in the proposition 1 is a solution to problem (15) and collusion has no impact on the optimal contract to the CEO and the value of the firm.

For (ii): if \( \frac{B/\Delta}{B_x} \leq \psi(\lambda) \) does not hold, the contract considered in (i), namely \( \{w_h = \frac{B/\Delta}{x_{s_y} - (1-\lambda)s_h}, w_f = w_l = 0, x_h = \frac{B_x}{p_h}, x_f = 0, x_l = B_x\} \), satisfy all the constraints except (7): \( x_l + w_l \geq p_l(x_h + w_h) + (1 - p_l)(x_f + w_f) \). This constraint is thus binding now. Three claims that follow immediately will be used in the transformed optimization problem later.

Claim (i): The constraint (6): \( p_h(x_h + w_h) + (1 - p_h)(x_f + w_f) \geq x_l + w_l \) is non-binding. This constraint, with the binding (7), becomes \( p_h(x_h + w_h) + (1 - p_h)(x_f + w_f) \geq p_l(x_h + w_h) + (1 - p_l)(x_f + w_f) \iff (p_h - p_l)[(x_h + w_h) - (x_f + w_f)] \geq 0 \), which holds because \( p_h > p_l \) and \( x_h + w_h > x_f + w_f \) (the two are paid more when the project succeeds than when it fails and to be verified later as well).

Claim (ii): The constraint (4), \( x_l \leq p_h x_h + (1 - p_h) x_f \) is binding. To slack the binding constraint (7), namely, \( x_l + w_l \geq p_l(x_h + w_h) + (1 - p_l)(x_f + w_f) \), the firm wants \( x_l \) to be as large as possible. That is, to encourage disclosure of bad news, the firm wants to pay the expert as much as possible upon the disclosure because this loosens the constraint to its maximum extent. Besides (6), which was judged non-binding, the
only constraint that places an upper bound on \(x_l\) is (4), which is therefore binding, namely,

\[ x_l = p_h x_h + (1 - p_h) x_f. \]  (16)

Claim (iii): \(x_f = w_f = 0\): With (16) substituted, the binding constraint (7) becomes

\[ w_l \geq p_l w_h - (p_h - p_l) x_h + (1 - p_l) w_f + (p_h - p_l) x_f. \]

To relax the constraint to the maximum extent, \(x_f = w_f = 0\). Moreover, setting \(w_f = 0\) relaxes the binding constraint (3).

Using the last result, \(x_f = w_f = 0\), (16) becomes

\[ x_l = p_h x_h. \]  (17)

Then, the binding (7) becomes

\[ w_l = p_l w_h - (p_h - p_l) x_h. \]  (18)

Substitute these two equations and \(w_f = 0\) into (14) (namely the IC constraint for the CEO to exert effort), and it becomes

\[ \tilde{A} w_h + (2\lambda - 1)(p_h - p_l) x_h \geq B/\Delta, \]  (19)

where \(\tilde{A} = \lambda s_g - (1 - \lambda)s_b - p_l(2\lambda - 1)\). Substitute (17) and \(x_f = 0\) into (8) and note

\[ e_h + q_l p_h = q_l p_h + q_l p_h = p_h, \]

and (8) then become:

\[ p_h x_h \geq B_x. \]  (20)

And substitute (17), (18), and \(x_f = w_f = 0\) into the objective function of problem (15), and note that

\[ e_h + q_l p_l = e_h + e_l \] and that

\[ e_h + q_l p_h - q_l (p_h - p_l) = e_h + e_l. \]

The objective becomes \((e_h + e_l)(w_h + x_h)\).

Therefore, problem (15) becomes:

\[
\min_{\{w_h, x_h\}} w_h + x_h, \ s.t. \ (19), \ (20), \text{ and } w_h, x_h \geq 0.
\]  (21)
Since $\lambda > (2\lambda - 1)(p_h - p_l)$ (which is equivalent to $\lambda s_b - (1 - \lambda)s_b > (2\lambda - 1)p_h \iff \lambda s_g - (1 - \lambda)s_b > (2\lambda - 1)e_h/g_h \iff [\lambda s_g - (1 - \lambda)s_b]q_h > (2\lambda - 1)e_h$, which has been shown in the proof of Claim A), the solution to problem (21) is

$$x_h^* = \frac{B_x}{p_h}; \quad w_l^* = \frac{\tilde{A}^{-1} \cdot (B/\Delta - (2\lambda - 1)\frac{(p_h - p_l)}{p_h}B_x)}{B_x},$$

which makes both (19) and (20) binding.

We explain the intuitive meaning of this solution here. To contain the collusion, the firm should encourage the collusive alliance to report the bad news. To do that it has two ways. One is to pay the CEO upon the reporting of the low signal, namely, $w_l > 0$. The other is to increase the payment to the expert on this contingency, namely $x_l$. Both ways are reflected in the binding collusion proof constraint, (18). Given that the incentive wage, $w_h$, cannot be too low, this constraint asserts that if $x_h$ which is proportional to $x_l$ by (17) is not big enough, then $w_l > 0$. The solution shows that at the optimum, the firm should not pay the expert more and makes the IR to her binding; and instead the firm should make the severance payment to the CEO.

Substituting this into (17) finds $x_l^* = B_x$ and into (18) finds

$$w_l^* = \frac{\tilde{A}^{-1} \cdot (p_h B/\Delta - (\lambda s_g - (1 - \lambda)s_b)\frac{(p_h - p_l)}{p_h}B_x)}{B_x}. \quad (23)$$

All this together with $x_l^* = w_l^* = 0$ gives the full optimal contract.

The IR constraint to the expert, to which (20) is equivalent, is binding. Thus the expected payment to the expert is $B_x$. The expected payment to the CEO is to be find by substituting the optimal contract to him and thus equals $C^* = \{q + (1 - q)s_b\}B/\Delta - (\lambda s_g - (1 - \lambda)s_b)\frac{(p_h - p_l)}{p_h}B_x \sqrt{\tilde{A}}$. As $C_{nc} = \frac{\sqrt{\tilde{A}}}{\Delta} [q + \frac{(1 - \lambda)s_b}{\lambda s_g - (1 - \lambda)s_b}]$ from Proposition 1, $C^* - C_{nc} = \frac{\lambda^2 s_g - (1 - \lambda)^2 s_b}{\lambda s_g - (1 - \lambda)s_b} \cdot w_l^* > 0$.

(ii) If $\lambda^* < 1$, it is the root of $\psi(\lambda) = \frac{B/\Delta}{B_x}$. Note that $\psi(\lambda) = (1/p_l - 1/p_h)(\lambda s_g - (1 - \lambda)s_b)$ increases with $\lambda$ because $p_l (p_h)$ decreases (increases) with $\lambda$. Therefore, $d\lambda^*/d\frac{B}{\Delta} = \frac{1}{B_x \psi'} > 0$. 

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(iii) If $\lambda < \lambda^*$, the collusion-proof constraints are not binding, and thus $w^x_h = w^\text{ncx}_h = \frac{B/\Delta}{\lambda s_g - (1 - \lambda) s_b}$ by Proposition 1(i), which decreases with $\lambda$. If $\lambda \geq \lambda^*$, $w^x_h$ is given by (22) and decreases with $\lambda$ too. To see this, note that $B/\Delta - (2\lambda - 1)\frac{(p_h - p_l)}{p_h}B_x$ decreases with $\lambda$ because both $(2\lambda - 1)$ and $(p_h - p_l)/p_h = 1 - p_l/p_h$ increases with $\lambda$. 

If $\lambda < \lambda^*$, $w^x_h$ is given by (22) and decreases with $\lambda$ too. To see this, note that $B/\Delta - (2\lambda - 1)\frac{(p_h - p_l)}{p_h}B_x$ decreases with $\lambda$ because both $(2\lambda - 1)$ and $(p_h - p_l)/p_h = 1 - p_l/p_h$ increases with $\lambda$. 

$\tilde{A} = \lambda s_g - (1 - \lambda) s_b - p_l(2\lambda - 1)$ increases with $\lambda$ because $\tilde{A}_\lambda = s_g + s_b - 2p_l - (2\lambda - 1)p_l^l > 0 \iff \lambda s_g + (1 - \lambda) s_b > p_l \iff \frac{\lambda}{1 - \lambda} > \frac{\sqrt{q}}{\sqrt{1 - q}}$, which follows from (2) and $s_g y - L > L - s_b y$, both assumed by the paper. (The subscript denotes the variable with respect to which the function is differentiated.) Similarly, $w^x_l = 0$ if $\lambda < \lambda^*$ and is given by (23) otherwise. The latter value decreases with $\lambda$ because $\tilde{A}$ increases with $\lambda$ (as we just have shown); and $p_l B/\Delta - (\lambda s_g - (1 - \lambda) s_b)\frac{(p_h - p_l)}{p_h}B_x$ decreases with $\lambda$: $p_l$ decreases with $\lambda$ and both $(\lambda s_g - (1 - \lambda) s_b)$ and $(p_h - p_l)/p_h$ increases with it.

Q.E.D.

Appendix B

For the proofs of Propositions 3 to 5, we let $T^* \equiv p_l w^\text{ncx}_h - (p_h - p_l)x^*_h$, then $T^*$ is the highest amount of the value that the expert can receive from the CEO by mis-reporting the bad news as the good news: the mis-reporting causes the expert to lose $x_l - p_l x_h = (p_h - p_l)x^*_h$ and the most the CEO is willing to compensate her is $p_l w^\text{ncx}_h$.

And let $z_h \equiv p_h z_s + (1 - p_h) z_f$ (the probability of the expert being re-hired if she has truthfully reported $\tilde{m} = h$). With $w^\text{ncx}_h$ given by Proposition 1 and $x^*_h = B_x/q_h$ given by Proposition 2 (ii), $T^* = \frac{p_l}{\lambda s_g - (1 - \lambda) s_b} \cdot B/\Delta - \frac{p_h - p_l}{p_h} \cdot B_x$.

The Proof of Proposition 3:

Let $\lambda^*$ be the root of

$$\frac{B/\Delta}{B_x} = \psi(\lambda) \frac{1}{1 - p_h(\lambda)}$$

if it exists between $[1/2, 1]$, otherwise define $\lambda^* \equiv 1$. As $\psi(\lambda)\frac{1}{1 - p_h} = 0$ at $\lambda = 1/2$, 

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and increases with $\lambda$, if such a root exists it must be bigger than $1/2$ and unique. With $\lambda^{**}$ so defined, $\psi(\lambda) \frac{1}{1-p_h} \leq \frac{B_y}{B_x}$ for $\lambda \in [1/2,1]$ if and only if $\lambda \leq \lambda^{**}$ because $\psi(\lambda)/(1 - p_h(\lambda))$ increases with $\lambda$: We demonstrated that $\psi(\lambda)$ increases with $\lambda$ in the previous section; and that $\frac{1}{1-p_h(\lambda)}$ increases with it as well.

To prove the proposition, we show that if $\psi(\lambda)/(1 - p_h) \leq \frac{B_y}{B_x}$, then there exists no values for $(z_0, z_l, z_s, z_f)$, between 0 and 1, that satisfies both (9) and (10). By (9), $z_h \equiv p_hz_s + (1-p_h)z_f \geq z_l$, which together with (10) implies:

$$\beta(p_h - p_l)(z_s - z_f)(V_1 - V_0) \geq T^*.$$ (25)

In a typical setting of Folk Theorem, $V_1 - V_0$ is in the order of $\frac{1}{1-\beta}$ and hence (25) will be satisfied as $\beta$ approaches 1. However, in the setting of this paper, since the signal is noisy and hence the expert has to be wrongly punished, we are going to show, $V_1 - V_0$ is in the order of $\frac{1}{1-p_h\beta}$, which is upper bounded even at $\beta = 1$. For that purpose, we set off to find the following upper bound:

**Claim B:** $\frac{B_y}{1-p_h} > (z_s - z_f)(V_1 - V_0)$ for any $\beta < 1$.

**Proof:** Let us go to calculate $V_1 - V_0$ for a given $(z_0, z_l, z_s, z_f)$. At a period where state 1 prevails, the expert gets $B_x$ at the end of the period, and she will be rehired next period with probability $q_hz_h + (1-q_h)z_l$ (recall $q_h \equiv q\lambda + (1-q)(1-\lambda)$ is the probability of $\bar{m} = h$). Therefore,

$$V_1 = B_x + \beta\{[q_hz_h + (1-q_h)z_l]V_1 + [1-q_hz_l - (1-q_h)z_l]V_0\}. \quad (26)$$

We saw that (9) implies $z_h \geq z_l$. Thus, $q_hz_h + (1-q_h)z_l \leq z_l$. Then, $V_1 \leq B_x + \beta[z_hV_1 + (1 - z_h)V_0] \Rightarrow V_1 \leq \frac{B_x + \beta(1-z_h)V_0}{1-\beta z_h}$. It follows that $V_1 - V_0 \leq \frac{B_x(1-\beta)V_0}{1-\beta z_h} < \frac{B_x}{1-z_h}$, where $V_0 \geq 0$ because of limited liability.

Therefore, $(z_s - z_f)(V_1 - V_0) < B_x \cdot \frac{z_s - z_f}{1-z_h} = B_x \cdot \frac{z_s - z_f}{1-p_hz_s - (1-p_h)z_f} \equiv B_x \cdot f(z_s, z_f)$. Note that $f$ increases with $z_s$, which is no bigger than 1. Hence, $f(z_s, z_f) \leq f(1, z_f) = \frac{1}{1-p_h}$. So is the claim proved. Q.E.D.
By the claim above, \( \frac{p_b-p_l}{1-p_h} B_x \) is bigger than the left hand side of (25) for any \( \beta < 1 \). Therefore, if \( \frac{p_b-p_l}{1-p_h} B_x \leq T^* \Leftrightarrow \frac{p_b-p_l}{1-p_h} B_x \leq \frac{p_l}{\lambda s_0-(1-\lambda)s_0} \cdot B/\Delta - \frac{p_b-p_l}{p_h} \cdot B_x \Leftrightarrow \psi(\lambda) \frac{1}{1-p_h} \leq \frac{B/\Delta}{B_x} \), then no \( (z_0, z_l, z_s, z_f) \) satisfies (25), to which the IC constraint (10) is equivalent, for any \( \beta < 1 \). Q.E.D.

The Proof of Proposition 4:

Constraints (9) and (10) are respectively equivalent to:

\[
\begin{align*}
& p_h z_s + (1-p_h) z_f \geq z_l; \quad (27) \\
& \beta(z_l - p_l z_s - (1-p_l) z_f)(V_1 - V_0) \geq T^*. \quad (28)
\end{align*}
\]

We proceed with the following steps: first we find out \( V_1 - V_0 \) as a function of \( (z_0, z_l, z_s, z_f) \), then solve the maximization problem for \( \Pi_1 \) and \( \Pi_0 \), and finally characterize the Pareto dominant equilibrium.

Step 1: \( V_1 - V_0 \) as a function of \( (z_0, z_l, z_s, z_f) \).

The equation for \( V_1 \) has been given by (26) above, replicated below,

\[
V_1 = B_x + \beta [QV_1 + (1-Q)V_0]. \quad (29)
\]

If she is not hired this period, she gets 0 now and will be rehired with probability \( z_0 \). Therefore,

\[
V_0 = \beta [z_0 V_1 + (1-z_0)V_0]. \quad (30)
\]

From these two equations,

\[
V_1 - V_0 = \frac{B_x}{1 - \beta Q + \beta z_0}.
\]

Substitute it into (28), which then becomes:

\[
\beta(z_l - p_l z_s - (1-p_l) z_f) \frac{B_x}{1 - \beta Q + \beta z_0} \geq T^*. \quad (31)
\]

Step 2: Solve problem (11).
The problem is equivalent to maximizing $Q$, the rehiring probability, subject to the same constraints. With (9) equivalent to (27), (10) to (28) and then (31), the transformed optimization problem is:

$$
\max_{\{z_l, z_s, z_f\}} Q(z_l, z_s, z_f), \text{s.t. (27) and (31)}.
$$

To solve this problem, first note that $\frac{\partial Q}{\partial z_l} > 0$. Thus, an increment in $z_l$ both improves the objective function and slackens (31), which means $z_l$ should be increased until (27) is binding. Hence,

$$
z_l = z_h = p_h z_s + (1 - p_h) z_f.
$$

Then, $Q = p_h z_s + (1 - p_h) z_f$ and (31) becomes:

$$
\beta (p_h - p_l) B_x \frac{z_s - z_f}{1 + \beta z_0 - \beta p_h z_s - \beta (1 - p_h) z_f} \geq T^*.
$$

An increment in $z_s$ both improves the objective as $\frac{\partial Q}{\partial z_s} > 0$ and slackens (32), and therefore should be carried out to its maximum. That is, $z_s = 1$.

By substituting $z_s = 1$ into the objective function and (32), we see the above optimization problem in (22) now becomes:

$$
\max_{\{z_0, z_f\}} Q = p_h + (1 - p_h) z_f,
$$

$$
\text{s.t.} \beta (p_h - p_l) B_x \frac{1 - z_f}{1 - \beta + \beta z_0 + \beta (1 - p_h)(1 - z_f)} \geq T^*.
$$

The objective function increases with an increase in $z_f$, which tightens the constraint. Therefore, the constraint is binding, which gives:

$$
\frac{1 - z_f}{1 - \beta + \beta z_0} = \frac{T^*}{\beta (p_h - p_l) B_x - \beta T^*(1 - p_h)}.
$$

This equation defines $z_f$ as a function of $z_0$, denoted by $z_f = h(z_0)$.

To sum up, the solution to the problem (22), namely, the equilibrium strategy of the firm in state $1$, is:

$$
z_s = 1, z_f = h(z_0), z_l = p_h + (1 - p_h) z_f.
$$
Step 3: Solve problem (12).

Following the analysis above, it is equivalent to:

$$\max_{\{z_0, z_f\}} z_0, \text{ s.t. (33)}. \quad (33)$$

Again, the constraint is binding, which gives the same equation, (34), whereby $z_0$ is a function of $z_f$, that is, $z_0 = h^{-1}(z_f)$, where

$$h^{-1}(z_f) = (1 - z_f)\left[\frac{(p_h - p_l)B_x}{T^*} - (1 - p_h)\right] - \frac{1 - \beta}{\beta}. \quad (34)$$

That means any combination of $z_f$ and $z_0$ forms an equilibrium, so long as it satisfies (34), namely, $z_f = h(z_0)$ or $z_0 = h^{-1}(z_f)$.

Therefore, the reputational equilibria exist, if and only if there exists a profile of $(z_0, z_l, z_s, z_f)$ of which the elements are all between 0 and 1 and satisfy (35). So long as $(z_f, z_0)$ is between $[0, 1]$, then so are all the rehiring probabilities. Thus, the reputational equilibria exist if we can find a $z_f \in [0, 1]$ such that $z_0 = h^{-1}(z_f)$ is between $[0, 1]$. Note that $h^{-1}(z_f)$ can be non-negative for $1 - z_f \geq 0$ only if $\frac{(p_h - p_l)B_x}{1 - p_h} - (1 - p_h) > 0 \iff \frac{(p_h - p_l)B_x}{1 - p_h} \ll \frac{p_l}{\lambda s_g - (1 - \lambda)s_h} \cdot B/\Delta - \frac{p_h - p_l}{p_h} \cdot B_x \iff \psi(\lambda) \cdot \frac{1}{1 - p_h(\lambda)} > \frac{B/\Delta}{B_x}. \quad (36)$

This inequality holds true if and only if $\lambda > \lambda^*$ when $\lambda^*$ (and therefore $\lambda^{**}$) is smaller than 1, because the two sides of it are equal at $\lambda = \lambda^{**}$ by the definition of $\lambda^{**}$ (given at the start of the proof of Proposition 3) and the left hand side of it, $\psi(\lambda)/(1 - p_h(\lambda))$, increases with $\lambda$ while the right hand side stays constant.

On the other hand, if $\lambda > \lambda^{**}$ so that (36) holds true, then $h^{-1}(z_f)$ decreases with $z_f$ and for $z_f \in [0, 1]$, $h^{-1}(z_f) \in \{-\frac{1 - \beta}{\beta}, \frac{(p_h - p_l)B_x}{1 - p_h} - (1 - p_h) - \frac{1 - \beta}{\beta}\}$. Therefore, if $\beta$ is close to 1 enough so that $\frac{(p_h - p_l)B_x}{1 - p_h} - (1 - p_h) - \frac{1 - \beta}{\beta} > 0$, there exists a $z_f \in [0, 1]$ so that $z_0 = h^{-1}(z_f)$ is between $[0, 1]$ and thus the reputation equilibria exist. Actually, any pair of $(z_f, z_0)$ such that $z_0 \in [0, \min(1, \frac{(p_h - p_l)B_x}{1 - p_h} - (1 - p_h) - \frac{1 - \beta}{\beta})]$ and $z_f = h(z_0)$,
together with $(z_s = 1, z_l = p_h + (1 - p_h)z_f)$ gives a reputation equilibrium. Thus, there is a continuum of such equilibria.

Substitute the equilibrium strategy (35), into the formula for $\Pi_1, \Pi_0, V_1,$ and $V_0,$ namely (11), (12), (29), and (30). And we find

$$\Pi_1 = \frac{v^{nce}}{1 - \beta} \left[ 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} \right] \quad \text{and} \quad \Pi_0 = \frac{\beta z_0}{1 - \beta(1 - z_0)} \Pi_1;$$

$$V_1 = \frac{B_x}{1 - \beta} \left[ 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} \right] \quad \text{and} \quad V_0 = \frac{\beta z_0}{1 - \beta(1 - z_0)} V_1. \quad (37)$$

It follows that in all the reputational equilibria, $\Pi_1$ and $V_1$ are the same, but $\Pi_0$ and $V_0$ both increase with $z_0$.

Note that $T^* = \frac{p_l}{\lambda s_y - \lambda s_b} \cdot B / \Delta - \frac{p_h - p_l}{p_h} \cdot B_x$ decreases with $\lambda$, because $\frac{p_l}{\lambda s_y - \lambda s_b}$ decreases with it and $\frac{p_h - p_l}{p_h}$ increases with it. And at $\lambda = \lambda^*$, the deduction leading to (36) indicates $\frac{(p_h - p_l)B_x}{1 - p_h} = T^*$ and therefore the discount factor $1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} = 0$. And at $\lambda = \lambda^*$, $T^* = 0$ – thus the discount factor equals 1 – because $T^* = 0 \iff \frac{p_l}{\lambda s_y - \lambda s_b} \cdot B / \Delta = \frac{p_h - p_l}{p_h} \cdot B_x \iff \lambda = \lambda^*$ if $\lambda^* < 1$, by the definition of $\lambda^*$.

This proves Proposition 4(i). We move on to find the most efficient equilibrium, namely the one with the highest value of $z_0$.

Step 4: The equilibrium with the highest $\Pi_0$ and $V_0$

We saw $z_0 = h^{-1}(z_f) = (1 - z_f)\left[ \frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) \right] - \frac{1 - \beta}{\beta}$. The function $z_0 = h^{-1}(z_f)$ is maximized at $z_f = 0$ and the maximal equals $\frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) - \frac{1 - \beta}{\beta}$. If this maximal is no bigger than 1, it gives the highest possible $z_0$. On the other hand, if the expression exceeds 1, the highest $z_0 = 1 \Rightarrow z_f = 1 - \frac{T^*}{\beta(p_h - p_l)B_x - p_h(1 - p_h)T^*}$. Therefore, in the unique Pareto dominant equilibrium, $(\Pi_1, \Pi_0)$ is given by (37) and the rehiring probabilities
are:

\[
\begin{align*}
    z_0 &= \min\left(\frac{(p_h - p_l)B_x}{T^*} - (1 - p_h) - \frac{1 - \beta}{\beta}, 1\right); \\
    z_f &= \max\left(0, 1 - \frac{T^*}{\beta(p_h - p_l)B_x - \beta(1 - p_h)T^*}\right); \\
    z_l &= p_h + (1 - p_h)z_f; \\
    z_s &= 1.
\end{align*}
\]

We have shown \( T^* \) decreases with \( \lambda \). Then, a direct differentiation of \((\Pi_1, \Pi_0, V_1, V_0)\) and \((z_l, z_f)\) with respect to \( \lambda \) proves that they all increase with \( \lambda \). Moreover, at \( \lambda = \lambda^* \), as \( T^* = 0 \), all the rehiring probabilities equal 1.

Q.E.D.

The Proof of Proposition 5:

If \( \lambda^* < 1 \) the firm can use the reputation mechanism to contain the collusion problem for \( \lambda > \lambda^* \). If it does so, by Proposition 4, the value of the firm is \( \Pi_1^R = \frac{\nu^{ncx}}{1 - \beta} \left[ 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} \right] \). But if it gives up the reputation mechanism and repeats the static contracts each period, it gets \( v^x \) each period, where \( v^x \) is given by Proposition 2. The value of the firm is \( \frac{v^x}{1 - \beta} \).

Let \( \lambda^{**} \) be the root of

\[
\frac{\nu^{ncx}}{1 - \beta} \left[ 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} \right] = \frac{v^x}{1 - \beta} \tag{39}
\]

if it is within \((1/2, 1)\), otherwise, let \( \lambda^{**} \equiv 1 \). Then, \( \lambda^{**} > \lambda^* \), because \( 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} \geq 0 \) if and only if \( \lambda \geq \lambda^* \): first, \( 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} = 0 \Rightarrow \frac{p_h - p_l}{1 - p_h}B_x = T^* \Leftrightarrow \psi(\lambda) \frac{1}{1 - p_h} = \frac{B/\Delta}{B_x} \Leftrightarrow \lambda = \lambda^* \) (if \( \lambda^* < 1 \)); and second, \( 1 - \frac{(1 - p_h)T^*}{(p_h - p_l)B_x} \geq 0 \) increases with \( \lambda \). At \( \lambda \) is bigger than \( \lambda^* \) but close to \( \lambda^* \), \( \Pi_1^R \approx 0 \) and thus less than the value by repeating the static contracts. Thus for \( \lambda \in (\lambda^*, \lambda^{**}) \), \( \Pi_1^R \) is still less. Therefore, the firm prefers using the reputation mechanism to repeating the static contracts only if \( \lambda \geq \lambda^{**} \). Q.E.D.
References


[23] Inderst, Roman and Holger Muller, forthcoming, CEO Replacement under private Information, Review of Financial Studies.


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