This course covers several topics in probability theory and mathematical statistics which serve as a basis for understanding the literature in mainstream and financial econometrics. Emphasis is also on computer programming, for which we currently use Matlab.

**Instructor:** Prof. Dr. Marc Paolella, email: paolella@isb.unizh.ch.
**Time and Location:** Tuesday, 14:15-15:45, Plattenstr. 32, Thursday, 10:15, PhD student office.
**Textbook:** Class notes to be given out.
**Grading:** The final grade will be based on in-class and take-home quizzes, and homework assignments.

**Course Outline:** The following is the table of contents from the class notes.

## Probability Theory

- Generating Functions
  - The Moment Generating Function
    * Moments and the m.g.f.
    * The Cumulant Generating Function
    * Uniqueness of the m.g.f.
    * Vector m.g.f.
  - Characteristic Functions
    * Complex Numbers
    * Laplace Transforms
      - Existence of the Laplace Transform
      - Inverse Laplace Transform
    * Basic Properties of Characteristic Functions
    * Relation between the m.g.f. and c.f.
      - If the m.g.f. exists on a neighborhood of zero
      - m.g.f. and c.f. for non-negative $X$
    * Inversion Formulae for Mass and Density Functions
    * Inversion formulae for the c.d.f.
  - Use of the Fast Fourier Transform
    * Fourier Series
    * Discrete and Fast Fourier Transforms
    * Applying the FFT to C.F. inversion
- Multivariate Case
- Problems

- Sums and Other Functions
  - Weighted Sums of Independent Random Variables
  - Exact Integral Expressions
  - Approximating the Mean and Variance
  - The Digamma Function
  - Problems
• The Multivariate Normal Distribution
  – Vector Expectation and Variance
  – Basic Properties of the Multivariate Normal
  – Density and Moment Generating Function
  – Simulation and c.d.f. Calculation
  – Marginal and Conditional Normal Distributions
  – Partial Correlation
  – Joint Distribution of $\bar{X}$ and $S^2$ for i.i.d. Normal Samples
  – Appendix: Matrix Algebra
  – Problems

• Convergence Concepts
  – Inequalities for Random Variables
  – Convergence of Sequences of Sets
  – Convergence of Sequences of Random Variables
    * Convergence in Probability
    * Almost Sure Convergence
    * Convergence in $r$–Mean
    * Convergence in Distribution
  – The Central Limit Theorem
  – Problems

• Saddlepoint Approximations
  – Univariate
    * Density Saddlepoint Approximation
    * C.D.F. Saddlepoint Approximation
    * Detailed Illustration: The Normal–Laplace Sum
  – Multivariate
    * Conditional Distributions
    * Bivariate c.d.f. Approximation
    * Marginal Distributions
  – Problems

• Order Statistics
  – Distributional Theory for i.i.d. Samples
    * Univariate
    * Multivariate
    * Sample Range and Midrange
  – Further Examples
  – Distribution Theory for Dependent Samples
  – Problems
• Generalizing and Mixing
  – Basic Methods of Extension
    * Nesting and Generalizing Constants
    * Asymmetric Extensions
    * Extension to the Real Line
    * Transformations
    * Invention of Flexible Forms
  – Weighted Sums of Independent Random Variables
    * Weighted Sums of Independent $\chi^2$ Random Variables
  – Mixtures
    * Countable Mixtures
    * Continuous Mixtures
  – Problems

• The Stable Paretian Distribution
  – Symmetric Stable
  – Asymmetric Stable
  – Moments
    * Mean
    * Fractional Absolute Moment Proof I
    * Fractional Absolute Moment Proof II
  – Simulation
  – Generalized Central Limit Theorem

• GIG and GHyp Distributions
  – Introduction
  – The Modified Bessel Function of the Third Kind
  – Mixtures of Normal Distributions
    * Mixture Mechanics
    * Moments and Generating Functions
  – The Generalized Inverse Gaussian Distribution
    * Definition and General Formulae
    * The Subfamilies of the GIG Distribution Family
  – The Generalized Hyperbolic Distribution
    * Definition, Parameters and General Formulae
    * The Subfamilies of the GHyp Distribution Family
    * Limiting cases of GHyp
  – Properties of the GHyp Distribution Family
    * Location-Scale Behavior of GHyp
    * The Parameters of GHyp
    * Alternative Parameterizations of GHyp
    * The Shape Triangle
    * Convolution and Infinite Divisibility
  – Problems
• Noncentral Distributions
  – Noncentral Chi Square
    * Derivation
    * Moments
    * Computation
    * Weighted Sums of Independent $\chi^2 (n_i, \theta_i)$ Random Variables
  – Singly and Doubly Noncentral F
    * Derivation
    * Moments
    * Exact Computation
    * Approximate Computational Methods
      - Matching the first Three Moments
      - c.d.f. Saddlepoint Approximation
      - Density Saddlepoint Approximation
  – Noncentral Beta
  – Singly and Doubly Noncentral $t$
    * Derivation
      - Singly Noncentral $t$
      - Doubly Noncentral $t$
    * Saddlepoint Approximation
    * Moments
  – Appendix
    * Saddlepoint Uniqueness for the doubly noncentral F
    * The Hypergeometric Functions $1 \, F_1$ and $2 \, F_1$
  – Problems

• Quadratic Forms in Normal Variables
  – Computation of the Distribution and Moments
    * Density and Distribution
    * Moments
    * Generalized Quadratic Form
  – Basic Distributional Results
  – Ratios of Quadratic Forms in Normal Variables
    * Calculation of the c.d.f.
    * Calculation of the p.d.f. of $R$
  – Moments of Ratios
    * For $X \sim N (0, \sigma^2 \mathbf{I})$ and $B = \mathbf{I}$
    * For $X \sim N (0, \Sigma)$
    * For $X \sim N (\mu, \mathbf{I})$
    * For $X \sim N (\mu, \Sigma)$
Mathematical Statistics

• Random Sampling and Parameter Estimation
  – Introducing Estimation: Sampling from the Urn
    • Binomial Model
    • Hypergeometric Model
    • Negative Binomial Model
  – Further Inferential Methods
    • Nonparametric p.d.f. and c.d.f. Estimation
    • Sample Moments
    • Method of Moments Estimator
  – The Likelihood Function
    • Basic Definitions and m.l.e. Mechanics
      - Scalar Parameter Case
      - Vector Parameter Case
    • Asymptotic Behavior of the m.l.e.
  – Problems

• Likelihood: Numerical Methods
  – Root Finding
    • One Parameter
    • Several Parameters
  – Approximating the Distribution of the m.l.e.
  – Numerical Likelihood Maximization
    • Newton Raphson and Quasi–Newton Methods
    • Imposing Parameter Restrictions
    • Potential Numerical Problems
    • The EM Algorithm
  – Model Under– Over– and Misspecification
  – Problems

• Likelihood and Bayesian Inference

• Unbiased Point Estimation
  – Sufficiency
    • Introduction
    • Factorization
    • Minimal Sufficiency
    • The Rao–Blackwell Theorem
  – Completeness and u.m.v.u.e.
    • Completeness
    • Lehmann–Scheffé u.m.v.u.e. Theorem
    • Ancillarity
  – Cramér–Rao Inequality
    • Univariate Case
Multivariate Case
- c.r.l.b. and the m.l.e.
  - An Example with i.i.d. Geometric Data
- Methods of Bias Reduction
  - The Bias–Function Approach
  - Median Unbiased Estimation
  - Case Study: The Correlation Coefficient
  - The Jackknife
- Problems

• Confidence Intervals
  - Definitions
  - Pivotal Method
    - Asymptotic Pivots
  - Intervals Associated with Normal Samples
    - Single Sample
    - Paired Sample
    - Two Independent Samples
    - Welch’s Method for a c.i. of \( \mu_1 - \mu_2 \) when \( \sigma_1^2 \neq \sigma_2^2 \)
    - Satterthwaite’s Approximation
  - c.d.f. Inversion
    - Continuous Case
    - Discrete Case
  - Bootstrap Confidence Intervals
  - Problems

• Hypothesis Testing

• The Linear Model
  - Introduction
    - Ordinary Least Squares
    - Generalized Least Squares
    - Stochastic Regressors
    - Robust Estimation
  - The Geometric Approach to Least Squares
    - The Projection Theorem
    - Implementation
  - Linear Parameter Restrictions
    - Estimation
    - Testing With \( h = 0 \)
    - Testing With Nonzero \( h \)
    - Examples
    - Confidence Intervals
  - Alternative Residual Calculation
  - Problems
• Fixed Effects ANOVA Models
  – Two Sample $t$ Test for Differences in Means
  – The Two Sample $t$ Test with Ignored Block Effects
  – One Way ANOVA with Fixed Effects
    * The Model
    * Estimation and Testing
    * Determination of Sample Size
    * The ANOVA Table
    * Computing Confidence Intervals
    * A Word on Model Assumptions
  – Problems

• Introduction to Random and Mixed Effects Models
  – One-Factor Random Effects Model
    * Satterthwaite’s Method
  – Two-Factor Nested Random Effects Model
    * All Effects Random
    * Two-Factor Mixed Nested REM
  – Three-Factor Nested Random Effects Model
    * All Effects Random
    * Classes Fixed
    * Classes and Subclasses Fixed
  – Two-Factor Crossed Random Effects Model
    * All Effects Random
    * Mixed Effects
  – Three-Factor Crossed Random Effects Model
    * All Random Effects
    * Three Factor Crossed and Nested Model
  – Improving the Satterwaite Approximation
  – Using the Bootstrap for Inference with Heterogeneous Error Variance and/or Non-normality
  – Problems

• Introduction to Time Series: The AR(1) Model
  – Moments and Stationarity
  – Least Squares and ML Estimation
    * Likelihood Derivation I
    * Likelihood Derivation II
    * Likelihood Derivation III
    * Asymptotic Distribution
  – Forecasting
  – Small Sample Properties
    * Distribution of the O.L.S. and M.L. Point Estimators
    * Alternative Point Estimators of $a$
    * Confidence Intervals for $a$
Regression Models with AR(1) Disturbances
• The AR(1) Model with Exogenous Regressors
• Point and Interval Estimation
• Problems

• AR and MA Models

• AR(p) Processes
  • Stationarity and Unit Root Processes
  • Moments
  • Estimation
    • Without Mean Term
    • Starting Values
      1. Least Squares
      2. Yule–Walker
    • With Mean Term
    • Approximate Standard Errors

• Moving Average Processes
  • MA(1) Process
  • MA(q) Processes
• Problems

• Autoregressive Moving Average Processes

• Basics of ARMA Models
• Infinite AR and MA Representations
• Closed–Form (Initial) Parameter Estimation
• Likelihood–Based Estimation
  • Covariance Structure
  • Point Estimation
  • Interval Estimation
• Forecasting
• Problems

• Correlograms

• Theoretical and Sample Autocorrelation Function
  • Definitions
  • Marginal Distributions
  • Joint Distribution
    • Support
    • Asymptotic Distribution
    • Small–Sample Joint Distribution Approximation
  • Conditional Distribution Approximation
• Theoretical and Sample Partial Autocorrelation Function
• Problems
• Identification
  – Correlogram Analysis
  – Significance Tests
  – Pattern Identification
  – Penalty Criteria
  – Use of the Conditional SACF
  – Other Methods
  – Problems